

# GLOBAL COLLAPSE OF DETERIORATING MDOF SYSTEMS

# Luis F. IBARRA<sup>1</sup> and Helmut KRAWINKLER<sup>2</sup>

# SUMMARY

Global collapse in earthquake engineering refers to the inability of a structural system to sustain gravity loads in the presence of seismic effects. The research summarized here proposes a methodology for evaluating global sidesway collapse based on a <u>relative intensity measure</u>. The relative intensity is the ratio of ground motion intensity to a structure strength parameter, which is increased until the response of the system becomes unstable. The largest relative intensity is referred to as "collapse capacity".

Deteriorating hysteretic models are developed to represent the monotonic and cyclic behavior of structural components at large inelastic deformations. Parameter studies that utilize these deteriorating models are performed to obtain collapse capacities and quantify the effects of system parameters that strongly influence the collapse for multi-degree-of-freedom (MDOF) frame structures with concentrated plasticity. The studies reveal that softening of the post-yield stiffness in the backbone curve of the plastic hinge moment – rotation relationships and the displacement at which this softening commences (defined by the ductility capacity) are the two system parameters that most influence the collapse capacity of a frame structure. Cyclic deterioration appears to be an important but not dominant issue for collapse evaluation. P- $\Delta$  effects greatly accelerate collapse of deteriorating systems and may be the primary source of collapse for flexible structural systems. Applications of the proposed collapse methodology are presented for developing collapse fragility curves and evaluating the mean annual frequency of collapse.

# INTRODUCTION

In this study, global collapse implies dynamic instability in a sidesway mode, usually triggered by large story drifts that are amplified by P- $\Delta$  effects and deterioration in strength and stiffness. Several investigations have focused on global collapse due to P- $\Delta$  effects. For instance, Bernal [1] evaluated the safety against dynamic instability of two-dimensional non-deteriorating frames based on the reduction of a multi-story building to an equivalent SDOF system. The results indicated that the minimum base shear capacity needed to withstand a given ground motion without collapse is strongly dependent on the shape of the controlling mechanism. Gupta [2] evaluated the performance of steel moment resisting frames. They concluded that P- $\Delta$  effects could lead to global collapse of the frames due to the development of a

<sup>&</sup>lt;sup>1</sup> Senior Research Engineer, Southwest Research Institute, USA. Email: libarraolivas@swri.org

<sup>&</sup>lt;sup>2</sup> Professor, Dept. of Civil and Environmental Engineering, Stanford University, USA. Email: krawinkler@stanford.edu

negative post-yielding stiffness in specific stories of the frame. Medina [3] reached the same conclusions by evaluating non-deteriorating generic frames with different number of stories and different periods.

Few systematic studies have taken into account deterioration of strength and stiffness in the nonlinear range. For example, Lee [4] evaluated the performance of new steel moment resisting frames utilizing analytical models that included a fracturing element. For evaluating the global drift capacity of the buildings, they utilized the Incremental Dynamic Analysis (IDA) approach (Vamvatsikos [5]). The onset of global dynamic instability was defined as the point at which the local slope of the IDA curve decreased to less than 20% of the initial slope of the IDA curve. Jalayer [6] also employed the IDA concept for estimating the global dynamic instability capacity of a reinforced concrete frame. She included strength deterioration caused by shear failure of the columns.

In this paper, global collapse is evaluated for MDOF frame structures including nonlinear behavior by means of concentrated plasticity. The frames utilize rotational springs that include deterioration of strength in the backbone curve and cyclic deterioration. To evaluate collapse capacity, the relative intensity is increased until the relative intensity – EDP curve becomes flat. The collapse capacity is expressed in terms of the maximum relative intensity the frame can withstand prior to collapse. System collapse is evaluated in a probabilistic format that considers the uncertainty in the frequency content of the earthquake ground motions (GM), as well as the deterioration characteristics of each structural element. Parameter studies are carried out in which the period (number of stories) of the structural system and the deterioration properties of the component models are varied. The collapse capacity results are used for computing the mean annual frequency of collapse, which is obtained by combining collapse fragility curves with the hazard information at a given site.

