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SEISMIC PERFORMANCE BASED LOSS ASSESSMENT

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SUMMARY

The paper describes procedures adopted to develop and implement building vulnerability curves to relate damage ratio (defined as dollar loss / replacement value) to spectral acceleration for individual building and portfolio loss assessment. The Performance Based Engineering framework developed by the PEER researchers is implemented through the use of Incremental Dynamic Analyses to develop the building vulnerability functions. Representative structure models are subjected to a carefully selected suite of ground motions, which are scaled so that their elastic spectral accelerations at the fundamental period of the structure are equal to a target value. The maximum inter-story drift of each story from time-history analysis is computed and related to a damage state and an associated damage ratio for both structure and non-structural components. Details of the procedure and results are presented for low and mid-rise Steel Perimeter Moment Frame buildings.

The final section of the paper highlights the impact of implementing performance based vulnerability functions for portfolio loss assessment. Losses computed using performance based loss assessment (PBLA) vulnerability functions are compared to losses using MMI based curves for the same hazard characteristics. The comparisons are done for both scenario events and for annualized and return period losses of interest to the insurance industry. The results of sensitivity analyses show that the spectral response based results are qualitatively more consistent with damage patterns observed during past events.

INTRODUCTION

Over the last several years, the Modified Mercalli Intensity (MMI) has been used to measure the overall performance of buildings for the purpose of risk assessment. In the ATC-13 (1985) publication [1], which has been used widely during the last two decades to estimate economic losses during earthquakes, building vulnerability assessments are based on damage probability matrices developed using expert

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opinions for different building characteristics with MMI employed as the measure of ground motion intensity. ATC-13 has served as the basis for the first generation of catastrophic modeling software, which is still widely used for earthquake insurance portfolio analysis.

A shortcoming of the use of MMI as an intensity measure is that it is a subjective indicator of ground motion intensity, which depends on observed damage and it does not appropriately account for the performance of buildings of different characteristics. In recent years, a new approach has been proposed for performance evaluation of buildings at the regional level that attempts to use earthquake seismic demand parameters and building capacity characteristics in such evaluations. In 1997, the National Institute of Building Sciences (NIBS) [8] completed the development of a geographic information system (GIS) based regional loss assessment tool to facilitate these performance evaluations. The primary purpose of that project was to develop a framework and procedures for earthquake loss estimation at the regional scale using the HAZUS software. The FEMA/NIBS earthquake loss estimation methodology utilized a quantitative procedure based on characteristics of the buildings involved for damage assessments due to earthquake ground motions. This FEMA/NIBS capacity spectrum-based approach is superior to the older MMI methodology for building damage assessments as it explicitly takes into consideration both the building capacity (using pushover procedures) and seismic demands to develop necessary fragility functions.

The use of the capacity spectrum approach to quantify building damage for regional-scale loss assessments was an important first step in the right direction. However, the method has limited accuracy because the response of a structure during an earthquake is only approximately computed using an incremental static nonlinear analysis. In this approach, the dynamic response of buildings is not considered explicitly.

During the last few years, significant advances have been made to link building performance to ground motion characteristics and to both qualitatively and quantitatively describe the building performance for different hazard levels. The FEMA356 guidelines provide a framework and procedures for a performance-based seismic engineering evaluation of buildings. Structural element capacity data and seismic demand parameters such as inter-story drift are used to describe the building performance for different hazard levels. Most importantly, these guidelines document information related to the damage threshold capacities of various structural elements corresponding to different levels of seismic performance. The same procedure was introduced in the probabilistic form by Luco and Cornell (1988) [6] to measure the seismic performance of structures. Probabilistic seismic demand estimates, along with information on capacities of elements and structures can be used to determine probabilities of exceeding certain performance levels. The Pacific Earthquake Engineering Research (PEER) center has developed a framing equation (Cornell and Krawinkler, 2000) [2] for the performance evaluation of buildings. In the present study, the PEER framework is utilized for economic loss assessment based on building performance at the level of a single site as well as at a regional scale. The study is presented from an insurance perspective, focusing on estimation of the dollar cost of repairs for damaged buildings.

FRAMEWORK FOR EARTHQUAKE LOSS ASSESSMENT METHODOLOGY

The performance of buildings during strong earthquakes depends mainly upon the characteristics of the ground motions and the energy absorption and dissipation capacity of the structures. A systematic study of those parameters that influence both ground motion and structural response characteristics is required in order to be able to accurately evaluate the expected building performance during earthquakes.

The ability of structures to dissipate seismic input energy is related to the inelastic behavior of the structures. Inelastic response is affected by global dynamic characteristics (e.g., periods, damping, etc.) as well as by material properties, yield strength, ductility capacity, and detailing of the elements in the structures. The nonlinear analytical modeling of a structure is used to evaluate its global response characteristics in order to estimate the degree of damage sustained during cyclic loading. An accurate assessment of the seismic response of buildings is a complex problem due to the significant number of parameters that affect building performance during an earthquake. These parameters are also subject to uncertainty – for example, arising from the evaluation of material properties from tests, from simplified analytical modeling of building, from the effects of nonstructural components, from definition of building damage thresholds, and finally from inherent variability in ground motion characteristics. In this study, the PEER framework has been adopted for seismic Performance-Based Loss Assessment (PBLA) of buildings from an insurance perspective for site-specific as well as portfolio analysis.

