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DEPENDENCE OF ACCIDENTAL TORSION ON STRUCTURAL SYSTEM PROPERTIES

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SUMMARY

Studied are maximum displacements at the edges of the plan of elastic structural systems subjected to seismic motions that include accidental inputs: rotational seismic component and variations of center of mass location. The system properties that influence the responses are identified and assessed. Evaluated are the torsional provisions of Venezuelan and Mexican Seismic Codes, International Building Code, and De la Llera and Chopra Procedure; they can be either conservative or unconservative depending on the values of the system properties. A proposal that improves the mentioned procedure leading to a better agreement with the theoretical values, and including limitations concerning torsional stability, is presented.

INTRODUCTION

Seismic codes prescribe torsional effects to be incorporated in addition to translational effects in building design. Overall torsion can be split into inherent torsion derived from nominal dynamic properties of the system, and accidental torsion due to random variations of the stiffness and mass distributions and to the rotational seismic excitation; for inelastic response accidental torsion also comes from variations of the strength distribution. The effects of the accidental torsion are usually incorporated by means of an additional static torque, calculated as the product of the shear storey force by an accidental eccentricity, Newmark [1], just as it is specified in several seismic codes [2-4]. Recently, a more accurate procedure has been proposed, consisting in multiplying the inherent torsion effects by an amplification factor, De la Llera [5], Lin [6].

Emphasis has been made in the need of designing buildings for different performance levels according to seismic intensity. Besides the inelastic torsional response, it is therefore important the appropriate estimate of the elastic torsional response corresponding to the established performance for moderate earthquakes, Goel [7]. The accuracy of the procedures for incorporating the accidental torsion has been usually evaluated by means of elastic analysis, taking into account variations of the mass and stiffness distribution in the building plan, and adding rotational seismic excitation at the fixed base. This evaluation satisfies the requirements of elastic performance and has been extrapolated for inelastic performance. However, in our

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knowledge the elastic calibration has not covered all the range of systems parameters. For instance, in De la Llera [5] the design recommendation takes into account the torsional stiffness and the plan aspect ratio but not the eccentricities between the centers of mass and rigidity, neither the system periods; this procedure is based on the response of symmetric systems having a relatively long period (1 sec.). Later on in Lin [6] adjustments for smaller periods (from 0.1 sec.) were presented. In this paper, the influence of the system properties in the accidental torsion is examined, including the cases of two-way eccentricities and different periods in the two orthogonal directions. The analyses are made with single storey systems that are known to represent the response of regular multistorey buildings, Kan [8]. The seismic excitation is established by means of: i) response spectra for the two principal horizontal components of ground motion, López [9], which can act along any direction with respect to structural axes, Smeby [10], Menun [11], López [12]; ii) a smoothed response spectrum for the rotational motion [5]. An evaluation of the torsional provisions of the Venezuelan [2] and Mexican [3] seismic codes, the International Building Code [4] and of De la Llera and Chopra Procedure [5, 6] is also presented. Finally, a proposal in order to improve the mentioned procedure is suggested.

OVERVIEW OF TORSIONAL PROCEDURES

In static analysis, the usual method for incorporating torsional effects consists in the modification of nominal eccentricities and the addition of accidental torques, in order to determine two torsional design moments that control the design of elements on the flexible and stiff sides of the building plan (Figure 1).

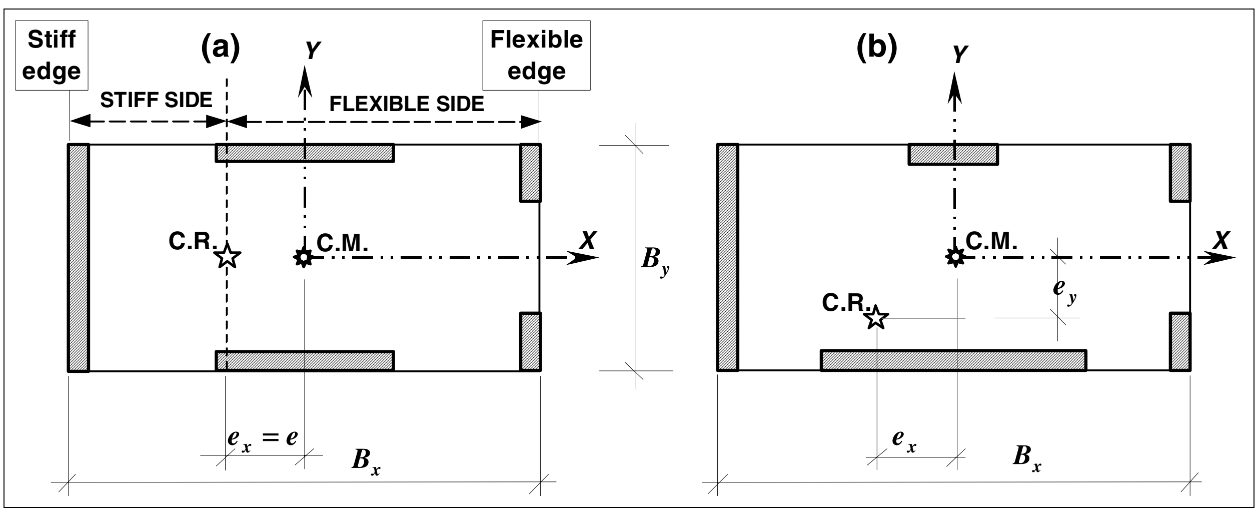

Figure 1. Single storey systems: (a) One-way asymmetric plan; (b) Two-way asymmetric plan.

