13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 1035

YIELD DISPLACEMENT ESTIMATES FOR DISPLACEMENT-BASED SEISMIC DESIGN OF DUCTILE REINFORCED CONCRETE STRUCTURAL WALL BUILDINGS

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SUMMARY

This paper presents the results of an analytical study on yield curvature for estimating effective yield displacements for the design of ductile reinforced concrete cantilever structural wall buildings. Improved estimates of effective yield curvature were obtained for rectangular and barbell cross sections based on standard moment curvature analyses. Variables considered include boundary and web reinforcement ratios, concrete compressive and reinforcing steel strengths, and axial load ratio. Formulas for estimating effective yield curvatures and displacements are also presented, and an example is provided to illustrate the application of these estimates to the design of ductile reinforced structural wall buildings.

INTRODUCTION

New displacement-based methods for seismic design [1, 2, 3] rely on an estimate of the effective yield displacement corresponding to the fundamental mode response of the structure to be designed. The yield displacement is a relatively stable parameter that can be estimated based on kinematic relationships early in the design process, accounting for the structural geometry, approximate distribution of mass, material properties, and the nominal member dimensions of the structural system [1]. In contrast, conventional design approaches are based on the period of vibration, which is prone to greater variation because the stiffness of the structure is not known initially and typically will vary as the strengths of the structural members are adjusted to obtain a structure having the intended seismic performance characteristics. The variation of the period with changes in strength tends to cause period-based seismic design approaches to require a larger number of iterations than are needed with yield displacement approaches.

Estimates of yield displacement used for the design of ductile reinforced concrete structural wall buildings are currently based on yield curvature estimates that range between $0.0025/l_w$ and $0.0035/l_w$ or higher, where l_w is the length of the wall in plan [4]. An analytical study was undertaken in order to improve the precision of this estimate for rectangular section and barbell section walls having a large range of boundary and web reinforcement ratios, concrete compressive strengths, and axial load ratios. This paper

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presents the results of this analytical study on yield curvature. Formulas for estimating effective yield curvatures and displacements are presented together with an example illustrating the application of these estimates to the design of ductile reinforced concrete structural wall buildings. The discussion is limited to full-height prismatic cantilever wall systems exhibiting ductile flexural response, for which only flexural deformations of the walls are considered.

YIELD DISPLACEMENT OF DUCTILE REINFORCED CONCRETE WALLS

Fig. 1(a) shows a typical cantilever wall having height h_w and length l_w. Under lateral inertial forces (Fig. 1(b)), the roof displacement of this wall at yield, Δ_y, accounting for flexural deformations only (Fig. 1(c)) can be expressed by

$$\Delta_y = \kappa_\Delta \phi_y h_w^2, \tag{1}$$

where κ_Δ = yield displacement coefficient, and ϕ_y = effective yield curvature of the wall cross section at the base. The yield displacement coefficient, κ_Δ, depends on the curvature distribution along the height of the wall (Fig. 1(e)), which in turn depends on the lateral load distribution and stiffness distribution over the wall height. It also depends on secondary effects, such as foundation rotation, shear distortion, and tension shift mechanism. The curvature distribution over the height of the wall can be estimated using simple beam theory as $M(x)/E_c I(x)$, where $M(x)$ = moment at a section located *x* from the base (Fig. 1(d)), and $E_c I(x)$ = flexural rigidity at a section located *x* from the base.

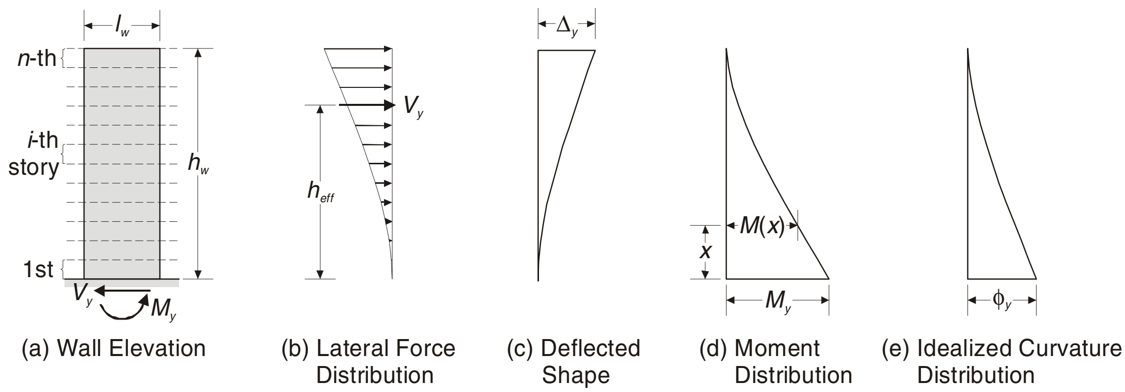


Fig. 1 Response of a single wall system at yield.

Values of κ_Δ, computed assuming uniform $E_c I(x)$ over the height of the wall, uniform floor masses, uniform story heights, and response in the fundamental mode, are given in Table 1. For irregular wall systems, κ_Δ may be determined by an elastic structural analysis using cracked section properties with lateral forces applied in proportion to the fundamental mode shape amplitude and mass at each floor.