The PEER framing equation used as a basis for the seismic performance evaluation of buildings is as follows:

$$\lambda(DV) = \iiint G(DV \mid DM)\,dG(DM \mid EDP)\,dG(EDP \mid IM) \mid d\lambda(IM) \mid \qquad (1)$$

in which *G(DV|DM)*, *G(DM|EDP)*, and *G(EDP|IM)* are conditional probabilities that the decision variable *(DV),* the damage measure *(DM)*, and the engineering demand parameters *(EDP)* are such that exceedance of a specified performance level results, given the intensity measure *(IM)*. The term, $\mid d\lambda(IM) \mid$, in Eq. (1) is the absolute value of the derivative of the hazard curve that defines the annual probability of exceeding a given value of *IM*.

A modified form of Eq. (1) as presented by Miranda & Aslani (2003) [7] is more appropriate for the objectives of this study and is presented below.

$$P[DR > dr] = \sum_{i=1}^{m} \iint P[DR > dr \mid DS = ds_i]\,P[DS = ds_i \mid IDR = idr] \mid dP[IDR > idr \mid SA = S_a(T_1)] \mid \left|\frac{d\nu(SA)}{dSA}\right| dSA \qquad (2)$$

where:

$P\left[DR > dr\right]$ is the annual probability of exceeding a specified building damage ratio, *dr*,

$P\left[DR > dr \middle| DS = ds_i\right]$ is the conditional probability of exceeding the damage ratio, *dr*, given that the structure is in damage state ds_i,

$P\left[DS = ds_i \middle| IDR = idr\right]$ is the conditional probability of being in damage state ds_i given that the maximum inter-story drift demand is *idr*,

$P[IDR > idr \mid SA = S_a(T_1)]$ is the probability of exceeding a maximum inter-story drift demand, *idr*, given that the spectral acceleration due to the input ground motion at the fundamental period of the structure is $S_a(T_1)$, and

$\left|\frac{d\nu(SA)}{dSA}\right|$ is the absolute value of the slope of the hazard curve for spectral acceleration at the fundamental period of the structure.

Comparing Eq. (2) with Eq. (1), it may be seen that the ground motion intensity measure (*IM*) chosen for this study is spectral acceleration at the fundamental period of the structure. Similarly, the maximum inter-story drift demand was used as the engineering demand parameter (*EDP*). Using different drift threshold values for the different performance levels, inter-story drift demands can be mapped to different damage states (*DS*). The Decision Variable (*DV*) used for this study is the damage ratio (*DR*) computed as the repair cost for the building normalized by the replacement cost.

In the software implementation, loss computations as indicated by Eq. (2) are performed using a vulnerability module and a hazard module. The hazard module computes the hazard at each building location using a set of predetermined events, taking into account the effects of attenuation and surface geology. The vulnerability module contains a set of pre-computed vulnerability functions for different types of construction and coverage. These functions express the probability of exceeding a given damage ratio for specified level of spectral acceleration, i.e. $P\left[DR > dr \middle| SA = sa\right]$

This can be computed as

$$P[DR > dr \mid SA = sa] = \sum_{i=1}^{m} \int P[DR > dr \mid DS = ds_i] P[DS = ds_i \mid IDR = idr] \mid dP[IDR > idr \mid SA = sa] \mid$$

The Incremental Dynamic Analysis (IDA) (Vamvatsikos and Cornell, 2001) [13] procedure is utilized to compute the vulnerability functions. In the IDA procedure, the structure is subjected to a series of time histories of increasing intensity. The hazard parameter chosen for this study, spectral acceleration at the fundamental period, is used to scale the ground motions to the different levels of intensity. The total number of unscaled records at each level of spectral acceleration is insufficient; hence, scaling of the ground motions is necessary in order to be able to evaluate the seismic performance of buildings for the different intensity levels. The effects of scaling of ground motion records using PGA as well as $S_a(T_1)$ have been examined by several researchers, and the extent of scaling and its limitations depend on the characteristics of the ground motions (i.e., taking into consideration near-source effects, directivity, soil conditions, etc.) as well as on structural properties (such as period and strength). Therefore, careful review of the scaled ground motions is necessary to ensure that important characteristics of the unscaled ground motions such as frequency content are preserved.

Earthquake Ground Motions

A total of 480 ground motions recorded at stiff soil sites during seven earthquakes with magnitudes greater than 6.2 are selected for the time history analyses. Table 1 summarizes the list of the earthquakes and the number of strong ground motion records used in this study. The normalized elastic strength demand spectra for these ground motions as well as mean and mean $\pm\sigma$ spectra are shown in Figure 1.

