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FINITE ELEMENT RELIABILITY METHODS IN EARTHQUAKE ENGINEERING PRACTICE

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SUMMARY

The field of earthquake engineering is rapidly moving towards the use of sophisticated computational methods in engineering practice. This development is motivated by improved capabilities to numerically simulate nonlinear structural behavior during strong ground motion and the observation from recent earthquakes that the actual performance implied by simplified design guidelines is unclear. In this paper, the use of finite element reliability methods to further the transition towards performance-based engineering is discussed. Possibilities and remaining difficulties are discussed. Emphasis is put on the need to take all sources of uncertainty into account. A key element in the methodology is the use of newly developed techniques for time-variant reliability analysis to compute the probability of excursion of nonlinear response over a threshold during a time interval. This problem is of primary concern in earthquake engineering. Furthermore, the benefits of the parameter sensitivity and importance measures available in finite element reliability analysis are stressed.

INTRODUCTION

Earthquake engineering has traditionally been based on simplified regulations. Typically, a building is designed to resist a seismic lateral load with magnitude determined by factors that account for the type of building, the natural period(s) of the structure, the foundation type, etc. For instance, the 1995 version of the National Building Code of Canada (NBCC) prescribes the design base shear equal to

$$V = \frac{v \cdot S \cdot I \cdot F}{R} \cdot U \cdot W \qquad \text{Eq. (1)}$$

where v is a zonal velocity factor depending on the amplitude of the selected response spectrum at the first natural frequency of vibration of the structure, S is a seismic response factor depending on the shape of the response spectrum, I is an importance factor to increase the seismic load for critical structures, F is a foundation factor, R is a force reduction factor, U is a calibration factor to produce a base shear close to values in previous codes and W is the seismic weight of the structure. A similar but modified equation is found in the 2005 version of the NBCC. It is noted that the factor R in Eq. (1)

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reduces the base shear compared to the elastic force. The force redistribution implied by this assumption increases the possibility of unexpected structural performance. This is emphasized by the fact that the structural response in the nonlinear response range is significantly more sensitive to perturbation in parameter values than in the elastic response range [1]. In fact, observations from recent earthquakes have shown that the implication of the current earthquake engineering practice may be at odds with the expectations. This motivates the development of design and engineering procedures that rather focus on the performance of the structure.

Performance-based earthquake engineering (PBEE) is based on the notion that decisions should be made with regard to the predicted performance of the structure. This approach is enabled by state-of-the-art capabilities to predict structural performance. Several initiatives towards PBEE have emerged during the recent years. Programs such as SAC [2] and strategies outlined in FEMA-349 [3] represent attempts to incorporate performance-based engineering in earthquake engineering practice. The magnitude of the NEES project [4] indicates the research community's commitment to develop tools and knowledge to support this trend. Another significant research initiative to develop formalized PBEE procedures is presented by the Pacific Earthquake Engineering Research (PEER) Center [5,6] and other NSF-funded earthquake engineering research centers in the US. The methodology suggested by PEER assumes that the decision maker (client, owner, society) is primarily concerned with *decision variables, DV*, such as downtime or monetary loss. These measures are related by loss models to *damage measures, DM*, such as amount of concrete spalling or gaps at bridge abutments, which again are related by damage models to *engineering demand parameters, EDP*, such as strain or displacement, which are related by, e.g., finite element analysis to *intensity measures, IM*, such as peak ground acceleration or other characteristic measures of strong ground motion.

Common for the performance-based engineering methodologies is the need for state-of-the-art technology to predict structural behavior. The finite element method is presently the dominant tool for this purpose. This technique allows realistic simulation of the nonlinear behavior of complex structures – involving columns, girders, slabs, shear walls, foundations, etc. – subjected to loading such as strong ground motion. However, predictions about structural performance can only be made in a probabilistic sense. Unavoidable uncertainties are present in material properties, cross-sectional geometry, nodal coordinates and loading, as well as in the model and analysis procedures. Hence, it is critical for the advancement of PBEE that uncertainties are appropriately accounted for. Modern reliability methods combined with the finite element method; so-called finite element reliability methods, address this issue. Here, the model parameters are considered random variables and probability estimates for achieving pre-defined performance criteria are computed along with valuable parameter importance measures. This methodology has previously not been extensively used in structural engineering practice. An objective of this paper is to contribute to the current shift in this trend by demonstrating some of the capabilities of finite element reliability methods. First, the PBEE procedures envisioned with this methodology is discussed, followed by a review of the current capabilities of finite element reliability methods. Next, the use of time-variant reliability analysis to address a key issue in earthquake engineering is presented. Subsequently, the advantages of the parameter importance measures that are available as a by-product of a reliability analysis are emphasized. Finally, strategies for addressing outstanding difficulties are presented.

