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PROBABILISTIC PREDICTION OF SURFACE EARTHQUAKE FAULT BEHAVIOR BY USING ADVANCED NUMERICAL ANALYSIS METHOD

Hidenori NAKAGAWA[1] and Muneo HORI[2]

SUMMARY

Non-linear spectral stochastic finite element method (NL-SSFEM) has been developed for probabilistic estimate of surface earthquake fault behavior. Since the structures and material properties of ground layers are not fully measured, a non-linear elasto-plastic stochastic model is used to simulate the rupture processes which often produce large variability, and NL-SSFEM numerically analyzes the stochastic model when the movement of the base rock mass is given as boundary conditions. The validity of the NL-SSFEM simulation is verified by reproducing results of model experiments. It is shown that the formation of Riedel shears due to uniform lateral sliding is computed in the simulation. The formation of the Nojima Earthquake Fault, a source fault for the Hanshin-Awaji Earthquake, is reproduced to demonstrate the usefulness of the NL-SSFEM simulation. It is shown that the simulation provides fairly good probabilistic estimate of the observed configuration of the surface earthquake fault.

INTRODUCTION

The authors have developed a new analysis method, called non-linear spectral stochastic finite element method (NL-SSFEM), in order to numerically simulate the formation processes of surface earthquake faults in unconsolidated layers when the base rock mass is moved due to faulting; see Anders and Hori (2001) and Hori *et al*. (2002). The simulation is aimed at evaluating a possible damage of near-by structures which is caused by large displacement of faulting; see, for instance, Konagai (2000, 2003). Even though unconsolidated layers are much shallower than the base rock through which the rupture processes propagate, the ground structures and the material properties are not fully measured, and the layers have uncertainty. Thus, stochastic modeling is used. By prescribing the amount of the base rock movement as an input parameter, the stochastic responses of faulting within the unconsolidated layers are computed; with the modern geological knowledge, it is impossible to accurately evaluate the amount of the base rock movement at a specific cite, and hence the amount is used as a control parameter of the NL-SSFEM simulation.

It should be noted that the fault displacement on the surface often has large variability. When fault appears on the ground surface, it produces larger ground deformation or displacement gap, but no deformation is

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[1] Assistant Professor, Humanities and Natural Sciences, Ibaraki National College of Technology, Nakane 866, Hitachi, Ibaraki, 312-8508, Japan, hide@ge.ibaraki-ct.ac.jp.
[2] Professor, Earthquake Research Institute, University of Tokyo, Yayoi 1-1-1, Bunkyo, Tokyo, 113-0031, Japan, hori@eri.u-tokyo.ac.jp.

observed in near-by places. One cause of such large variability in ground deformation due to faulting is the complicated ground structures through which the rupture processes of faulting pass. The evaluation of the variability is important in order to consider rational countermeasures for faulting. Thus, when the underground structures are not fully known, stochastic modeling is a reasonable choice to evaluate large variability of faulting behavior.

The NL-SSFEM simulation provides a probability that fault appears on the ground surface for a given amount of the base rock movement. Also, it provides the most probable fault configuration such as width, orientation, or fault displacement, and the range of these configuration parameters. The major advantage of NL-SSFEM is that it is able to evaluate the stochastic responses in a fully descriptive manner; the full descriptiveness means that NL-SSFEM computes the mean, variance, covariance or probability density function (PDF). With this advantage, NL-SSFEM is different from ordinary stochastic finite element method which uses perturbation expansion of stochastic quantities, and NL-SSFEM needs much less numerical computational cost than Monte-Carlo simulation; see Bray et al., (1994); see also Konagai (2000). The full descriptiveness of NL-SSFEM is based on abstract but rigorous treatment of the model stochasticity and the probabilistic responses; as will be shown, they are expanded (or discretized) in an abstract stochastic space such that the probabilistic responses can be numerically computed.

This paper briefly presents the basic formulation of NL-SSFEM and the results of two numerical simulations of faulting. The first simulation is for model experiments to verify the basic validity of NL-SSFEM as a numerical analysis method of faulting; the results of simulation are compared with experimentally observed data. The second simulation is for actual faulting, to show the usefulness of the NL-SSFEM simulation which predicts faulting behavior with variability due to complicated ground structures. The fault configuration parameters which are computed by NL-SSFEM are compared with observed data, and some discussion is made on the probability of the fault appearance on the ground surface which is obtained by the NL-SSFEM simulation.