The effective yield curvature, ϕ_y, in Eq. (1) ideally corresponds to $M_y / E_c I_{cr}$, where M_y = effective yield moment at the base of the wall, and $E_c I_{cr}$ = cracked-section flexural rigidity at the base of the wall, rather than the instant when the yield strain is reached at the extreme tension reinforcement. Various estimates of the effective yield curvature, ϕ_y, have been proposed [e.g., 4, 5, 6], often in the form

$$\phi_y = \frac{\kappa_\phi}{l_w}, \quad (2)$$

where κ_ϕ = effective yield curvature coefficient that depends primarily on the cross sectional shape of the wall, axial load level, and the amount, configuration, and yield strength of the longitudinal reinforcement.

Table 1 Properties of Prismatic Walls Responding in the Fundamental Mode

n	κ_Δ	Γ_1	α_1	h_{eff}/h_w
1	0.333	1.000	1.000	1.000
2	0.316	1.197	0.791	0.879
3	0.308	1.291	0.727	0.833
5	0.300	1.384	0.679	0.794
8	0.295	1.445	0.653	0.770
10	0.293	1.467	0.645	0.761
15	0.290	1.498	0.634	0.750
20	0.289	1.514	0.629	0.744

For rectangular cross sections, Wallace and Moehle [4] have recommended values of κ_ϕ in the range of 0.0025 to 0.0035 for Grade 60 (414 MPa) steel, for typical levels of axial load and reinforcement ratio. Values for other cross sections have also been suggested by other researchers [e.g., 5, 6]. More generally, κ_ϕ can be obtained from a moment-curvature analysis by linearly extrapolating the yield curvature corresponding to the first yield of longitudinal reinforcement to the effective yield moment, M_y, which is the moment resistance corresponding to a predetermined curvature, as illustrated in Fig. 2. The predetermined curvature could correspond to the development of the nominal flexural strength, as determined using building codes such as ACI-318 [7].

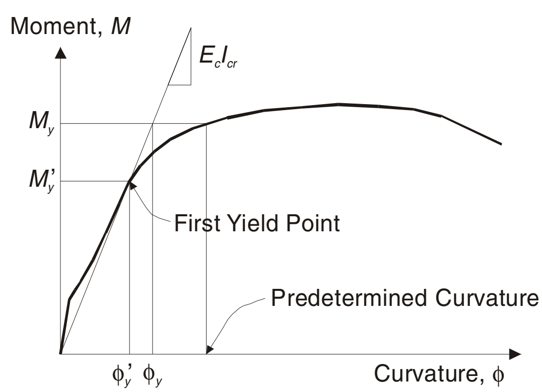


Fig. 2 General definition of effective yield curvatures.

ESTIMATES OF EFFECTIVE YIELD CURVATURES

Methodology

An analytical study was conducted to improve the precision of effective yield curvature estimates for rectangular and barbell wall cross sections (Fig. 3(a)). Variables considered include axial load level, longitudinal boundary reinforcement ratios, longitudinal web reinforcement ratio, specified concrete compressive strength, and specified steel reinforcement yield strength. Common ranges of variables were selected for the study, as summarized in Table 2.

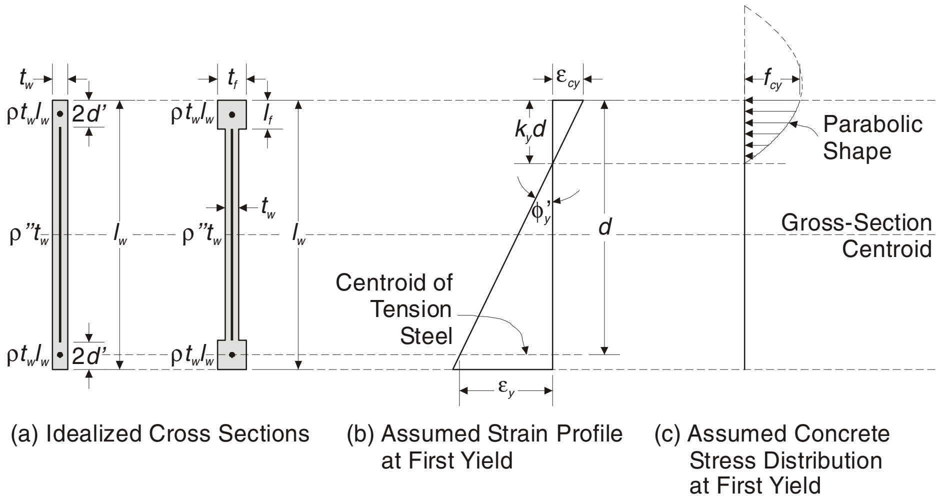


Fig. 3 Idealization and assumptions used for determining effective yield curvatures.

Table 2 Summary of the Variables Used (1 ksi = 6.895 MPa)

Parameter	Value
Cross Section	Rectangular, Barbell
End (Boundary) Longitudinal Steel Ratio, ρ	Rectangular: 0.25%, 0.5%, 1%, 2%, 3% Barbell: 1%, 2%, 3%, 4%, 5%
Web Reinforcing Steel Ratio, ρ''	0.25%, 0.3%, 0.4%, 0.5%
Specified Compressive Strength of Concrete, f_c' (ksi)	4, 5, 6
Specified Yield Strength of Reinforcing Steel, f_y (ksi)	40, 60, 75
Axial Load Ratio, $P/(f_c'A_w)$	0 to 0.2
Normalized Centroid of Boundary Reinforcement, d'/l_w	0.05, 0.1, 0.15
Normalized Flange Thickness, t_f/t_w (for Barbell only)	2
Normalized Flange Depth, l_f/t_f (for Barbell only)	1, 1.5, 2

The effective yield curvature estimates were determined using standard moment-curvature analyses satisfying strain compatibility, material stress-strain relationships, and equilibrium. The longitudinal boundary reinforcement was assumed to be lumped at its centroid, and the longitudinal web reinforcement was assumed to be uniformly distributed as a thin sheet (Fig. 3(a)). For barbell sections, the centroid of boundary reinforcement coincides with the centroid of the boundary region. The strain distribution was assumed to be linear across the section (Fig. 3(b)).

