

APPLICATION OF HEURISTIC SEARCH METHODS TO PHASE VELOCITY INVERSION IN MICROTREMOR ARRAY EXPLORATION

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SUMMARY

Microtremor array exploration has been recognized as one of powerful techniques to know S-wave profile of sedimentary layers. In the exploration, phase velocity of Rayleigh wave is estimated from array records of vertical microtremors. The phase velocity is inverted to an S-wave profile using optimization techniques. Originally, least square methods developed in seismological community were applied in the inversion. Recently Genetic Algorithms which are regarded as one of the heuristic search methods are tried to be used in inverting phase velocity from microtremor exploration, because of robustness of the algorithms.

In this study we compared performances of heuristic optimization techniques in phase velocity inversion. The heuristic methods examined are Genetic Algorithms, Simulated Annealing, and Tabu Search. We inverted synthetic phase velocity in numerical experiments. The Genetic Algorithms and Simulated Annealing show the rapid convergence of the misfits. However, the Simulated Annealing can find models with the smallest misfits. We also found the same conclusions in application of the methods to actual data.

INTRODUCTION

Assessment of local site effects is one of the most important subjects in engineering seismology. There are many kinds of techniques to the assessment. Probably, the most inexpensive technique is a method using microtremors. Applications of microtremors in estimating site effects can be mainly classified into two approaches. One is methods to estimate directly site effects or ground motion characteristics, such as predominant period, spectral ratio and horizontal-to-vertical ratio. Although these techniques are easy in field operation and data processing, we still have some uncertainty in results or interpretation of data. The second approach is a microtremor array technique. In the technique, an S-wave profile is estimated from an inversion of Rayleigh wave phase velocity obtained from an analysis of array data of vertical microtremors (Horike [1], Okada et al. [2]). Once an S-wave profile is deduced, we can estimate site effects with various numerical simulations. Field operation and data processing are much more complex

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than that of the direct estimation. However, there is no ambiguity in the approach and allow us a quantitative evaluation of site effects and its accuracy.

Estimation of phase velocity from array data and its inversion to an S-wave profile are the two major tasks in the microtremor array exploration. Array data analyses using frequency-wavenumber spectrum and spatial autocorrelation function are often used in estimation of phase velocity (Okada et al [2]). Inversions of surface wave phase velocity are one of the most common techniques for a velocity profile in seismology. Least square methods are usually used to invert observed phase velocity to a 1D profile of Swave velocity. However, we have some difficulties in practical applications of linearized least square inversions of phase velocity. Numerical instability in calculating an inverse matrix is one of the difficulties in inversions. Preparation of an appropriate initial is also one of the difficulties in practical domain. Since the misfit function defined as L2-norm of difference between observed and calculated phase velocities is non linear, the misfit surface is multi-modal in parameter space. This arises the dependency of inverted results on initial model assumed. When an inappropriate initial model is used, we cannot estimate an S-wave profile with the global minimum of the misfit function.

In this study, we apply heuristic search methods in an inversion of Rayleigh-wave phase velocity estimated from analysis of microtremor array data. In particular, we pay our attention to phase velocity inversion for S-wave profile of deep sedimentary layers. From numerical experiments and application to actual data, convergence speeds of the misfits between phase velocities for the true and inverted models are discussed.

PROBLEM SETTING

Phase velocity of fundamental mode of Rayleigh wave is assumed in the inversion. We calculate phase velocity for a horizontally layered model using the method by Haskell [3]. The misfit E(m) for the model, m, is calculated from

$$E(\mathbf{m}) = \frac{1}{N} \sum_{j=1}^{N} \left| \frac{C^{c}(t_{j}) - C^{o}(t_{j})}{\sigma(t_{j})} \right|$$
(1)

where $C^{c}(t_{j})$ and $C^{o}(t_{j})$ are calculated and observed phase velocities at a period of t_{j} . $\sigma(t_{j})$ and N are the standard deviation and number of the observed phase velocity. We choose S-wave velocity and thickness of each layer as the model parameter, m. P-wave velocity is calculated from that of S-wave using an empirical equation by Kitsunezaki et al [4]. Density is fixed and given before the inversion. The number of the model parameters is 2M-1 for M-layer model. The problem is simply to find model parameters with the minimum misfit in given search areas of the parameters.