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In the NL-SSFEM simulation, ground layers are modeled as a non-linear elasto-plastic body which has spatially and stochastically varying material properties. For simplicity, it is assumed that only Young's modulus is given as a random function, $E(\boldsymbol{x},\omega)$, where $\boldsymbol{x}$ is a position vector and ω represents a stochastic event and that other material parameters are deterministic and uniform. It should be mentioned that when the argument ω is given, the value of E changes depending on $\boldsymbol{x}$ and the body is heterogeneous. For a fixed $\boldsymbol{x}$, on the other hand, E changes its values depending on ω. When suitable boundary conditions are given, the resulting displacement field becomes a vector-valued random function, and we denote it by $u_i(\boldsymbol{x},\omega)$ with the argument ω emphasizing that it varies stochastically.

Assuming the associated flow rule, the NL-SSFEM approximates the instantaneous elasto- plastic tensor, which becomes stochastic as well, as $c_{ijkl}(\boldsymbol{x},\omega)=E(\boldsymbol{x},\omega)h_{ijkl}(\boldsymbol{x})$, where $h_{ijkl}(\boldsymbol{x},\omega)$ is determined by the mean of stress at $\boldsymbol{x}$ and a yield function; this approximation means that the variability of the elasto-plastic tensor due to the stochastically varying stress is fully neglected. Then, for given boundary conditions, the increment of $u_i(\boldsymbol{x},\omega)$ is computed as solution of a stochastic variational problem of the following functional:

$$J(\dot{u}) = \int_{\Omega} \int_{V} \tfrac{1}{2} c_{ijkl}(\boldsymbol{x},\omega)\dot{u}_{i,j}(\boldsymbol{x},\omega)\dot{u}_{k,l}(\boldsymbol{x},\omega)\,dv\,dP(\omega) \quad (1)$$

where $dP(\omega)$ stands for the stochastic measure and the integration with respect to $dP(\omega)$ in Ω gives the probabilistic mean; Ω is the space of stochastic events which includes all possible ω's. The integration with respect to dv is for the physical domain and it gives the elasto-plastic strain energy stored in the body. Thus, the functional gives the stochastic mean of the total elasto-plastic strain energy, and the random field of displacement which minimizes this mean is the one that is the solution of the stochastic model; see Hori *et al*. (2003).

The NL-SSFEM applies the spectral decomposition of $E(\boldsymbol{x},\omega)$ for efficient computation of random functions of $u_i(\boldsymbol{x},\omega)$; see Ghanem and Spanos (1991). When the correlation function of $E(\boldsymbol{x},\omega)$ is given, $E(\boldsymbol{x},\omega)$ and $u_i(\boldsymbol{x},\omega)$ are expanded as

$$
\begin{aligned}
E(\boldsymbol{x},\omega) &= \overline{E}(\boldsymbol{x}) + \sum \lambda^{\alpha}\phi^{\alpha}(\boldsymbol{x})\xi^{\alpha}(\omega),\\
\dot{u}_i(\boldsymbol{x},\omega) &= \overline{\dot{u}}_i(\boldsymbol{x}) + \sum \dot{u}_i^{\beta}(\boldsymbol{x})\Gamma^{\beta}(\omega).
\end{aligned} \tag{2}
$$

Here, bar on symbol stands for mean, and the values of stochastic functions $\xi^{\alpha}(\omega)$ and $\Gamma^{\beta}(\omega)$ are explicitly computed by using the correlation function of $E(\boldsymbol{x},\omega)$; see Hori *et al*. (2003) for more detailed derivations.

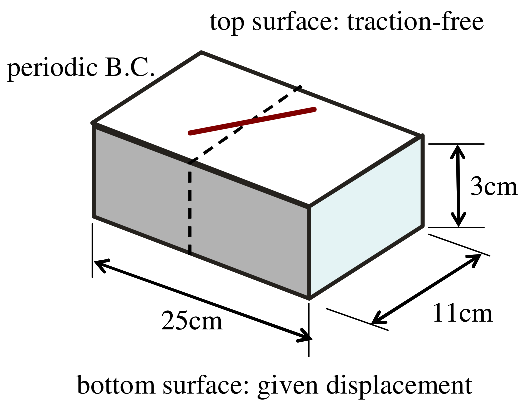

Fig. 1. SSFEM model for model

Table 1. Parameters of model experiment

mean of E (kgf/cm^2)	12.5
ν	0.25
friction angle (°)	51
SD of E (kgf/cm^2)	0.15
corelation length of E (cm)	50

