

# CONSIDERATION OF TORSIONAL EFFECTS IN THE DISPLACEMENT CONTROL OF DUCTILE BUILDINGS

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# SUMMARY

Torsional effects may significantly modify the seismic response of buildings, and they have caused severe damage or collapse of structures in several past earthquakes. These effects occur due to different reasons, such as no uniform distribution of the mass, stiffness and strength, torsional components of the ground movement, etc. In ductile structures, the main consequence of floor twist is an unequal demand of lateral displacements in the elements of the structure. As a result, the lateral ductility capacity of the system may be smaller than the lateral ductility capacity of the elements. Design codes incorporate especial requirements to take into account the torsional effects, which usually imply de amplification of eccentricity and the consideration of an accidental eccentricity. These requirements are mainly based on elastic considerations developed several decades ago.

This paper summarizes the main aspects of the torsional response of ductile structures and presents a modified design procedure. In this procedure, the ductility capacity of the system is reduced, in order to account for the torsional strength of the structure, and the yield displacement of the elements in order to yield simultaneously. The proposed criterion is compared with analytical results obtained from a parametric study. The influence of several variables involved in the problem was evaluated using static and dynamic nonlinear analysis. Uncertainties of the problem are discussed and recommendations for further research are presented.

# **INTRODUCTION**

The seismic response of buildings subjected to ground motions may be significantly modify due to the occurrence of torsional effects. As a result, the floors of the building not only translate laterally but also rotate along a vertical axis. This effect produces an uneven distribution of the lateral displacements at the same level (with an increase of the displacement at some points of the perimeter of the building), and a modification of the internal actions. The main reasons for the occurrence of torsional effects are two. First, lack of symmetry of the structural system due to a non-uniform distribution in plan of the stiffness, mass or strength. This lack of symmetry may be evident or accidental (unpredicted variation of the properties of the structural system). The asymmetric configuration of the building results in a coupling of the translational and rotational mode of vibration of the structures. Second, asynchronic movement of the

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foundation of the building due to characteristics of the seismic excitation. As a result, torsional vibrations may also occur in symmetric buildings. This effect depends on the type of foundation and on the dimensions of the building compared to the length of the seismic wave. Damage of buildings observed in past earthquakes indicated that one mayor cause may be the torsional vibration induced by the ground motion. It is worth noting that the severe damage occurred during the Michoacán Earthquake, Mexico, 1985, showed the importance of torsional effects and pointed out the need to understand the problem and improve the design requirements [1].

In the last four decades, numerous researchers have conducted extensive studies to investigate the torsional response of asymmetric buildings considering elastic and inelastic behaviour [2]. Despite this effort, design provisions include in most of the seismic codes [3] are based on elastic concepts developed in the 1960s and 1970s for elastic systems. This criterion considers that the torsional effect induced by the earthquake can be represented in the static analysis of the building by applying a torque at each floor level. The torque depends on a design eccentricity, which considers the stiffness eccentricity of the system (distance between the centre of mass, CM, and the centre of stiffness, CK) modified by a dynamic amplification factor, and the accidental eccentricity. When dynamic analysis is performed, only the accidental eccentricity is considered.

In the last decade, there has been a renovated interest in the evaluation of the torsional effect in building subjected to earthquakes based on the need to revise and improve torsional provisions [4, 5]. Paulay [6, 7, 8, 9] has shown that the traditional criterion is not adequate since building structures located in seismic areas rely on inelastic response when subjected to large earthquakes. Furthermore, he pointed out that the strength eccentricity is a better parameter to evaluate the torsional effects in ductile systems. Crisafulli and Formica [10] conducted a large parametric study in which simple asymmetric buildings where analyzed using static linear analysis and applying usual torsional provisions included in seismic codes. The results confirmed Paulay's conclusions and indicated that the application of these provisions originates an increase of the required lateral resistance of the structure, with uncertain effectiveness to control the torsional problem in the inelastic range. Furthermore, an increase of the final strength eccentricity was observed in some cases, which is obviously an unfavourable condition. Other researchers have continued the investigation of this topic [11, 12, 13].