The stress-strain curve for concrete in compression was assumed to be parabolic (Figs. 3(c) or 4(a)). The modulus of elasticity of concrete in compression, E_c, was $57{,}000\sqrt{f_c'}$ (in psi), as defined in ACI 318-02 [7]. This modulus is a secant modulus representing the slope of line passing through the concrete stress-strain curve at $0.45 f_c'$, as illustrated in Fig. 4(a). The strain at the instant that the stress reaches f_c', denoted ε_c', that corresponds to this definition is 0.00193, 0.00216, and 0.00237 for f_c' equal to 4, 5, and 6 ksi (28, 35, 42 MPa), respectively. The concrete tensile strength was neglected. An elasto-plastic stress-strain curve was used for reinforcing steel in both compression and tension (Fig. 4(b)). The modulus of elasticity of the steel, E_s, was taken as 29,000 ksi (200,000 MPa).

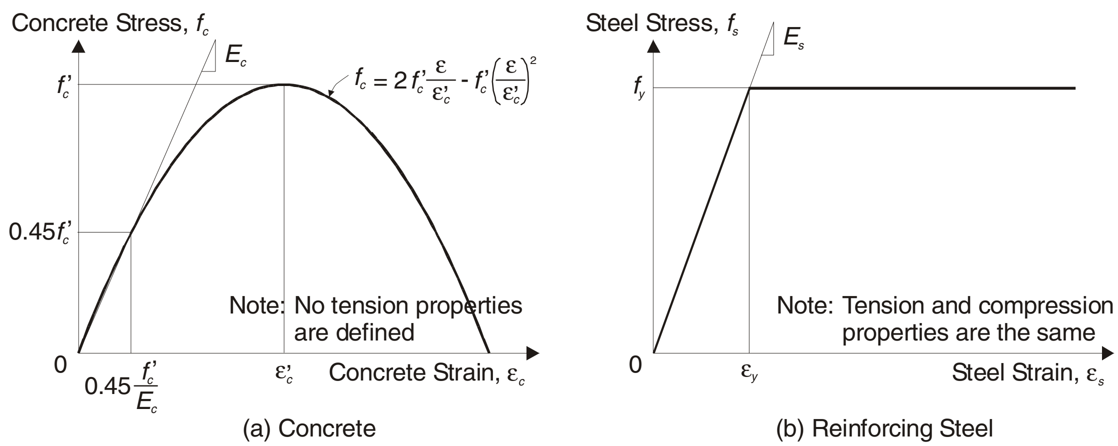


Fig. 4 Stress-strain relationships for concrete and reinforcing steel used in this study.

The first-yield curvature, ϕ'_y, was established based on the strain condition shown in Fig. 3(b) as

$$\phi'_y = \frac{\varepsilon_{cy} + \varepsilon_y}{d}, \tag{1}$$

where $\varepsilon_y = f_y / E_s$ = steel yield strain, ε_{cy} = concrete strain at the extreme compression fiber at the time longitudinal boundary reinforcement strain reaches ε_y, and d = the distance from the extreme concrete compression fiber to the centroid of the boundary reinforcement. For higher axial load levels, the steel yield strain, ε_y, may occur at a very high value of ε_{cy}. For these cases, the first-yield curvature is defined as curvature corresponding to ε'_c, i.e., the strain at the time the stress reaches f'_c. The effective yield curvature, ϕ_y, was obtained from extrapolating the first-yield curvature to a point where the moment reaches ultimate strength, M_u, assuming elasto-plastic response (Fig. 5), or

$$\phi_y = \frac{M_u}{M'_y}\phi'_y, \tag{2}$$

where M'_y = moment resistance when longitudinal boundary reinforcement strain reaches ε_y. The ultimate flexural strength, M_u, is defined as the moment resistance corresponding to a concrete strain of 0.003 at the extreme compression fiber.

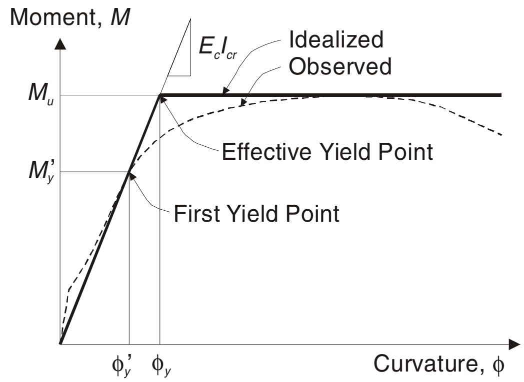


Fig. 5 Definition of effective yield curvatures used in this study.

Results

Charts were generated to allow the effective yield curvature of rectangular and barbell wall cross sections to be estimated directly. Each chart plots the yield curvature coefficient as a function of the axial load level in a dimensionless form for a number of longitudinal boundary reinforcement ratios, a specified longitudinal web reinforcement ratio, specified concrete compressive strength, and steel reinforcement yield strength. Typical charts are given in Fig. 6 for both rectangular and barbell cross sections. Given the cross sectional shape, the axial load level, the longitudinal boundary reinforcement ratio, ρ, and the longitudinal web reinforcement ratio, ρ'', the effective yield curvature coefficient can be interpolated between the curves.