Table 1 List of events whose strong ground motion records are used in this study

Date	Event Name	Magnitude	# of records used
1983	Coalinga	6.5	90
1984	Morgan Hill	6.2	46
1987	Whittier-Narrows	6.0	104
1989	Loma Prieta	7.0	72
1992	Big Bear	6.4	48
1992	Landers	7.3	34
1994	Northridge	6.7	86
	Total Number of records		480

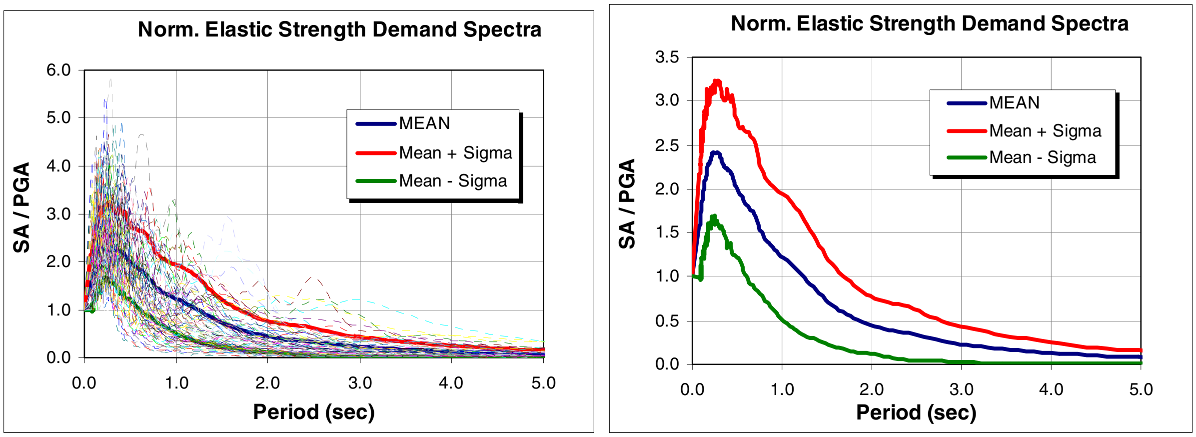

Figure 1. Normalized Elastic Strength Demand Spectra

Structural Models for the Incremental Dynamic Analyses

Three steel moment resisting frame structure models (corresponding to 3, 9, and 20-story buildings), which were designed as part of the SAC steel project, are utilized for the nonlinear time history analyses. Details of these structures and a comprehensive evaluation of their seismic response are documented in Gupta and Krawinkler (1999) [4].

In the PBLA approach, the damage assessment for a selected building is based on the results from nonlinear dynamic analyses. The Drain-2DX analysis program was used to perform a comprehensive set of nonlinear time history analyses of the SAC steel buildings using appropriate bilinear hysteresis models for each structure. A total of 480 ground motions were used for the time history analyses at each scaled level of spectral acceleration. Additionally, for each ground motion considered, permutations on the structural properties were introduced to account for uncertainty associated with the capacity evaluation of the structural elements.

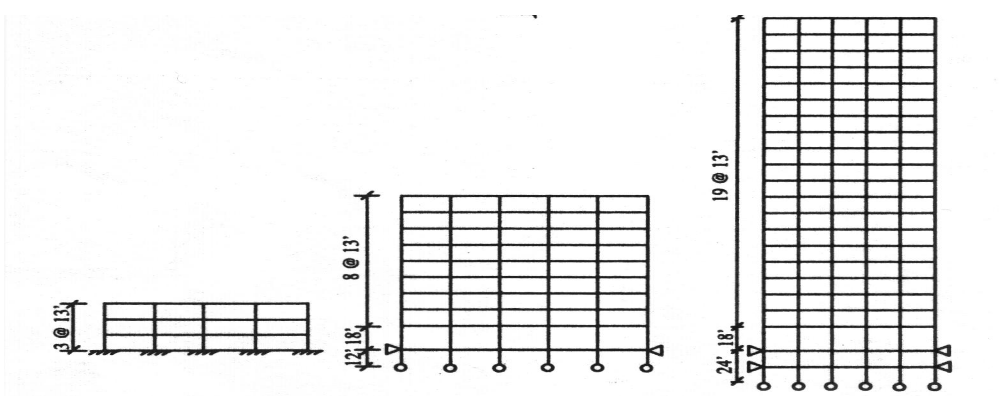

Figure 2. Elevation views of the Three SAC Steel Moment Resisting Frames used in the Nonlinear Dynamic Analyses (Gupta and Krawinkler, 1999)

The performance of a building during an earthquake depends on the vulnerability of the building and the characteristics of the ground motion excitation. The vulnerability is related to the capacity of the building, which may be a function of the global inter-story drift, plastic rotations or other seismic demands. Among all the seismic demand parameters, the inter-story drift ratio, which is the relative displacement at two consecutive story levels normalized by the story height, is the most convenient parameter for measuring the performance of buildings. For the SAC moment-resisting steel frame structures, the inter-story drift results from the lateral deformations caused by flexure in the beams and columns, and shear deformations at the joints. In this study, the maximum inter-story drift is used to assess the building damage state while developing the building vulnerability functions.

