

SEISMIC RESPONSE SPECTRA HAVING UNIFORM MEAN FAILURE RATES: SYSTEMS WITH DISSIPATING DEVICES AND WITHOUT THEM

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SUMMARY

An approach is presented for calculating seismic response spectra with uniform annual failure rates, applicable to conventional systems as well to those provide with energy-dissipating devices. The approach is based on the analysis of single-degree-of-freedom systems with uncertain mechanical properties subjected to simulated ground motions. The effect of uncertainties related to the structural stiffness, the yield displacement and the ductility capacity on the response spectra with uniform annual failure rate is analyzed. Based on this, the importance of taking into account the uncertainty implicit in the available ductility is underlined.

INTRODUCTION

Seismic response spectra with uniform expected annual failure rates can be used for reliability-based seismic design and for the reliability assessment of structures. Several authors have treated this subject (Collins et al [1], Mendoza et al [2], etc); however, none of them has dealt with the response of systems considering uncertainties about their mechanical properties and having hysteretic dissipating devices.

The response spectra with uniform failure rates contain uncertainties that may be significant in the design process. Those uncertainties are related, among others, to the algorithms used during the numerical process, the equivalence between the actual multi-degree-of-freedom (MDOF) structure and a single-degree-of-freedom (SDOF) system, the external loads acting on the systems and the mechanic properties of the structures.

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In this study three types of uncertainties are considered: a) those related to the excitation (seismic ground motions), b) those related to the uncertainties about the structural elements, and c) that related to the ductility capacity of the structural system

The uncertainty related to the seismic excitation is introduced here by means of simulated accelerograms with statistical properties similar to those of the ground motion recorded during the September 19, 1985 seismic event, at the Ministry of Communications and Transportation, East-West component (SCT-EW, 1985), in Mexico City. The mathematical model used in this study for the numerical simulation is described in the next section.

The uncertainty related to the mechanical properties of the structure, including its deformation capacity, is taken into account by means of Monte Carlo simulation analysis. The parameters considered as random variables are: a) the structural stiffness, b) the yield displacement, and c) the available ductility of the SDOF systems taken an equivalent to the detailed systems.

The first part of the paper refers to response spectra with main failure rates associated with conventional systems. In the second part, a structural element that represents an energy dissipating device is (EDD) added in parallel to the SDOF system. The EDD considered in this study presents hysteretic behavior (load versus deformation) which does not depend on the velocity or on the frequency of the excitation, but on the relative displacement between its ends.

The spectra obtained in this study takes into account all the possible intensity motions that can occur at the site of interest (SCT) by means of seismic hazard curves that are supposed to be known. For the SCT site (located at the lake bed zone in Mexico City) those curves were obtained by Alamilla [3].

SIMULATION OF GROUND MOTION

The method followed to simulate the ground motions used for the analyses is based on that proposed by Grigoriu et al [4], and modified by Yeh and Wen [5]. The steps followed to generate the motions are:

1. The Fourier amplitude spectrum of the original accelerogram (SCT-EW, 1985) is divided into several frequency bands. Each band is separately transformed into the time domain through the inverse fast Fourier transform. The record was divided into the three following bands (see Figure 1): from 0.0Hz to 0.48Hz, from 0.48Hz to 1.32Hz and from 1.32Hz to 12.5Hz (Rivera [6]).

2. For each band a squared amplitude modulation function $c^2(t)$ is determined. The expressions for $c^2(t)$ are:

$$c_1^2(t) = 52 \left[0.0185 \,\mathrm{e}^{-\frac{(t-62)^2}{5}} + 0.0019 \,\mathrm{e}^{-\frac{(t-45)^2}{100}} + 0.00074 \,\mathrm{e}^{-\frac{(t-66)^2}{130}} + 1.68 \times 10^{-5} \right]$$
(1)

$$c_{2}^{2}(t) = 25 \left[0.0340 \,\mathrm{e}^{-\frac{(t-58)^{2}}{5}} + 0.0065 \,\mathrm{e}^{-\frac{(t-45)^{2}}{100}} + 0.0024 \,\mathrm{e}^{-\frac{(t-66)^{2}}{130}} + 3.36 \times 10^{-5} \right]$$
(2)

$$c_{3}^{2}(t) = 138 \left[0.000303 \,\mathrm{e}^{-\frac{(t-61.4)^{2}}{10}} + 0.0001 \,\mathrm{e}^{-\frac{(t-48)^{2}}{80}} + 1.68 \times 10^{-6} \right]$$
(3)



Figure 1. Fourier spectrum divided in three bands, and their corresponding accelerograms obtained by applying the inverse fast Fourier transform.