ENVISIONED ENGINEERING PROCEDURES

In this paper we envision an engineering procedure that begins with the establishment of a detailed finite element model of the specific structure at hand. In the past, this has often been considered infeasible in common engineering practice due to the engineering cost and the computational cost involved. However, over the past years these costs have shown a diminishing trend. This is partly due to improved user-interfaces for the software and increased expertise in finite element modeling in the engineering community. Furthermore, it is frequently observed that advanced 3-D computer aided drawing (CAD) models of a structure is made by the architect for the purpose of visualization, coordination and drawing production. Tools to transform CAD models to finite element models already exist. This point towards a future where refined finite element models of ordinary buildings are readily available. It is emphasized that the finite element reliability methodology discussed in this paper is applicable also to simplified structural models, such as models with one or two degrees of freedom per storey. However, such models inherently possess significant model uncertainty.

Provided the availability of detailed finite element models and nonlinear dynamic analysis options the question may be raised as to which approach will advance the objective of probabilistic PBEE. Recent research has outlined alternative procedures to utilize the available analysis results. An example is the Incremental Dynamic Analysis approach [7] where the structure is subjected to a series of nonlinear time-history analyses of increasing intensity. Probabilistic guidelines can be developed based on a series of IDA analyses of groups of similar buildings. In this paper an alternative probabilistic methodology is discussed where time-variant reliability is employed. Here, the ground motion is modelled as a filtered, modulated train of random pulses. This latter approach is an example of so-called synthetic generation of ground motion while IDA analysis is based on scaling of recorded ground motion. Benefits and pitfalls are present with both approaches.

Two observations are made with regard to the consequences on structural engineering practice of the visions outlined in this paper. Firstly, PBEE broadens the involvement of the structural engineer in management of earthquake risk and use of advanced performance prediction tools. This is in contrast to the trend of the past century where the profession grew dependent on prescriptive design guidelines. Instead of the detailed deemed-to-comply rules of earlier design codes one may envision design guidelines that require satisfaction of prescribed performance criteria during earthquakes of certain intensity. In this situation it is evident that advanced performance prediction tools are needed and that the involvement of the structural engineer increases. Figure 1 schematically describes the historical development in this regard, from the viewpoint of the author.

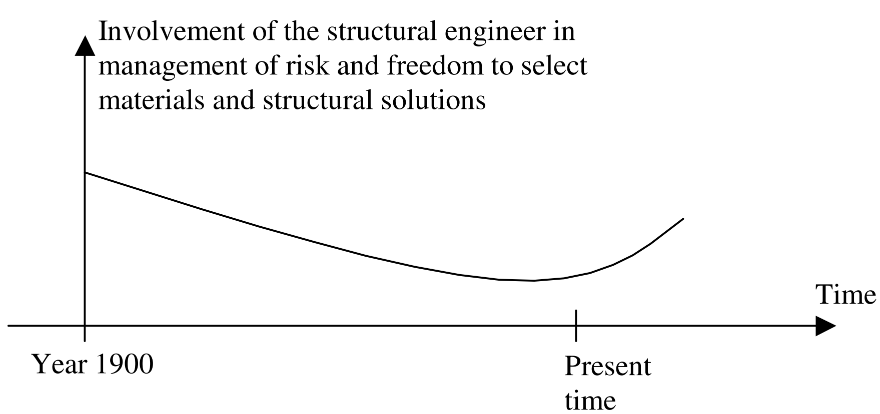

Figure 1: Envisioned development in involvement of the structural engineer in risk management.

A symptom of the valley in Figure 1 is the inability of present design guidelines to allow structural engineers to make use of new solutions. For instance, the introduction of base isolators or new materials has been hindered by the fact that they are outside the scope of current design codes. An advantage of the envisioned PBEE approach is the freedom of choice in this regard. In performance-based guidelines it is the performance that is of interest.

The development outlined in Figure 1 may be welcomed by some, while it may ignite reservations by other. One concern is the possibility of inconsistent practice. Moreover, a concern raised in the engineering community in California during the introduction of PBEE is the issue of liability. It is indeed clear that implication of a certain structural performance during an impending earthquake may trigger liability if the performance is not met during an expected event. However, this concern further motivates the use of probabilistic methods to account for the uncertainties and provide probabilistic predictions of structural performance.