The center of shear forces (C.S.), point of application of the storey shear forces, weights the positions of the mass centers above the floor in consideration, according to the force applied at each one. To estimate the effects of inherent torsion an amplification factor (τ) and a reduction factor (τ') are prescribed, which modify the eccentricity between the C.S. and the center of rigidity (C.R.), for both directions of analysis. The first one controls the design of the flexible side of the building plan and the second that of the stiff side. The accidental torsion is estimated by prescribing an accidental eccentricity of the storey shear force V equal to a percentage (β) of the plan width that is orthogonal to the direction of analysis, B (= B_x or B_y), with the ± signs in order to increase the response on the flexible and stiff sides, respectively. Therefore, two torsional design moments are specified: $M_{T,1} = V(\tau\, e + \beta\, B)$ and $M_{T,2} = V(\tau'\, e - \beta\, B)$, where e and V are taken positive. When using dynamic analysis, the effects of accidental eccentricities, $\pm\beta\, B$, are statically added to the effects of the inherent torsion.

The appropriate values of the parameters (τ and τ') corresponding to the inherent torsion can be determined by matching the dynamic amplification of the displacements (basic design parameters), and can be expressed in terms of few nondimensional parameters. When investigating a displacement along certain direction, we denote e as the eccentricity orthogonal to that direction, e_o the eccentricity in that direction, ω the uncoupled circular frequency of the system in the direction of the displacement, ω_o that for the orthogonal direction and ω_θ the uncoupled circular torsional frequency; uncoupled frequencies refer to natural frequencies of the system if it were torsionally uncoupled ($e_x = e_y = 0$) but with the same translational and rotational stiffnesses. For example, if we study displacements along direction Y: $e = e_x$, $e_o = e_y$, $\omega = \omega_y$, $\omega_o = \omega_x$. The required nondimensional parameters are: (a) the normalized eccentricities $\varepsilon = e/\rho$ and $\varepsilon_o = e_o/\rho$; and (b) the frequency ratios $\Omega = \omega_\theta/\omega$ and $\Omega_o = \omega_o/\omega$, where ρ is the inertial radius of gyration of the system. In the case of one-way asymmetric plans $\varepsilon_o = 0$, Ω_o can be left out, and we only need ε and Ω.

In the Venezuelan Seismic Code [2] the static analysis method is only allowed for regular buildings that comply with $\Omega \geq 0.5$ and $\varepsilon \leq 0.2$. The parameters of the method are:
$\tau = 1 + [4\text{-}16\varepsilon]\Omega$, if $0.5 \leq \Omega \leq 1$; $\tau = 1 + [4\text{-}16\varepsilon\,(2-\Omega)](2-\Omega)^4$, if $1 \leq \Omega \leq 2$; $\tau = 1$, if $2 \leq \Omega$;
$\tau' = 6\,(\Omega - 1) - 0.6$, but $-1 \leq \tau' \leq 1$; $\beta = 0.06$. The responses to the orthogonal seismic components must be combined by means of either the SRSS-rule, the 30%-rule or the CQC3-rule, including the accidental torsion for each component.

The Mexican Seismic Code [3] sets $\tau = 1.5$, $\tau' = 1$ and $\beta = 0.10$; the responses for the orthogonal horizontal components are combined by means of the 30%-rule. In the International Building Code [4] the maximum and the average of the displacements (δ_{max} and δ_{avg}) of the resistant elements for each storey and direction, subjected to the storey shear forces plus the torsional moments defined by $\tau = 1$ and $\beta = 0.05$, are first calculated. Then the torsional amplification factors A are determined by means of the expression $1 \leq A = (\delta_{max}/1.2\,\delta_{avg})^2 \leq 3$. The structure must be designed using $\tau' = 1$, $\tau = A$ and $\beta = 0.05A$ for each storey and direction. The responses to the two orthogonal seismic components are combined using either the SRSS-rule or the 30%-rule.

In De la Llera and Chopra Procedure, De la Llera [5], with the modification indicated in Lin [6], the effects of inherent torsion are calculated first and later amplified by: (1) a factor A for $0 \leq \Omega \leq 1$; (2) a value determined by a linear interpolation between A and 1 for $1 \leq \Omega \leq 1.8$, where $A = 1 + 0.0475\,(B/\rho)^{\alpha}$ with $\alpha = -0.066(0.7/T_y)^2 + 0.69(0.7/T_y) + 1.38$ if $T_y < 0.7$ sec. and $\alpha = 2$ if $T_y \geq 0.7$ sec. This procedure contains two debatable elements: i) the expression for α, which was evaluated for $T_y = 0.1$, 0.3 and 0.5 sec. in Lin [6], reaches its maximum for $T_y = 0.134$ sec. (but α should have a smaller value than it has for $T_y = 0.1$ sec.) and it even takes negative values for $T_y < 0.058$ sec; ii) the structure would be unstable for the included value of $\Omega = 0$; in addition, the minimum allowed value of Ω must be made dependent on the value of the eccentricities, as discussed below.

METHOD OF ANALYSIS

We consider single storey structural models with the center of mass (C.M.) at the geometric center of the plan and eccentricities (e_x and e_y) between the C.M. and center of rigidity (C.R.), (Figure 1). The plans are called symmetric if both of the eccentricities are null ($e_x = e_y = 0$), one-way asymmetric if only one of them is null, (for instance, $e_x \neq 0$; $e_y = 0$), or two-way asymmetric if none of them are null ($e_x \neq 0$; $e_y \neq 0$). We assume that for a certain probability of occurrence the variability of the position of the C.M. can be defined by means of the statistical mean of the value Δe that the C.M. can shift along each one of the principal orthogonal directions, which represents a determined proportion (ζ) of the plan width B. That is to say, $\Delta e_k = \zeta B_k$, k = X or Y. We also assume that a similar probability of occurrence is associated with the translation of the C.M. along an ellipse that matches the points of maximum variations of eccentricity (Figure 2). Several positions of the C.M. are considered for the analysis: the ellipse center (point 0 = nominal position), the intersection points of the ellipse and the diagonals of its enveloping rectangle (points 2, 4, 6 and 8) and the points of maximum eccentricity (points 1, 3, 5 and 7), making nine cases of analysis for each nominal position (Figure 2).