Another criterion proposed to improve torsional provisions is based on the use of corrective eccentricities able to equate the maximum lateral displacement obtained from static and dynamic analyses [14, 15, 16, 17]. Despite the accuracy of the proposed expressions to evaluate the torque, these procedures are conceptually equivalent to those developed several decades ago and included in most of the seismic codes. In the authors opinion this is an inappropriate method to face the problem. It must be noted that a refined evaluation of the displacements based on elastic considerations is not important, since the design procedure relies on inelastic response of the building to dissipate energy. Furthermore, the extensive use of computers and the development of software for structural analysis allow the designer to conduct an "exact" elastic dynamic analysis very easily, if required.

The torsional response of ductile buildings represents a challenging topic, which still requires extensive research. The problem, however, is not simple due to the complexity of the inelastic behaviour under seismic attack. Consequently, the main objectives of this paper are:

- 1. To discuss conceptual issues related to the torsional response in the inelastic range.
- 2. To analyse symmetric and asymmetric simple buildings in order to evaluate the influence of several variables which may affect the response.
- 3. To propose a rational methodology for the consideration of torsional effects in the design process.

In this paper the term "system" refers to the complete three-dimensional structure, whereas the term "element" indicates the different planar structures, such as frames, walls, etc.

#### CLASIFICATION OF THE STRUCTURAL SYSTEMS

Structural systems can be classified in two groups, namely torsionally restrained or unrestrained systems, depending on their capacity to restrain the torsional rotation under unidirectional seismic attack in the principal directions of the building [8] (see Figure 1). It must be noted that this classification takes into account the inelastic response of the structure, which is adequate for ductile design. Consequently, torsionally unrestrained systems, such as illustrated in Figure 1 (b), are not considered a good structural solution, even though the building has torsional stiffness and strength in the elastic range.



Figure 1. Classification of structural systems: a) Torsionally restrained system and b) Torsionally unrestrained system

The degree of torsional restraint,  $\lambda_{tx}$  and  $\lambda_{ty}$  (in the x and y direction, respectively) can be evaluated as the ratio of the torsional stiffness of the elements located along the perpendicular direction to the total stiffness of the system, K<sub>t</sub> [8]:

$$\lambda_{tx} = \frac{\sum k_{yi} x_i^2}{K_t}, \qquad \lambda_{ty} = \frac{\sum k_{xi} y_i^2}{K_t}$$
(1)

where  $k_{xi}$  and  $k_{yi}$  are the stiffness of the elements located along x and y axes, respectively,  $y_i$  and  $x_i$  are the coordinates of these elements (measured from the centre of stiffness, CK), respectively. The torsional stiffness  $K_t$  is defined by:

$$K_{t} = \sum k_{xi} y_{i}^{2} + \sum k_{xi} x_{i}^{2}$$
(2)

The parameter  $\lambda_t$  defined in Eq. 1 considers the residual torsional stiffness of the system, due to the elements located along the perpendicular direction, which are intended to remain elastic while the other elements are yielding. In order to establish a quantitative limit between both types of systems, it can be considered that torsionally restrained systems verify the condition  $\lambda_t > 0.15$  or 0.20. It can be demonstrated from Eqs. 1 and 2 that:

$$\lambda_{tx} + \lambda_{ty} = 1 \tag{3}$$

Torsionally unrestrained systems are simpler to analyze from the conceptual point of view because they represent statically determinate structures. However, they are more sensitive to dynamically induced twist

[8] and results obtained from static and dynamic analyses may present significant differences [11, 18]. Furthermore, unrestrained systems are seldom used in real constructions. For these reasons, the work presented here focuses only on torsionally restrained systems.

# TORSIONAL RESPONSE OF DUCTILE BUILDINGS

The inelastic response of asymmetric buildings subjected to earthquakes is a complex phenomenon, in which many variables are involved. The understanding of the problem is a fundamental step in order to develop rational procedures and to propose design recommendations. In the following sections different aspects of this problem are studied, based on results obtained from static and dynamic analyses of inelastic systems.