HEURISTIC SEARCH METHODS

A heuristic search method is one of optimization methods that can find models near the global minimum solution with reasonable computational costs (Reeves [5]). The major advantages of the heuristic search methods are no requirements of calculation of derivatives of misfit functions and specific initial models. The heuristic search methods have been applied in many engineering optimal designs since 1980. Recently, the methods were examined in geophysical inversions (e.g., Sen et al. [6]). The heuristic search methods that we examined are Genetic Algorithms (GA), Simulated Annealing (SA), and Tabu Search (TS).

Simulated Annealing

The SA is based on the idea of thermodynamics where melted metal reaches to low-energy state with gradual decrease of temperature (Metropolis [7]). Kirkpartrick et al [8] applied the idea to optimization problems with an analogy between the thermodynamics and optimization as shown in Table 1. The misfit to be minimized in inversion corresponds to energy in thermodynamics, and parameter change does to move of material state. This move of parameters is controlled by cooling schedule of the system with temperature decrease.

Table 1 Analogy between thermodynamics and optimization		
Thermodynamics	optimization	
Material state	Possible model	
Energy	Objective function	
Change of material state	Move to neighbor model	
Temperature	Control parameter	
Freezing state	Heuristic solution	

The algorithm of the used SA is shown in Fig.1. First, we define a cooling schedule and an initial temperature, T_0 . Search areas for all the unknown parameters are also defined before calculation. Then, an initial model, m_0 is randomly generated within the defined parameter spaces. The misfit, $E(m_0)$ for the initial model is calculated using the equation (1). Next, we add a random perturbation to the initial model for generation of a neighbor solution, m_1 . We again calculate the misfit, $E(m_1)$ for the neighbor model. If the difference of the misfits of the two models, $\Delta E = E(\mathbf{m}_1) - E(\mathbf{m}_0)$, is negative, \mathbf{m}_1 becomes the present model. If the difference is positive (m_1 is worse model), m_1 is still chosen as the present model with a probability $P = \exp(-\Delta E/T)$. Because of the temperature-dependent probability, a model with high misfit is frequently chosen at high temperature. At low temperature, bad model is not often selected and only good model becomes the present model. After these processes have been repeated for all the parameters at predetermined times, temperature is decreased according to the cooling schedule. The present model can be modified to the neighbor model near the global solution by repeating the above processes. Although the SA is one of local search methods using a perturbation of the model parameters, the SA works as a global search method at high temperature, because it allow to climb up a hill of the misfit surface. It, however, works as a local search method at low temperature. This feature is different from pure random search methods, such as the Monte Carlo search method.

There are several algorithms in the SA. We examined the Metropolis algorithm (Metropolis [7]) and the Very Fast Simulated Annealing (Ingber [9]) in this paper. In the Metropolis algorithm, a new model, m_1 , is generated from a perturbation of the i-th parameter of the present model, m_0 , that is

$$m_1^i = m_0^i + u_i \left(m_{\max}^i - m_{\min}^i \right)$$
(4)

where u_i is a random number with uniform distribution between 0 and 1, and m_{max}^i and m_{min}^i are the predetermined upper and lower limits of the search area for the i-th parameter. The cooling schedule used are defined using

$$T_k = \frac{T_0}{\ln(k+2)} \tag{5}$$

where T_k is the temperature at the k-th iteration.

In the Very Fast Simulated Annealing (VFSA in the following), a perturbation is also generated using equation (4). However, u_i in the equation is replaced by y_i that is defined as

$$y_i = \operatorname{sgn}(u_i - 0.5)T_k \left[\left(1 + \frac{1}{T_k} \right)^{|2u_i - 1|} - 1 \right].$$
(6)

The cooling schedule is based on

$$T_k = T_0 \exp\left(-ck^a\right) \tag{7}$$

where a, c are constant.



Fig.1 Algorithm of inversion based on Simulated Annealing

Genetic Algorithms

The GAs are simulation algorithms based on evolution strategy in biology. The algorithms were developed to study artificial intelligence by Holland [10]. Applications of GAs in many optimization problems were examined by Goldberg [11]. Details of the application of GA to inversion of surface wave phase velocity can be explained by Yamanaka and Ishida [12]. In this study, we used a binary-code GA, where each parameter in defined search area is digitized using a binary code. A model is expressed with a sequence of 0 or 1 (gene-type) by connecting all binary parameters. We call this bit string chromosome. First, L models in the genetic type are randomly generated for the initial population. The three genetic operations that are called selection, crossover, and mutation, are applied to the gene-type parameters of the models in the initial population. In the selection, new L models in the next generation are chosen using the probability determined with misfits for the models in present generation. The probability to be chosen in the next generation for the i-th model in the present generation is given from

$$P_i = \frac{E(m_i)}{\sum_{k=1}^{L} E(m_k)}$$
(2).