From the moment-curvature analyses, it is observed that effective yield curvature is nearly insensitive to concrete strength and web reinforcement ratio. It is also observed that both the rectangular and barbell cross sections exhibit similar results for the range of values covered in the study. At early stage of the design process, the boundary and web steel reinforcement steel is usually not known yet. For design purposes, it is therefore practical to define effective yield curvature as a function of axial load level and steel reinforcement grade; at a later stage the necessity of a confined boundary element can be addressed. The effective yield curvature coefficient for rectangular or barbell cross sections may be estimated with the following formula with errors typically less than 5 to 10%:

$$\kappa_\phi = 1.8\varepsilon_y + 0.0045\frac{P}{f_c' A_w}. \tag{5}$$

It should be noted that Eq. (5) is valid for cases where the axial load ratio, $P/(f_c' A_w)$, is not more than 0.2. In addition, Eq. (5) is applicable for barbell cross sections in which $1 \le l_f / t_f \le 2$ and $t_f / t_w \le 2$. The same equation proposed for both the rectangular and barbell cross sections provides flexibility in satisfying the detailing requirements for special transverse boundary reinforcement prescribed in the performance objectives or required by code strain compatibility analysis provisions.

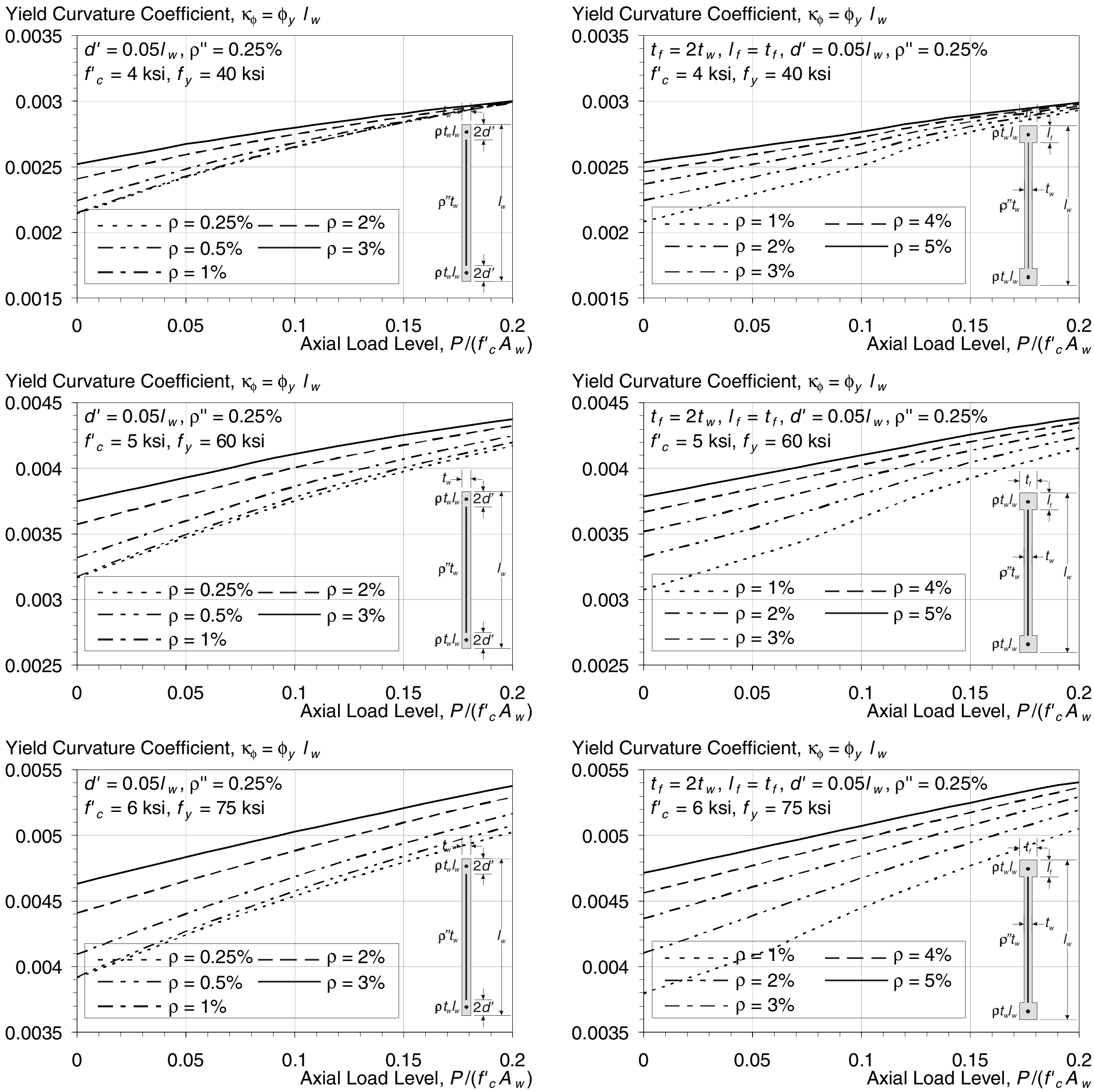


Fig. 6 Typical charts of effective yield curvature coefficients (1 ksi = 6.895 MPa).