The second observation is made with regard to the increasing use of advanced reliability methods in engineering practice. Due to the current leading role of the finite element method, the envisioned use of finite element reliability methods has the potential of bringing sophisticated reliability methods into common use. This is a long-awaited development. Over the past thirty years, a rapid development in our capabilities to assess structural reliability has occurred. However, these developments have only to some extent been manifested in engineering practice. In fact, applications of advanced reliability methods have been mostly limited to calibration of partial safety factors in prescriptive design guidelines. Figure 2 depicts the envisioned development. The indicated trend towards increased use of reliability methods is supported by observations from the offshore industry, the nuclear industry, the space industry, etc. In these fields the importance of the performance of the structure have motivated and advanced the use and development of structural reliability methods.

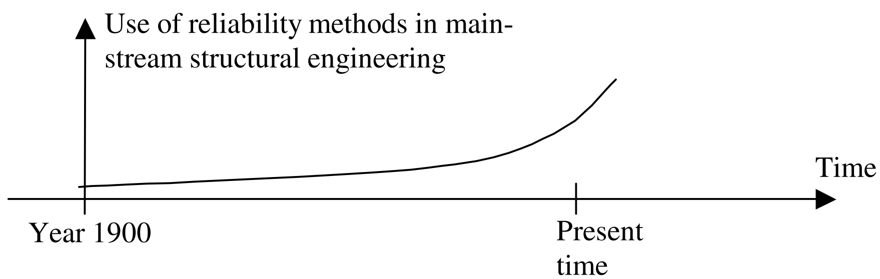

Figure 2: Envisioned development in the use of reliability methods in engineering practice.

CURRENT CAPABILITIES OF FINITE ELEMENT RELIABILITY AND SENSITIVITY ANALYSIS

The fundamental objective in finite element reliability analysis is to compute the probability of performance events that are specified in terms of finite element response. For example, in applications to earthquake engineering the probability of exceeding a displacement threshold during a strong ground motion is sought. To facilitate the computation of such quantities we consider the response from a finite element analysis as a function of a vector of random variables $\mathbf{x}$ and a vector of random processes $\mathbf{y}(t)$. The components of $\mathbf{x}$ typically represent material parameters, nodal coordinates and cross-sectional geometry, while $\mathbf{y}(t)$ describe the loading. It is instructive to first consider the case of a static pushover analysis. An example of a sought probability is $P[u(\mathbf{x}) > u_o]$, where the performance event is the displacement response $u(\mathbf{x})$ exceeding a threshold u_o. The corresponding performance function $g(u(\mathbf{x}))$ is formulated under the syntax rule g = {$g \leq 0$: failure, $g>0$: safe, $g=0$: limit-state} as $g = u_o - u(\mathbf{x})$. By denoting by $f(\mathbf{x})$ the joint probability density function of the random variables the reliability problem for the case of one performance function reads:

$$\mathrm{P}[u(\mathbf{x}) > u_o] \equiv p_f = \int_{g(u(\mathbf{x}))\leq 0} f(\mathbf{x})\, \mathrm{d}x \qquad \text{Eq. (2)}$$

The probability of failure p_f is related to the so-called reliability index β by the relation $p_f = \Phi(-\beta)$, where $\Phi()$ denotes the standard normal cumulative distribution function. Exact solutions to the multiple integral in Eq. (2) are generally not available. Approximate methods such as the first and second-order reliability methods (FORM and SORM) and sampling techniques have been developed to address Eq. (2). FORM and SORM approximates the integration boundary by a hyper-plane and a hyper-paraboloid in the transformed standard normal space of the random variables $\mathbf{y} = \mathrm{T}(\mathbf{x})$, respectively. This approximation is centered at the so-called design point $\mathbf{y}^*$, which is the solution to the constrained optimization problem $\mathbf{y}^*$= argmin{$\|\mathbf{y}\| \mid g=0$}. Of essence in finite element reliability analysis is to limit the number of evaluations of the performance function, due to computational cost. Determination of the design point requires in the order of 10 evaluations of the performance function $g(\mathbf{x})$ and its gradient vector

$$\nabla_{\mathbf{y}} G(\mathbf{y}) = \frac{\partial g}{\partial u} \frac{\partial \mathrm{u}}{\partial \mathrm{x}} J_{\mathbf{y},\mathbf{x}}^{-1} \qquad \text{Eq. (3)}$$