# COLLAPSE CAPACITY METHODOLOGY

#### **Collapse Capacity**

In the proposed methodology, global collapse is described by a <u>relative intensity measure</u> instead of an engineering demand parameter (EDP<sup>3</sup>), such as roof drift, which becomes very sensitive when the system is close to collapse. Global collapse is obtained by increasing the relative intensity,  $[S_a(T_I)/g]/\gamma$ , of the structure until the response of the system becomes unstable.  $S_a(T_I)$  is a measure of the intensity of the GM and corresponds to the 5% damped spectral acceleration at the fundamental period of the structure  $(T_I)$ , without P- $\Delta$  effects. The parameter  $\gamma$  is a measure of the structure and is defined as the base shear coefficient,  $\gamma = V_y/W$ , where  $V_y$  is the yield base shear without P- $\Delta$  effects and W is the weight of the structure.  $[S_a(T_I)/g]/\gamma$  represents the ductility dependent response modification factor, which in present codes is equal to the R-factor if no overstrength is present.  $[S_a(T_I)/g]/\gamma$  may be interpreted in two ways; either keeping the GM intensity constant while decreasing the base shear strength constant while increasing the intensity of the GM, i.e., the IDA perspective (Vamvatsikos [5]).

An illustration of  $[S_a(T_1)/g]/\gamma$ - EDP curves is shown in Fig. 1 for a frame subjected to a set of 40 GMs. The description of the system and GMs is given in the next section. Fig. 1 shows individual and statistical curves for a deteriorating frame and statistical curves for an equivalent non-deteriorating frame. The systems differ only on the properties of the springs at the end of the beams. In this example, the EDP is the normalized maximum roof drift angle,  $\theta_{r,\max}/[S_d(T_1)/H]$ , where  $\theta_{r,\max}$  is the maximum roof drift angle,  $S_d(T_1)$  is the spectral displacement, and H is the total height of the frame.

<sup>&</sup>lt;sup>3</sup> EDPs are the output of response prediction, such as story drift, ductility, and hysteretic energy dissipation



Fig. 1. Individual and Statistical  $[S_a(T_1)/g]/\gamma$ - EDP Relationships for the Reference Frame 0918 and Statistical Relationships for the Non-Deteriorating Frame

The statistical curves of the non-deteriorating frame exhibit stable behavior even for large relative intensities. In the case of the deteriorating frame, since only bending elements are utilized (i.e., shear and axial failures are not modeled), collapse implies that the interstory drift in a specific story grows without bounds (sidesway collapse), i.e., the  $[S_a(T_I)/g]/\gamma$  - EDP curve becomes horizontal. This large EDP increase with a minute increase in the relative intensity is associated with a state (not a mechanism in the classical sense) in which P- $\Delta$  effects become equal to the first order story shear resistance provided by the structural elements. Collapse may occur in a single story or in a series of stories. Note that collapse is not associated with attaining zero strength in any of the structural elements. At some plastic hinge locations the bending resistance merely means that the "plastic hinge" responds like a natural hinge, since it is assumed that the element has sufficient shear capacity to prevent local gravity load collapse.

The relative intensity associated with the last point of the  $[S_a(T_I)/g]/\gamma$  - EDP curve is the "collapse capacity" for a given structural system and GM. Collapse capacity is expressed as  $\{[S_a(T_I)/g]/\gamma\}_c$  or in a simpler form as  $[S_{a,c}(T_I)/g]/\gamma$ . For a system of given strength  $\gamma$ ,  $S_{a,c}(T_I)/g$  represents the ground motion intensity leading to collapse, and for a given  $S_a$  value,  $\gamma_c$  represents the strength at incipient collapse.

Because of the large scatter in the data, the collapse capacity is evaluated statistically. The median collapse capacity is obtained from "vertical" statistics, in which a distribution is fitted to the collapse data. For the EDP given  $[S_a(T_I)/g]/\gamma$ , "counted horizontal" statistics is employed because of the incompleteness of the data set after some GMs produce collapse. For a set of 40 GMs, the average of the 20<sup>th</sup> and 21<sup>st</sup> sorted value is taken as the median and the 34<sup>th</sup> sorted value is taken as the 84<sup>th</sup> percentile. The median EDP curve at different intensity levels terminates when 50% of the GMs have led to collapse of the frame.

#### **Deterioration Models**

Replication of collapse necessitates modeling of deterioration characteristics of structural components. The literature on this subject is extensive, (e.g., Kunnath [7] and Song [8]) but few simple deterioration models exist, and little systematic research on the effects of component deterioration on the collapse potential has been performed in the past. Refined component models that incorporate deterioration characteristics are being developed as part of the PEER OpenSees effort [9]. These models are detail-specific and cannot be employed for general sensitivity studies. Thus, a general deterioration model had to be developed as part of one of the PEER demand studies, Ibarra [10].