The models chosen in the selection are applied to crossover. Two models which are randomly selected exchange their bit strings to generate two new models. An example of calculation of the crossover is shown in Fig.3. The location of cutting the bit strings is chosen using a random number. Then, the models after the crossover are modified in the mutation where a randomly chosen bit is reversely changed as shown in Fig.3. Mutation works as a local or global search according to the occurrence location of bit reverse. Although the crossover and mutation might generate models with large misfits, such models have low possibility to be chosen in the selection in the next generation. The calculation is terminated at a predetermined generation.



Fig.2 Algorithm of inversion based on genetic algorithms Fig.3 Example of crossover and mutation

Tabu Search

The TS is a kind of iterative local search methods using an idea of flexible memory (Glover, [13]). The method is not well known in seismology and earthquake engineering, though it is regarded as one of major approaches in heuristics. The algorithm of the used Tabu search method is shown in Fig.4. Model space for each parameter is equally divided in N_t possible values. The initial model is randomly generated and its misfit is calculated. Neighbor models are generated with moves of all the parameters in increase and decrease directions. Namely, misfits for 2(2M-1) neighbor models are calculated for choosing a move to a new model. We select a model with the minimum misfit among the neighbor models. If the attribute from the present model to the minimum model is not in the Tabu list, this model is accepted as the present model. If not, the minimum neighbor model is not used to renew the present model. Instead, the 2nd minimum model is compared with the Tabu list. The 3rd minimum neighbor model is examined, when the 2^{nd} model is also in the Tabu list. The attribute used here is defined by move and parameter. For example, the attribute in the Tabu list (30,5) means the 30th value of the 5th parameter. The attributes for the selected neighbor model are stored in the Tabu list whose memory is defined by Tabu length. Therefore, attributes for only recent moves are memorized in the Tabu list, and attributes for old moves are forgotten. The Tabu list can avoid cycling near the local minimum model. We used two additional operations as shown in Fig.4. We select a model that is listed in the Tabu list, if the aspiration rule is satisfied. We make the aspiration rule on, when a model has the minimum misfit among the models that have been visited before. Another operation which is included in our algorithm is the re-active rule. When the misfits are not improved in certain moves, we randomly generate a new starting model and clear the Tabu list. The calculation is terminated, when predetermined number of moves is executed.



Fig.4 Algorithm of inversion based on Tabu Search

NUMERICAL EXPERIMENTS

The performance of the above-mentioned heuristic search methods in the phase velocity inversion is examined in numerical experiments. A four-layer model is used in the numerical experiments to generate synthetic data of phase velocity. Table 2 shows the model assumed. Since we are, here, interested in a microtremor array exploration of deep sedimentary layers, the synthetic phase velocity in a period range form 0.5 to 8 seconds are used in the experiments. We used the noise-free synthetic phase and the standard deviation in equation (1) is neglected in calculation of misfit. The search areas for the unknown parameters (Vs and H) are shown in Table 2.

Tuble 2 Subsulface substantial model used in numerical test				
No	Vp(km/s)	Vs(km/s)	H(km)	$\rho(g/cm3)$
1	1.96	0.6	0.4	1.8
2	2.40	1.0	0.5	2.0
3	2.96	1.5	0.6	2.3
4	4.84	3.2	-	2.5

Table 2 Subsurface structural model used in numerical test

Table 3 Search areas in numerical test				
No	Vs(km/s)	H(km)		
1	0.4-0.9	0.2-1.0		
2	0.7-1.3	0.2-1.0		
3	1.2-1.8	0.2-1.0		
4	2.6-3.6	-		