Static analyses were performed with the computer program "TORSION" [19]. This software considers the response of the system assuming one-story elements located along two perpendicular directions; these elements are linked by a rigid diaphragm. The inelastic behaviour of the elements is represented by a bilinear relationship between the lateral force and displacement. The failure of the system occurs when the ductility available of one or more elements is achieved. Dynamic time history analyses were conducted using the program "RUAUMOKO" [20] with an acceleration record obtained in the Kobe earthquake, 1995. The cyclic behaviour of the elements was represented by the Takeda model with a bilinear envelop and no limitation in their ductility capacity was imposed.

# Stiffness and strength eccentricity

Stiffness eccentricity,  $e_k$ , has been considered one the most important parameters in the evaluation of the torsional response of buildings, and this fact is clearly reflected in existing design provisions. However, this parameter depends on the location and stiffness of the elements of the system. Other important parameter is the strength eccentricity,  $e_v$ , defined as the distance between the centre of strength, CV, and the centre of mass, CM. The centre of strength is the point of application of the resultant force of the lateral resistance of all elements. The use of the strength eccentricity was introduced in the Mexican Code after the Michoacán Earthquake, 1985 [21], however it was not until the last decade that the importance of this parameter was recognized.

It must be noted that the exact position of the centre of stiffness cannot be found for multi-story threedimensional structures [22]; similar considerations could apply regarding to the position of the center of strength. However, both centers can be located in practical cases and represent important concepts that the structural engineer should consider in the design process.

In order to show the importance of both type of eccentricity, results obtained from the static inelastic analysis of a simple structure, named Building A (see Figure 2 (a)), are presented here. Case 1 considers the situation in which the stiffness and strength of the elements is assigned such as  $e_{kx}=2.4m$  and  $e_{vx}=0$ , whereas in Case 2  $e_{kx}=0$  and  $e_{vx}=2.4m$ . Figure 3 illustrates the response, in terms of base shear and floor rotation (twist) versus lateral displacement, for both cases. It is shown that the influence of each eccentricity is quite different. In the first case, the influence of the stiffness asymmetry induces a significant twist in the elastic range, which almost disappears during the ductile response of the structure. Contrarily, the effect of the strength eccentricity is null during the elastic response and increases to a large value when the structure behaves in the inelastic range.

The non-linear response of Building A has been evaluated to considered the combined effect of both eccentricities, which is the usual case in asymmetric buildings. Case 3 evaluates the situation in which strength is directly proportional to stiffness and then,  $e_{kx} = e_{vx} = 2.4m$ . An intermediate situation is analysed in Case 4, in which  $e_{kx} = 2.4m$  and  $e_{vx} = 1.2m$ . Results, in terms of floor rotation, are presented in Figure 4 where the previous cases are also included for the sake of comparison.



Figure 2. Schematic representation of simple buildings considered in the examples.



**Figure 3**. Response of Building A, (a) Case 1:  $e_{kx}=2.4m$  and  $e_{vx}=0$ , and (b) Case 2:  $e_{kx}=0$  and  $e_{vx}=2.4m$ .

In the previous examples, the stiffness and strength of the elements has been assigned independently in order to obtain different values for the eccentricities  $e_{kx}$  and  $e_{vx}$ . It must be noted, however, that in real structures stiffness and strength are related. For example, when the strength of the elements has been assigned "exactly" as derived by the elastic analysis without considering an accidental eccentricity it is found that [9]:

$$\mathbf{e}_{\mathbf{v}} = \lambda_{\mathbf{t}} \, \mathbf{e}_{\mathbf{k}} \quad \rightarrow \quad \mathbf{e}_{\mathbf{v}} \le \mathbf{e}_{\mathbf{k}} \tag{4}$$

When the accidental eccentricity is considered in the design process, as required by most of the codes, it is difficult to derive a general equation between both types of eccentricity. However, numerical results obtained from different structures indicates that  $e_v < e_k$ .

Torsional response during the ductile response can be evaluated only after the strength of the elements has been established, due to the significant influence of the strength eccentricity. This fact indicates that the design process should be iterative, as explained later in more detail.