where $\partial g / \partial u$ is readily obtained since g usually is a simple algebraic function of u, $\partial u / \partial x$ signifies the need for response sensitivities and $J_{\mathbf{y},\mathbf{x}}$ is the Jacobian of the probability transformation $\mathbf{y} = \mathrm{T}(\mathbf{x})$. For this reason; namely, limited number of evaluations of g, FORM analysis is preferred over, e.g., the Monte Carlo sampling scheme that require orders of magnitude more evaluations of the performance function. However, it is noted that Importance Sampling around the design point subsequent to a FORM analysis represents an effective approach to update the FORM estimate of the failure probability.

As an example of finite element reliability analysis for a static push-over problem, consider the truss structure in Figure 3. This example was analyzed in [1] to demonstrate some of the reliability analysis options available in the modern and comprehensive software framework OpenSees [1,8,9]. The cross-sectional area of the vertical truss members are modeled as independent lognormal random variables with a mean of 7450 mm^2 and a coefficient of variation (c.o.v.) of 5%. The mean value of the cross-sectional area of all the other truss members is 4350 mm^2. Bi-linear material models are used for all truss elements. The Young's moduli are modeled as lognormal random variables with a mean of 210 GPa, c.o.v. 5% and inter-correlated by a correlation coefficient of ρ=0.3. The yield stress is modeled as lognormal random variables with a mean of 300 MPa, c.o.v. 10% and ρ=0.3. The second stiffness ratio is modeled as lognormal random variables with a mean of 1%, c.o.v. 10% and ρ=0.3. All the nodal coordinates are modeled as independent normal random variables with mean equal to the nodal locations shown in Figure 3 and standard deviation equal to 10 mm. Concentrated nodal loads are applied in the x-direction at the four top nodes, each with a deterministic magnitude of 120 kN. While the load is applied along the x-axis direction, the truss is rotated 30 degrees around its vertical axis compared to being aligned with the axis directions. Hence, the example is not symmetric and must be analyzed by a 3-D model. In total, 392 random variables are present.

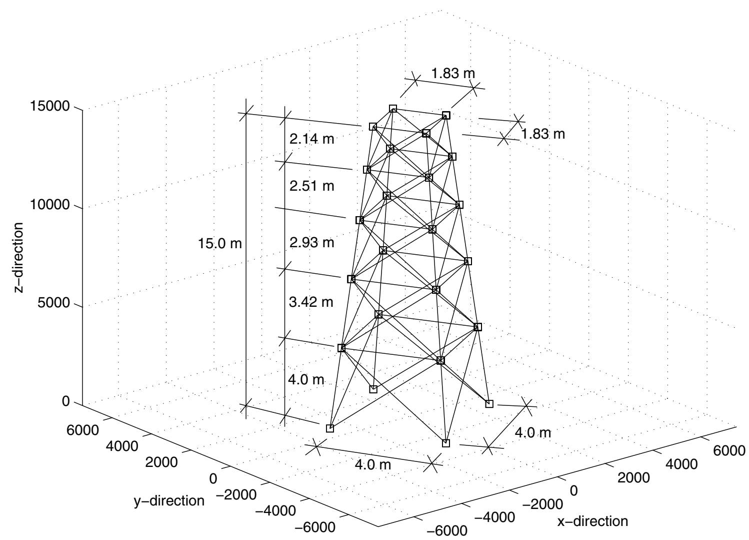

Figure 3: Truss example.

We decide to seek the probability that the top displacement (at node 21; see Figure 5) in *x*-direction exceeds 500 mm. The performance function for this event is formulated as

$$g = 500\,\mathrm{mm} - u_{x,21} \qquad \text{Eq. (4)}$$

7 evaluations of the performance function and its gradient vector was needed to determine the design point. The recorded displacement response at each trial point of the search algorithm is shown in Figure 4.