3. The motion obtained for each band is transformed into a stationary process. This is done by dividing it by its amplitude modulating function to get an "amplitude-modulated accelerogram".

4. A time scale (t to ϕ) transformation is done. Notice that this transformation is not always necessary (Yeh and Wen 1989 [5]), since if the Fourier amplitude spectrum is divided into a large enough number of bands, the change in the frequency content will not be significant; otherwise it will be necessary to do the transformation using the following equation:

$$\phi(t) = \mu_0(t) / \mu'_0(t_0) \tag{4}$$

here $\mu'_0(t) = r_1 + r_2 t^2 + r_3 t^3$, where r_1 , r_2 y r_3 are parameters that depend on the evolution in time of the rate of zero crossings, t_0 is an arbitrarily chosen instant, and the prime stands for the derivative with respect to time.

5. For each band the spectral density $S(\omega)$ of the amplitude modulated accelerogram is calculated. Then, through a least-squares fitting between $S(\omega)$ and the Clough and Penzien filter the parameters ω_g , ω_f , ζ_g , ζ_f and S_0 corresponding to each frequency band are determined (Silva et al [7]). The form of that filter is:

$$S_{CP}(\omega) = S_0 \left[\frac{\omega_g^4 + 4\zeta_g^2 \omega_g^2 \omega^2}{(\omega_g^2 - \omega^2)^2 + 4\zeta_g^2 \omega_g^2 \omega^2} \right] \left[\frac{\omega^4}{(\omega_f^2 - \omega^2)^2 + 4\zeta_f^2 \omega_f^2 \omega^2} \right]$$
(5)

where ω_g , ω_f , ζ_g and ζ_f are parameters associated with the equivalent frequencies and damping of the soil. S_0 is the white noise intensity.

6. The filtered white noise is generated using the following equation:

$$Y(t) = \sum_{j=1}^{N} \sigma_j (a_j \cos \omega_j t + b_j \operatorname{sen} \omega_j t)$$
(6)

In this equation, a_j and b_j are independent random variables with zero mean and unit standard deviation, $\sigma_j^2 = 2 S_{CP}(\omega) \Delta \omega$ is the variance of the excitation process or spectral energy, N is the number of intervals in which $S(\omega)$ is divided, and $\Delta \omega$ is the amplitude of these intervals.

7. To get a simulated accelerogram, a transformation into the real time scale (ϕ to t) should be done, and afterwards the result obtained is multiplied by its corresponding intensity function $c_i(t)$. Finally, the corresponding signals of all bands are added up.

SEISMIC RESPONSE SPECTRA WITH UNIFORM EXPECTED ANNUAL FAILURE RATES FOR CONVENTIONAL SYSTEMS

Analytical fundamentals

The methodology followed in this study for calculating the spectra with uniform failure rates is based on the following expression:

$$V_{F} = \int P(structural \ failure|y) \left| \frac{\partial v}{\partial y} \right| \partial y$$
(7)

Where v_F represents the structural failure rate, $\left|\frac{\partial v}{\partial y}\right|$ is the absolute value of the derivative of the seismic hazard curve (v) with respect to the intensity y and *P(structural failurely)* is the probability that the structural failure occurs for a given seismic intensity y.

In this study it is assumed that the structure fails when its ductility demand (μ_d) is larger than or equal to its ductility capacity (μ_c) . These variables are defined in what follows.

The ductility demanded by the system is:

$$\mu_d = \frac{\delta_u}{\delta_y},\tag{8}$$

where δ_u represents the maximum displacement demanded by the SDOF system, and δ_y is the yield displacement. A vector containing 100 values of μ_d was generated by means of the Monte Carlo simulation technique.

The probability $P(Q = \mu_d / \mu_c \ge 1)$ for different intensity values was obtained. In this case, equation 7 is expressed as follows:

$$\boldsymbol{v}_{F} = \int \boldsymbol{P}(\boldsymbol{Q} \ge 1|\boldsymbol{y}) \left| \frac{\partial \boldsymbol{\nu}}{\partial \boldsymbol{y}} \right| \partial \boldsymbol{y}$$
(10)

Notice that instead of Q it is possible to propose another measure to describe the structural damage; for example, the Park and Ang index [8] I_D . For this case, equation 10 is transformed into:

$$v_F = \int P(I_D \ge 1|y) \left| \frac{\partial v}{\partial y} \right| \partial y$$
(11)

Collins [9] has obtained spectra using this expression. Rivera [10] is working in this direction.