This hysteresis model reproduces all important modes of deterioration that are observed in experimental studies. An example of a monotonic load-displacement response and a superimposed cyclic response of "identical" plywood shear wall panels is illustrated in Fig. 2. The monotonic test result shows that strength is "capped" and is followed by a negative tangent stiffness. The cyclic hysteresis response indicates that the strength in large cycles deteriorates with the number and amplitude of cycles. Deteriorating models that replicate this behavior have been developed for bilinear, peak-oriented, and pinching hysteretic models. The monotonic backbone curve of these systems consists of an elastic branch, a strain hardening branch, a negative tangent (post-capping) stiffness branch, and in some cases a residual strength branch of zero slope (Fig. 3a). Cyclic deterioration are included: basic strength (1), post-capping strength (2), unloading stiffness (3), and accelerated reloading stiffness (4) deterioration (Figs. 2 and 3b). The deteriorating models have been incorporated in the analysis program DRAIN-2DX [11], and can be used to analyze deteriorating MDOF frames with concentrated plasticity.



Fig. 2. Experimental Load-Displacement Relationships for a Wood Shear Wall Panel Subjected to Monotonic and Cyclic Loading



**Fig. 3. Deterioration Modeling** 

In this research the term "deteriorating models" is used for MDOF structures having components that possess a post-capping stiffness branch in the backbone curve and/or are subjected to cyclic deterioration. The term "non-deteriorating" is used for systems with components that do not have a post-capping branch and that are not subjected to cyclic deterioration, although geometric nonlinearities (P- $\Delta$  effects) could be present.

#### Deterioration Based on Hysteretic Energy Dissipation

The cyclic deterioration rates are controlled by the rule developed by Rahnama [12], which is based on the hysteretic energy dissipated when the component is subjected to cyclic loading. It is assumed that the hysteretic energy dissipation capacity is a known quantity that is independent of the loading history. The cyclic deterioration in excursion *i* is defined by the parameter  $\beta_{i_i}$ 

 $\sim c$ 

$$\boldsymbol{\beta}_{i} = \left(\frac{E_{i}}{E_{t} - \sum_{j=1}^{i} E_{j}}\right)^{T}$$
(1)

where

 $E_i$  = hysteretic energy dissipated in excursion *i* 

- $\Sigma E_i =$  hysteretic energy dissipated in all previous excursions (both positive and negative)
- $E_t$  = hysteretic energy dissipation capacity,  $E_t = \gamma F_y \delta_y$ . The parameter  $\gamma$  is calibrated from experimental results and can be different for each deterioration parameter<sup>4</sup>.
- c = exponent defining the rate of deterioration of the hysteretic parameter (strength or stiffness). In this study, c = 1, which implies a linear rate of deterioration of the hysteretic parameter.

Throughout the time history analysis,  $\beta_i$  must be within the limits  $0 < \beta_i \le 1$ . If this inequality does not hold ( $\beta_i \le 0$  or  $\beta_i > 1$ ), the hysteretic energy capacity is exhausted and collapse takes place. Mathematically,

$$\gamma F_{y} \delta_{y} - \sum_{i=1}^{i} E_{i} < E_{i}$$
<sup>(2)</sup>

The parameters  $\gamma_i$  and  $\beta_i$  are individualized for the four modes of deterioration. For example, the unloading stiffness ( $K_u$ ) is deteriorated in accordance with the following equation,

$$K_{u,i} = (1 - \beta_{k,i}) K_{u,i-1}$$
(3)

where

 $K_{u,i}, K_{u,i-1}$  = deteriorated unloading stiffness for excursion *i* and *i-1*  $\beta_{k,i}$  is associated with an appropriate cyclic deterioration parameter  $\gamma_k$ .

Details of the deterioration model are presented in Ibarra [10].

#### Additional Observations on Deterioration Model

The versatility of this deterioration model makes it feasible to represent many deterioration modes in a transparent and physically justifiable manner. If all cyclic deterioration modes can be represented by the same  $\gamma$  value, and the exponent 'c' is taken as 1.0, then all cyclic deterioration modes are controlled by a single parameter,  $\gamma_{s,c,k,a}$ . There is no simpler way to describe cyclic deterioration.

<sup>&</sup>lt;sup>4</sup> The nomenclature for the cyclic deterioration parameter  $\gamma_{s,c,k,a}$  should not be confused with that of the base shear coefficient  $\gamma$ .