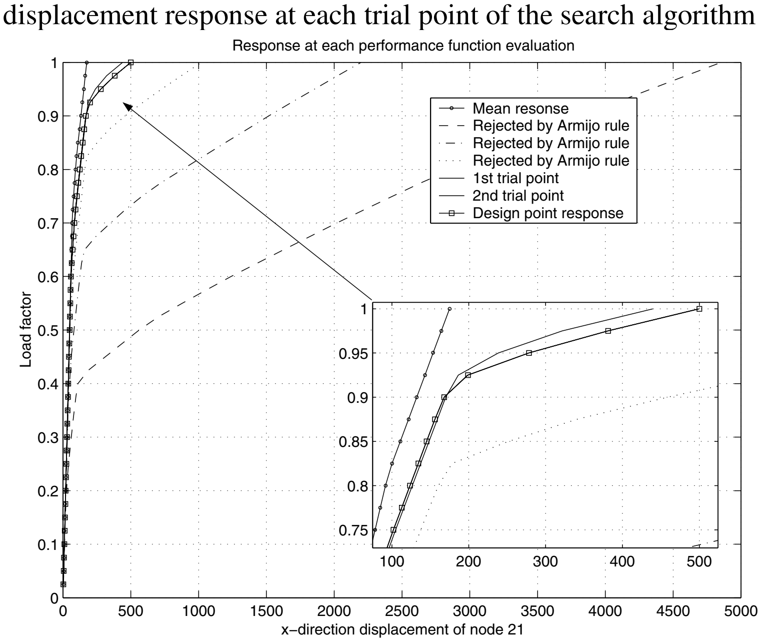

Figure 4: Response at each realization of the random variables in the search for the design point.

The results in Figure 4 clearly show a problem commonly encountered in nonlinear finite element reliability analysis when searching for the design point; namely, the start point is linear and the search algorithm tends to vastly overshoot the design point far into the nonlinear domain in the first step. This is one of the issues discussed towards the end of this paper.

The resulting reliability index of β=1.89 has a corresponding FORM estimate of the failure probability of p_{f1}=0.029026. Importance sampling around the design point gives the result $p_{f,sim}$=0.0447 (β=1.70) with 6.5% coefficient of variation on the failure probability estimate after 1000 random samples of the performance function.

Parameter importance measures represent an important by-product of FORM reliability analysis. These measures are useful in identifying structural elements and model parameters, which have significant influences on the performance of the structure. A ranking of the random variables according to their relative contribution to the variance of the performance function in Eq. (4) reveals that the yield strengths of the vertical elements 4, 20 and 2 (see Figure 5) are most important, followed by the yield strengths of the surrounding cross braces. Based on physical intuition it is reasonable that elements 2, 4 and 20 are important for the given performance function. This exemplifies the usefulness of parameter importance measures from reliability analysis to gain physical insight into a problem. It is also interesting to observe that the uncertainty in the nodal coordinates is quite important; the *x*-direction coordinate of node 4 ranks as 21st most important of the 392 random variables. In fact, 12 of the nodal coordinates enter the list of the 40 most important random variables, though the standard deviations are only 10 mm.

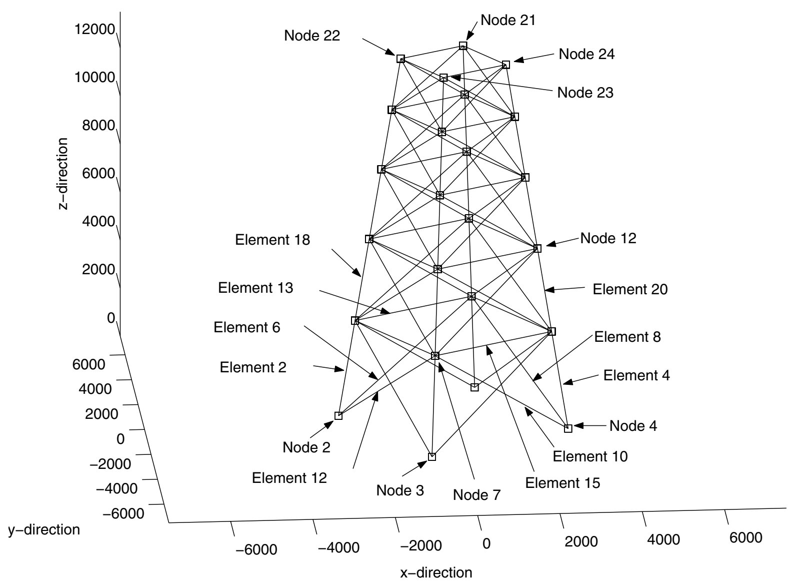

Figure 5: Identification of element and node numbers in truss example.