APPLICATION TO DESIGN APPROACHES BASED ON YIELD DISPLACEMENT

ESDOF System for Approximating Dynamic Response of Structures

"Equivalent" Single-Degree-of-Freedom (ESDOF) representations have been used in seismic evaluation procedures, such as the displacement coefficient and capacity spectrum methods [8, 9], to estimate inelastic seismic demands of the structure under consideration. In these procedures, the response of a multi-degree-of-freedom (MDOF) model of the structure is assumed to be predominantly in a single deflected shape throughout the response history. ESDOF representations have also recently been employed for determining the required strength and stiffness to limit the maximum displacement response to a desired value. The latter use will be demonstrated by a design example presented in the next section.

Fig. 7 summarizes an ESDOF formulation adapted from ATC-40 [8]. This formulation uses the elastic first mode shape considering cracked-section flexural stiffness properties, resulting in a match between the fundamental period of the wall system, T, associated with the cracked section stiffness, k, (Fig. 7(a)) and the initial period of the ESDOF model, T^*. Also, the height of the lateral force resultant of the MDOF system, h_{eff}, is the same as the effective height of the ESDOF system.

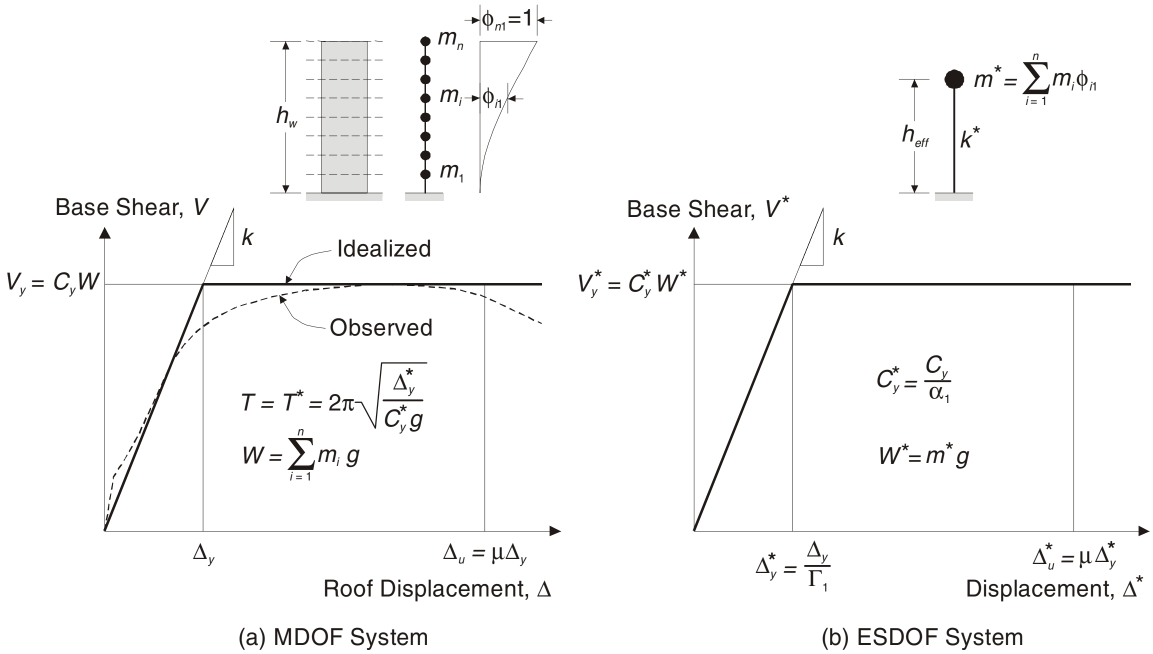


Fig. 7 Idealized load-deformation responses of a wall system and the ESDOF system.

As indicated in Fig. 7, the relationship between the roof displacement of a MDOF model of the wall system at yield, Δ_y, and the yield displacement of the corresponding ESDOF system, Δ^*_y, is given as

$$\Delta^*_y = \frac{\Delta_y}{\Gamma_1}, \tag{6}$$

where Γ_1 = the first mode participation factor, calculated with the mode shape normalized such that the mode shape amplitude at the roof, ϕ_{n1}, is equal to one.

The base shears of the ESDOF system, V^*_y, and the MDOF system, V_y, at yield are related by

$$V^*_y = \frac{V_y}{\alpha_1}, \tag{7}$$

where α_1 is the first mode effective mass coefficient. Normalizing Eq. (7) by the corresponding weights of the systems gives

$$C_y^* = \frac{C_y}{\alpha_1}, \tag{8}$$

where $C_y^* = V_y^* / W^*$ = the yield strength coefficient of the ESDOF system and $C_y = V_y / W$ = the base shear coefficient of the MDOF system.

For buildings consisting of prismatic walls having uniform story heights and uniform floor masses and responding in flexure only, Γ_1, α_1, and h_{eff} have the values given in Table 1. For other cases, the values of these terms can be determined using standard formulas.

While not required for design, the period of vibration of the wall system often is of interest. Corresponding to cracked-section stiffness of the wall system, *k*, as defined in Fig. 7(a), this period may be calculated based on the properties of the ESDOF system as

$$T = T^* = 2\pi\sqrt{\frac{\Delta_y^*}{C_y^* g}}. \tag{9}$$

The displacement response and peak drift of an ESDOF system, Δ_u^*, subjected to ground motion excitation may be estimated using a dynamic analysis using a simplified hysteretic model. For slender reinforced concrete structural wall buildings, an elasto-plastic model with stiffness degradation often is sufficiently accurate. For cases where only smoothed code spectra are available, the peak drift demand may be estimated using a suitable R-μ-T relationship developed for stiffness-degrading oscillators.

