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COMPONENT-LEVEL AND SYSTEM-LEVEL SENSITIVITY STUDY FOR EARTHQUAKE LOSS ESTIMATION

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SUMMARY

A probabilistic methodology is presented to estimate economic losses resulting from earthquake damage in buildings. The methodology allows the identification of different sources that contribute to economic losses in buildings both at the component-level and at the system-level. Sensitivity analyses are then performed to identify and quantify those that contribute the most to the overall uncertainty in loss estimates. The expected annual loss and the mean annual frequency of exceedance of a certain dollar loss are computed both at the component-level, for a slab-column connection, and at the system-level. The methodology is illustrated by applying it to a seven-story reinforced concrete building.

INTRODUCTION

Recent earthquakes in the United States have highlighted the importance of controlling damage in buildings in order to limit economic losses. Designing structures in which economic losses are controlled requires reasonable estimations of losses in earthquakes with different levels of intensity. Different approaches can be used to estimate economic losses in a building. In the proposed approach losses are estimated as a function of the damage in individual structural and nonstructural components.

As part of the research being conducted at the Pacific Earthquake Engineering Research (PEER) center a methodology is developed to estimate economic losses in buildings. The methodology is developed on the basis of the PEER's probabilistic framework. One primary advantage of the proposed methodology is its transparency in identifying the sources of uncertainty that contribute to probability parameters of economic losses both at the component-level and at the system-level. Once different sources of uncertainty are identified, sensitivity analyses can be performed to quantify which sources are significant and which ones are not significant. This information can then be used to introduce simplifications in the procedure of loss estimation.

Presented in this paper is a sensitivity study on different sources of uncertainties that affect economic losses at the component-level and at the system-level in buildings. First, we develop the formulation to estimate the expected annual loss, *EAL*, and the mean annual frequency, *MAF*, of exceeding a certain

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dollar loss in an individual component. The methodology is then illustrated by applying it to a reinforced concrete slab-column connection. Sensitivity studies are performed on different sources of uncertainty to identify the significance of each source in the *EAL* and *MAF* of the loss of the case study component. The results show that the modeling uncertainty in the seismic hazard is the most important source of uncertainty to estimate *EAL* of a slab-column connection. The *MAF* of the component is mainly sensitive to the uncertainty in the repair cost of the component.

Similarly, at the system-level, the formulation to estimate the *EAL* and *MAF* of the loss of the system is developed. The loss estimation methodology is then applied to an existing seven-story reinforced concrete building. The effects of different sources of uncertainty on the estimation of the system *EAL* are investigated. For the case of *MAF* the effects of correlation between losses in individual components are investigated. Our studies show that the *EAL* of the building is mainly sensitive to the uncertainty in the damage estimation. For the *MAF* of the system, we found that the effect of correlation is significant for low probability events, i.e. global collapse, and can be ignored for high-probability events. The later result is promising since it allows for significant simplifications in the loss estimation of buildings when damage control is the main seismic performance objective.

LOSS ESTIMATION AT THE COMPONENT-LEVEL

Estimation of the expected annual loss (*EAL*) of a component

Using the total probability theorem, the average annual loss in an individual component, $E[L_i]$ can be computed as

$$E[L_i] = \int_0^{\infty} E[L_i \mid IM] \left| d\nu(IM) \right| \tag{1}$$

where $d\nu(IM)$ is the derivative of the seismic hazard curve evaluated by performing a probabilistic seismic hazard analysis at the site as a function of a ground motion intensity measure, *IM*. $E[L_i \mid IM]$ is the expected loss in a component conditioned on *IM*. L_i is a random variable that represents the loss in the *i*th component normalized by the original cost of that component. For example, a realization of L_i equal to 0.7 means that the loss in the component is 70% of its original cost.

Similarly to Eq. (1), the expected loss in the *i*th component for a given scenario can be computed as

$$E[L_i \mid IM] = \int_0^{\infty} E[L_i \mid EDP_i] \left| d\,P(EDP_i \mid IM) \right| \tag{2}$$

where $d\,P(EDP_i \mid IM)$ is the probability density function of the engineering demand parameter in the *i*th component, EDP_i, conditioned on *IM*. $E[L_i \mid EDP_i]$ is the average loss in component *i* as a function of the level of seismic demand in that component, EDP_i.

The deformation parameter, EDP_i, in the $P(EDP_i \mid IM)$ is the structural response parameter which is closely correlated with the seismic damage in the component. For example, damage in almost all types of structural components, such as columns and slab-column connections is closely correlated with the level of interstory drift ratio, *IDR*, in the component. For some of the non-structural components, however, peak floor acceleration, *PFA*, is the primary deformation parameter which is closely correlated with the seismic damage in some nonstructural components.

Different approaches can be used to estimate the probability density function of EDP_i conditioned on *IM*. One feasible way is through response history simulations for a suite of earthquake ground motions scaled to certain levels of intensity, Miranda and Aslani [1].

Using the total probability theorem for discrete random variables, we can expand $E\left[L_i \mid EDP_i\right]$ as follows

$$E\left[L_i \mid EDP_i\right]=\sum_{k=1}^{m} E\left[L_i \mid DM_k = dm_k\right] P\left(DM_k = dm_k \mid EDP_i\right) \tag{3}$$

where $P\left(DM_k = dm_k \mid EDP_i\right)$ is the probability of being in the *k*th damage state, DM_k, of the *i*th component when the structure is subjected to a deformation level equal to edp_i. $E\left[L_i \mid DM_k = dm_k\right]$ is the expected loss in the *i*th component given that the component is in its *k*th damage state. The summation is on all the possible damage states, *m*, that a component can experience before loosing its vertical carrying capacity.

The probability of being in a certain damage state, DM_k, conditioned on EDP_i, $P\left(DM_k \mid EDP_i\right)$, can be estimated as

$$P\left(DM_k = dm_k \mid EDP_i\right)=P\left(DM_k > dm_{k+1} \mid EDP_i\right)-P\left(DM_k > dm_k \mid EDP_i\right) \tag{4}$$

where functions $P\left(DM_k > dm_{k+1} \mid EDP_i\right)$ and $P\left(DM_k > dm_k \mid EDP_i\right)$ are the probability of exceeding $k+1$th and *k*th damage states, respectively, conditioned on EDP_i , and are known as fragility functions for those damage states. Damage states in a component are defined based on the required courses of action to repair a seismically damaged component, Aslani and Miranda [2].