TIME-VARIANT RELIABILITY ANALYSIS

While static finite element reliability analysis is useful to incorporate uncertainties in traditional pushover analysis, and to identify the important parameters of a model, it does not answer the key question in earthquake engineering. That is, in reality we seek the probability of exceeding a predefined performance threshold during an impending earthquake event. This is a so-called time-variant reliability problem. Its solution, for inelastic structures, is a challenging problem in structural reliability analysis. Here we will discuss a newly developed methodology to addresses it. A noteworthy feature of the method is its applicability to nonlinear structural problems.

A key component of this approach is the modeling of the ground motion as a filtered and modulated white noise process, which can be used in FORM analysis to estimate the mean out-crossing rates of selected response quantities. In the most general case, a process $x(t)$ with non-stationary frequency content is modeled with multiple filters with different modulating functions in the form [10]

$$x(t) = \mu(t) + c\sum_{k=1}^{K} q_k(t) \sum_{i=1}^{N} y_{ik} h_k(t - t_i) \qquad \text{Eq. (5)}$$

where $\mu(t)$ is the time-varying mean, c is a coefficient that governs the variance, $q_k(t)$ are modulating functions, y_{ik} are standard normal random variables and $h_k(t\text{-}t_i)$ represent the filters. In this most general case the number of random pulses (random variables) is equal to the number of time steps, N, multiplied by the number of filters, K. A simpler alternative is

$$x(t) = \mu(t) + c\sum_{k=1}^{K} q_k(t) \sum_{i=1}^{N} y_i h_k(t - t_i) \qquad \text{Eq. (6)}$$

where the stochastic model is physically interpreted as a single train of white noise pulses filtered through a model with K peaks in the transfer function. The magnitudes of these peaks are controlled by the user's selection of the modulating function for each filter.

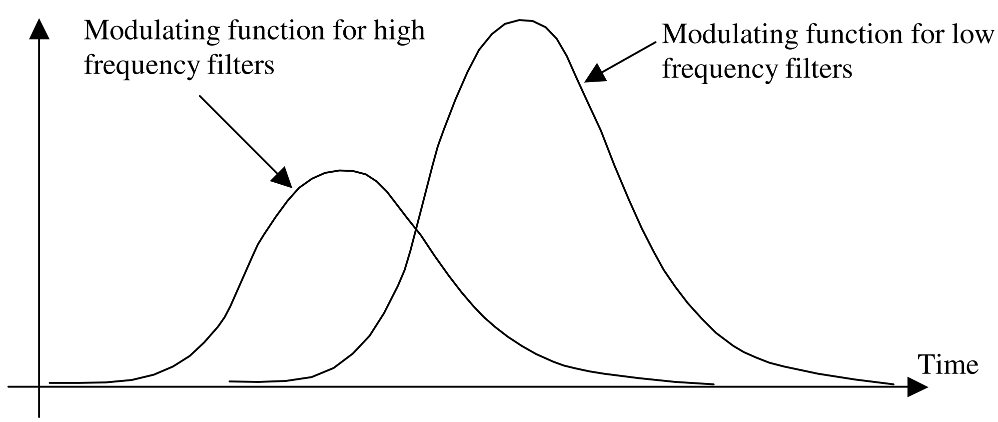

Figure 6: Schematic demonstration of the flexibility in choice of ground-motion characteristics.

Figure 6 shows schematically and example of selection of filter properties and modulating functions. As seen, the analyst does not select particular recorded ground motions but rather describes the characteristics of an impending earthquake event. This type of synthetic ground motion generation has a physical interpretation in earthquake engineering if the rupture during an earthquake event is seen as a train of random pulses. The pulses are filtered and modulated through soil layers before reaching the structure.

In time-variant reliability analysis a fundamental problem is the first-excursion problem, where we seek the probability of a time-varying response process $\mathbf{u}(t)$ entering the failure domain $g(\mathbf{u}(t), t)<0$ during a time interval T:

$$p_f = \mathrm{P}\left(\left\{\min_{0\le t\le T} g(\mathbf{u}(t),t)\right\}\le 0\right) \qquad \text{Eq. (7)}$$

One solution to this problem can be found by defining a limit-state function at every time step of the analysis and solve it as a series system reliability problem. At each time instant the problem is reduced to a time invariant component problem, which can be treated with methods such as FORM and IS analysis. However, it is readily seen that this represents a costly analysis approach in a nonlinear finite element context. This approach has been used in [11, 12]. An alternative is to employ an Active Set Gradient Projection scheme, as suggested by [13]. The approach presented here is based on the earlier works presented in [10, 12, 14, 15] and estimates the mean out-crossing rate, which is a critical response statistic for time-variant reliability analysis. Various reliability measures, such as the upper bound to the exceedance probability during a time interval T, are available.