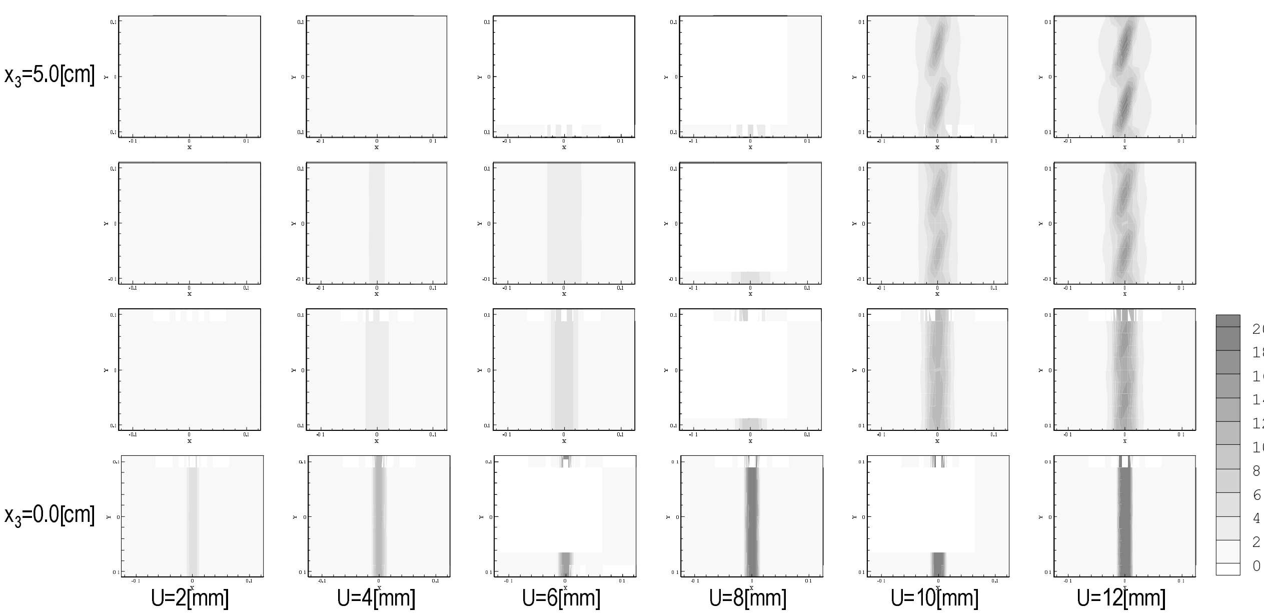

Fig. 2. Evolution of Riedel shears

Table 2. Configuration of Riedel shears

	orientation [deg]			interval [cm]			width [cm]		
depth [cm]	3	5	7	3	5	7	3	5	7
experiment	26	31	27	11			2.5～5.0	2.5～11.0	2.5～10.0
SSFEM	23	29	31	11			1.8	3.0	3.0

Table 3. Parameters of Nojima Earthquake Fault

mean Young modulus [kN/m^2]	6125
Poisson ratio	0.25
density [g/cm^3]	2.1
friction angle [deg]	51
cohesion [kN/m^2]	38
standard deviation of Young modulus [%]	30
correlation length of Young modulus [m]	2

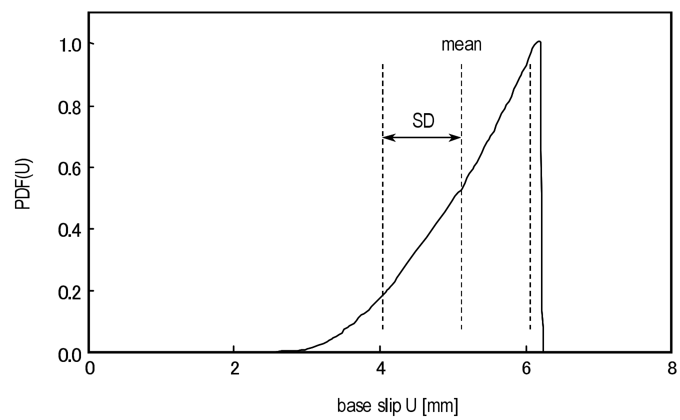

Fig. 3. PDF of failure for model experiment

Substituting (2) into (1) and discretizing spatial functions, we can reduce the variational problem of J to a matrix equation for the discretized spatial function in the same manner as ordinary FEM. Indeed, the discretization of spatial displacement increment function uses element-wise shape functions in NL-SSFEM. It is shown that when the expansions of (2) are truncated at six terms, the accuracy of NL-SSFEM is at the same level as the Monte-Carlo simulation that requires more than 1,000 simulations of randomly generated non-linear elasto-plastic models.