Estimation of the mean annual frequency of exceeding a certain level of loss

The approach used to derive Equations (1) through (4) for the average expected loss of a component can be applied to estimate the mean annual frequency of exceedance of the loss in an individual component, by simply replacing the expected values with probability distribution of the component loss in a given damage state

$$\nu\left[L_i > l_i\right]=\sum_{k=1}^{m} \int_0^{\infty} \int_0^{\infty} P\left[L_i > l_i \mid DM_k = dm_k\right] P\left(DM_k = dm_k \mid EDP_i\right)\left| d\, P\left(EDP_i \mid IM\right)\right|\left| d\, \nu\left(IM\right)\right| \tag{5}$$

where $P\left[L_i > l_i \mid DM_k = dm_k\right]$ is the probability of exceeding a certain level of loss given that component *i* is in the *k*th damage state.

If the loss curve of the component has been developed, using Eq. (5) , an alternative approach to estimate the expected annual loss of the component instead of using Eqs. (1) - (4) is to compute the area underneath the loss curve

$$E\left[L_i\right]=\int_0^{\infty} \nu\left[L_i > l_i\right]\left|d\, l_i\right| \tag{6}$$

Loss estimation of a reinforced concrete slab-column connection

To exemplify the formulation of loss estimation at the component-level, we have applied it to a reinforced concrete slab-column connection located in the third story of a testbed structure. The testbed is a seven - story reinforced concrete structure. It was designed in 1965 and built in 1966. The structural system of the building consists of moment-resisting perimeter frames and interior gravity-resisting frames (flat slabs and columns). The structure is nominally symmetric with the exception of an infill wall in the first floor of the north frame of the building. A detailed description of the testbed building has been presented in Browning et al. [3].

Four basic ingredients are required to evaluate Eqs. (1)–(4): (1) seismic hazard curve at the site, $\nu(IM)$; (2) probability distribution of EDP_i conditioned on IM, $P(EDP_i|IM)$(3) fragility functions corresponding to different damage states that a component can experience as a function of the level of EDP_i in that component, $P(DM_k > dm_k | EDP_i)$; and (4) loss functions corresponding to the cost of repair of the component in a given damage state, $P[L_i > l_i | DM_k = dm_k]$. Figures 1 through 4 presents each of the above basic ingredients for loss estimation of a reinforced concrete slab-column connection in the testbed building.

Figure 1 presents the seismic hazard curve at the site of the testbed structure. In this study the spectral displacement of a linear elastic single-degree-of-freedom system evaluated at the first period of vibration of the multi-degree-of-freedom model of the structure, S_d, is selected as the ground motion intensity measure. Information shown in Figure 1 is used to estimate $d\nu(IM)$ in Eqs. (1) and (5).

Figure 2 presents the variations of the median interstory drift ratio along the height of the building at different levels of intensity. The results in Figure 2 are obtained using a series of non-linear time history analyses of the testbed structure , Miranda [1]. Corresponding to the median response at each floor level and each level of intensity, shown in Figure 2, a probability distribution exists. An example is presented for the *IDR* in the third story when the structure is subjected to a spectral displacement of 20 cm. The information presented in Figure 2 is used to estimate $d\,P\,(EDP\,|\,IM)$ in Eqs. (2) and (5).

Fragility functions corresponding to different damage states of a slab-column connection are shown in Figure 3. Four damage states are defined for this component. The first damage state, DM_1, occurs when the small levels of cracking are observed in the connection. The required repair action is cosmetic repairs such as pasting and painting. The next damage state, DM_2 , is defined when the cracks are wide enough to require epoxy injection as the repair action. The third damage state, DM_3 , is defined when the component has experienced a punching shear failure. At this stage the damaged concrete needs to be removed and additional reinforcement may be required before pouring new concrete. The last damage state, DM_4, corresponds to the loss of vertical carrying capacity, *LVCC,* in the component. At this damage state the component collapses. As a simplifying assumption in this study we assume that loosing the vertical carrying capacity at the component level results in a system failure. Information presented in Figure 3 is used to estimate $P\,(\,DM\,|\,EDP)$ in Eqs. (3) and (5).

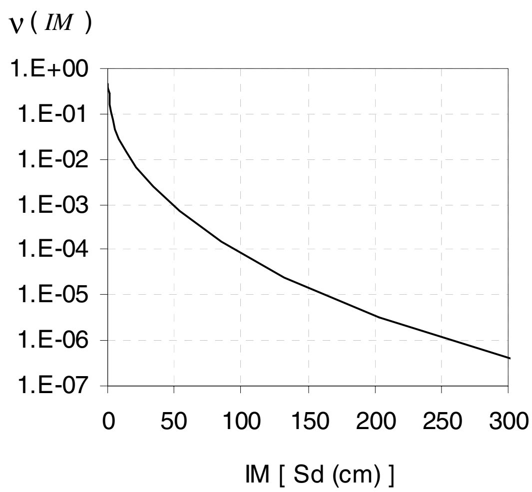


Fig. 1. Seismic hazard curve at the site of the case study building.

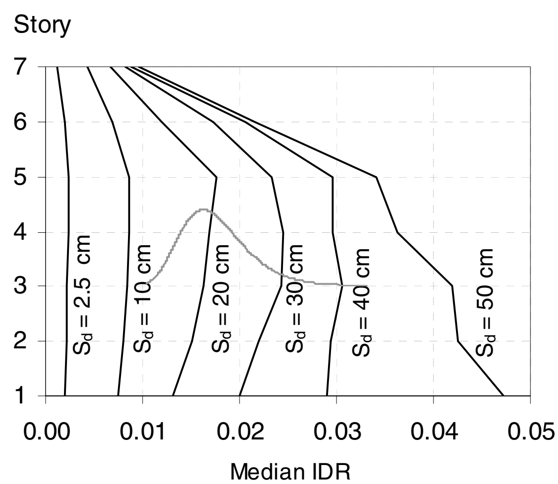


Fig. 2. Variations of the median interstory drift ratio (*IDR*) along the building height at different levels of intensity.

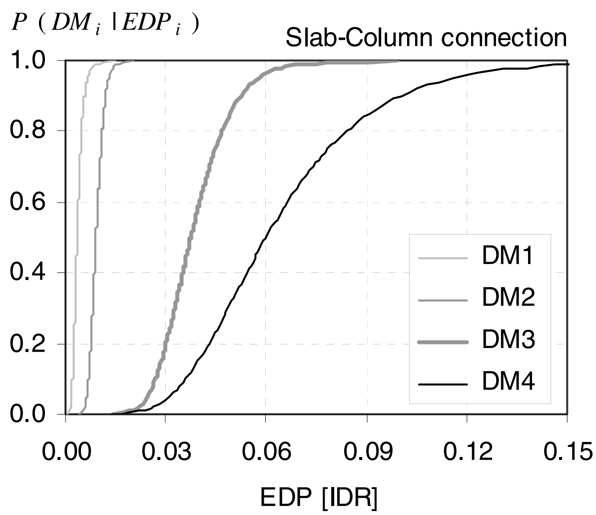


Fig. 3. Fragility functions for different damage states of a slab-column connection located in the third story of the building.