Spectra with uniform failure rates, using deterministic structural properties

In what follows different values of v_F were calculated for SDOF systems with vibration periods between 0 and 4s. For these cases several values of the seismic coefficients (C_e or C_y) and of the nominal ductility values (μ^*) were assumed. From the analyses, demand hazard curves as those shown in Figure 2, which correspond to $\mu^*=1$, were constructed.



Figure 2. Failure rates (v_F) of SDOF systems with different periods (T), designed with $\mu^*=1$

Figure 2 indicates that the annual failure rates for a given seismic coefficient (C_e) are much larger for systems with vibration period T = 2s than for the other periods. This is because the dominant period of the ground motion at the SCT site is $T_s = 2s$ (see Figure 1).

Based on Figure 2 several spectra associated with different annual failure rates ($v_F = 0.001, 0.005, 0.01, 0.05$ and 0.1) were determined. These are shown in Figure 3a. Similarly, spectra for different nominal ductility values were also obtained. The vertical axis of Figures 3a and b corresponds to the seismic coefficient, which is equal to the strength (F) divided by the weight of the structure, C = F / W. In Figure 3a the vertical axis is indicated as C_e , where the sub-index refers to an elastic system. In Figure 3b C_y is used instead, corresponding to a non-linear system with design ductility $\mu^* = 2$. For all the cases studied here, the critical damping of the system was assumed $\xi = 0.05$.

Notice that the spectra corresponding to the elastic system ($\mu^* = 1$, Figure 3a) indicate larger seismic design coefficients for systems with vibration periods equal to 2s; while the spectra associated with $\mu^* = 2$ have their peak ordinates for periods slightly smaller than 2s (about 1.98s). This is a consequence of the structural nonlinear behavior ($\mu^* = 2$). Another consequence of this behavior is that the spectral ordinates are smaller for systems associated with $\mu^* = 2$ than for those associated with $\mu^* = 1$, as expected.



Figure 3. Response spectra associated with different uniform annual failure rates.

Figures 3a and b may have different interpretations. One of them is the following: the smaller the structural failure rate, the larger the seismic coefficient that should be used for structural design.

The curves shown in figures 2 and 3 do not take into account the uncertainties implicit about the mechanical properties of the structural systems. The influence of those uncertainties is analyzed in the next section.

Spectra with uniform failure rates, using uncertain structural properties

In this section the ductility capacity (μ_c), the structural stiffness (*K*), and the yield displacement (d_y) are considered as random variables. It was supposed that these variables were governed by a lognormal probability density function (pdf).

For multi-degree-of-freedom (MDOF) systems the ductility capacity μ_c can be obtained by means of nonlinear static analysis ("push-over analysis) [11] or either by incremental dynamic analysis (IDA's) [12]; however, for SDOF systems it is not necessary to perform this type of analysis to estimate the structural ductility capacity. For the SDOF systems, the model proposed by Esteva and Ruiz [13] was adopted. These authors assume that μ_c has lognormal distribution with mean value equal to:

$$\mu_c = \mu^* \exp(\alpha \beta V_{\mu}) \tag{12}$$

Were μ^* is the nominal design ductility, β is Cornell's index (assumed equal to 3), V_{μ} is the coefficient of variation of μ , and $\alpha = 0.55$. Similarly, the mean values of the structural stiffness (*K*) and of the yield displacement (d_{ν}) were calculated using the following expressions.

$$\overline{K} = K^* \exp(\alpha \beta V_k), \qquad (13)$$

$$\overline{d_y} = d_y^* \exp(\alpha \beta V_{d_y}) \tag{14}$$

Montiel et al [14] obtained values of the coefficients of variation of V_{μ} , V_K and V_{dy} (see equations 12-14) based on IDA's for the global response of 5-. 10- and 15-story reinforced concrete frames subjected to ground motions recorded at the SCT site. The results of these authors are shown in Table 1.

	5-story frame	10-story frame	15-story frame
V_{μ}	0.168	0.180	0.266
V_{K}	0.119	0.082	0.142
V_{d_y}	0.113	0.095	0.115

Table 1. Coefficients of variation of of μ_c , *K* and d_y (Montiel et al [14])

Based on the ranges of values that appear in Table 1, and on equations 10, 12 -14, the spectra on Figures 4, 5 and 6 were calculated. In each case, only one of the variables mentioned in Table 1 were considered uncertain, while the other two were taken as deterministic.