The major modification of the original SSFEM of Ghanem and Spanos (1991) to develop the present NL-SSFEM is the approximate evaluation of the instantaneous elasto-plastic tensor, i.e., the stochastic variability due to the stress which also changes stochastically is neglected. This approximation does not allow NL-SSFEM to simulate faulting processes which involve brunching or bifurcation. This limitation, however, is not critical for the estimate of the surface earthquake fault, since the primary objective of the NL-SSFEM is the probabilistic evaluation of the fault appearance and configuration, not the stochastic evaluation of entire faulting processes.

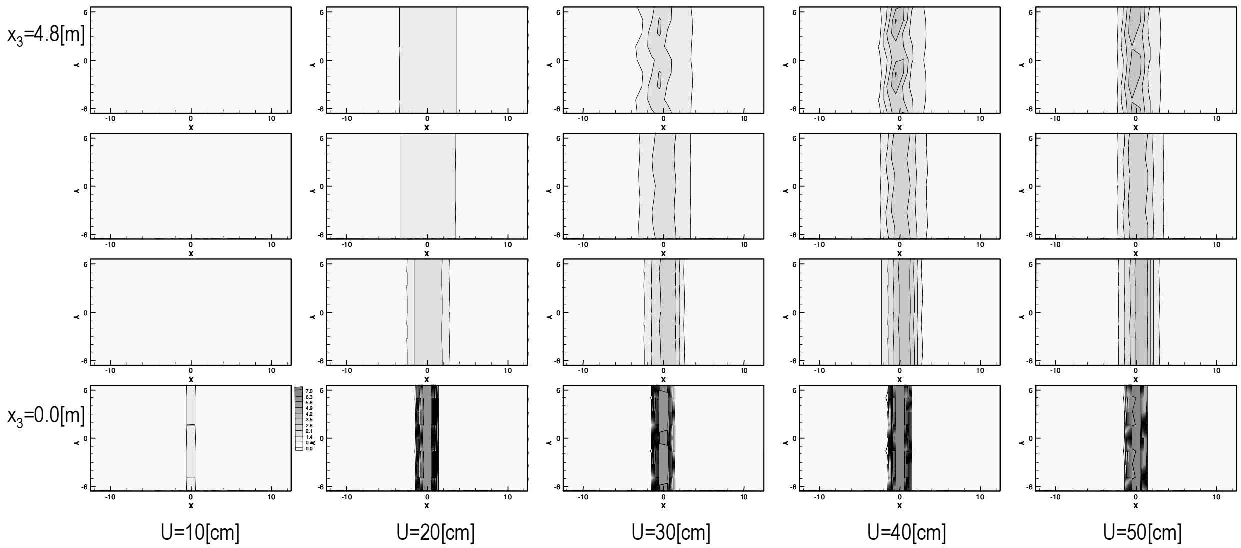

Fig. 4. Rupture process of Nojima Earthquake Fault

RESULTS OF NUMERICAL SIMULATION

Model Experiment

In order to verify the basic validity of the SSFEM simulation, we first seek to reproduce results of model experiments made by Tani *et al*. (1997); see also Taniyama and Watanabe (1998). They use a sand box in which Toyoura sand is loosely settled, and move the base of the sand box to simulate faulting; faulting is modeled as growing shear bands in a sand layer. Three height of the sand box, 3, 5, 7[cm], are used. For simplicity, we use the Drucker-Prager yield function and assume that the correlation function of stochastic E is given by $\sigma\exp(-r/R)$, with σ and R being the standard deviation and the correlation length of the Young modulus.

Figure 1 shows a stochastic model of the sand box for lateral sliding. The periodicity along the sliding direction is assumed, and the interval is determined to minimize the external work which is needed to make the shear bands reach the surface; the external work equals the total strain energy stored in the body, and hence the solution that minimizes the external work minimizes the functional J for each increment. The material parameters used in the present NL-SSFEM simulation are shown in Table 1. For the sand box of height 5[cm], the evolution of the shear band as the base movement U is increased is displayed in Fig. 2; each column shows the distribution of maximum shear strain on horizontal cross sections. It is shown that the formation process of the Riedel shears due to uniform sliding, which accompanies bifurcation from the uniform shearing, is reproduced by the present NL-SSFEM simulation. Finding such bifurcated solution is not trivial in numerical simulation of a homogeneous non-linear body. Since the present stochastic model is spatially heterogeneous, such a bifurcated solution is found without using any special numerical techniques.