There are fundamental differences between this deterioration model and cumulative damage models. The latter merely count cumulative damage and use a counter to indicate degree of damage and complete "failure". They do not consider that cumulative damage causes a decrease in strength and stiffness and therefore lead to an increase in deformations. However, it is the loss of strength and the increase in deformation that ultimately will cause collapse of a structure. Moreover, cumulative damage models apply to components and not to structures. Many attempts are reported in the literature to extrapolate from component cumulative damage models to structure damage models, but none of these attempts are believed to have been successful in tracing damage close to collapse.

The proposed deterioration model incorporates cyclic deterioration controlled by hysteretic energy dissipation as well as deterioration of the backbone curve. This dual deterioration behavior is equivalent to the two-part damage concept of some cumulative damage models such as the Park-Ang model [13].

# GROUND MOTIONS AND SYSTEMS USED FOR PARAMETER STUDY

# **Selection of Ground Motions**

Time history analysis requires the specification of input GMs. In this study the MDOF systems are subjected to a set of 40 "ordinary" GMs recorded in California (LMSR-N, Large Magnitude Small Distance-New, Medina [3]). The records do not exhibit pulse-type near-fault characteristics and are recorded on stiff soil or soft rock, corresponding to soil type D according to NEHRP (FEMA-356, 2002). The source-to-site distance ranges from 13 to 40 km. and the moment magnitude from 6.5 to 6.9.

The selected intensity measure, IM, is the 5% linear elastic spectral acceleration at the fundamental period of the MDOF systems,  $S_a(T_I)$ . The use of  $S_a$  as IM implies that all the GMs are scaled to a common  $S_a$  at the fundamental period of the MDOF system. Thus, the frequency content of the GM is not considered explicitly.

# **Structural Systems**

The MDOF systems are two-dimensional regular generic frames of a single bay and 3, 6, 9, 12, 15 and 18 stories (Medina [3]). Their basic characteristics are the following:

- The fundamental period of the structure is associated with the number of stories, *N*. "Stiff frames" have a fundamental period  $T_1 = 0.1N$ , whereas "flexible frames" have  $T_1 = 0.2N$ .
- The same moment of inertia is assigned to the columns in a story and the beam above them.
- Relative element stiffnesses are tuned to obtain a straight line deflected shape for the first mode. Absolute stiffnesses are tuned to obtain the aforementioned periods.
- Plastic hinges are permitted to form only at the beam ends and the base of the columns. The columns are assumed infinitely strong at all other locations.
- The strength design of the frames is such that simultaneous yielding is attained at all plastic hinge locations under a parabolic load pattern (NEHRP, k = 2).
- The ratio of span to story height is 2.0
- The same mass is used at all floor levels
- The effect of gravity load moments on plastic hinge formation is not included
- Global P- $\Delta$  is included, whereas member P- $\delta$  is disregarded. The mass used to consider P- $\Delta$  effects is 1.4 times larger than the seismically effective mass.
- Axial deformations and P-M-V interaction are not considered
- For nonlinear dynamic analyses, 5% Rayleigh damping is assigned to the first mode and to the mode at which the cumulative mass participation exceeds 95%.

The frames include rotational springs at beam ends and column bases with deterioration in strength and stiffness. It is assumed that every plastic hinge in the structure can be described by the same hysteresis model. The main parameters of the model are:

- The strain hardening ratio,  $\alpha_s = 0.03$
- The ductility capacity  $\delta_c/\delta_y$ . Values of 2, 4, and 6 are used.
- The post-capping tangent stiffness ratio  $\alpha_c$ . Values of -0.1, -0.3, and -0.5 are used.
- The cyclic deterioration parameter  $\gamma = E_t/F_y \delta_y$ . Values for  $\gamma_{s,c,k,a}$  of  $\infty$  (no cyclic deterioration), 100, 50, and 25 are used.  $\gamma_{s,c,k,a}$  indicates the same  $\gamma$  for the four modes of deterioration.
- No residual strength  $(\lambda = 0)$

The following sections summarize the salient findings for generic frames. A reference frame with springs having a peak oriented hysteretic model,  $\delta_c/\delta_y = 4$ ,  $\alpha_c = -0.1$  and  $\gamma_{s,c,k,a} = \inf$  infinite is used as a starting point in the discussion of results. Also, a four-digit code is used to identify the generic frames. The first two digits correspond to the number of stories and the other two to the first mode period. For instance, 0918 means a 9-story frame with  $T_I = 1.8$  s.