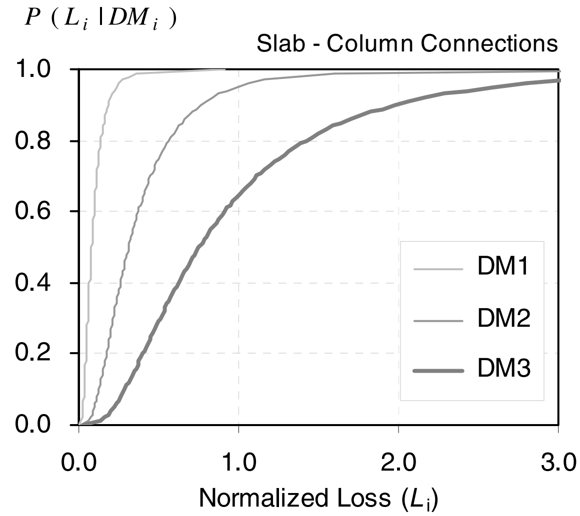


Fig. 4. Loss functions for a slab-column connection.

Fragility functions developed for slab-column connections are based on experimental research on this type of component. A summary of the experimental data used to develop these fragility functions is presented in Aslani and Miranda [2]. Further, we assume that the fragility functions follow a cumulative lognormal distribution. This assumption introduces some approximation in estimating the *EAL* at the component-level and system-level.

Figure 4 presents the loss functions associated with the repair cost for each of the first three damage states of a slab-column connection. Loss functions are calculated by itemizing the tasks required to repair a component at different damage states. Information presented in Figure 4 is used to estimate $E\,[L_i\,|DM_k\,]$ in Eq. (3) and $P\,[\,L_i > l_i\,|\,DM_k\,]$ in Eq. (5).

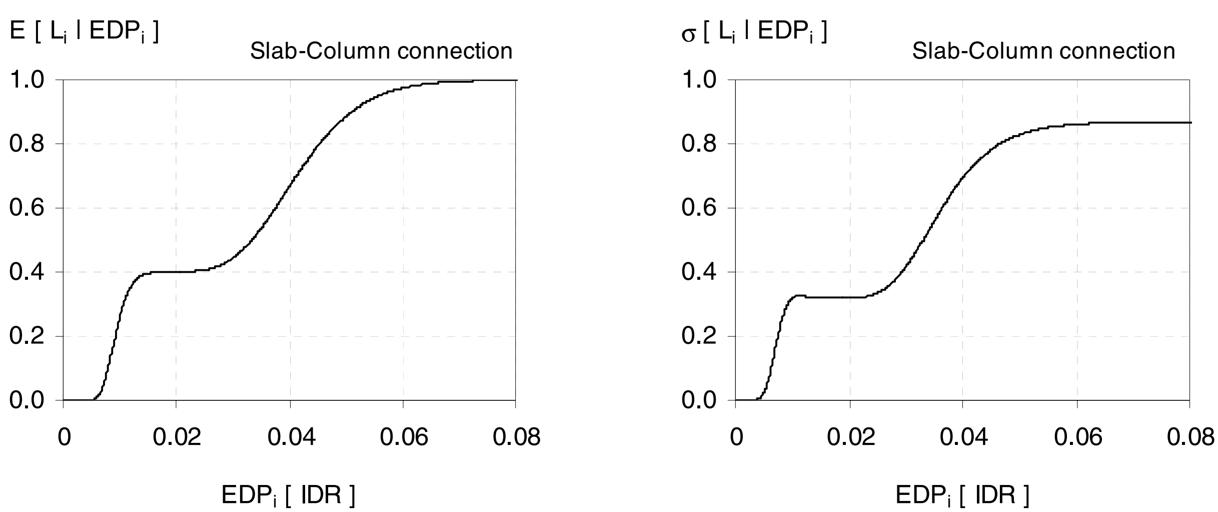
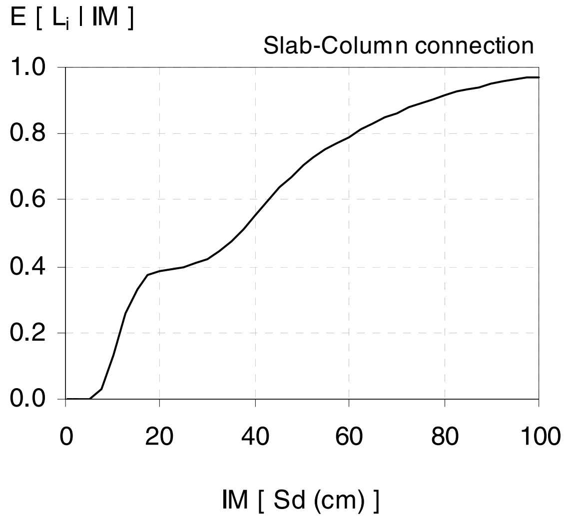


Fig. 5. Variations of the mean, $E\,[\,L_i\,|\,EDP_i\,]$, and the standard deviation of the loss, $\sigma[\,L_i\,|\,EDP_i]$, for a slab-column connection located in the third story of the building with changes in the interstory drift ratio, *IDR*, in that component.



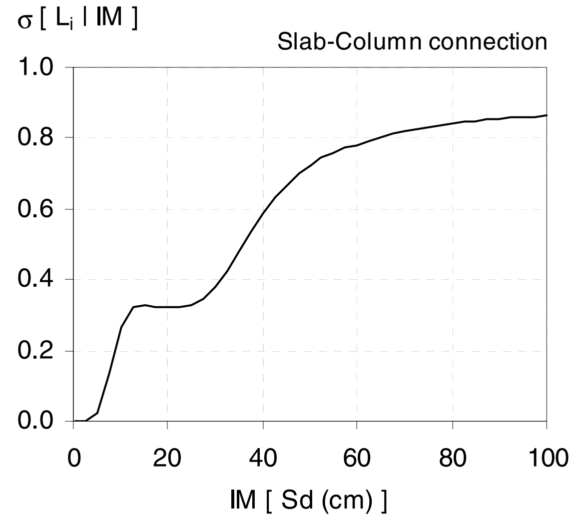


Fig. 6. Variations of the mean, $E\ [\ L_i \mid IM\ ]$, and the standard deviation of the loss , $\sigma[\ L_i \mid IM\ ]$, for a slab-column connection located in the third story of the building with changes in the level of seismic hazard intensity, *IM*.

Information presented in Figures 1 to 4 are used in Eqs. (1) to (4) to estimate the average economic losses in a slab-column connection. Beyond the expected loss , variations of the standard deviation of the loss for an individual component are investigated using the same approach presented in Eqs. (1) to (4).