Given the ESDOF drift estimate, the roof drift of the wall system can be estimated as $\Delta_u = \Gamma_1 \Delta_u^*$. This implies that the displacement ductility of the ESDOF and MDOF systems is equal and is given as

$$\mu = \frac{\Delta_u}{\Delta_y} = \frac{\Delta_u^*}{\Delta_y^*}. \tag{10}$$

Yield Point Spectra for Estimating Seismic Demands and Required Lateral Strengths

Yield Point Spectra (YPS) [10] plot curves of constant displacement ductility on the axes of yield strength coefficient and yield displacement, for SDOF oscillators having a range of initial (elastic) periods of vibration, a specified hysteretic property, and a specified level of viscous damping (Fig. 8). Any one point on a YPS plot represents four quantities: Δ_y^*, C_y^*, T^*, and μ. The first three of these quantities are related according to Eq. (9). Furthermore, knowledge of the ESDOF ductility, μ, allows the peak displacements, Δ_u^* and Δ_u, to be estimated using Eq. (10). YPS may directly be computed for specific ground motion records or estimated by applying previously established R-μ-T relationships to elastic design response spectra.

YPS may be used to estimate the peak displacement response of a SDOF system (or an ESDOF system) having a known yield point. YPS may also be used for an inverse process, i.e., to determine the yield point for a new design or to determine the level of strengthening and stiffening required to rehabilitate an existing structure to have acceptable seismic performance. The inverse process, for determining acceptable yield points for a desired performance, is schematically illustrated in Fig. 8.

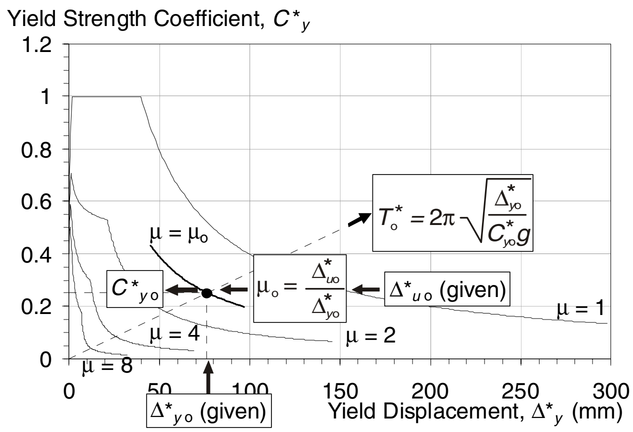


Fig. 8 Example of YPS and its use for estimating yield strength coefficient (25.4 mm = 1 in.).

Design Example

Building Description

The design of two barbell walls in an eight-story structural wall building is presented to illustrate the use of the estimated yield displacement in design. Two identical barbell section walls are used as the lateral load carrying system for the N-S direction, as indicated in Fig. 9. All floor-to-floor heights are 12 ft (3.65 m). The concrete compressive strength, $f_c^{'}$, and the reinforcing steel yield strength, f_y, are 5 ksi (35 MPa) and 60 ksi (414 MPa), respectively. Floor dead and live loads are 175 psf (8.4 kN/m^2) and 50 psf (2.4 kN/m^2), respectively. The axial load at the base of the walls was estimated to be $0.15 f_c^{'} A_w$. Seismic weight, *W*, was assumed to come from the dead load only, which is 8(175 psf)(180 ft)(60 ft) = 15,120 kips (67,254 kN).

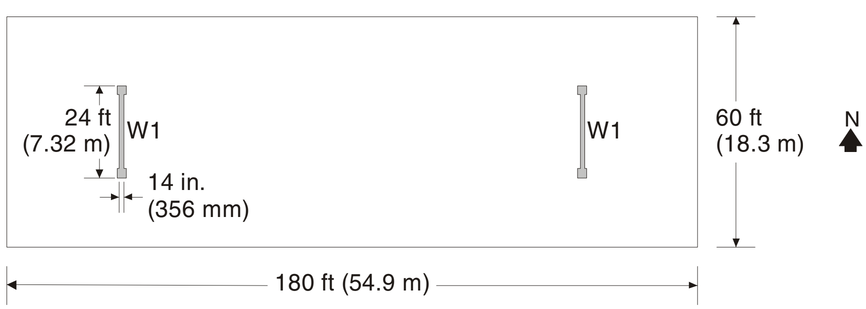
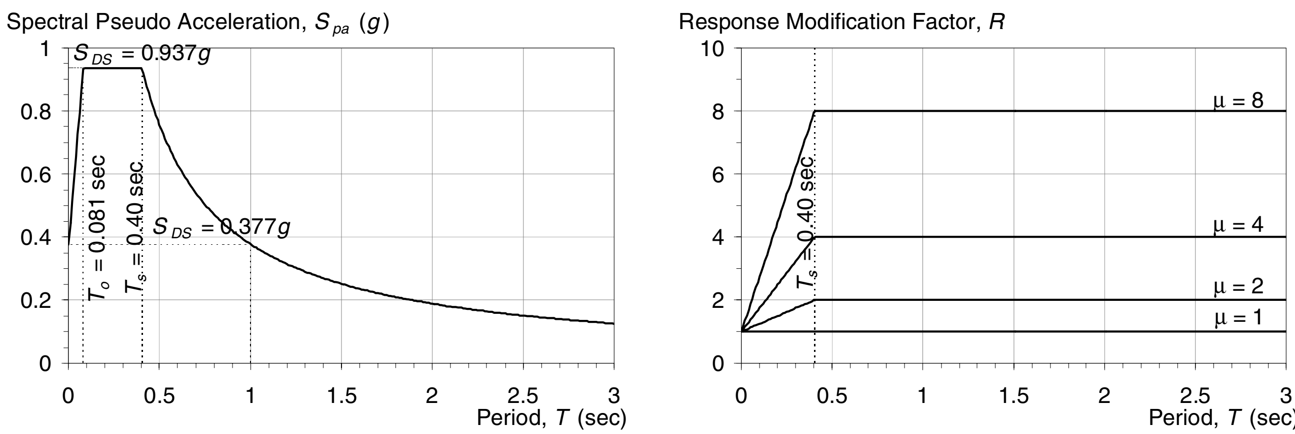


Fig. 1 Typical floor of the design example.