# **RESULTS OF PARAMETER STUDY**

#### **Global Pushover Curves for Deteriorating Systems**

The pushover method estimates force and deformation demands using a static incremental, inelastic analysis. In this study, the method is used to evaluate the behavior of deteriorating systems. Fig. 4 presents a comparison of global pushover curves for the non-deteriorating and the deteriorating reference frame 0909 when a parabolic lateral load pattern (NEHRP, k = 2) is applied to the structure. For both systems the global strain hardening stiffness is larger than in the beam springs ( $\alpha_s = 0.04$  vs. 0.03). For the deteriorating system the global "ductility capacity" (drift at onset of deterioration divided by yield drift) is smaller (3.0 vs. 4.0) and the global post-capping stiffness is much steeper ( $\alpha_c = -0.34$  vs. -0.10) than in the beam springs. Also, for large deformations there is a third post-yielding branch that indicates an apparent "recovery" in the stiffness of the global pushover curve.



Fig. 4. Global Pushover Curves for the Non-Deteriorating and Deteriorating Ref. Frame 0909

A major contributor to the differences between the properties of the global pushover curve and those of the springs is the fact that columns remain elastic throughout the pushover analysis. After the simultaneous yielding of the beam springs in the pushover analysis (because of the assumptions discussed previously), the incremental displacements in the global pushover curve contain inelastic deformations from beams and elastic deformations from columns. These elastic deformations in columns account for the increase in the effective  $\alpha_s$ , part of the increase in  $\alpha_c$ , and the decrease in  $\delta_c/\delta_y$  of the global pushover curve.

Several studies have shown that the story drifts in the lower stories are amplified when P- $\Delta$  effects are important (i.e., when they lead to a negative story tangent stiffness), see Medina [3]. This results in highly nonlinear deflected shapes with very large drifts in the lower stories and possibly unloading (decrease in story drifts) in the upper stories. Post-capping strength deterioration has a similar effect as P- $\Delta$ . Fig. 5 shows deflected shapes associated with the roof drifts indicated with dotted vertical lines in Fig. 4. Without post-capping strength deterioration, the deflected shapes remain essentially linear (Fig. 5a) because P- $\Delta$  effects are small for this frame. When strength deterioration is included, great amplifications of drifts in the lower stories are observed in the deflection profiles (Fig. 5b). This radical change in deflected shape occurs when the loading path is on the descending branch of the pushover curve. This change in deflective  $\alpha_c = -0.34$  seen in the global pushover. The same phenomenon also accounts for the "recovery" portion of the pushover curve at very large drifts. This "recovery" portion comes from the fact that the lower portion of the frame has undergone very large horizontal displacements and the global bending mode (i.e., the cantilever bending mode) of the infinitely strong columns starts to dominate over the effect of frame action.



Fig. 5. Deflected Shapes From Pushover Analyses for Frame 0909

The column moments can become very large compared to the value when the beam reaches its bending strength (Ibarra [10]). The implication is that the strong column – weak beam factor would have to be very large (larger than 3.0) in order to avoid column plastic hinges at very large lateral displacements.

## **Collapse Capacity for the Set of Reference Frames**

Fig. 6a shows median and 16<sup>th</sup> percentile collapse capacity spectra for the set of reference frames. Collapse capacities are grouped in stiff ( $T_I = 0.1N$ ) and flexible ( $T_I = 0.2N$ ) frames. The collapse capacity strongly depends on the first mode period  $T_I$ . This is expected in the short period range, but the large decrease in collapse capacity for long period structures is striking, indicating that the "period independent *R*-factor concept" may not be appropriate for long period structures. The reason is that P- $\Delta$  effect is more important than might be expected. The large effect of P- $\Delta$  is also exposed in Fig. 6b, which shows collapse capacities versus the number of stories for flexible and stiff frames. Except for short period frames, collapse capacities of stiff frames are larger (especially at long periods) because P- $\Delta$  effects are smaller for these frames. The trend is reversed for the 3-story frames because  $P-\Delta$  effects are small and the displacements are more sensitive to the strength of the system, causing the structure with shorter fundamental period to have the smaller collapse capacity.



Fig. 6. Median and 16<sup>th</sup> Percentile Collapse Capacities for Set of Reference Frames

# **Effect of Post-Capping Stiffness**

The effect of the post-capping stiffness is isolated in Fig. 7, which shows median collapse capacities for similar frames but with different  $\alpha_c$  values. There is a large difference between the collapse capacities for systems with  $\alpha_c = -0.1$  and -0.3. However, any further increase in post-capping slope has a small effect because the component reaches zero strength soon after  $\delta_c$  is reached. Fig. 7b presents the collapse capacity ratios with respect to the most ductile system. The larger differences arise in the medium period range, where ratios smaller than 0.7 are reported.