Figure 5 presents the variations of the mean, $E\ [\ L_i \mid EDP_i\ ]$, and the standard deviation of the loss, $\sigma\ [\ L_i \mid EDP_i]$, with changes in the interstory drift ratio, *IDR*, for the slab-column connection located at the third story of the testbed structure. As shown in the figure the loss in the component starts at around 0.6% drift. At around 2% drift the slope of both mean and standard deviation of the loss becomes almost 0, caused by the relatively large demand required to experience the third damage state compared to the drift level at which the second damage state occurs, Figure 3.

Integrating results presented in Figure 5 over all possible *EDP*'s , Eq. (2), we can now estimate the mean, $E\ [\ L_i \mid IM\ ]$, and the standard deviation of the loss , $\sigma\ [\ L_i \mid IM\ ]$, for a given scenario, *IM*. Figure 6 presents the variations of $E\ [\ L_i \mid IM\ ]$ and $\sigma\ [\ L_i \mid IM\ ]$ for the slab-column connection located in the third story of the testbed structure. As can be seen in the figure, loss in the component initiates at $IM = 5$ cm, whereas the standard deviation of the loss starts at around 2.5 cm. Comparing the initiation of standard deviation of the loss with the expected loss shows that standard the coefficient of variation (c.o.v.) at small levels of intensity is significantly larger than 1 since $\sigma\ [\ L_i \mid IM\ ]$ approaches toward 0 more slowly than $E\ [\ L_i \mid IM\ ]$. This observation is of high importance since the initiation of loss in a component plays a significant role both in component expected annual loss and the whole building (system) expected annual loss.

At $IM = 20$ cm the average loss in the component is around 40% of the component original cost. At this level the standard deviation of the loss is significant, leading to c.o.v.'s around 0.8. The main reason of the relatively large values of the c.o.v. at this level and in general in this example is the significant uncertainty for the repair costs, uncertainty at $DV \mid DM$ level. Later, in the sensitivity analysis it is shown how this source of uncertainty dominates the results of estimating the mean annual frequency of exceeding a certain level of loss.

The results from Figure 6 can be integrated together with the seismic hazard curve at the site to estimate the mean annual frequency of exceedance of the loss in the component, Eq. (1). The curve labeled "uncertainty at all levels" in Figure 7 presents the loss curve for the slab-column connection example. Shown in the figure is the annual rate of exceeding a certain level of loss in the component. For example, the mean annual frequency of exceeding 50% of the component original cost is 0.005.

Sensitivity study on the *EAL* and *MAF* of loss of a slab-column connection

A main advantage of the formulation presented to evaluate the expected annual loss of a component, Eqs. (1) - (4), is its transparency in identifying different sources of uncertainty that effect *EAL*. Two different sources of uncertainty are investigated: uncertainty stemmed from the estimation of the seismic response at different levels of intensity, $P\,(EDP \mid IM)$; and uncertainty stemmed from the incurred seismic damage in the component caused by the imposed seismic demand, $P\,(DM \mid EDP)$.

To investigate the effects of each of the above sources of uncertainty in estimation of the *EAL*, we have estimated the changes in average annual loss by assuming uncertainty at one level and certainty in another level. For example, we estimated the *EAL* of the component when there is no uncertainty at $DM \mid EDP$ level but there is uncertainty at $EDP \mid IM$ level and compared it to the case when uncertainties at both levels are taken into account. Our observations show that for the type of uncertainties considered in this study and for this component the uncertainty at $DM \mid EDP$ level is more significant than the one of $EDP \mid IM$ in estimating the *EAL*.

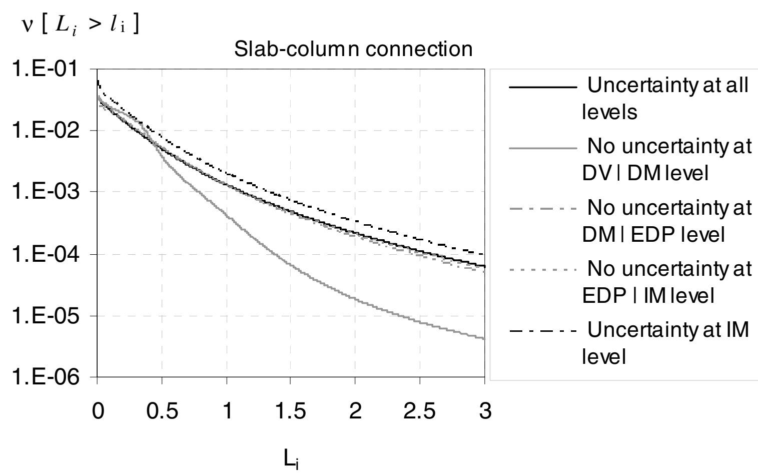


Fig. 7. Effects of different sources of uncertainty on the estimation of the component loss curve.

For the case of *MAF* of the loss at the component-level three sources of uncertainty are investigated. The first two sources of uncertainty are the same as what considered in *EAL* sensitivity analysis. The third source is the uncertainty corresponding to the $DV \mid DM$ level. The results of the sensitivity analysis on the loss curve of the slab-column connection are presented in Figure 7. As shown in the figure the dominating source of uncertainty in this case is the one corresponding to that associated with repair costs of the component.

Effects of the seismic hazard uncertainty

The sensitivity of the *EAL* and *MAF* to the assumptions made to estimate the seismic hazard at the site is investigated. The uncertainty in the seismic hazard is accounted for by computing the 84th percentile hazard curve, Jalayer [4]. Our observations show a 50% increase in the expected annual loss of the component because of the modeling uncertainty at the seismic hazard level. The changes in the loss curve of the slab-column connection are shown in Figure 7.

LOSS ESTIMATION AT THE SYSTEM-LEVEL

Estimation of the expected annual loss of the system

The *EAL* of the system can be expanded as

$$E[L_T] = \int_0^{\infty} E[L_T \mid IM] \left| d\nu(IM) \right| \tag{7}$$

where $E[L_T \mid IM]$ is the average loss of the system conditioned on *IM*.

The scenario-based expected loss of the system can be computed as

$$E[L_T \mid IM] = E[L_T \mid IM, NC][1 - P(C \mid IM)] + E[L_T \mid C] P(C \mid IM) \tag{8}$$

where $E[L_T \mid IM, NC]$ is the expected loss of the system conditioned on *IM* when the structure does not collapse, $E[L_T \mid C]$ is the expected loss of the system when the structure collapses, $P(C \mid IM)$ is the probability that structure collapses in a given earthquake scenario.

The non-collapse system expected loss conditioned on *IM*, $E[L_T \mid IM, NC]$, can be estimated by simply summing over all component losses in the building.

$$E[L_T \mid IM, NC] = \sum_{i=1}^{n} a_i \, E[L_i \mid IM] \tag{9}$$

where a_i is the original cost of the *i*th component, and $E[L_i \mid IM]$ is estimated from Eq. (2). The summation is over all non-rugged components in the building, *n*.