The seismic excitation consists of two horizontal components and one rotational component. The principal horizontal components 1 and 2 are described by pseudo-acceleration response spectra, A_1 and A_2 (Figure 3). Denoting the period T in seconds, the assumed values A_1 of the major component 1 are: 0.40 g for $T = 0$; 1.00 g for $0.15 \leq T \leq 0.7$; a linear variation between $0 \leq T \leq 0.15$; values corresponding to a constant pseudo-velocity for $0.7 \leq T \leq 2.2$ and to a constant deformation for $T \geq 2.2$. The values A_2 of the minor component 2 are obtained as $A_2 = \gamma A_1$, taking $\gamma = 0.7$ according to recent studies, López [9]. The normalized rotational spectrum ρA_θ is a smoothed version of that presented in De la Llera [5] for ρ = 20.41 m (corresponding to a square plan with $B_x = B_y = 50$ m); ρA_θ takes the value 0.10 g for $T = 0$; 0.25 g for $0.1 \leq T \leq 0.3$; values corresponding to a constant pseudo-velocity for $0.3 \leq T \leq 0.9$ and to a constant rotation for $T \geq 0.9$.

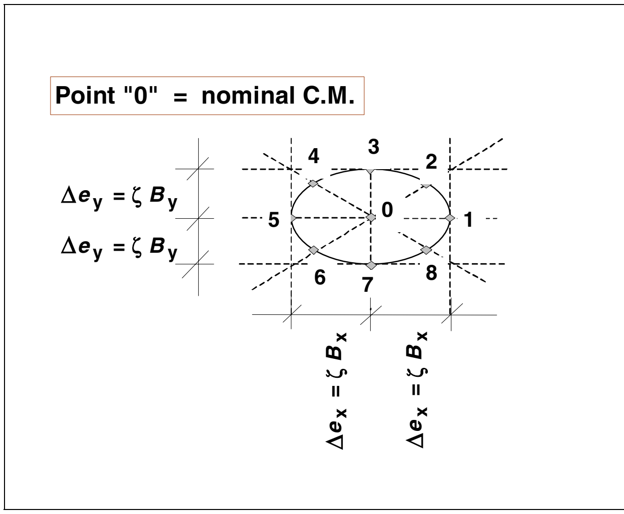

Figure 2. Points (0 to 8) along elliptic pattern for the center of mass (C.M.).

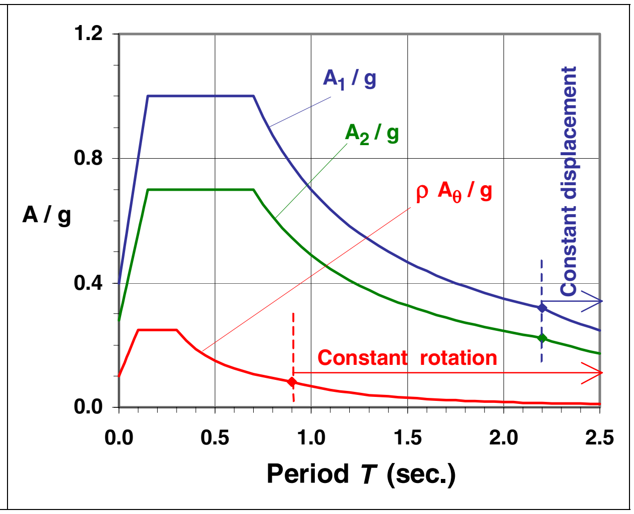

Figure 3. Normalized pseudo-acceleration spectra. A_1/g: major horizontal; A_2/g: minor horizontal; ρA_θ/g: rotational component.

Under each seismic component the responses are calculated by combining the modal responses using the CQC-rule, Rosenblueth [13], Der Kiureghian [14], that takes into account the correlation among the responses of modes with closed periods. We compute the critical value of a determined response to the rotational seismic component and the two horizontal seismic components by considering all the possible incidence angles, by means of the CQC3-rule, Smeby [10], Menun [11], López [12]. When using the CQC3-rule, we consider that the rotational component is not correlated with the horizontal components.

INFLUENCES OF STRUCTURAL PROPERTIES

The structural response is expressed in terms of the ratio r_{max}/r_{nom} between the maximum response (r_{max}) and the nominal response (r_{nom}). We define the maximum response as the largest of the critical responses to the three seismic components for the 9 positions of the C.M. in the ellipse showed in Figure 2. The nominal response is the critical response to the two horizontal seismic components with the nominal position of the C.M. only (point 0 of Figure 2), leaving out the two sources of accidental effects: the rotational seismic component and the variation of the position of the C.M. The investigated responses are the displacements along direction Y at the flexible and stiff plan edges. For $\Omega < 0.8$ the system is called torsionally flexible and for $\Omega > 1.2$ torsionally stiff.

Figure 4 shows the variation of ratio r_{max}/r_{nom} with the system properties. The effect of $\Delta e/B$ is shown in Figure 4(a). We can observe the quasi-proportionality of the increment of the ratio with an increment in $\Delta e/B$. The small increment for $\Delta e/B = 0$, points out the slight influence of the rotational seismic component in absence of variations of the positions of the C.M. Figure 4(b) presents values of the ratio r_{max}/r_{nom} for different aspect ratios, B_x/B_y, fixing $\Delta e/B = 0.05$ and $T_y = 0.5$ sec.; these results are similar to those presented in De la Llera [5] for $\Omega > 0.58$ and $T_y = 1$ sec. However, when Ω is close to 0.25 the uniform tendency of the curves, observed for $0.5 < \Omega < 0.8$, is lost and a significant increase in r_{max}/r_{nom} is observed, specially for the larger values of B_x/B_y. In Figure 4(c) we observe the influence of the uncoupled translational period T_y. An increment of r_{max}/r_{nom} takes place for $\Omega \le 0.8$ when $T_y \le 0.3$ sec. For example, r_{max}/r_{nom} is close to 2 for $\Omega \cong 0.3$ when $T_y = 0.3$ sec., but the same occurs for $\Omega \cong 0.55$ when $T_y = 0.1$ sec. However, for all values of periods we observe similar increments in systems of moderate torsional stiffness, $0.9 \le \Omega \le 1.1$, while the increments grow gradually as Ω falls from about 0.9; this fact indicates that is conservative to keep a constant value of A for $\Omega \le 1$ when the period is small. In Figure 4(d) we observe the influence of the period in the orthogonal directions; if the system is more flexible for the direction perpendicular to the examined displacement, large increments are possible for values the values of Ω that correspond to $\omega_o \cong \omega_\theta$.