Results of SA inversion

First results of the inversion based on the VFSA are shown. From trial execution of the programs with small number of iterations, we determined the parameters ($T_0=1.0$, a=0.5, c=1.0) in the cooling schedule of the VFSA in equation (7). The temperature decrease is shown in Fig.5. We examined 5 iterations at each temperature. Therefore, 35 neighbor models are examined at one temperature, because we have 7 unknown parameters. The variations of the minimum misfit and the misfit for the present model are also shown in the figure. Models with large misfits are often accepted as the current model at high temperature. However, models with small misfits are chosen at low temperature, when the number of examined models exceeds over 1000. This figure clearly shows the temperature-dependent features of the SA in the inversion. The variations of the unknown parameters are shown in Figs.6a and 6b. Similar to the variations of the misfits, the parameters reach to the true ones beyond 1000 moves. Since we used many random numbers in the calculation of the SA inversion, the results of the inversion more or less depend on random numbers. Therefore, 10 inversions were conducted with different initial numbers of a random number generator. Fig. 7 shows the variations of the misfits in the 10 inversions with different initial numbers of the random number generator. Although the convergence speed is slow in some of the results, most of the results show a similar decay of the misfit against the number of model moves. The variations of average and individual misfits for the SA inversion based on the Metropolis algorithms are shown in Fig.8. The average misfit quickly decreases within the first 200 iterations, and converges at 500 iterations. As compared with the variation of the misfits for the VFSA inversion, most of the VFSA inversions exhibit the better performance in finding small misfit model than that of the Metropolis algorithms.



Fig.5 Variations of misfits of current and minimum models in a VFSA inversion of synthetic phase velocity data. Crosses and solid line show current and minimum misfits and dotted line does temperature.



Fig.6 Variations of a) S-wave velocity and b) depth for each layer of models accepted in a VFSA inversion of synthetic phase velocity data.



Fig.7 Plot of misfits of individual and average minimum models in 10 VFSA inversions of synthetic phase velocity data with different initial values in random number generators.



Fig.8 Plot of misfits of individual and average minimum models in 10 metropolis SA inversions of synthetic phase velocity data with different initial values in random number generators.

Results of GA inversion

The search area for each unknown parameter is linearly divided with an 8-bit binary code. Therefore, total length of the bit string (chromosome) is 56 bits for a model. The parameters in the GA inversion are determined with a few test run of the program. We used 20 models for the population. Probabilities for crossover and mutation were set to be 0.7 and 0.01, respectively. This mutation probability means the occurrence of the mutation every two model. The calculations were terminated at the 300th generation.

Fig.9 shows the variation of the minimum misfit in a GA inversion of the synthetic data. Since the misfits for 20 models are calculated in each generation, the averaged misfit of the 20 models is also shown in the figure. The misfits rapidly decrease within the first 20 generations. Then, the minimum misfit is not significantly reduced, while the average misfit is fluctuated. Probably this can be due to bad models generated in mutation and crossover operations. The model parameters obtained in some of the generations are shown in Fig.10. The parameters are randomly distributed at the initial generation. They are concentrated in a few clusters after the 25th generation. Because of the mutation and crossover, jump of the parameters can be seen in the distribution of the parameters in the later generations. We conducted 10 GA inversions with different initial values of the random number generator. The average misfit and individual misfits for the inversions are shown in Fig.11.



Fig.9 Variation of the minimum and average misfits with increasing generation in a GA inversion of synthetic data.



Fig.10 Distributions of S-wave velocity and thickness of the models examined in GA inversion of synthetic data at initial, 2nd, 10th, 25th, 50th, and 200th generations..



Fig.11 Plot of misfits of individual and average minimum models for 10 GA inversions of synthetic phase velocity data with different initial values in random number generators.

Results of TS inversion

We conducted test executions of the program to determine the length of memory in the Tabu list, and found that the best choice of the Tabu length is 10 for this numerical experiment. The variation of the current and the minimum misfits are shown in Fig. 12. The misfits rapidly decrease in the first 100 moves. Beyond the 100th iteration, models with large misfits are also chosen, because of the Tabu criteria. It is notes that models with the smaller misfit can be found after selection of the worse models. This indicates the effectiveness of the Tabu list in global search algorithms. The parameter moves for the current and minimum models in the TS inversion are displayed in Fig.13. The S-wave velocity and thickness for the first layer smoothly approach near the true values, while those for the second layer start at a location far from the true ones and jump to the best model. The parameters for the third layer travel in a long path of the model space to reach to the best location. In particular, the moves for the thickness of the third layer distribute in wider space than that for the S-wave velocity. This suggests that the thickness of the third layer is the most sensitive in the misfit. The misfits for 10 TS inversions using different initial values of the random number generator are shown in Fig.14 together with their average misfit. Similar to the above results, the convergence for the some of the inversions are not well. This again indicates needs of several runs with different random numbers for stable results.