The probability of collapse considered in this study takes into account the collapse caused by the loss of vertical carrying capacity, *LVCC*. To estimate $P(C \mid IM)$, we assume, Aslani [4], that no re-distribution of vertical loads occurs at the system-level and the probability of collapse due to *LVCC* would be equal to the largest probability of any individual structural element that can loose its vertical carrying capacity

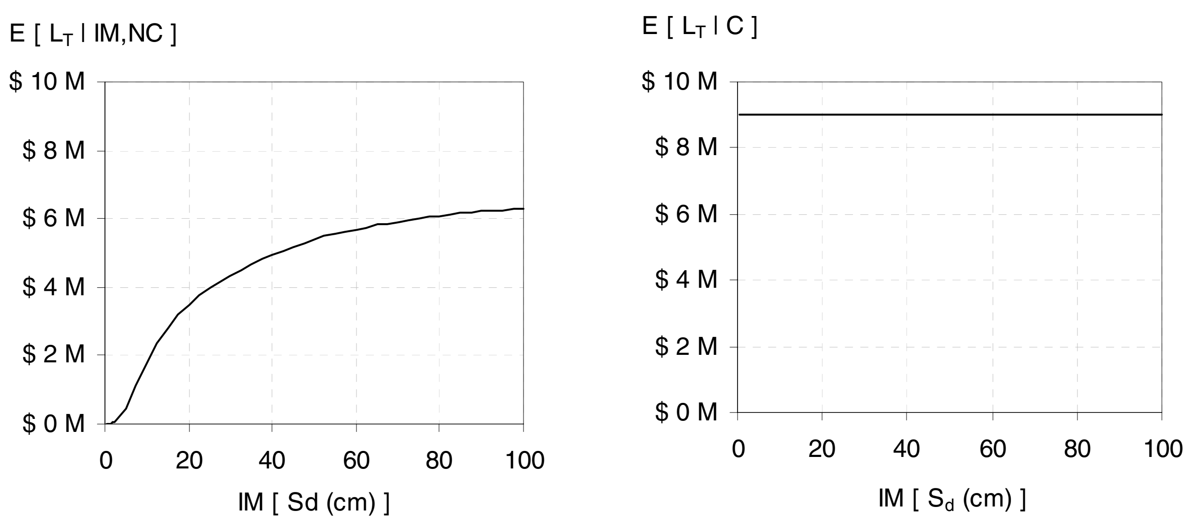


Fig. 8. Variation of the system expected loss with changes in the level of intensity, *IM*, for both cases of occurrence and non-occurrence of the structural collapse.

$$P\left(C \mid IM\right) = \max_{\forall i_s}\left[P\left(LVCC_i \mid IM\right)\right] \tag{10}$$

where $P\left(LVCC_i \mid IM\right)$ is the probability of losing the vertical carrying capacity in the *i*th component conditioned on *IM* and is computed as

$$P\left(LVCC_i \mid IM\right) = \int_0^{\infty} P\left(LVCC_i \mid EDP_i\right) \left| d\, P\left(EDP_i \mid IM\right)\right| \tag{11}$$

where $P\left(LVCC_i \mid EDP_i\right)$ is the probability of the ith component losing its vertical carrying capacity given that it is subjected to a deformation level equal to edp_i. $P\left(LVCC_i \mid EDP_i\right)$ is computed from fragility surfaces, Aslani and Miranda [5], developed for *LVCC* damage states on the basis of experimental studies on structural components. In a fragility surface the mean and standard deviation of *EDP* corresponding to the *LVCC* damage state are evaluated as a function of a new parameter, α, which allows the incorporation of additional information. The parameter α can incorporate information on the element (e.g., geometry, detailing, etc.), the loading and or a combination of the two. The probability of exceeding the *LVCC* damage state is then estimated as a function of the level of *EDP* in the component but also as a function of the parameter α.

Estimation of expected annual loss for the reinforced concrete testbed structure

The formulation presented in the previous section is applied to the seven-story testbed structure. All structural and non-structural components in the building are carefully identified. For each component fragility functions and loss functions corresponding to different damage states in the component are developed, Aslani and Miranda [2], Taghavi and Miranda [6]. Further, for certain damage states of structural components fragility surfaces are developed for a reliable estimate of that damage state.

Figure 8 presents the expected loss of the system as a function of the level of intensity in the building both for non-collapse and collapse cases. The average loss of the system for the case of non-collapse starts at around 2.5 cm linear spectral displacement. The main source of loosing money at such small levels of intensity stems from the facts that the loss in non-structural components initiates at low levels of

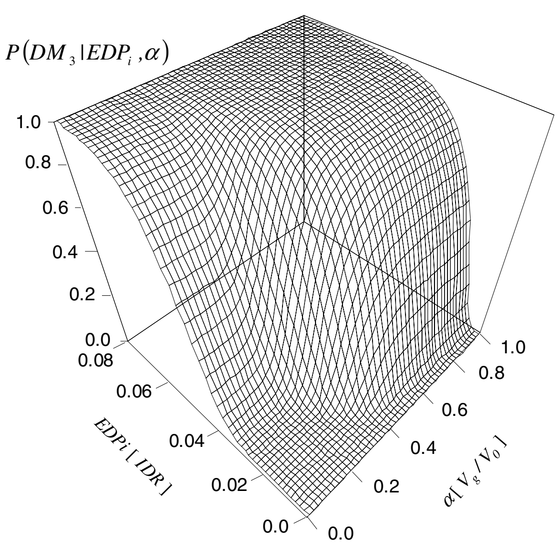


Fig. 9. Fragility surface for loss of vertical carrying capacity (*LVCC*) in slab-column connection as a function of the *IDR* and α.

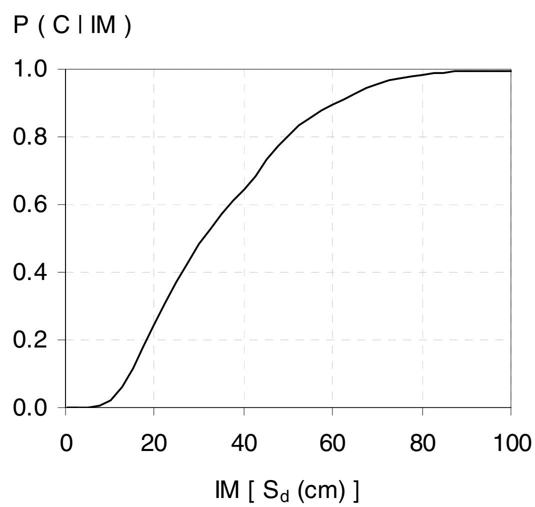


Fig. 10. Probability of system collapse caused by loosing the vertical carrying capacity at the component level.

intensity and the fragility functions are assumed lognormal. If the structure collapses at any level of intensity the loss of the system is equal to the total cost of the building, as shown on the right-hand side graph of Figure 8.