The out-crossing rate, denoted η, marks the number of times per unit time interval that the response vector process $\mathbf{u}(t)$ makes a transition from the safe state into the failure state. For a stochastic input, this number is random and varies with time. Our interest is in computing its mean value, $\nu(t)=\mathrm{E}[\eta(\mathrm{t})]$, as a function of time.

Consider the two events $g(\mathbf{u}(t), t)>0$ and $g(\mathbf{u}(t+\delta t), \mathrm{t}+\delta t)\le 0$, indicating the occurrence of one or more out-crossings into the failure domain during $(t, t+\delta t)$. Two auxiliary limit-state functions are now defined: $g_1=-g(\mathbf{u}(t),t)$ and $g_2=g(\mathbf{u}(t),t)+\dot{g}\,\delta\mathrm{t}$ where $\dot{g}=\frac{\partial g}{\partial t}$. g_2 is the linear Taylor expansion of $\mathrm{g}(\mathbf{u}(t+\delta t),\mathrm{t}+\delta t)$. The probability of an out-crossing can be written as the probability of the intersection of the failure events of g_1 and g_2:

$$p_f(t,t+\delta t) = \mathrm{P}[\mathrm{g}_1 \le 0 \cap \mathrm{g}_2 \le 0] \qquad \text{Eq. (8)}$$

The mean rate of out-crossings is written as [16]

$$\nu(t) = \lim_{\delta t\to 0}\frac{\mathrm{P}[\mathrm{g}_1 \le 0 \cap \mathrm{g}_2 \le 0]}{\delta t} \approx \frac{\mathrm{P}[\mathrm{g}_1 \le 0 \cap \mathrm{g}_2 \le 0]}{\delta t},\ \delta t \text{ small} \qquad \text{Eq. (9)}$$

The numerator in Eq. (9) represents a parallel system reliability problem with two components. This problem represents a special case with high negative correlation between g_1 and g_2 and $\beta_1 \approx -\beta_2$. In [12] an approximate expression to solve this problem is developed, given probability estimates for g_1 and g_2. Two approaches are available to obtain these probabilities. One alternative is to separately obtain the design point of g_1 and g_2. The expression for g_2 is found by $g_2=g(\mathbf{u}(t),t)+\dot{g}\,\delta\mathrm{t}$. For example, for a performance function expressed in terms of displacement quantities, the chain rule of differentiating is applied: $\dot{g}\delta t=\frac{\partial g}{\partial u}\frac{\partial u}{\partial t}\delta t$, effectively introducing velocity response quantities in the expression for the second performance function. The alternative method proposed in [12] circumvents the cost of searching for the design point of g_2 by the following observation. Assume the performance function g is of the threshold type, i.e., $g=u_o-u$, where u_o is the threshold and u is a response quantity. When the design point is found for g_1, the design point realization of the response shows a displacement u equal to u_o at time t. Similarly, the design point realization for g_2 must show a displacement u equal to u_o at time $t+\delta t$. A simple approximation to the design point of g_2 is obtained by simply shifting the input time series (the realization of random pulses defined in Eq. (6) of the design point of g_1 along the time axis by a value of δt. The error of this approximation is small for small δt and is mainly present at the beginning of the time series, which is not a critical segment of the excitation for systems with damping. Numerical examples have shown that for δt small in relation to the predominant period of the response, this method provides sufficiently accurate results for all practical purposes.

Upon determination of the mean out-crossing rate $\nu(t)$ at discrete points along the time axis, the upper bound to the probability of excursion into the failure domain during time interval T is:

$$p_f(T) < \int_0^T \nu(t)\,\mathrm{d}t \qquad \text{Eq. (10)}$$

In cases where the out-crossing events may be assumed independent, an approximation to the true failure probability is derived by the assumption of Poisson distributed out-crossing events [14]:

$$p_f(T) = 1 - e^{-\int_0^T \nu(t)\,\mathrm{d}t} \qquad \text{Eq. (11)}$$

Caution must be exercised when using Eq. (11). For narrow-band processes, the out-crossings occur in clusters. This violates the assumptions of independence between out-crossing events. Furthermore, the assumption may be invalid if structural properties are assumed random in addition to the discretized stochastic input. In these cases, the upper bound in Eq. (10) should be used.