Fig. 7c shows the dispersion in collapse capacity due to RTR variability in terms of the standard deviation of the log of  $[S_{a,c}(T_1)/g]/\gamma$ . The dispersion is not sensitive to the fundamental period of the frame or changes in the  $\alpha_c$  value of the springs of the reference frame. The same conclusion is obtained when the ductility capacity and the cyclic deterioration parameters are modified.

## Effect of Ductility Capacity on Collapse Capacity

The effect of this parameter is illustrated in Fig. 8 for systems with  $\alpha_c = -0.10$ . The effect of  $\delta_c/\delta_y$  is significant and, as observed in the collapse capacity ratios of Fig. 8b, essentially independent of the fundamental period of the structure or the number of stories of the frame. Fig. 9 presents the same information for systems with  $\alpha_c = -0.30$ . Although the collapse capacity decreases for all the systems, compared to  $\alpha_c = -0.10$ , the effect of ductility capacity now is larger because the strength capacity deteriorates faster after the cap displacement  $\delta_c$  is reached, i.e., collapse occurs soon after  $\delta_c$  has been reached. This illustrates that any one parameter cannot be evaluated independently of the others.



a) Median Collapse Capacity Spectra



b) Median Ratios of Col. Capacity Spectra



Fig. 7. Effect of  $\alpha_c$  of Springs on Collapse Capacity of Generic Frames,  $\delta_c/\delta_y = 4$ ,  $\gamma_{s,c,k,a} = \text{Inf}$ 



a) Median Collapse Capacity Spectra

b) Median Ratios of Collapse Capacity Spectra

Fig. 8. Effect of  $\delta_c/\delta_y$  of Springs on Collapse Capacity of Generic Frames,  $\alpha_c = -0.1$ ,  $\gamma_{s,c,k,a} =$  Infinite





# Effect of Cyclic Deterioration on Collapse Capacity

The effect of CD on collapse capacity is shown in Fig. 10. The effect is evident, although not overpowering, indicating that the combination of ductility capacity and post-capping stiffness is more important than the CD effect. The effect diminishes for long period structures because of the dominant importance of  $P-\Delta$  effects.



![](_page_10_Figure_5.jpeg)

## Effect of Hysteretic Models on Collapse Capacity

Fig. 11 shows collapse capacity spectra for frames with peak-oriented, pinching and bilinear models in the plastic hinge springs. Collapse capacities for the frames with peak oriented and pinching models are similar. On the other hand, for MDOF systems with a short fundamental period the collapse capacity of frames with bilinear springs is slightly larger than that of frames with peak oriented or pinching springs, whereas for frames with medium and long fundamental periods the trend reverses because the branch with negative slope in the plastic hinge springs has a larger "ratcheting" effect in bilinear models (Ibarra [10]).

![](_page_11_Figure_0.jpeg)

Fig. 11. Effect of Hysteretic Models on Median Collapse Capacity of Generic Frames

#### COLLAPSE FRAGILITY CURVES AND MEAN ANNUAL FREQUENCY OF COLLAPSE

In seismic performance assessment, collapse constitutes one of several limit states of interest. Collapse contributes to the cost of damage, if monetary losses or downtime are the performance targets. More important, collapse is the main contributor to casualties and loss of lives. To assess the contributions of collapse to these performance levels, it is necessary to quantify collapse in a probabilistic format. This can be achieved by means of collapse fragility curves (FCs), which can be derived from the collapse capacity data, and the computation of a mean annual frequency (MAF) of collapse.

#### **Collapse Fragility Curves (FCs)**

A fragility function for a limit state expresses the conditional probability of exceeding the limit state capacity for a given level of GM intensity. In this study, the limit state of collapse is evaluated by using the spectral acceleration at the fundamental period of the system as the GM intensity. Thus, the FC for these conditions is,

$$F_{C,S_{a,c}}(x) = P[S_a \ge S_{a,c} | S_a = x] = P[S_{a,c} \le x]$$
(4)

 $F_{C,S_{a,c}}(x)$  corresponds to the value of the FC at spectral acceleration, *x*, for the limit state of collapse, i.e., the "collapse fragility curve". By considering that the demand  $(S_a = x)$  is statistically independent of the capacity of the system  $(S_{a,c})$ , the FC can be expressed as the probability that  $S_{a,c}$  is less than or equal to *x*. The collapse FC can be viewed also as the cumulative distribution function (CDF) of a random variable, the collapse capacity,  $S_{a,c}$ . Data of the type shown in Fig. 1 can be utilized to develop FCs.