Building Fragility Functions

The economic loss assessment in earthquake events is based on a probabilistic approach that takes into consideration inherent uncertainties in the prediction of ground motions, in seismic demands on buildings, and in damage assessment of the buildings. The probabilistic relationship between structural damage and seismic demands is characterized by a fragility function, which expresses, for example, how a building damage ratio is related to the intensity of the ground motion.

Fragility curves express the probability of reaching or exceeding specific structural damage states as a function of seismic demand parameters such as spectral acceleration or displacement, or the inter-story drift ratio. Building fragility curves for any damage state are obtained by evaluating the conditional probabilities of being in, or exceeding, that damage state, given different levels of the seismic demands. The computed fragility curves take into account variability and uncertainty associated with the ground motions, with the structural elements' capacity evaluations, and with the selected thresholds that define the building damage states. In this study, four discrete damage states (corresponding to slight, moderate, extensive, and complete damage) are utilized to characterize the physical condition and state of the building damage.

The PBLA procedure involves quantification of the seismic demands for structures subjected to a suite of ground motion records. These seismic demands are utilized along with the specifed performance criteria to define the building damage states and subsequently measure the performance of structural and nonstructural components in the buildings. Several guildelines such as FEMA 356, and the 1999 SEAOC Blue Book [10] provide recommendations regarding building damage thresholds expressed in terms of inter-story drift ratios, plastic rotations, and other seismic demands to characterize building performance. These structural and nonstructural damage thresholds have generally been based on results from experimental tests on building components as well as from observations during post-earthquake building surveys.

Several thousand nonlinear analyses are routinely performed (as is done here) in order to take into the account the variability of the ground motions, the structural response, and the defined damage thresholds while developing the fragility functions. These fragility curves are then used to develop building vulnerability damage functions, which express the expected losses (normalized by the total replacement value) in term of ground motion intensity. These vulnerability functions are then used to assess economic losses due to earthquake events.

Figure 3 illustrates fragility functions for the 9-story SAC steel moment frame building. Each data point in the four plots shown describes the empirical (from analyses) probability of non-exceedance of the building damage ratio (*DR*) when subjected to the ground motion scaled to spectral acceleration (S_a) levels of 0.75, 1.0, 1.5 and 1.75g at the building's fundamental period. A lognormal cumulative distribution function (*CDF*) is fit to the data. This *CDF* describes the probability of non-exceedance of a given damage ratio, which is computed using the maximum inter-story drift as a measure of the story damage. The damage ratio (*DR*) used here is for the building and, as such, it is taken to be the average of the story damage from all the stories.