Figure 4. Comparison of the floor rotation of Building A for different cases.

It is convenient to express the strength eccentricity,  $e_v$ , as a relative value,  $\varepsilon_v$ , in order to generalize the results. A first option, usually found in the literature, is to define the ratio between the eccentricity and the length of the floor plant of the building (both in the same direction):

$$\varepsilon_{v} = \frac{e_{v}}{L}$$
(5a)

Alternatively, the following definition can be used:

$$\varepsilon_{v} = \frac{e_{v} V_{u}}{M_{m}}$$
(5b)

where  $V_u$  is the lateral strength in the direction perpendicular to  $e_v$ , and  $M_{tu}$  is torsional strength of the complete system. The comparison of Eqs. 5a and 5b indicates that both produce similar result in building with a good distribution of elements around the perimeter of the building. In other cases, Eq. 5b results in greater values of the relative eccentricity. In the authors' opinion the latter definition seems to be more general because the ratio  $M_{tu}/V_u$  represents the "internal arm" for torsional effects.

#### **Torsional restraint**

The degree of torsional restraint is a very important parameter because controls the twist at the ultimate stage,  $\theta_{tu}$ . Paulay [9] has shown that  $\theta_{tu}$  can be evaluated as:

$$\theta_{tu} = \frac{e_{vy} V_{ux}}{\lambda_{tx} K_{t}}, \quad \text{or} \quad \theta_{tu} = \frac{e_{vx} V_{uy}}{\lambda_{ty} K_{t}}$$
(6a)

where  $V_{ux}$  and  $V_{uy}$  are the base shear in each direction. The previous equation is valid for elasto-plastic systems, but it can be also applied to bilinear systems with small post-yielding stiffness. The elastic twist,  $\theta_{te}$ , when the structure develops its lateral strength, is given by:

$$\theta_{te} = \frac{e_{ky} V_{ux}}{K_t}, \quad \text{or} \quad \theta_{te} = \frac{e_{kx} V_{uy}}{K_t}$$
(6b)

Eq. 6a indicates that both strength eccentricity and torsional restraint have a direct influence of the twist when the structure behaves inelastically. Therefore, the optimal values for the restraint cannot be determined as a general rule because they depend on the strength eccentricities. It must be also considered that the increase in the torsional restraint in one direction will decrease the restraint in the other direction (see Eq. 3). In the particular case in which  $e_{vx} = e_{vy}$ , the optimal condition is achieved when  $\lambda_{tx} = \lambda_{ty} = 0.50$ .

The influence of the torsional restraint is illustrated in Figure 5 by comparing the response of Building A, Case 3 ( $\lambda_{tx} = 0.52$ ) with a new case in which the stiffness of the elements located along x-direction has been reduced, Case 4 ( $\lambda_{tx} = 0.28$ ). It is observed that the reduction of the torsional restraint results in a considerable increase of the twist (2.8 times, in this example). This fact can be explained taking into account Eq. 6a, since in Case 4 both  $\lambda_{tx}$  and K<sub>t</sub> were reduced. The base shear versus displacement curves are not included in Figure 5 because they are very similar in this comparison.



Figure 5. Comparison of the response of Building A, for Cases 3, 4 and 5.

The effectiveness of the torsional restraint provided by elements located along the perpendicular direction is valid whether these elements remain elastic. Otherwise the system loses its capacity to restrain the twist resulting in a very unfavourable condition. In order to investigate this effect, another example, Case 5, is analyzed considering that the torsional restraint is equal to that of Case 3 ( $\lambda_{tx} = 0.28$ ), but the strength of the elements located along y-direction is reduced to allow yielding under seismic attack in x-direction. The results, included in Figure 5, indicate that the elastic twist in Case 3 and 5 is the same (because the torsional restraint is the same). However, the twist significantly increases when the elements located along y-direction yield, resulting in values larger than in Case 4 (with smaller torsional restraint,  $\lambda_{tx} =$ 0.28). Therefore, it can be concluded that the torsional restraint defined in Eq. 1 is not enough to control the twist and to assure an adequate response of asymmetric ductile systems. Consequently, the designer should also verify that the torque induced by the earthquake in a given direction during the ductile response of the building is smaller than the torsional resistance provided by the elements located along the perpendicular direction, M<sub>tux</sub> and M<sub>tuy</sub>. This condition can be expressed as:

$$M_{tux} \ge 1.25 V_{ux} e_{vy}$$
, and  $M_{tuy} \ge 1.25 V_{uy} e_{vx}$  (7)

where  $V_{ux}$  and  $V_{uy}$  are the nominal lateral strength in x and y directions. The factor 1.25 in Eq. 7 has been included to consider a probable overstrength, using principles of capacity design. The torsional resistance in each direction depends on the nominal strength of the elements,  $V_{xi}$  and  $V_{yi}$ , and their position measured from the centre of strength,  $x_i$  and  $y_i$ :

$$\mathbf{M}_{\text{tux}} = \sum_{i=1}^{nx} \mathbf{V}_{xi} |\mathbf{y}_i| \quad \text{and} \quad \mathbf{M}_{\text{tuy}} = \sum_{i=1}^{ny} \mathbf{V}_{yi} |\mathbf{x}_i|$$
(8)

where nx and ny are the number of elements in each direction.

The validity of Eq. 7 has been tested for different configurations. However, it can be not conservative in buildings in which the number of elements located along the perpendicular directions is reduced to two or three and the yield displacement and strength of these elements is very different. The previous conclusions were based on equilibrium condition and verified using static nonlinear analyses. The dynamic nature of the problem may result in a more favourable situation, since the lost of the torsional restraint due to yielding usually occurs in a very short time and then the system regains the restraint. Consequently, Eq. 7 can be used until more research is accomplished to find a final conclusion.

#### Angle of attack

Seismic design provisions usually consider that the earthquake attack occurs along the two principal directions of the building. However, it is interesting to investigate the effect of the angle of seismic attack in the torsional response of the system, because skew attack may produce simultaneous yielding in both directions. For this reason, the structure represented in Figure 2 as Building B was analysed using static and dynamic procedures and considering variable angle between  $0^{\circ}$  and  $180^{\circ}$ . In the first steep, the stiffness and strength of the elements was assigned to obtain a symmetric structure, without any eccentricity. In this case, the angle of attack has no significant influence, except for a certain angle, which originates that elements yield in both directions. As a result, the story twist and the lateral displacement increase, but the lateral strength of the system also increase because the resistance of all elements is developed. This angle depends on the relative lateral strength  $V_{nx}/V_{ny}$  (for example, the angle is equal to  $45^{\circ}$  when  $V_{nx} = V_{ny}$ ). In the second step, asymmetric systems were considered and the stiffness and strength of the elements was assigned to obtain a strength and stiffness eccentricity equal to 2.4 m in both directions and a torsional restraint equal to 0.50. The results obtained in this example, see Figure 6 (a), indicate that the floor rotation increases when the angle of attack approaches to 45° and decrease to zero when the angle is 135°, because in the latter case the seismic action pass through the centre of mass. Similar trends are obtained from the static and dynamic analyses. In general, the most adverse condition occurs when both components of the seismic action produce twist in the same sense.

The effect on the ductility is variable depending on whether the results are obtained from static or dynamic analyses. In the former case, the ductility capacity of the system, see Figure 6 (b), is smaller than the maximum ductility allowed in the elements (in this case,  $\mu_{max}=5$ ) as a result of the twist, which imposes a non-uniform lateral displacement in the perimeter of the building. The decrease in the ductility capacity of the system (35% at 45°) is accompanied with an increase of the lateral resistance (14%). However both effects are not proportional and the skew attack produces an unfavourable condition. Paulay [6] obtained an opposite conclusion for a restrained structure formed by four elements. On the other hand, the ductility demand on the critical elements (located on perimeter of the building) obtained from dynamic analyses, see Figure 7, changes as a function of the angle of attack. For elements 1 and 5 the maximum values are well predicted by seismic attack along the principal directions, whereas for elements 2 and 7 the maximum demand occurs under skew attack. The example showed here suggests that the usual seismic provisions could be not conservative for asymmetric structures and that skew seismic attack may increase the effects of torsional response.