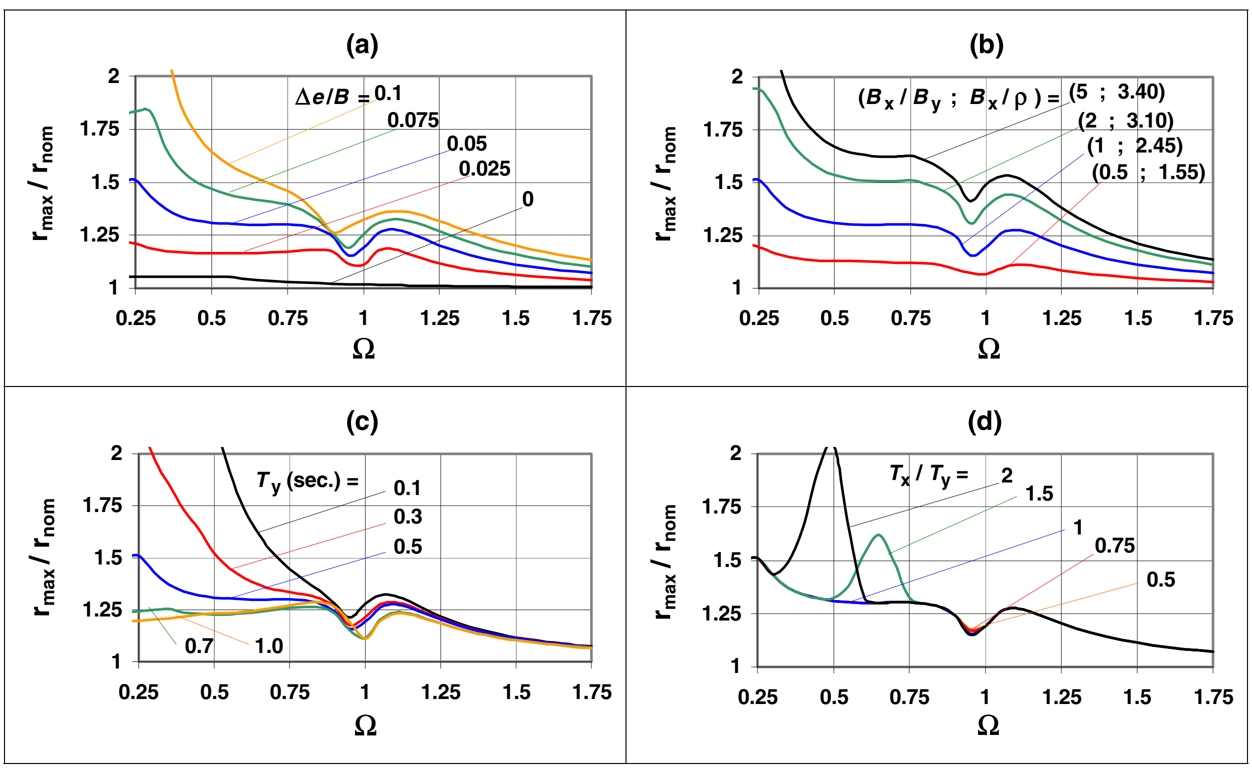

Figure 4. Values of r_{max}/r_{nom} for the displacements along direction Y at both plan edges of symmetric systems, $e_x = e_y = 0$; $B_x = 50$ m: (a) $B_x/B_y = 1$, $T_x = T_y = 0.5$ sec., for several values of $\Delta e/B$; (b) $T_x = T_y = 0.5$ sec., $\Delta e/B = 0.05$, for several values of B_x/B_y; (c) $B_x/B_y = 1$, $\Delta e/B = 0.05$, $T_x/T_y = 1$, for several values of T_y; (d) $B_x/B_y = 1$, $\Delta e/B = 0.05$, $T_y = 0.5$ sec., for several values of T_x/T_y.

Figure 5 shows the influence of the normalized eccentricities on the ratio r_{max}/r_{nom}. In Figures 5(a), 5(c) and 5(e) we can notice that the response at the stiff edge can be affected by small eccentricities depending on the degree of torsional stiffness. In general, the values of r_{max}/r_{nom} increase with the eccentricity for systems of high-moderate torsional stiffness, $1 \leq \Omega \leq 1.2$, but drop for systems of low-moderate torsional stiffness, $0.8 \leq \Omega \leq 1$. In the torsionally flexible systems, $\Omega < 0.8$, the eccentricity does not lead to more unfavorable effects except for the very torsionally flexible ones, $\Omega < 0.5$. On the contrary, in Figures 5(b), 5(d) and 5(f) we observe that the accidental effects at flexible edge increase with the eccentricity for torsionally flexible systems, $\Omega < 0.8$. As expected, very large increments of responses are found when the system is close to the torsional instability condition; it can be proved that this condition is reached when $\Omega = (\varepsilon_o^2\,\Omega_o^2 + \varepsilon^2)^{1/2}$, being impossible a lower value of Ω. For one-way asymmetric systems this condition simplifies to $\Omega = \varepsilon$. In order to guarantee the stability we have to add the accidental variations of nominal eccentricities in ε and ε_o. In addition, to avoid excessive displacements it is also necessary to move away from the instability limit. For example, if $\varepsilon_o = 0.2$ and $\varepsilon = 0.5$, being $\Omega_o = 1$ and $\Delta\varepsilon = 0.12$ (for $\Delta e/B = 0.05$ and $B_x/B_y = 1$), we strictly need that $\Omega > 0.65$, but it is advisable that at least $\Omega > 0.75$ to prevent very large displacements or a fortuitous instability due to uncertainties in the parameters of the system.