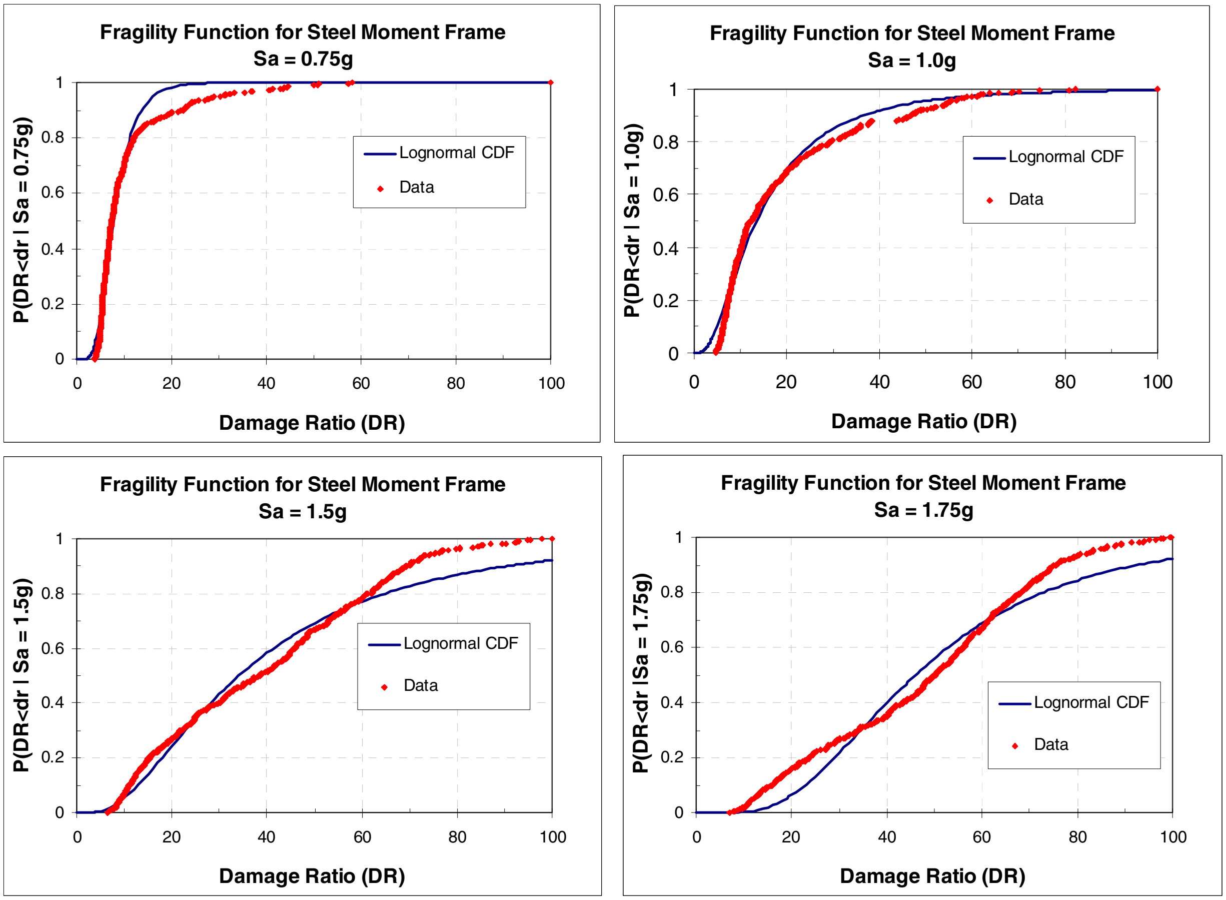

Figure 3. Fragility functions for the 9-story SAC building for different spectral acceleration levels

In order to investigate whether simplified single degree of freedom (SDOF) models might be adequate for the development of building vulnerability functions, similar types of SDOF model analyses were carried out as with the MDOF systems. The equivalent SDOF systems were selected to have the same natural period as the fundamental period of the corresponding MDOF system. These SDOF systems were then subjected to the same set of scaled ground motions and the same damage thresholds were utilized to develop the fragility functions and, subsequently, the building vulnerability functions. The SDOF- and MDOF-based vulnerability functions for the 9-story SAC buildings are shown in Figure 4. The plots show the mean structural damage ratio expressed as a function of spectral acceleration. As indicated in the figure, the SDOF-based damage ratios are typically underestimated when compared with the damage ratios based on the full MDOF analyses. This is likely due to the contribution of higher modes that are not accounted for in the SDOF analyses.

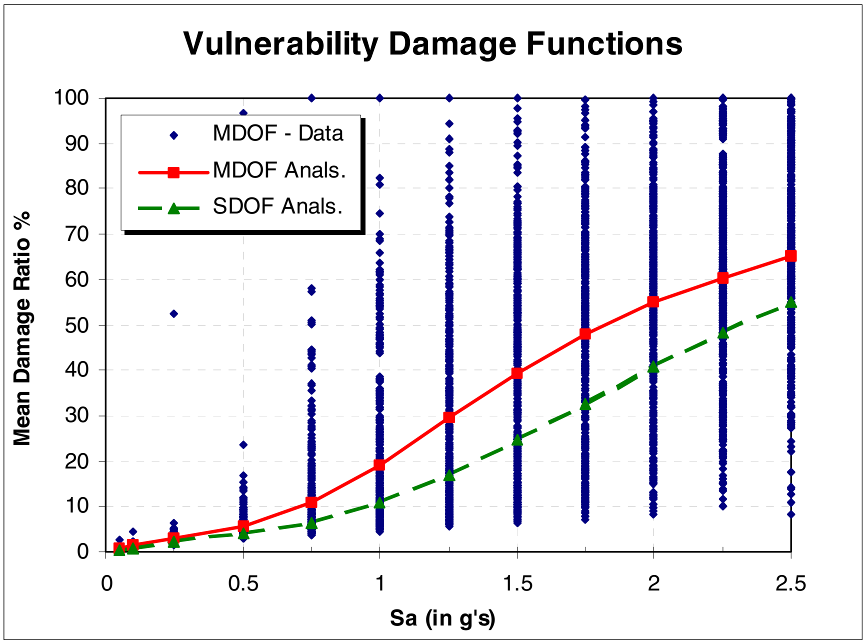

Figure 4. Building vulnerability function 9-story SAC buildings

Treatment of Uncertainty

One of the most difficult tasks in earthquake catastrophic modeling is the treatment of uncertainty. Several sources of uncertainty arise in such modeling. Two key sources are in the estimation of possible ground motion intensity levels at the site (i.e., the hazard) and in the evaluation of building vulnerability.

Hazard analysis requires the characterization of seismic sources (including location, magnitude, frequency of occurrence, etc.), that are likely to give rise to potentially damaging earthquakes at a site. Other critical elements in seismic hazard assessment include knowledge of the attenuation of ground motion or seismic intensity with distance from the source to the site, and knowledge of the local geology and soil conditions. Uncertainty in ground motion attenuation is a significant source of uncertainty as is soil amplification due to site effects. Earthquake hazard uncertainty, as incorporated in the RMS models, arises from uncertainties in the characterization of sources, in the ground motion attenuation functions, and in the soil amplification. Collateral hazards such as landslide and liquefaction are also treated with uncertainty when they affect building damage evaluations.