#### **Post-yield stiffness**

The nonlinear response of structural elements is commonly represented by a perfect elasto-plastic or a bilinear approximation. In the latter case, it is assumed that the element exhibits some stiffness after yielding. Usual values of the post-yield stiffness may range from 1 to 10% of the elastic stiffness, depending on the material and characteristics of the element.



Figure 6. Results corresponding to Building B under skew seismic attack: (a) floor rotation from dynamic and static analyses, and (b) system ductility.



Figure 7. Ductility demand in critical elements of Building B, obtained from dynamic analysis.

The influence of this parameter on the torsional response of ductile structures is variable, consequently the use of the perfect elasto-plastic model should be use with care. In the case of unrestrained systems or systems that become unrestrained (for example, all the elements of the system yield under skew seismic attack), the use of the perfect elasto-plastic model may result in a significant increase of the lateral displacement or twist due to the lack of lateral or torsional stiffness when the elements have yielded. Numerical results obtained by the authors from dynamic nonlinear analyses of torsionally restrained systems indicate that, in general, the effect of the post-yield stiffness on the torsional response is similar to that reported for planar structures and is not important for usual values of this parameter

## Mass rotational inertia

Useful conclusions may be obtained from the static analysis of ductile systems; however, a verification based on dynamic analysis should be always conducted, since the actual problem of buildings subjected to earthquakes is a dynamic phenomenon. A significant difference between the static or dynamic consideration of the problem arises from the influence of the mass rotational inertia, which can modify the torsional response. For torsionally unrestrained systems this influence is very important and usually has a beneficial effect, according to the results reported by Castillo et al. [11] and Moller et al. [18]. Restrained systems seem to be more complex due to larger number of elements involved in the problem. A series of examples were analysed in order to investigate this effect, using the simple buildings sketched in Figure 8. The stiffness of the element 2, in Building C, and element 1, in Building D, was changed from zero to three times the stiffness of the other elements; the strength of all elements was assigned in direct proportion to their stiffness. The models for the dynamic analyses considered two situations: with and without mass rotational inertia. Results of these examples are presented in Figure 9, in terms of maximum floor rotation versus the strength eccentricity of the system for Building C. It can be observed that the inclusion of the mass rotational inertia increases the values of the twist, as expected from rational considerations; the increase is more significant for systems with larger strength eccentricities. The results corresponding to other cases are not presented here due to lack of space, although they followed a similar trend to that showed in Figure 9.

It is interesting to investigate not only the twist induced in the systems, but also the ductility imposed on the critical elements. The results showed an erratic trend, with some cases in which the ductility demand decreased even though the maximum twist increased due to the consideration of the rotational inertia. In the authors' opinion this is due to the fact that the maximum floor rotation does not usually occur simultaneously with the maximum lateral displacement of the centre of mass. Furthermore, the changing of the stiffness and mass of the system modifies its dynamic properties and, consequently, the seismic demand. In order to investigate this effect a parametric study using a number of representative earthquake records should be performed.

Despite the results reported here, the authors have not been able to obtain a general conclusion regarding the effect of the mass rotational inertia, nor to define the cases in which has a negative influence. Consequently, it is recommended that in future research the mass rotational inertia be always considered in the dynamic nonlinear analysis of ductile structures.



Figure 8. Schematic representation of buildings considered in the examples.



Figure 9. Effect of the mass rotational inertia on the floor rotation.

## **Element Yield displacement**

Element yield displacement, defined as the ratio of the lateral strength to the stiffness, is a very important parameter to understand the plastic mechanism of three-dimensional structures. Systems incorporating elements with different yield displacements will experience a progressive incursion in the inelastic range. As a result, the elements that yielded first may develop their ductility capacity while the others are still elastic or yielding with smaller ductilities, producing in that way a reduction of the system ductility. Paulay [6, 7, 8, 9] has contributed to the understanding of this problem and no further explanations are presented here.