In earlier sections, collapse capacity is used as the parameter for collapse evaluation. Thus, "normalized collapse fragility curves" can be generated by using this parameter instead of the GM intensity,

$$F_{C,(S_{a,c}/g)/\gamma}(x) = P[(S_{a,c}(T_1)/g)/\gamma \le x]$$
(5)

Figure 12 presents normalized FCs for reference frames. The curves show a pattern equivalent to that exhibited in the median collapse capacity spectra of MDOF systems (Fig. 6), of high fragility (small collapse capacity) for short period structures ( $T_1 = 0.3$  s.), a large decrease in the fragility for medium period structures ( $T_1 = 0.6$  and 0.9 s.), and then an increase in fragility for long period structures ( $T_1 = 1.8$  and 3.6 s.) because of the predominance of P- $\Delta$  effects.

![](_page_12_Figure_0.jpeg)

Fig. 12. Normalized Fragility Curves for Reference Frame Structures

The collapse fragility curves of Fig. 12 are obtained by fitting a lognormal distribution to the collapse capacity data, which facilitates numerical and analytical calculations. The lognormal distribution is a logical selection because most of the individual collapse capacity data has a skewed distribution with a longer tail for higher values and because previous studies have associated the distribution of spectral acceleration and the response of a nonlinear structure (in terms of EDPs) to lognormal distributions.

In concept, all observations made for median collapse capacities hold true for the FCs. The value of these curves lies in their probabilistic nature that permits probabilistic expressions of performance and design decisions. For instance, if for a given long return period hazard (e.g., 2/50 hazard) a 10% probability of collapse could be tolerated, then the intersections of a horizontal line at a probability of 0.1 with the individual FCs provides targets for the *R*-factor that should be used in design. In addition, FCs are employed for computing in a rigorous manner the mean annual frequency of collapse.

## Mean Annual Frequency (MAF) of Collapse

If the collapse fragility curve for a given system has been determined, probabilistic collapse assessment can be carried out according to the following equation:

$$\lambda_f = \int F_{C_{Sa}}(x) | d\lambda_{S_a}(x) |$$
(6)

where

 $\lambda_f$  = mean annual frequency of collapse

 $F_{C_{S_a}}(x)$  = probability of the  $S_a$  capacity,  $S_{a,c}$ , (for a given  $\gamma$  or  $\eta$  value) exceeding x

 $\lambda_{Sa}(x)$  = mean annual frequency of  $S_a$  exceeding x (ground motion hazard)

Given the  $S_a$  hazard curve and fragility curves, the mean annual frequency of collapse can be computed by numerical integration. In this context, the structure strength parameter ( $\gamma$ ) is kept constant, i.e., the individual curves shown in Fig. 1 represent IDAs.

To provide an illustration of typical results, equal hazard spectra employed in PEER studies for a Los Angeles building (Fig. 13) are used for developing approximate  $S_a$  hazard curves for various periods. These hazard curves are combined with FCs developed for reference frames to obtain the MAF of collapse for various periods and selected strength levels  $\gamma$ . The resulting MAF curves (Fig. 14) illustrate general trends and permit a global quantification of the effect of several parameters on the MAF of

collapse. The MAF of collapse can increase by more than one order of magnitude due to the strength characteristics of the system. Also, there is a clear dependence of the MAF of collapse on  $T_1$ . However, for long period MDOF frames, the beneficial effects of smaller  $S_a$  values at a given hazard diminishes drastically. The cause is the P- $\Delta$  effect, which dominates the response of long period structures.

![](_page_13_Figure_1.jpeg)

Los Angeles (Somerville [14])

![](_page_13_Figure_3.jpeg)

## CONCLUSIONS

In MDOF systems incremental collapse is associated with a state (not necessarily a "mechanism" in a classical sense) in which P- $\Delta$  effects become equal to the deteriorated first-order story-shear resistance provided by the structural elements. Collapse may occur in a single story or in a series of stories. The use of deteriorating models permits simulation of the response until collapse occurs and redistribution of moments to less damaged components. To obtain a consistent evaluation of collapse capacity, it is assumed that every plastic hinge of the generic frames has the same deteriorating hysteretic model. The main findings are summarized below:

- The developed deteriorating models combine a cyclic deterioration parameter with a backbone curve that incorporates strength deterioration, which makes this model simple but versatile
- Material deterioration may produce highly nonlinear deflected shapes with very large drifts in the lower stories. This pattern is similar to that caused by large P- $\Delta$  effects
- The properties of the global pushover curves of deteriorating systems may be very different from those of the nonlinear springs at the end of the beams. This behavior is due to the combined effect of material and geometric nonlinearities
- The interdependence of the various system parameters must be kept in mind when the collapse capacity results are interpreted
- The collapse capacity strongly depends on the first-mode period  $(T_i)$ . The period-independent *R*-factor concept is not appropriate due to the large importance of P- $\Delta$  effects in the inelastic range
- The post-capping stiffness and the ductility capacity of the rotational springs are the two deterioration parameters that most influence the collapse capacity of MDOF systems. Cyclic deterioration appears to be an important but not dominant issue for collapse evaluation
- The collapse capacity of deteriorating frames is similar when a pinching model is used for the plastic hinge springs at the beam ends instead of a peak-oriented model. Except for short period structures, the collapse capacity of frames with bilinear springs is smaller than that of frames with peak oriented

• The evaluation of global collapse in terms of the relative intensity facilitates the computation of the mean annual frequency of collapse, which is obtained by integrating the collapse fragility curve for a given base shear strength over a spectral acceleration hazard curve pertaining to a specific site

An important aspect of this work is the development of a transparent methodology for the evaluation of incremental collapse, in which the assessment of collapse is closely related with the physical phenomena that lead to this limit state. The methodology addresses the fact that collapse is caused by deterioration in complex assemblies of structural components that should be modeled explicitly.

## ACKNOWLEDGEMENTS

This research was supported by the Pacific Earthquake Engineering Research (PEER) Center, an Engineering Research Center sponsored by the US National Science Foundation. The studies of the first author were mainly supported by the Consejo Nacional de Ciencia y Tecnologia (CONACYT) and complemented by funds from PEER. This support is much appreciated.

# REFERENCES

- 1. Bernal, D. "Instability of buildings subjected to earthquakes," *Journal of Structural Engineering*, 118, 8, pages 2239-2260, 1992
- 2. Gupta, A., Krawinkler, H. "Seismic demands for performance evaluation of steel moment resting frame structures." Report 132. John A. Blume EERC, Stanford University, 1999
- 3. Medina R. "Seismic demands for nondeteriorating frame structures and their dependence on ground motions." Ph.D. Dissertation, Department of CEE. Stanford: Stanford University, 2002
- 4. Lee, K. and Foutch, D. A. "Performance evaluation of new steel frame buildings for seismic loads," *Earthquake Engineering and Structural Dynamics*, 31, pages 653-670, 2001
- 5. Vamvatsikos, D. and Cornell, C. A. "Incremental Dynamic Analysis," *Earthquake Engineering and Structural Dynamics*, 31, 3, pages 491-514, 2002
- 6. Jalayer, F. "Direct probabilistic seismic analysis: implementing nonlinear dynamic assessments," *PhD. Dissertation*, Department of Civil Engineering, Stanford University, 2003
- 7. Kunnath, S. K., El-Bahy, A., Taylor, A. and Stone, W. "Cumulative Seismic Damage of Reinforced Concrete Bridge Piers," *Technical Report NCEER-97-006*, Sep. 1997
- 8. Song, J. and Pincheira, J. "Spectral displacement demands of stiffness and strength degrading systems," *Earthquake Spectra*, 16, 4, pages 817-851, 2000
- 9. OpenSees, Open System for Earthquake Engineering Simulation, *Pacific Earthquake Engineering Research Center*, <u>http://peer.berkeley.edu/</u>, 2002
- 10. Ibarra, L.F. "Global collapse of frame structures under seismic excitations." Ph.D. Dissertation, Department of Civil Engineering, Stanford University, 2003
- 11. DRAIN-2DX: Basic program description and user guide," Report No. UCB/SEMM-1993/17, by Prakash, V., Powell, G. H., and Campbell, S., University of California, Berkeley, CA, Nov. 1993
- 12. Rahnama, M. and Krawinkler, H. "Effect of soft soils and hysteresis models on seismic design spectra," *John A. Blume EERC*, Report No. 108, Stanford University, 1993
- 13. Park, Y. and Ang, A. "Mechanistic seismic damage model for reinforced concrete," *Journal of Structural Engineering*, 111, 4, Apr. 1985, pages 722-739, 1985
- 14. Somerville, P. and Collins, N. "Ground motion time histories for the Van Nuys building," *PEER Methodology Testbeds Project*, URS Corporation, Pasadena, CA. March 7, 2002.