Uncertainty in any developed earthquake vulnerability functions refers to the uncertainty in the performance of a building, for a single given characterization of the ground motion (e.g., using a spectral acceleration level). Sources of uncertainty in vulnerability functions include uncertainty in material properties, in the structural system, in the construction quality, in the modeling of the structure, in the definition of the damage thresholds, and in the inherent uncertainty in building response given the intensity and duration of the seismic event. The variability associated with vulnerability is computed using a logic tree approach, for different levels of ground motion intensity. The contributions from variability in building strength and from selection of damage thresholds for the different damage states are considered in the different branches of a logic tree. For each of the above permutations (i.e., branches in the logic tree), the structural response is computed for an ensemble of ground motions scaled to the same intensity to capture the variability introduced by the differences in frequency content and duration of the various ground motions considered.

RESULTS DISCUSSION

The new performance-based vulnerability functions have been implemented in the Risk Management Solutions (RMS) earthquake catastrophic loss assessment software, which provides comprehensive catastrophe modeling for either a single location or for an entire portfolio.

The characteristics of the earthquake ground motions that are considered in the model are source mechanism, source-site distance, orientation, travel path, and local geological and soil conditions. In the PBLA approach, consideration of soil amplification is linked to the building height and depth of the soil layer. In order to illustrate advantages of the performance-based risk assessment as compared to the MMI approach, several scenario analyses are performed for sites located at distances from 1 to 200 km of the San Andreas Fault. Economic losses, expressed as average annual loss (AAL), were evaluated for the 3- and 20-story steel moment frame buildings located on stiff and soft soils. Figure 5 shows the site locations considered in the scenario analyses relative to the trace of the San Andreas Fault.

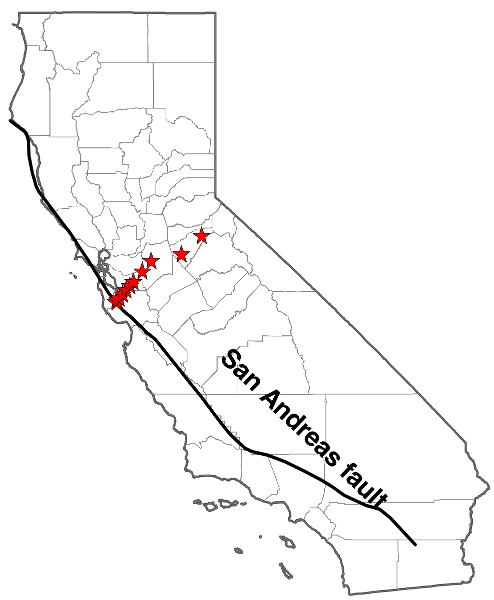

Figure 5. Selected sites and their locations relative to the San Andreas Fault.

Figure 6 illustrates the variation of AAL ratios (high-rise to low-rise) by magnitude, distance, and soil type. The charts on the left present the variation of AAL ratios (for magnitudes 8.0 and 6.5) with distance using the performance-based approach. The variation of AAL with distance for NEHRP class “C” and for soil class “E” is shown with light and dark bars, respectively. As seen in the figure, the variation of AAL with distance depends on magnitude as well as soil type. The chart on the right shows similar AAL ratios as those on the left but they are based on the MMI approach. The variation of AAL ratios by distance is constant regardless of magnitude and soil type, except for soft soil sites that are far from the source.

A comparison of these two approaches clearly demonstrates the significant advantages of the PEER methodology over the MMI methodology if one is interested in an accurate assessment of the impacts of distance, building height, soil type, and event magnitude on loss results.