To estimate the expected loss at the system level using Eq. (8), we need to estimate the probability of collapse as a function of the level of intensity in the building. For the testbed building we assume that the most probable mode of collapse is losing the vertical carrying capacity. To capture this mode we used fragility surfaces of different structural components in the building. Figure 9 presents an example of a fragility surface developed for the loss of vertical carrying capacity in a slab-column connection.

We used fragility surfaces developed for different structural components together with response simulation results of the testbed building, Figure 2, to estimate the probability of collapse in the *LVCC* mode, using Eqs. (10) and (11). Figure 10 presents the probability of collapse computed for the testbed building as a function of the ground motion intensity level. As shown in the figure, probability of collapse at the system level for intensities smaller than 7.5 cm is 0.

Incorporating the results of Figures 8, 9 and 10 in Eq. (8) we can estimate the conditional expected loss of the system at different levels of intensity for the case study building. The result is presented in Figure 11. The figure also shows the contribution of collapse and non-collapse loss to the total loss of the system. It can be seen that at small levels of intensity, less than 20 cm, the non-collapse losses is a major contributor to the total loss.

Integrating the results from Figure 11 with the seismic hazard at the site, Eq. (7) we can estimate the expected annual loss of the system. For the testbed structure the *EAL* is $ 146,000. It should be noted that this value is estimated based on two main assumptions; the most probable collapse mode of the system is *LVCC* and the components fragility functions are lognormally distributed. The sensitivity of the *EAL* to these assumptions is under investigation. Specially, for the case of second assumption, preliminary results show that the *EAL* is quite sensitive to the initiation of the loss in each individual component.

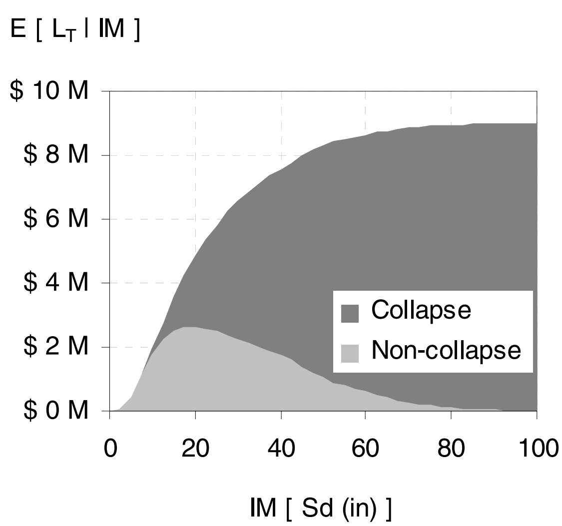


Fig. 11. Variations of the system expected loss with the level of intensity.

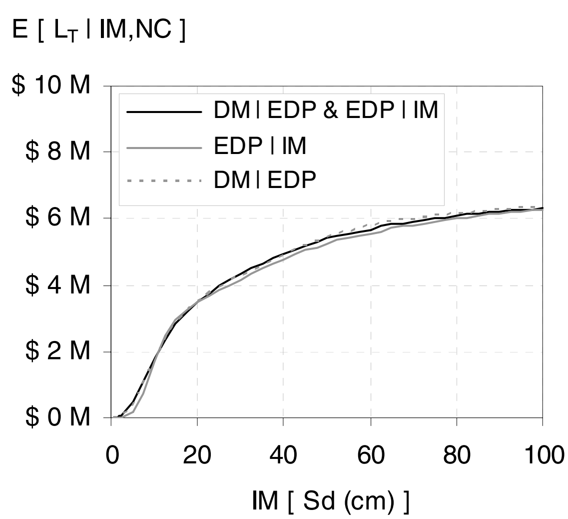


Fig. 12. Effects of uncertainty at *DM* | *EDP* and *EDP* | *IM* levels on the non-collapse expected loss of the system for a given *IM*.

Sensitivity study on the expected annual loss of the system

The effects of two sources of uncertainty on the *EAL* of the system are investigated; the uncertainty in estimating the damage as a function of *EDP*, *DM* | *EDP* level uncertainty, and the uncertainty in the estimation of seismic response as a function of the level of intensity, *EDP* | *IM* level uncertainty. It can be understood from Eq. (8) that these two sources of uncertainty affect both the $E\left[L_T \mid IM, NC\right]$ and $P\left(C \mid IM\right)$.

Figure 12 presents the sensitivity of the non-collapse average loss of the system conditioned on *IM* to uncertainty at *EDP* | *IM* and *DM* | *EDP* levels. As can be seen in the figure the non-collapse expected loss is not very sensitive to the uncertainties at these two levels. This observation is very important since it allows for significant simplifications in the estimation of $E\left[L_T \mid IM, NC\right]$, with relatively small amounts of error.

The effects of uncertainty at *EDP* | *IM* and *DM* | *EDP* levels on the probability of system collapse in the *LVCC* mode is shown in Figure 13. The labels on the figure shows which sources of uncertainty are considered in the estimation of the probability of collapse. For example, the curve with *EDP* | *IM* label presents the probability of collapse when the uncertainty at *EDP* | *IM* is considered and not the one of the *DM* | *EDP*. As can be seen in the figure the uncertainty at the *DM* | *EDP* level, plays a more significant role in the assessment of *P* (*C* | *IM*) compared to the effects of *EDP* | *IM* uncertainty .

The effects of the two sources of uncertainty at the response level and the damage level on the expected loss of the system are investigated, using Eq. (8). Figure 14 presents the results of this investigation. Our observations show that the uncertainty at the *DM* | *EDP* is very important since it can change the initiation of the total loss at the system level, which has a significant effect on the expected annual loss. For example, for the case of the testbed structure when certainty is assumed at the damage level the expected loss starts at 10 cm elastic spectral displacement, while when the *DM* | *EDP* uncertainty is accounted for, the initiation of loss starts at around 2 cm. Our investigations show that not considering the uncertainty at the damage level can introduce 17% difference in the *EAL* of the system, while. assuming certainty at the *EDP* | *IM* level introduces a 10% error in the system *EAL*.

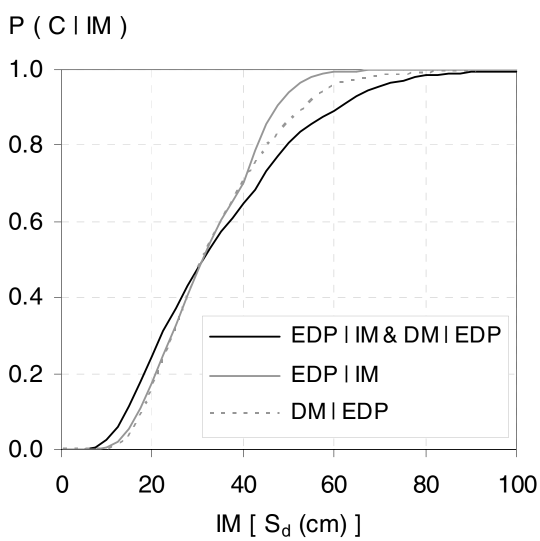


Fig. 13. Effects of uncertainty at *DM* | *EDP* and *EDP* | *IM* levels on the probability of collapse of the system in *LVCC* mode.