The configuration parameters of the simulated Riedel shears are compared with the ones which are observed in the model experiment. Table 2 summarizes the orientation, interval, and length of one Riedel shear for the three cases of the sand box experiment. As is seen, the agreement is satisfactory. We assume that the failure is as the instance that the Riedel shears reach the surface of the sand box, and compute the probability of failure as a function of the amount of the base movement U. This failure probability is of primary importance, since, if predicted values of the bed rock mass movement is small, the failure

Table 4. Configuration of Nojima Earthquake Fault

	observed	SSFEM
fault	left echelon	left echelon
angle to strike direction [deg]	22-35	25
interval of echelon faults [m]	4.0-6.0	6
width of echelon faults [m]	0.5-1.5	1.4

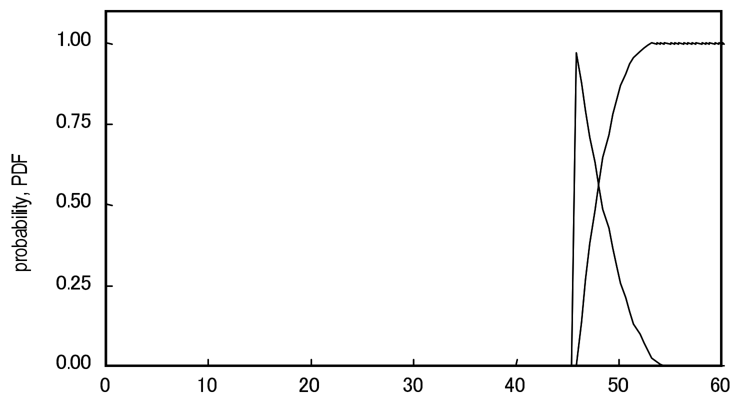

Fig. 6. PDF and CDF of failure for Nojima Earthquake Fault

probability becomes zero, which means that the threat of faulting on near-by structures is minimum. Figure 3 presents the PDF for the failure probability. The mean and variance of the computed PDF with the values obtained from the experiments though the number of experiment is two for each sand box height. It is seen that NL-SSFEM reproduces the failure probability fairly well.

Nojima Fault

We next examine the usefulness of NL-SSFEM to numerically simulate the actual surface earthquake fault, the Nojima Earthquake Fault, which is a source of the Hanshin Awaji Earthquake. The target is the one that appears in Nashimoto District, in which various measured geological and geotechnical data are available. Like the simulation of a sand box, a stochastic model is used for unconsolidated layers (which belong to the Osaka Layers). The flat single layer of the thickness 9[m] is used. The material parameters which are used in the present simulation are summarized in Table 3. According to the seismological analysis of the Hanshin Awaji Earthquake, the dip angle of the fault plane is set as 90[deg] and the direction of the fault displacement is set as 45[deg].

Figure 4 shows the mean rupture processes of the unconsolidated layers, by plotting the maximum shear strain distribution. In the simulation, formation of echelon faults is reproduced as observed, even though the vertical movement is included in the bed rock movement. We should emphasize that the reproduction of the NL-SSFEM is not perfect. The structure of the computed echelon faults is too simple compared with the actual one, mainly because coarse discretization and simple elasto-plastic constitutive relations are employed in the simulation. For the practical purpose of the probabilistic estimate of fault behavior, we may not need to use more elaborated computational efforts nor sophisticated constitutive relations.

In Table 4, we compare the configuration parameters of the simulated echelon faults with the observed ones. The agreement is satisfactory. This agreement is owing to the quality of the data which are input to the NL-SSFEM simulation. Next, we compute the failure probability and plot the PDF in Figure 5. The cumulative density function (CDF) of the failure is plotted as well. It is certainly true that there are no direct evidences that support or deny the accuracy of the computed PDF. However, the PDF estimates that the bed rock displacement that causes failure is around 0.5[m]. This value appears reasonable, based on the seismological analysis. If a bed rock movement is given, the simulated PDF can provide the probabilistic estimate of faulting, taking into account possible dissipation of rupture processes within soft unconsolidated layers. This is the usefulness of numerically simulating the rupture process with the aid of NL-SSFEM. The present result of the failure probability shows this usefulness.

CONCLUDING REMARKS

In the NL-SSFEM simulation of the model experiment and the Nojima Earthquake Fault, it is shown that the fault configuration can be reproduced to some extent. The probabilistic estimate of NL-SSEM is attractive since the actual fault behavior has large variability and modeling unconsolidated layers is not trivial. At least, we can make decision that the fear of faulting is minimum, based on the NL-SSFEM simulation, when the soft layers are sufficiently thick and possible bed rock mass movement is small. Therefore, the NL-SSFEM simulation will be a reliable tool to provide probabilistic estimate provided that further verification is made.

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