Fig.12 Variations of misfits of current and minimum models in a TS inversion of synthetic phase velocity data. Dotted and solid lines show current and the minimum misfits.



Fig.13 Variations of S-wave velocity and thickness for current (left) and the minimum (right) models in TS inversion for synthetic phase velocity



Fig.14 Plot of misfits of individual and average minimum models for 10 TS inversions of synthetic phase velocity data with different initial values in random number generators.

Comparison of results

The convergence speed of the misfits from the above inversions based on three heuristic search algorithms is compared with each other. Fig.15 shows the comparison of the average misfits for the 10 inversions based on the above algorithms. It is noted that the horizontal axis of the figure is the number of the examined models, because the GA inversion uses plural models at one time. We also depict the misfit for inversions using the Monte Carlo algorithm in the figure. At early stage of the searches, the GA and VFSA show a good convergence, and that for the Tabu search is worse than Monte Carlo method. However, the TS can find better models which have similar misfits as the GA finds, when the number of examined models exceeds over 1000 models. In the further iterations, only the VFSA is successful to find models with the smaller misfits. We also compare the variation of the misfits for the individual inversions in each algorithm. Fig.16 shows the standard deviation of the misfits for the algorithms, indicating the high stability of the results.



Fig.15 Comparison between misfits for genetic algorithms (GA), very fast simulated annealing (VFSA), Tabu search (TABU) and Monte Carlo (MC) methods.

Fig.16 Comparison between standard deviation of misfits for genetic algorithms (GA), very fast simulated annealing (VFSA), Tabu search (TABU) and Monte Carlo (MC) methods.

APPLICATION TO ACTUAL DATA

We applied the above inversion methods to actual phase velocity data obtained in the Kanto basin, Japan. The phase velocity data obtained at ASO in the western part of the basin are used. The location of the site is shown in Fig.17. The phase velocities were estimated from a frequency-wavenumber spectral analysis of array data of vertical microtremors by Yamanaka et al. [14] as shown in Fig, 18. The parameters used in each inversion are the same as the above numerical experiments.

The results of the 10 individual inversions based on each method are averaged to get final results. The misfits and their standard deviations for the GA, VFSA and TS inversions are shown in Fig.19. As expected from the numerical experiments, the GA and VFSA exhibit a similar good performance at early stage of iterations. However, only the VFSA can continue to search models with small misfit, because the VFSA has global and local search ability according to temperature decrease. Probably, the premature

convergence is one of the reasons for less performance of the GA (e.g., Goldberg, [10]). The high stability of the results of the VFSA can be also confirmed in the application to the actual data as shown in Fig.19. The inverted models from the inversions are displayed in Fig.20. The models inverted with the GA and VFSA are similar to each other. The model from the TS has a shallower basement depth than the others. The theoretical Rayleigh wave phase velocities for the models are also compared with the observation in Fig. 18. The theoretical phase velocity for the model from the TS is larger than the observed ones at period of longer than 3.5 seconds. This difference makes the shallow basement depth of the model from the Tabu search.



Fig.17 Location of observation site, ASO, in the Kanto basin, Japan.



Fig.18 Comparison between theoretical phase velocity of Rayleigh wave for the inverted models in Fig.19 with observed phase velocity.



Fig.19 Comparison between S-wave profiles derived from inversions based on Genetic Algorithm, Very Fast Simulated Annealing, and Tabu Search.



Fig.20 Comparison of the averaged minimum misfits and their standard deviation of 10 inversions based on based on Genetic Algorithm, Very Fast Simulated Annealing, and Tabu Search.

CONCLUSIONS

Three major heuristic search algorithms, Genetic Algorithms, Simulated Annealing, and Tabu Search, are implemented in an inversion of Rayleigh wave phase velocity estimated in microtremor array exploration. We examined the performance of these algorithms in the phase velocity inversion from the numerical experiments and the application to the actual data obtained in a microtremor array exploration in the Kanto basin, Japan. The GA and VFSA show similar performance in convergence speed to find models near the optimal solutions. However, VFSA is good at finding models with smaller misfits than that for the GA, because of local search features of the VFSA. On the other hand, it turned out that convergence speed for the TS is not so fast. Probably, the TS has a high ability in local search and does not effectively work as a global search. These results clearly indicate that the VFSA and GA are the promising tools in phase velocity inversion used in microtremor array explorations. Probably, this is also true for other inversion problems in earthquake engineering.

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