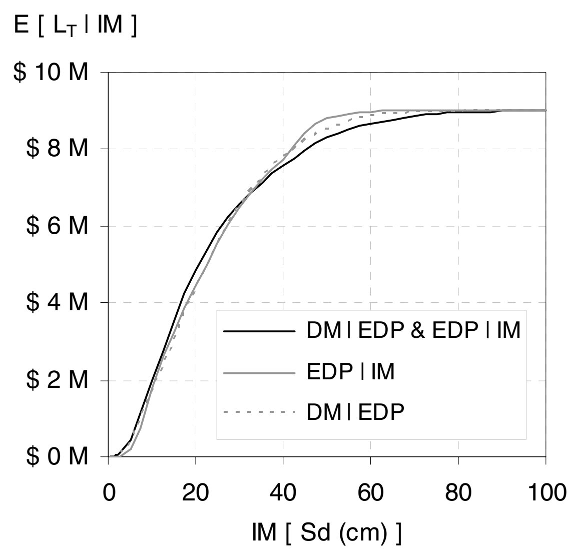


Fig. 14. Effects of uncertainty at *DM* | *EDP* and *EDP* | *IM* levels on the expected loss of the system conditioned on *IM*.

On the basis of above observations it can be concluded that for the case of the testbed structure when the expected annual loss is the target performance measure, a more accurate estimation of fragility functions, which results in decreasing the uncertainty at *DM* | *EDP* level, should be performed. The conclusion cannot be generalized, however, since the effects of modeling uncertainty at different levels, specifically at the *EDP* | *IM* level, has not taken into account and is currently under investigation.

Estimation of the mean annual frequency of the system loss

The *MAF* of the system can be computed as

$$\nu\left[L_T > l_T\right] = \int_0^{\infty} P\left[L_T > l_T \mid IM\right] \left| d\nu\left(IM\right)\right| \tag{12}$$

where $P\,[\,L_T > l_T \mid IM\,]$ is the probability of losing a certain level of loss, l_T, in a given scenario, *IM*.

The probability of exceeding a certain level of loss at a given scenario can be expanded as

$$P\left[L_T > l_T \mid IM\right] = P\left[L_T > l_T \mid IM, NC\right]\left[1 - P\left(C \mid IM\right)\right] + P\left[L_T > l_T \mid C\right] P\left(C \mid IM\right) \tag{13}$$

where $P\left[L_T > l_T \mid IM, NC\right]$ is the probability of losing money in the system conditioned on *IM* when the structure does not collapse, $P\left[L_T > l_T \mid C\right]$ is the probability of losing money in the system when the structure collapses. In this study we assume $P\left[L_T > l_T \mid C\right]$ is lognormally distributed.

From central limit theorem in the theory of probability, we can assume that the sum of *n* random variables is normally distributed for large enough *n*'s. Hence, non-collapse probability of losing money in the system at a given scenario when the structure does not collapse can be assumed normal

$$P\left[L_T > l_T \mid IM, NC\right] = 1 - \Phi\left[\frac{l_T - E\left[L_T \mid IM, NC\right]}{\sigma\left[L_T \mid IM, NC\right]}\right] \tag{14}$$

where Φ is the standard cumulative normal distribution of the loss at the system-level, $E\left[L_T \mid IM, NC\right]$ is estimated from Eq. (9) , $\sigma\left[L_T \mid IM, NC\right]$ is the standard deviation of the L_T ,when the structure does not collapse for a given scenario and can be expanded as

$$\sigma\left[L_T \mid IM, NC\right] = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j \rho_{L_i L_j \mid IM, NC}\, \sigma_{L_i \mid IM, NC}\, \sigma_{L_j \mid IM, NC}} \tag{15}$$

where $\rho_{L_i L_j \mid IM, NC}$ is the correlation coefficient between the losses in individual components as a function of the level of intensity, $\sigma_{L_i \mid IM, NC}$, $\sigma_{L_j \mid IM, NC}$ is the standard deviation of the loss in a given scenario for the *i*th and *j*th components and can be estimated as described in the component-level loss estimation section.

if the losses in individual components are not correlated, Eq. (15) simplifies to

$$\sigma\left[L_T \mid IM, NC\right] = \sqrt{\sum_{i=1}^{n} a_i^2 \sigma_{L_i \mid IM, NC}^2} \tag{16}$$

if the losses in individual components are correlated, then the correlation coefficient can be written as

$$\rho_{L_iL_j|IM,NC} = \frac{\sigma_{L_iL_j|IM,NC}}{\sigma_{L_i|IM,NC}\sigma_{L_j|IM,NC}} \tag{17}$$

where $\sigma_{L_iL_j|IM,NC}$ is the covariance between the losses in the *i*th and *j*th components for a given scenario, *IM*. Estimation of $\sigma_{L_iL_j|IM,NC}$ requires the knowledge of correlation between individual components at three different levels, $EDP\,|\,IM$, $DM\,|\,EDP$, and $DV\,|\,DM$. Detailed formulation on the evaluation of correlation between losses in individual components is presented in Aslani and Miranda [5].

Estimation of the mean annual frequency of the loss at the system-level for the testbed structure

The scenario-based probability of losing money for collapse and non-collapse cases, $P[L_T > l_T \,|\, IM, NC]$ and $P[L_T > l_T \,|\, C]$ respectively , are estimated for the testbed structure, using formulations and assumptions presented in previous section. The building loss curve, MAF of loss at the system level, is then computed using Eqs. (12) and (13). The results are shown in Figure 16 and are discussed in the next section.

Sensitivity study on the effects of correlation on the mean annual frequency of the system loss

To investigate the effects of correlation on the MAF of the loss for the testbed structure, two sets of loss estimations are accomplished. In the first set, we assume that the losses in individual components are not correlated and estimate the standard deviation of the loss at the system level, L_T, using Eq. (16). In the second set ,we assume that the losses in individual components are correlated. The correlation is computed using the methodology and data presented in Aslani and Miranda [5]. Figure 15 compares the standard deviation of L_T at different levels of intensity for correlated and non-correlated cases when the structure does not collapse (left graph) and when it collapses (right graph). As can be seen in the figure the effects of correlation on standard deviation of L_T is significant both for collapse and non- collapse cases. When the structure does not collapse the standard deviation of L_T almost triples for the case of correlated losses in individual components compared to the non-correlated case. When the structure collapses the standard deviation of L_T increases by 75% from correlated to non-correlated case.