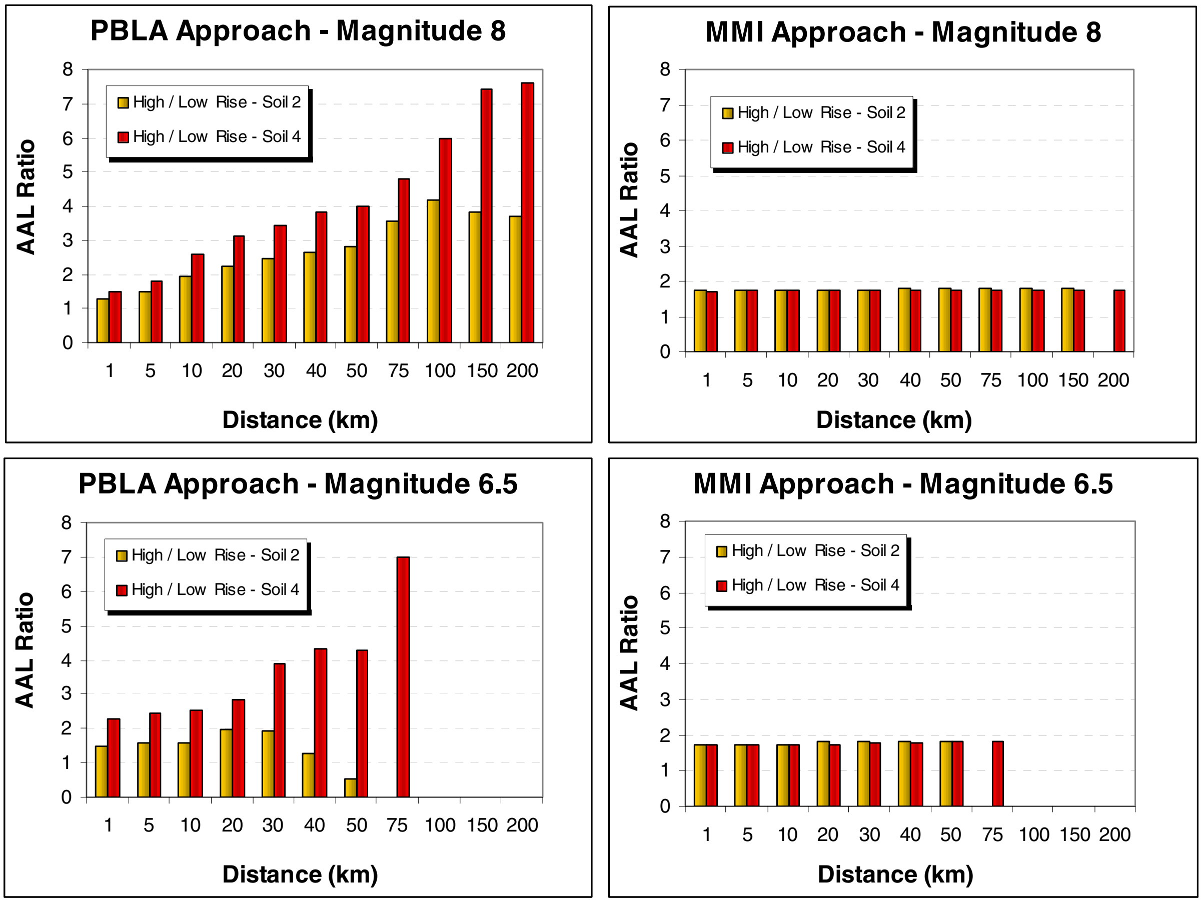

Figure 6 Variation of AAL ratios for PBLA and MMI approaches

Next, in order to illustrate the impact of the PBLA approach in loss estimation for different return periods, analyses are done for the 3- and 20-story steel moment frame buildings at a location in the San Francisco area with soft soil conditions. Figure 7 illustrates the probability of exceedance of different loss levels from either single or multiple occurrences. For the high-rise building, the losses at low return periods using the PBLA approach are less than those based on the MMI approach. These differences in loss results are due to the different treatment of soil amplification in these two approaches. The distance to the seismic source(s) and the height of the building play a role in the soil amplification and hence in the differences seen for the low- and high-rise buildings. As the ground motion intensity increases (which corresponds to larger, less frequent events), the PBLA loss estimates for these longer return periods are seen to be higher than is the case with the MMI approach. For tall buildings located at soft soil sites in San Francisco, the MMI approach underestimates losses from high return period events.