Design Performance Objective

The N-S direction was designed to satisfy a “life safety” performance objective, which is somewhat arbitrarily associated with a peak roof displacement limit equal to 0.83% of the height of the building. The design earthquake associated with this performance objective is represented by the smoothed elastic design spectrum shown in Fig. 10(a). This spectrum has the following parameters per IBC 2000 [11] terminology: $T_s = 0.40\,\text{sec}$, $T_o = 0.2T = 0.081$ sec, $S_{Ds} = 0.937g$, and $S_{D1} = 0.377g$. The simple *R*-μ-*T* relationships shown in Fig. 10(b) are used to derive inelastic seismic demands from the design spectrum. Only translational response is considered in this design; the potential effects of accidental eccentricity and rotational components of the ground motion on torsional response are neglected.



(a) Design Spectrum

(b) R-μ-T Relationships

Fig. 10 Design spectrum and R-μ-T relationships used in the design example.

Design Calculations

The design approach described in the previous sections is used to determine the seismic demand and required base shear for determining the required flexural strength at the base of each wall. For brevity, only calculations for required base shear and base moment of the walls are presented. The distribution of lateral forces over the height of the building and to each wall at each floor level may follow code or other established procedures [e.g., 6, 12]. The required shear and moment capacities over the height of each wall should be determined considering higher mode effects using any of several established procedures [e.g., 13-16].

The effective yield curvature coefficient for the walls is estimated using Eq. (5), as 1.8(60 ksi/29,000 ksi) + 0.0045(0.15) = 0.0044. By Eq. (2), the estimated yield curvature is 0.0044/(24 ft) = 15.3×10^{-6} rad/in. (0.60×10^{-6} rad/mm). For an eight-story building, the estimated yield displacement coefficient, κ_Δ, is 0.295 according to Table 1. Using Eq. (1), the yield displacement, Δ_y, is 0.295(15.3×10^{-6} rad/in.)(96 ft)^2 = 6.0 in. (152 mm). Based on the design performance objective, the roof displacement limit, Δ_u, is (0.83%)(96 ft) = 9.6 in. (244 mm). Thus, the displacement ductility limit, μ, is 9.6/6.0 = 1.6.

From Table 1, the modal participation factor, Γ_1, mass coefficient, α_1, and effective height, h_{eff}, for an eight-story building are 1.445, 0.653, and 0.770, respectively. The ESDOF displacement, Δ^*_y, is (6.0 in.)/1.445 = 4.15 in. (105 mm).

The YPS for this design are shown in Fig. 11, established by applying R-μ-T relationships to the design spectrum. The minimum yield strength coefficient, C^*_y, for μ = 1.6 and Δ^*_y = 4.15 in. is obtained from YPS as 0.131 (Fig. 11). By Eq. (8), the required base shear coefficient, C_y, is 0.653(0.131) = 0.0855. Therefore, the base shear of the building is 0.0855(15,120 kips) = 1293 kips (5752 kN), the required base shear of each wall is (1293 kips)/2 = 646 kips (2876 kN), and the required moment at the base of each wall is 0.770(1293 kips)(96 ft)/2 = 47,790 k-ft (64,790 kN-m).

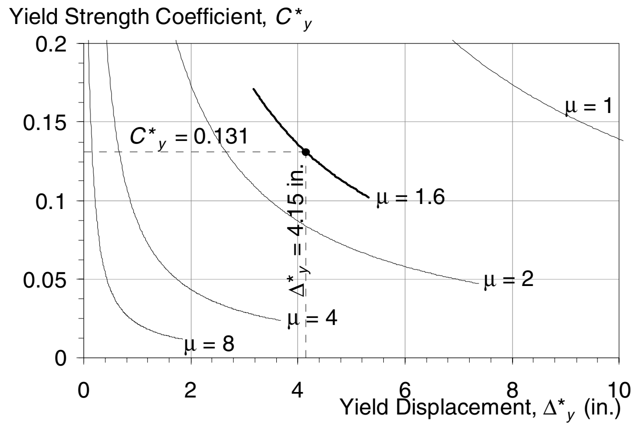


Fig. 2 YPS for the design example (1 in. = 25.4 mm).

CONCLUSION

Improved estimates of effective yield curvature for rectangular and barbell cross sections were presented for use in design of ductile reinforced concrete cantilever structural walls. These estimates were derived based on consistent standard moment curvature analyses covering variation in boundary reinforcement, web reinforcement, steel reinforcement yield strength, concrete compressive strength, axial load ratio, relative dimensions of the cross section. A set of charts was developed to allow the effective yield curvature to be estimated directly. Simple expression derived in terms of parameters usually known at the early design stage – axial load and steel reinforcement grade – was also presented. A design example based on "equivalent" single-degree-of freedom in conjunction with Yield Point Spectra was provided to illustrate the application of these estimates to the design of ductile reinforced structural wall buildings.