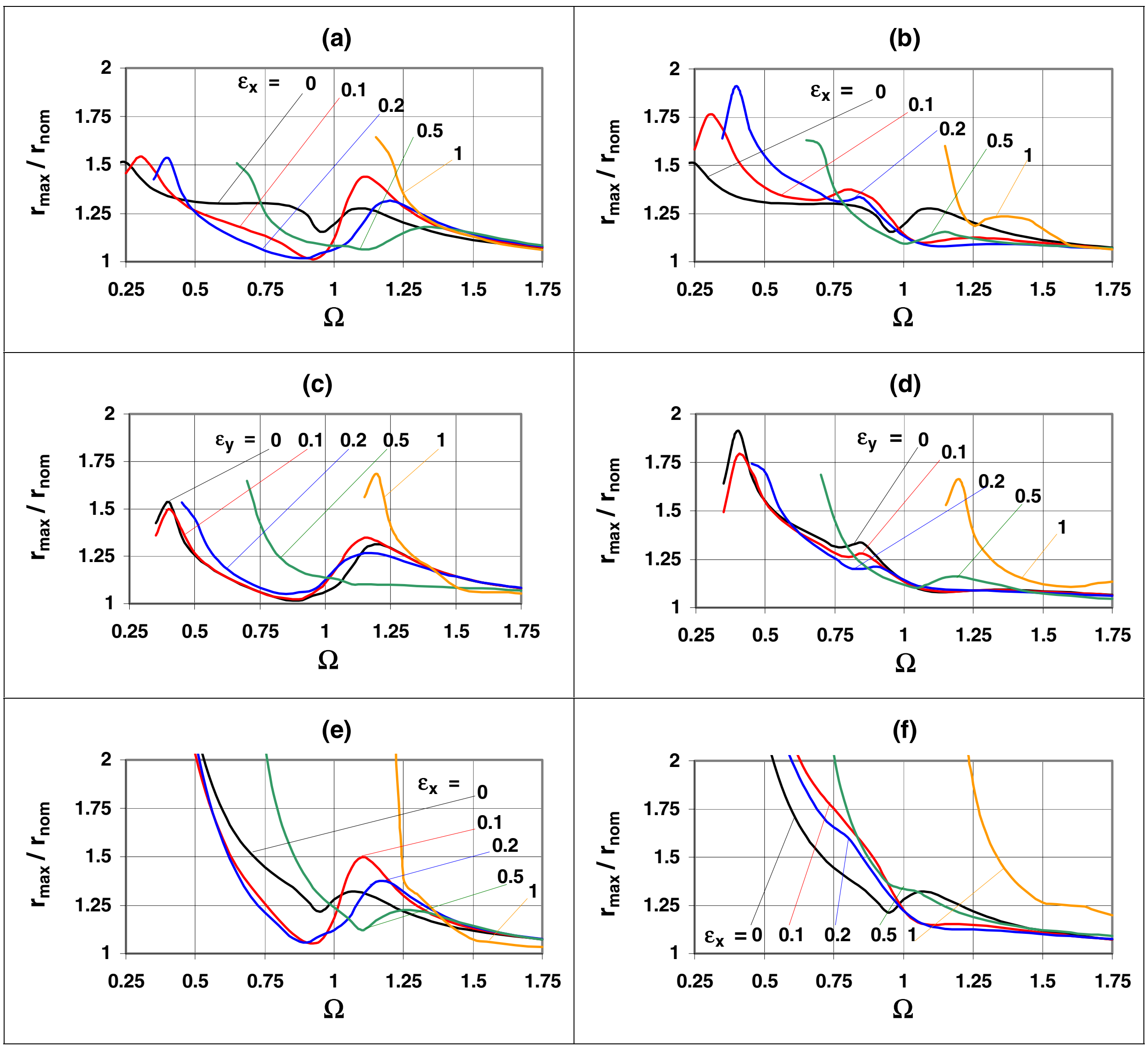

Figure 5. Values of r_{max}/r_{nom} for the displacements in direction Y at plan edges of asymmetric systems, $B_x = 50$ m; $\Delta e/B = 0.05$; $B_x/B_y = 1$. (a), (c) and (e): stiff edge; (b), (d) and (f): flexible edge; (a) and (b): $T_x = T_y = 0.5$ sec., for $\varepsilon_y = 0$ and several values of ε_x; (c) and (d): $T_x = T_y = 0.5$ sec., for $\varepsilon_x = 0.2$ and several values of ε_y; (e) and (f) $T_x = T_y = 0.1$ sec., for $\varepsilon_y = 0$ and several values of ε_x.