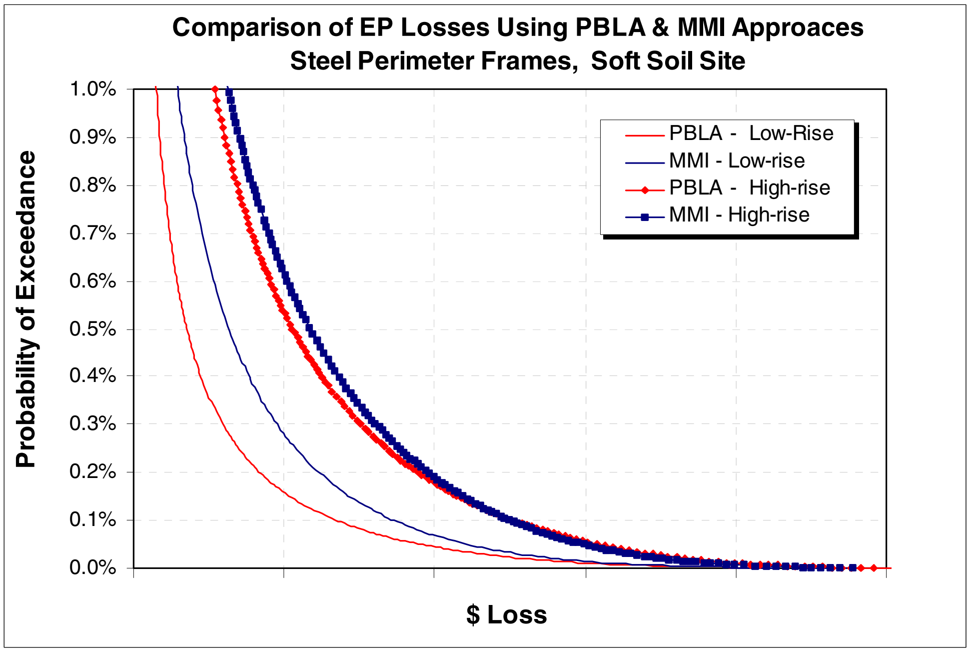

Figure 7 Variation of AAL ratios for PBLA and MMI approaches

As seen in Figure 7, for the low-rise building the PBLA-based approach generally yields lower loss estimates as compared to the MMI approach for most return periods. The figure shows the importance of building height and soil conditions and their impacts on loss results. The PBLA approach allows buildings of different characteristics at the same location to see different ground motion intensities, and consequently this leads to more accurate loss estimation by linking the ground motion characteristics to the building height, construction class, and soil type, whereas in MMI-based loss assessment, for a given MMI (Modified Mercalli Intensity), all the buildings, even if they have of different characteristics, at the same location are assumed to experience the same MMI intensity.

Finally, in order to further illustrate the benefits of the use of the PBLA approach in loss estimation, a series of scenario analyses were performed for low- and high-rise steel buildings located in the cities of Los Angeles, San Francisco, and San Jose, and considering soil classifications varying from stiff to soft soil. Figure 8 shows the variation in AAL ratio (high rise to low-rise) by soil type. The pattern is the same for all locations and the AAL ratio is seen to increase as the soil conditions change from stiff to softer soil. Losses for high-rise buildings located at stiff soil sites are lower than for low-rise buildings; while at softer soil sites, the trend changes and losses for taller buildings are greater than for low-rise buildings. This is another example of the capability of the PBLA approach to account for the effects of soil amplification, which is linked with structure height (period), in arriving at a more accurate loss assessment than is possible with the MMI approach.

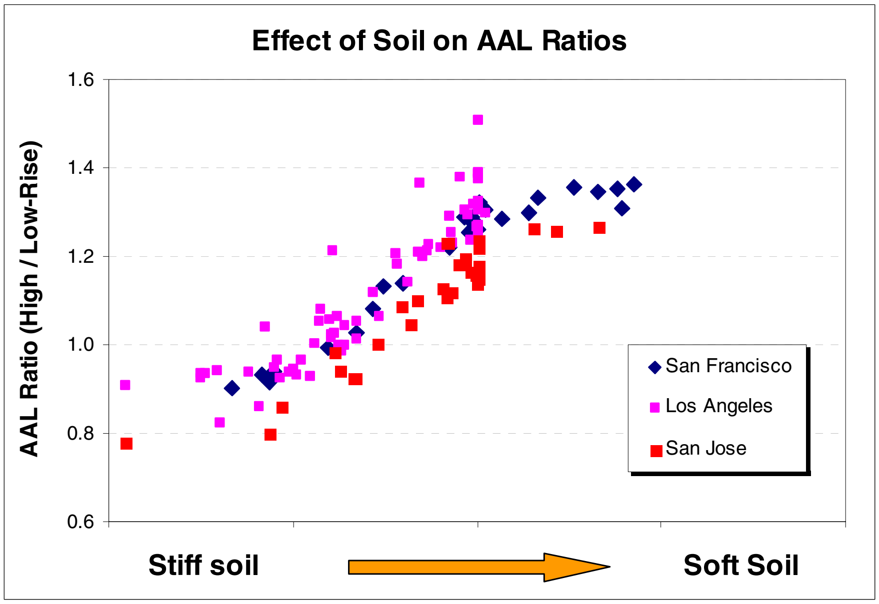

Figure 8 Variation of AAL by Soil and Building Height

SUMMARY AND CONCLUSIONS

The Performance Based Engineering framework developed by the PEER has been implemented for the application of earthquake risk assessment for a single location and portfolio analysis from the insurance perspective. The paper has discussed the methodology for developing building vulnerability functions using SAC steel moment frame structure models subjected to a suite of ground motions. The incremental nonlinear dynamic analysis approach is used to compute the seismic demands for increasing levels of intensity that are related to damage states and to damage ratios to develop vulnerability functions.

The earthquake loss assessment is based on a probabilistic approach and addresses several sources of uncertainty, which exist in the estimation of ground motion intensity (hazard), and vulnerability evaluation of buildings. The hazard side of this equation and uncertainty involved in prediction of ground motions are considered in the hazard module of RMS software. In order to account for the uncertainty related to the structural properties, building response evaluation, and building damage thresholds, a significant number of simulations were performed to capture all the variability in developing the building vulnerability functions.

The performance based loss assessment (PBLA) methodology for developing building vulnerability functions that link ground motion characteristics (hazard) to the building response (vulnerability) based on the PEER approach is very transparent and permits an objective evaluation of building performance as compared to the subjective ATC13 MMI approach. The PBLA approach allows consideration of the effects of earthquake characteristics including size, distance, soil condition, and frequency content as well as structure characteristics explicitly in the evaluation of losses.

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