Apparently, this aspect is not directly related to the torsional problems discussed previously, however their negative effects are similar to those induced by twist in asymmetric systems. Therefore, an adequate solution should include an integral treatment of both effects, as described in the following sections.

## **DESIGN RECOMENDATIONS**

## **Design strategy**

The principles of capacity design, widely used for the design of ductile structures, require the identification of the plastic mechanism. This should be done not only at the element level (frames, walls, etc) but also at the system level (the complete, three-dimensional structure). The plastic mechanism of the system can be modified by the torsional response induced by the earthquake in asymmetric structures, and the effect of elements having different yield displacement. As a result, the ductility of the system (measured as the ratio of the ultimate displacement to the yield displacement at the centre of mass) can be smaller than the ductility capacity of the elements, as shown in Figure 10. An adequate estimation of the system ductility is required in order to determine the strength reduction factor used to evaluate the base shear in force based design procedures. It must be also considered that story twist increases the lateral displacement in some locations, especially at the perimeter of the building. Following the criteria formulated by Paulay [6, 9], the principal objectives of the design strategy assumed in this work are:

- 1. Earthquake imposed deformations, resulting from the lateral translation and twist, should be limited in order that the ductility demand on any element does not exceed it assumed ductility capacity.
- 2. The interstory drift at any location should not be larger than acceptable values, typically 2 to 2.5% of the story height.



Figure 10. Reduction of the system ductility as a function of the strength eccentricity, Building C and D, obtained from static nonlinear analyses.

#### **Evaluation of lateral displacements**

The designer must carefully select the method to determine the lateral displacements at the ultimate limit state. Usually, they are obtained by increasing the elastic displacements in direct proportion to the force reduction factor (or deflection amplification factor [23]) used previously to calculate the required base shear. This method could not be strictly applied because the elastic twist changes when the structure goes into the inelastic range. From Eqs. 6a and 6b the ratio  $\theta_{tu} / \theta_{te}$  between the elastic and ultimate twist can be determined:

$$\frac{\theta_{tu}}{\theta_{te}} = \frac{e_{vy}}{e_{ky} \lambda_{tx}}$$
(9)

(valid for seismic attack along x-direction; a similar expression applies in the other direction). Eq. 9 is useful to determine whether  $\theta_{tu}$  is similar to  $\theta_{te}$  (in this case the simplified method is applicable) or a more detailed study is required. In the latter case, the lateral displacements can be calculated based on the displacement of the centre of mass plus the effect of the twist  $\theta_{tu}$ . Alternatively, a static nonlinear analyses can be performed in order to calculate the lateral displacements.

#### **Proposed procedure**

A rational procedure is proposed here, aimed at achieving the design strategy defined previously. The procedure can be summarized as follow:

- 1. Define the strength, stiffness and yield displacement of all elements. These parameters are reciprocally related and, consequently, they cannot be calculated at the beginning of the design process. The designer must assign values to some of these parameter based on rational considerations and experience. Then, Evaluate the position of the centre of strength and stiffness and their eccentricities referred to the centre of mass.
- 2. Estimate the system ductility,  $\mu$ , as a function of the ductility capacity of the element,  $\mu_e$ :

$$\mu = 1 + (\mu_e - 1) \rho_t \rho_d$$
 (10)

where  $\rho_t$  and  $\rho_d$  are factors introduce to considered the effect of the story twist at ultimate stage and the influence of different element yield displacement, respectively. Factor  $\rho_t$  is calculated as a function of the strength eccentricity and the torsional restraint,  $\lambda t$ . As a preliminary recommendation until more information be available, the following equation is proposed to estimate the value of  $\rho_t$ :

$$\rho_t = 1.0 - 1.5 \varepsilon_v (1.5 - \lambda_t) \le 0.5 \tag{11}$$

where  $\varepsilon_v$  is defined in Eq. 5b. The factor  $\rho_d$  is difficult to determine analytically, particularly for systems with numerous elements. Therefore, a conceptual evaluation is proposed according to the following values:

- 3. Elements with similar structural types and yield displacement,  $\rho_d = 1.0$
- 4. Elements with similar structural types and variations in the yield displacement up to 25% (in comparison to the larger values),  $\rho_d = 0.9$
- 5. Elements with different structural types and variations in the yield displacement up to 50% (in comparison to the larger values),  $\rho_d = 0.8$
- 6. Other cases,  $\rho_d = 0.7$
- 7. Calculate the base shear required by the seismic code, taking into account the system ductility, defined by Eq. 10, to estimate the response modification coefficient, R (force reduction factor). Then, calculate the internal forces in the components of the elements (beam, columns, etc) and assign the required strength according to capacity design.
- 8. Verify that the lateral strength of the system is equal or greater than the required shear base in each direction. Furthermore, verify the torsional strength provided by elements located in each direction, Eq. 7, in order to assure the torsional restraint at the ultimate limit state. Otherwise, improve the design and return to step 1.
- 9. Calculate the lateral displacement and story drifts at critical locations and verify that design limits are not exceeded. Otherwise, improve the design and return to step 1. In this step is recommended to use Eq.9 in order to select the proper method for the calculation of the displacements.

In the last decade, displacement based design procedures have been developed and improved for many researchers working in different countries. These procedures consider the lateral displacements of the structure as the main variable in the design process. However, displacement based design procedures are not widely used in practice by the structural engineers and most of the seismic code of the world still apply the traditional forced based design method. It is very probable that this trend will change in the next years. Consequently, the authors believe that the proposed procedure represents a simple and rational solution in the transition period between the use of force based design and displacement based design. Furthermore, some of the ideas presented here, for example to calculate the system ductility and lateral displacements, can be also applied in displacement based design

## CONCLUSIONS

Torsional effects significantly affect the seismic response of buildings, particularly for ductile structures, producing an uneven distribution of the lateral displacements. As a result, the ductility capacity of the system is reduced and the lateral displacement demand in some parts of the building is increased. Existing torsional requirements, which are based on elastic considerations developed several decades ago, are not effective to improve the response of the structure. For these reasons, a rational consideration of torsional effects needs to be included in seismic provisions. The torsional requirements should be coherent with existing design criteria, which relay on the ductile response of the structure to dissipate energy.

Based on the results discussed in this paper, the following conclusions can be presented, regarding the behaviour of torsionally restrained systems:

1. Both stiffness and strength eccentricity affect the torsional behaviour of the systems, although the latter has a more significant influence during the ductile response of the structure.

- 2. The degree of torsional restraint can be quantified based on the stiffness of the elements located along the perpendicular direction. These elements must behave elastically at any stage, in order to assure that the system remains effectively restrained.
- 3. The floor rotation at the stage when the structure achieves its ductility capacity depends primarily on the strength eccentricity and the degree of torsional restraint.
- 4. The maximum floor rotation and lateral displacement of the centre of mass do not occur simultaneously and, therefore, the ductility demand on the elements does not necessarily increase when the twist also increases.
- 5. The mass rotational inertia increases the twist, which may result in a larger ductility demand on some elements of the system.

A rational procedure is proposed in this paper to considered torsional effects in forced based design. The main feature of the procedure is the reduction of the system ductility based on the strength eccentricity, the degree of torsional restraint and the relative values of the element yield displacements. The proposed procedure does not considered the amplification the stiffness eccentricity nor the accidental eccentricity to increase the torque induced by the earthquake, since they are largely irrelevant to the problem of ductile behaviour.

The conclusions reported here are based on the results obtained by the authors from static and dynamic analysis of simple buildings, in which different conditions were considered regarding the configuration, number of elements and position of the centre of stiffness and strength. However, these preliminary conclusions are not general because a limited number of cases were considered. More research is required to improve the understanding of the problem; recommend topics for further investigation are skew seismic attack, complete parametric studies using a larger database of earthquake records and influence of the strength eccentricity, degree of restraint on the ductility demands of the elements and behaviour of multistorey asymmetric buildings.

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