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NOTATION

A_s = Area of longitudinal boundary (end) reinforcement of a wall.

A_w = Cross section area of a wall.

C_y = Base shear strength of a wall or a wall system, V_y, normalized by its seismic weight, W (usually termed base shear strength coefficient).

C_y^* = Yield strength coefficient of the ESDOF model of a wall system.

d = Distance from the extreme compression fiber to the centroid of longitudinal boundary reinforcement.

$d^{'}$ = Distance from the extreme concrete fiber to the centroid of longitudinal boundary reinforcement.

$E_c I(x)$ = Flexural rigidity at a wall section located at a distance *x* from the base.

$E_c I_{cr}$ = cracked-section flexural stiffness at the base.

E_s = Modulus of elasticity of steel reinforcement.

f_{cy} = Concrete compressive stress when the extreme compression fiber reaches ε_{cy}.

$f_c^{'}$ = Specified compressive strength of concrete.

f_y = Specified yield strength of longitudinal reinforcement.

g = Acceleration of gravity.

h_{eff} = Resultant of lateral forces measured from the base of a wall system (usually termed effective height).

h_w = Total height of a wall or a wall system.

k = Lateral elastic cracked-section stiffness of a wall or a wall system in elastic region when subjected to a lateral force distribution proportional to the product of the fundamental mode shape amplitude and floor mass at each floor.

$k_y d$ = Neutral axis or compression zone depth of a wall cross section at yield.

k^* = Lateral stiffness of the ESDOF model of a wall system.

l_f = Depth of the boundary part of a bar-bell wall cross section. This dimension is parallel to the horizontal length, l_w.

l_w = Horizontal length of a wall.

m_i = Lumped mass of the *i*-th floor of an *n*-story building.

m^* = Seismic mass of the ESDOF model of a wall system.

M_y = Moment resistance corresponding to effective yield curvature of a wall section at the base, ϕ_y.

$M_y^{'}$ = Moment resistance corresponding to first-yield curvature of a wall section at the base, $\phi_y^{'}$.

M_u = Moment resistance corresponding to a concrete strain of 0.003 at the extreme compression fiber.

$M(x)$ = Moment of a wall at a section located at a distance *x* from the wall base.

n = Number of stories.

P = Axial load of a wall.

R = Ratio of the elastic strength demand to the yield strength coefficient (also known as response modification coefficient in IBC 2000 [11]).

S_{D1} = Design spectral pseudo acceleration at 1 sec period (parameter in IBC 2000 [11]).

S_{DS} = Design spectral acceleration at short periods (parameter in IBC 2000 [11]).

t_f = Thickness of the boundary of a bar-bell wall cross section. This dimension is parallel to the web thickness, t_w.

t_w = Web thickness of a wall.

T = First mode period of vibration of a wall system in the design direction based on cracked-section stiffness, k.

T^* = Period of vibration of an ESDOF model of a wall system.

T_s = characteristic period (parameter in IBC 2000 [11]).

T_o = $0.2T_s$ (parameter in IBC 2000 [11]).

V_y = Lateral base shear of a wall or wall system at yield when subjected to a lateral force distribution proportional to the product of the fundamental mode shape amplitude and floor mass at each floor (also termed base shear strength).

V_y^* = Yield strength of the ESDOF model of a wall system.

W = Seismic weight of a wall system.

W^* = Seismic weight of the ESDOF model of a wall system.

x = Vertical distance of a wall cross section, measured from the wall base.

α_1 = First mode effective mass coefficient.

Δ_y = Lateral roof displacement of a wall or a wall system at yield when subjected to a lateral force distribution proportional to the product of the fundamental mode shape amplitude and floor mass at each floor.

Δ_y^* = Lateral displacement of the ESDOF model of a wall system at yield.

Δ_u = Maximum lateral roof displacement (also known as peak drift) of a wall system at ultimate.

Δ_u^* = Maximum lateral displacement of the ESDOF model of a wall system.

ε_{cy} = Concrete strain at the extreme compression fiber when longitudinal boundary reinforcement strain reaches ε_y .

ε_y = Yield strain of longitudinal reinforcement.

$\varepsilon_c^{'}$ = Strain corresponding to peak stress, $f_c^{'}$, in a uniaxial stress-strain curve of concrete.

ϕ_{i1} = First mode shape amplitude at the *i*-th floor of an *n*-story building.

$\phi_y^{'}$ = First-yield curvature of a wall section at the base.

ϕ_y = Effective yield curvature of a wall section at the base.

κ_Δ = Yield displacement coefficient of a wall.

κ_ϕ = Yield curvature coefficient of a wall.

Γ_1 = First mode modal participation factor.

μ = Displacement ductility factor of a wall or wall system.

ρ = Ratio of longitudinal boundary reinforcement of a wall = $A_s/(t_w l_w)$.

$\rho^{''}$ = Ratio of longitudinal web reinforcement of a wall = $A_s^{''}/(t_w l_w)$.

APPENDIX

Table 1 Conversion Factors from U.S. Customary to SI Units

To Convert	To	Multiply by
inch (in.)	millimeter (mm)	25.4
foot (ft)	meter (m)	0.3048
kilopound force (kip or k)	kilonewton (kN)	4.448
kilopound force per square inch (ksi)	megapascal (MPa)	6.895
pound per square foot (psf)	megapascal (MPa)	47.88