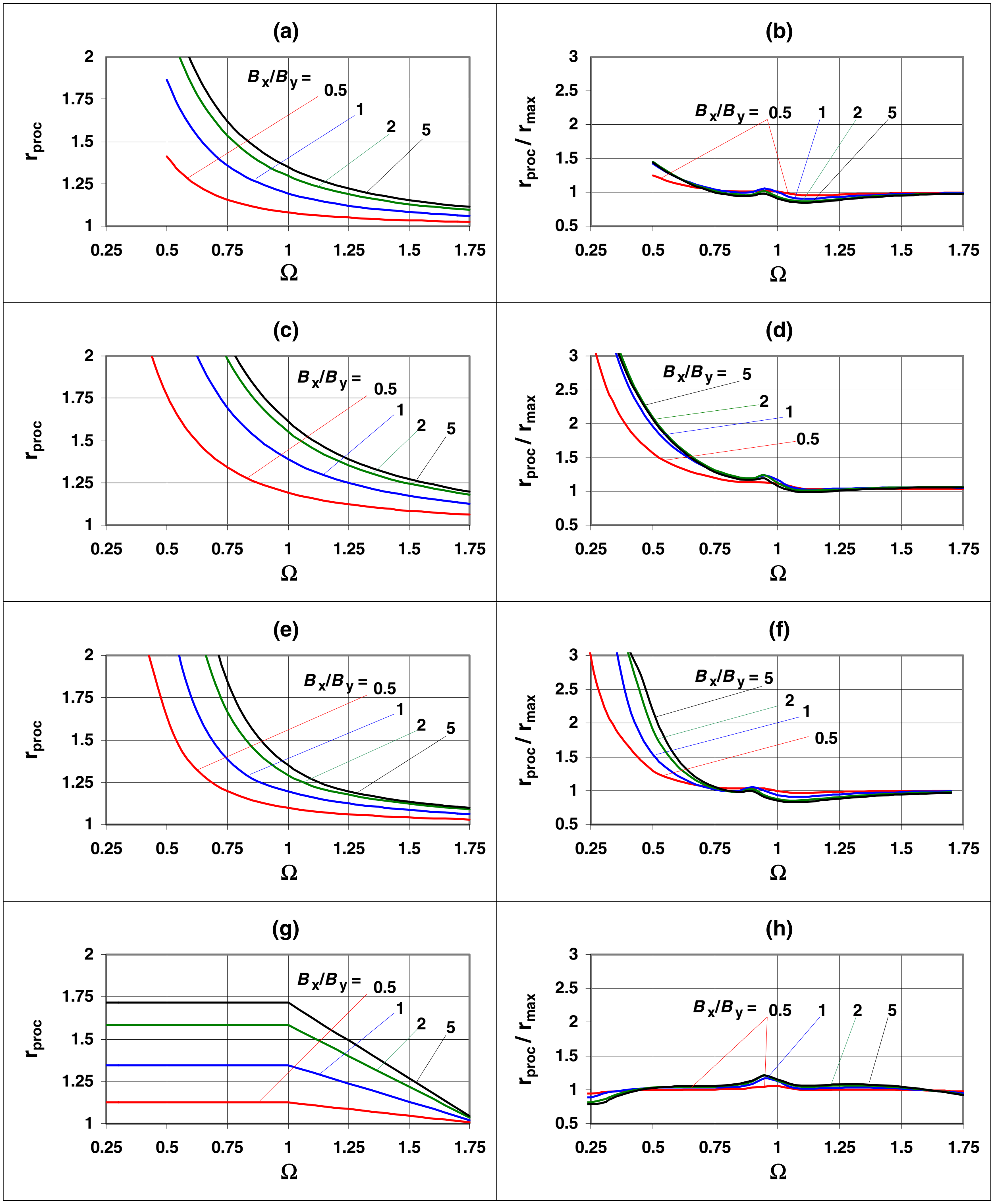

Figure 6. Values of r_{proc} and r_{proc}/r_{max} for displacements at plan edges in direction Y of symmetric systems, $\varepsilon_x = \varepsilon_y = 0$; $B_x = 50$ m; $\Delta e/B = 0.05$; $T_x = T_y = 0.5$ sec.; (a) and (b): Venezuelan seismic code [2]; (c) and (d): Mexican seismic code [3]; (e) and (f): International Building Code [4], (g) and (h): De la Llera and Chopra Procedure, De la Llera [5], Lin [6].

EVALUATION OF TORSIONAL PROCEDURES

Presented is an evaluation of the aforementioned torsional procedures [2-6] assuming as a reference value that $\Delta e/B = 0.05$. The response obtained from the application of each procedure, normalized to the response of the corresponding symmetric system with the same uncoupled periods, is defined as r_{proc}. The ratio r_{proc}/r_{max} measures the accuracy of the specific procedure for estimating the accidental effects. The evaluation is presented for B_x/B_y=5, 2, 1 and 0.5, assuming B_x=50 m.

Presented in Figures 6(a), 6(c), 6(e) and 6(g) are the values of r_{proc} corresponding to Venezuelan [2], Mexican [3] and International Building [4] codes, and to De la Llera and Chopra Procedure, De la Llera [5], as modified in Lin [6], respectively, for the displacement at the edges of a symmetric plan with $T_x = T_y$ = 0.5 sec. In Figures 6(b), 6(d), 6(f) and 6(h) the values of r_{proc}/r_{max} are presented. For systems of moderate and large torsional stiffness, $\Omega \geq 0.75$, all procedures provide a good estimate of the accidental effects. But for torsionally flexible systems, $\Omega < 0.75$, the seismic codes are conservative since the static accidental torque yields excessive displacements; on the contrary the De la Llera and Chopra Procedure provides a good estimate for all range of torsional stiffnesses, confirming for T_y = 0.5 sec. the results reported in [5] for symmetric systems with T_y = 1.0 sec.