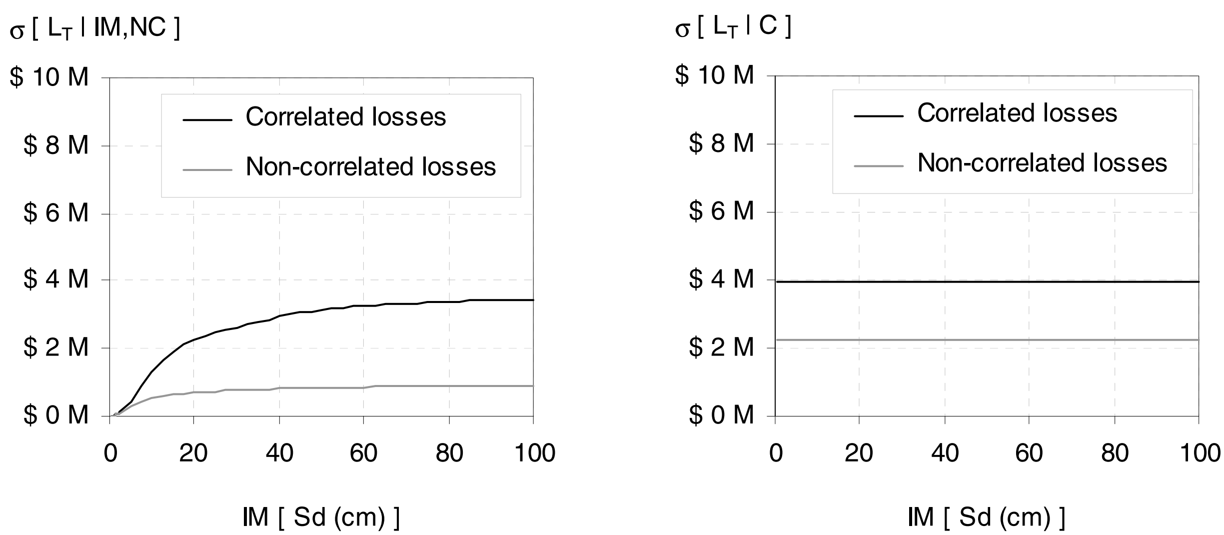


Fig. 15. Effects of correlation on the standard deviation of the loss at system-level at different levels of intensity , *IM*, when the structure does not collapse (left graph) and when it collapses (right graph).

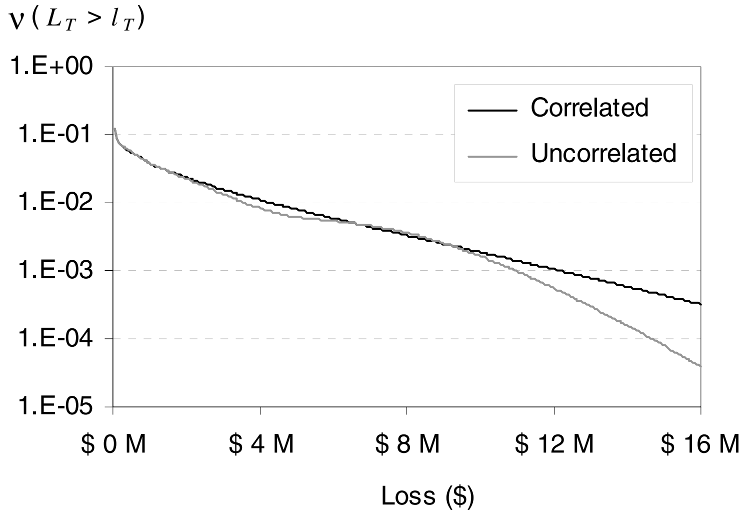


Fig. 16. Effects of correlation between losses in individual components on the loss curve of the testbed structure.

Presented in Figure 16 is the comparison between the loss curves of the testbed structure when the losses in individual components are correlated and when they are assumed non-correlated. It can be seen that for small levels of loss which can also be referred to as high-probability events, losses smaller than $ 2 M, the difference between the two curves is very small. For low-probability events, losses larger than $ 10 M, however, the difference between the correlated and non-correlated *MAF*'s are significant. For example, the *MAF* of losing more than $ 12 M when the losses at the component-level are assumed to be non-correlated is almost half of the *MAF* computed for the correlated case.

CONCLUSIONS

Identifying the important sources of uncertainty in estimating economic losses at the component-level and system-level in buildings is investigated. Four sources of uncertainty contribute to the loss estimation at the component-level; uncertainty in the seismic hazard, *IM*, uncertainty in the imposed seismic demand at a given scenario, $EDP \mid IM$ uncertainty, uncertainty caused by the damage in the component, $DM \mid EDP$ uncertainty, and the uncertainty in the incurred loss in the component , $DV \mid DM$ uncertainty. The contribution of each of these sources of uncertainty on the expected annual loss (*EAL*) and the mean annual frequency (*MAF*) of exceeding a certain level of loss is modeled by extending the PEER framing equation.

The extended methodology is then applied to a reinforced concrete slab-column connection. The results show that the most contributing source of uncertainty for the case of *EAL* is the uncertainty in the seismic hazard curve. For the *MAF* of loss, however, the effects of the $DV \mid DM$ uncertainty are the most on the loss curve compared to other sources of uncertainty.

At the system-level, the effects of $EDP \mid IM$ level uncertainty and $DM \mid EDP$ uncertainty are investigated on the *EAL*. The two sources of uncertainty effect both the scenario-based expected loss when the structure does not collapse, $E[L_T \mid IM, NC]$, and the probability of collapse at a given scenario, $P(C \mid IM)$. Our investigation shows that the *EAL* is mainly sensitive to the $DM \mid EDP$ uncertainty.

A sensitivity analysis is preformed on the effects of correlation on the building loss curve, *MAF*. First, we developed a formulation to account for the effects of correlation between losses in individual components. The formulation estimates the correlation between component losses as a function of the correlation at three levels of $EDP \mid IM$, $DM \mid EDP$ and $DV \mid DM$. The methodology is then applied to a reinforced concrete testbed structure. Our observations show that the correlation between component losses significantly increase the dispersion of the loss at the system-level, increase of more than 300% for the non-collapse dispersion of the loss at different scenarios. The effect of correlation on the building loss curve is then investigated. It is concluded that assuming non-correlated component losses is not conservative and for the case of low-probability events, e.g. collapse, can lead to underestimations of more than 50% in *MAF* in some cases. For the case of high probability events, however, assuming that the losses are non-correlated at the component-level does not introduce significant approximation to the loss curve. The results is promising since it allows for great simplifications in the loss estimation methodology and increasing its practicality for high probability events.

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