Figure 4. Influence of the yield displacement uncertainty on spectra with similar failure rates



Figure 5. Influence of the yield displacement uncertainty on spectra with similar failure rates



Figure 6. Influence of the ductility capacity uncertainty on spectra with similar failure rates

Figures 4, 5 and 6 indicate that the influence of the uncertainty about structural stiffness (K) is negligible on the response spectra, that about the yield displacement (d_y) has some significance, and that about the ductility capacity (μ_c) is very significant. This means that the uncertainty about the parameter μ_c should be taken into account in the design process.

SEISMIC RESPONSE SPECTRA WITH UNIFORM EXPECTED ANNUAL FAILURE RATES FOR SYSTEMS WITH DISSIPATING DEVICES

In order to take into account the contribution of the dissipating devices to the response of the combined system (CS), a dissipating element was added in parallel to the SDOF system, as shown in Figure 7. The combined system is defined here as the conventional system plus the dissipating element. Figure 7 indicates that the dissipating element has bilinear behavior, and the element presents stiffness degradation. In the figure the stiffness of the dissipating element is indicated as K_d , and that of the main system as K_c . The total stiffness of the CS is equal to $K_T = K_d + K_c$.



Figure 7. Main system plus an energy dissipating element

The ratio between the stiffness of the dissipating element and that of the main system is defined as:

$$\alpha = \frac{K_d}{K_c}$$

Another ratio of interest is that between the yield displacement of the dissipating element and that of the main system:

$$\gamma = \frac{d_{yd}}{d_{yc}}$$

In this study the stiffness and the yield displacement of the main system (K_c and d_{yc}) are considered as random variables; however, the stiffness and the yield displacement of the dissipators (K_d and d_{yd}) are considered deterministic parameters.

Equation 10 was applied to systems with dissipating devices, assuming different values of α and γ . Some results are shown in Figure 6, which presents spectra corresponding to different uniform failure rates ($v_F = 0.005, 0.01, 0.0145, 0.05, 0.1$), for the following design values: $\mu^* = 1$, $\alpha = 0.45$ and $\gamma = 1$.



Figure 8. Response spectra for different failure rates, $\mu^* = 1$, $\alpha = 0.45$ and $\gamma = 1$.

The curves in Figure 8 can be compared with those in Figure 3a, which correspond to the corresponding conventional system. For example, for an annual failure rate $v_F = 0.005$, the peak spectral ordinate for the conventional system (see Figure 3a) is equal to $C_e = 0.81$; however, the peak ordinate for the same v_F value, associated with the system with dissipators, is about $C_e = 0.65$. This means that it is necessary to use a larger seismic design coefficient for conventional systems than for those with dissipators, for structures with the same period of vibration. This conclusion is reasonable because the structural damping is larger in systems with dissipators, and, as a consequence, the spectral ordinate becomes smaller.

The ratio between the spectral ordinates of conventional systems and those of combined systems depends on v_F and on the period of vibration (T), as shown in Figure 9. This indicates that the ratios between the spectra of conventional systems and those of structures with dissipators, for the same annual failure rate, are larger than unity. This means that the annual failure rate of a system with dissipators is smaller than that of a conventional system with the same period of vibration. This is reasonable because of the larger structural damping on the system with dissipators.



Figure 9. Ratios of response spectra of conventional systems and systems with dissipators. $\mu^* = 1, \alpha = 0.45, \gamma = 1.$

Figure 9 also shows that the presence of the dissipating elements is more significant for systems with vibration periods longer than the dominant period of the soil ($T_s = 2s$), probably because the combined structure presents non-linear behavior (due to the dissipating element), and consequently the "degraded" vibration period is larger than the initial one.

CONCLUSIONS

The results show that the uncertainties related to the structural ductility capacity are more significant with respect to the response spectra with uniform failure rates than those related to the structural stiffness and to the yield displacement of the systems.

The results shown in this paper indicate that the uncertainty implicit in the ductility capacity should be taken into account in the design process.

The response spectra with uniform annual failure rates corresponding to conventional systems present larger values of seismic design coefficients than those associated with systems with dissipating elements. This occurs because of the seismic energy dissipated by these elements.

The ratio of spectra (corresponding to a given annual failure rate) associated with conventional systems and with systems with dissipating elements (associated with the same annual failure rate) is larger for systems with vibration period longer than the dominant period of the soil. This is related to the non-linear behavior of the dissipating elements.

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