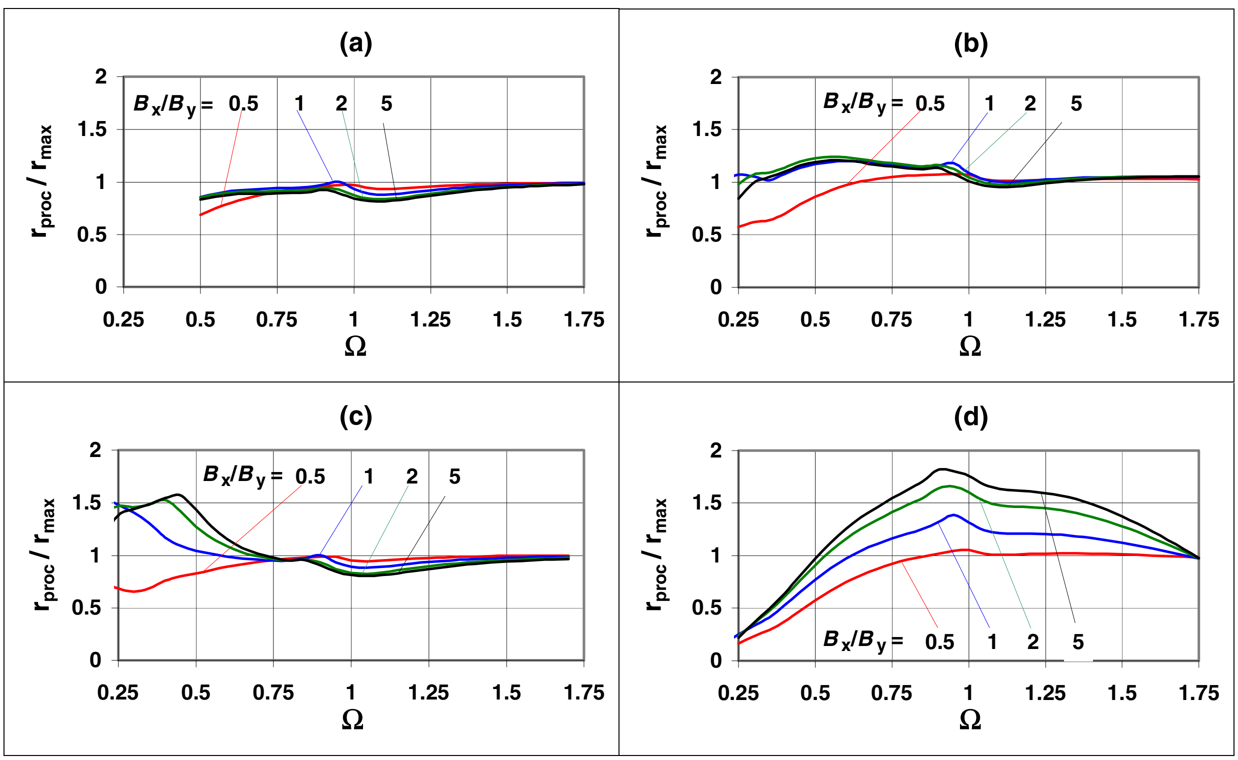

Figure 7. Values of r_{proc}/r_{max} for displacements at plan edges in direction *Y* of symmetric systems, $\varepsilon_x = \varepsilon_y = 0$; B_x = 50 m; $\Delta e/B = 0.05$; $T_x = T_y = 0.1$ sec. (a): Venezuelan Code [2]; (b): Mexican Code [3], (c): International Building Code [4], (d): De la Llera and Chopra Procedure, De la Llera [5], Lin [6].

Results for short-period symmetric systems with $T_x = T_y$ = 0.1 sec. are shown in Figure 7. For these systems disappear the great conservatism of the code provisions for torsionally flexible systems pointed out in the systems with $T_x = T_y$ = 0.5 sec. (Figure 6), moving into underestimation of the theoretical responses. In this region the Venezuelan code becomes a little unconservative for all aspect ratios, whereas the Mexican and International Building ones only become so for small aspect ratios (as long as B_x