Reliability analysis by FORM is employed to compute β_1 (and β_2, unless the "shift method" described above is used). For multi-degree-of-freedom structures with inelastic material behavior this may be a computationally challenging task [1]. Several strategies may be utilized to solve this problem, such as gradual increase of the performance function threshold or using the design point from a previous time instant as start point for the search for the design point at a subsequent time instant. Also, [12] have shown that the design point excitation is obtained by investigating the mirror image of the free vibration response of a system released from a displacement equal to the performance function threshold. Research is under way to explore these strategies.

Examples involving application of the methodology discussed above to problems in earthquake engineering is found in, e.g., [1, 17].

USE OF PARAMETER SENSITIVITY AND IMPORTANCE MEASURES

As exemplified previously, the random variables in a finite element model can be ranked by relative importance by means of reliability analysis [18, 19, 20]. Measures of sensitivity and importance have a central place in the envisioned PBEE approach. Such information is useful for several purposes, e.g., it can be used to identify locations where potential for design improvement exist and to guide the establishment of tolerance limits for the construction work. Parameter importance measures can also provide valuable physical insight into the behavior of a structure. In fact, it is broadly accepted that numerical studies of structural behavior should be complemented by parameter sensitivity studies [21].

A prerequisite to obtain sensitivity and importance measures is the availability of response sensitivities. In fact, finite element sensitivity analysis is an essential ingredient in finite element reliability analysis, as indicated in Eq. (3). The reliability and sensitivity implementations in OpenSees [1] include implementation of the Direct Differentiation Method (DDM) for this purpose. With this method the finite element response equations are differentiated and implemented alongside the response algorithm. Compared to finite difference procedures, the DDM is ideal in terms of efficiency, accuracy and consistency (consistent with the approximations inherent in the finite element response itself), at the cost of an initial investment of effort in deriving and implementing sensitivity algorithms into the finite element code.

Significant developments in the DDM technique have occurred during the recent years. It is now possible to obtain response sensitivities with respect to material, geometry and load parameters for both static and dynamic inelastic analysis [20, 21, 22]. However, examples of systematic use of response sensitivity results in practical applications are not abundant. Further research into the interpretation and practical use of response sensitivity measures are needed to encourage the methodical use of such results in PBEE.

From the above discussion it is clear that parameter sensitivity and importance measures can be utilized in "informal design optimization;" namely, by the analyst employing these measures to indicate areas of weakness in a design. However, formal reliability-based optimal design has an important role in the envisioned PBEE methodology. The objective of such methods is to minimize a total cost function, which may include initial cost, cost of maintenance and expected cost of failure, given reliability constraints and constraints on the design variables. While the design variables typically include geometry parameters, the randomness may include inherent variability in material properties and loads. Progress has recently been made in the development and practical use of this methodology, see, e.g., [23, 24]. Difficult issues remain, however, such as the estimation of cost of failure involving loss of lives.

STRATEGIES FOR ADDRESSING REMAINING DIFFICULTIES

The approach to probabilistic PBEE that is outlined and discussed above is founded on extensive use of finite element reliability methods. While the capabilities of this methodology have rapidly improved during the past twenty years, challenges still remain. As mentioned in the section about time-variant reliability analysis, it is not a trivial task to determine the design point for certain problems. One common problem is the occurrence of response sensitivity discontinuities stemming from the bi-linear material suddenly entering the yielding state. This problem is solved in [1] by use of so-called smooth material models. Such models are characterized by a smooth transition between the elastic and the plastic material states. Another strategy to improve the search for the design point is to modify the search algorithms. In particular, this strategy has proven useful in nonlinear finite element reliability applications where the following two problems are encountered: a) the response at the start point in the search for the design point is close to linear, while the response at the design point is characterized by significant inelastic behavior, b) the finite element analysis does not converge in certain regions of the outcome space of the random variables due, e.g., low stiffness. Strategies to address these issues are discussed in [1, 25].

It is observed that the uncertain model and analysis errors are not accounted for in the examples presented in this paper. To advance the use of finite element reliability analysis in PBEE it is imperative that these sources of uncertainty are taken into account. In fact, uncertain model errors are present in all the modeling assumptions made from a) reality to b) idealized Boundary Value Problem to c) discretized finite element model to d) finite element analysis results to e) the computation of event probabilities. An initial attempt to address these uncertain errors by the development of probabilistic models is presented in [26]. This work also involves collaboration with experimental researchers and the use of Bayesian techniques to incorporate experimental results and theoretical error bounds, as well as subjective information such as expert opinion.

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