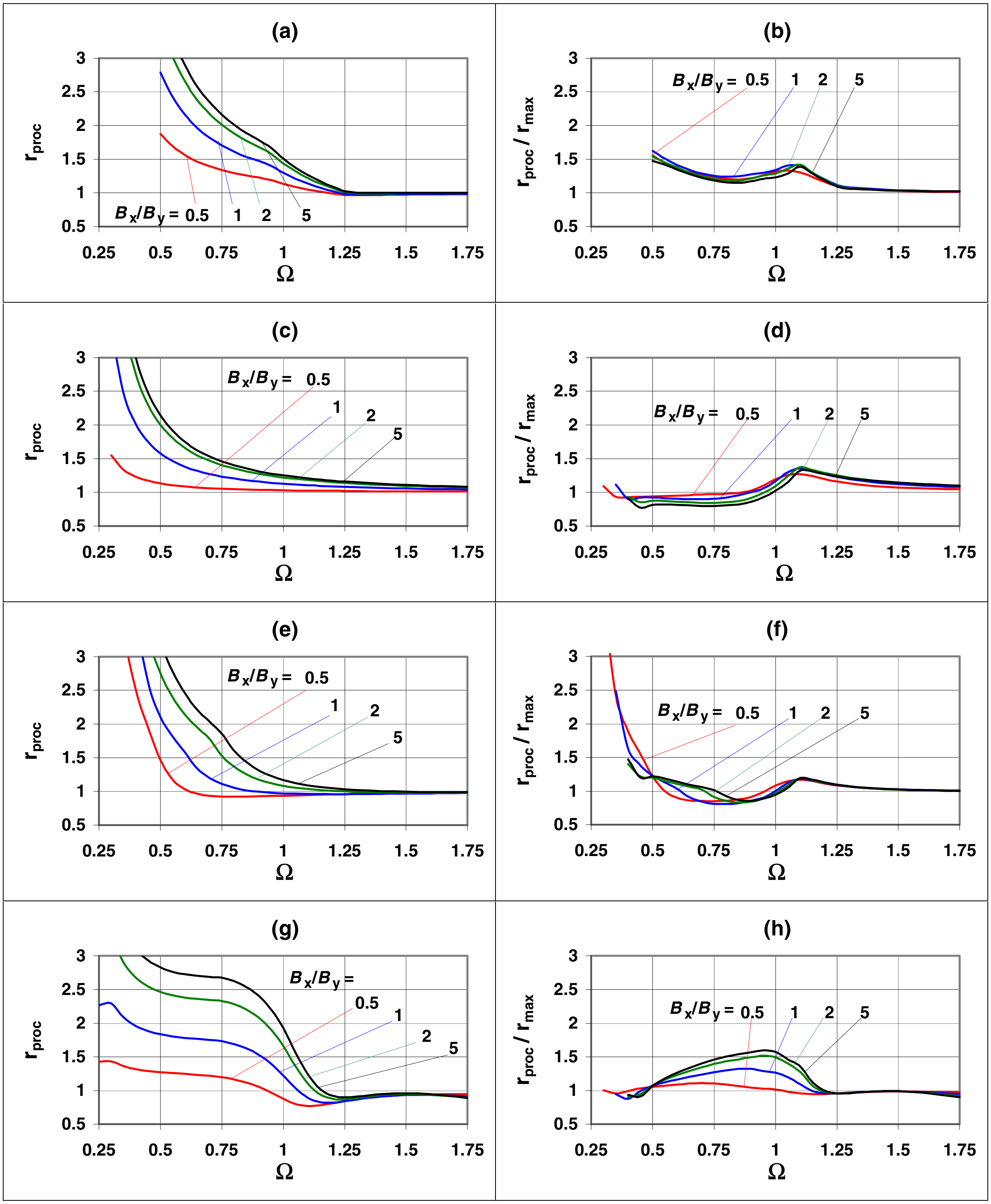

Figure 8. Values of r_{proc} and r_{proc}/r_{max} for displacements at the stiff edge of asymmetric systems with $\varepsilon_x = 0.2$, $\varepsilon_y = 0$, and $T_x = T_y = 0.5$ sec.; (a) and (b): Venezuelan Code [2]; (c) and (d): Mexican Code [3], (e) and (f): International Building Code [4], (g) and (h): De la Llera and Chopra Procedure, De la Llera [5], Lin [6].

= 50 m) because of the larger accidental eccentricities. The modification proposed in Lin [6] turns out to be unconservative for torsionally flexible systems but very conservative for other systems.

In the same format as presented for symmetric systems (Figure 6), Figure 8 presents results for asymmetric systems; results for the displacements at the stiff edge for systems with $\varepsilon_x = 0.2$, $\varepsilon_y = 0$, and $T_x = T_y = 0.5$ sec. are shown. The Venezuelan Code turn out to be conservative, but the Mexican and the International Building Codes are slightly unconservative for the region $0.5 < \Omega < 1$. On the contrary, De la Llera and Chopra Procedure is the most conservative in this region as observed in Figures 8(g) and 8(h).

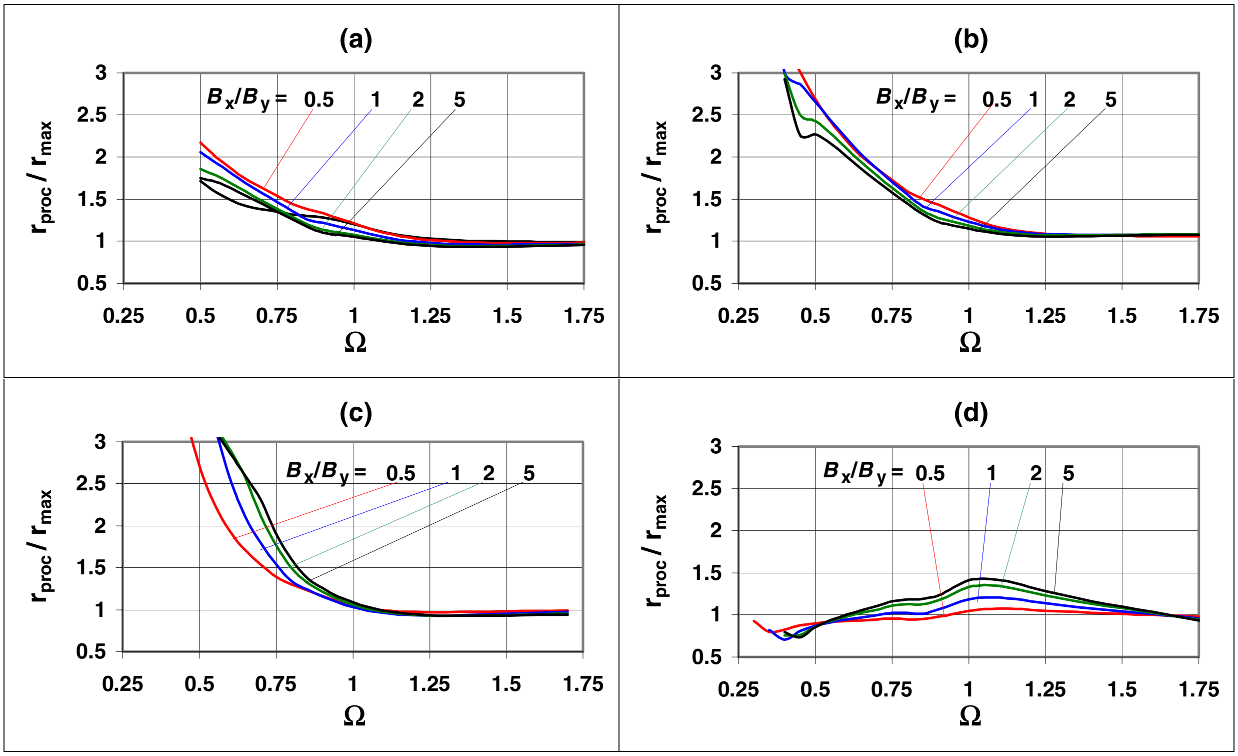

Figure 9. Values of r_{proc} / r_{max} for displacements at the flexible edge of asymmetric systems with $\varepsilon_x = 0.2$, $\varepsilon_y = 0$, and $T_x = T_y = 0.5$ sec.; (a): Venezuelan Code [2]; (b): Mexican Code [3], (c): International Building Code [4], (d): De la Llera and Chopra Procedure, De la Llera [5], Lin [6].

The ratio (r_{proc} / r_{max}) for the displacement at the flexible edge of the same asymmetric systems (Figure 8) is shown in Figure 9. We observe that the code-specified procedures provide good estimates of the response for $\Omega > 1$, whereas for the more torsionally flexible systems, $\Omega < 1$, turn out to be quite conservative. De la Llera and Chopra Procedure has a more uniform behavior, keeping certain conservatism except for the region of very large torsional flexibility, $\Omega<0.5$.

As a summary, the current torsional procedures do not keep a uniform accordance with the theoretical values, being conservative in some cases and unconservative in other ones. Although a deeper study can set a more accurate value of $\Delta e/B$ and its variance, the general nature of the presented results and especially the regions of dissimilar behavior will be probably the same.

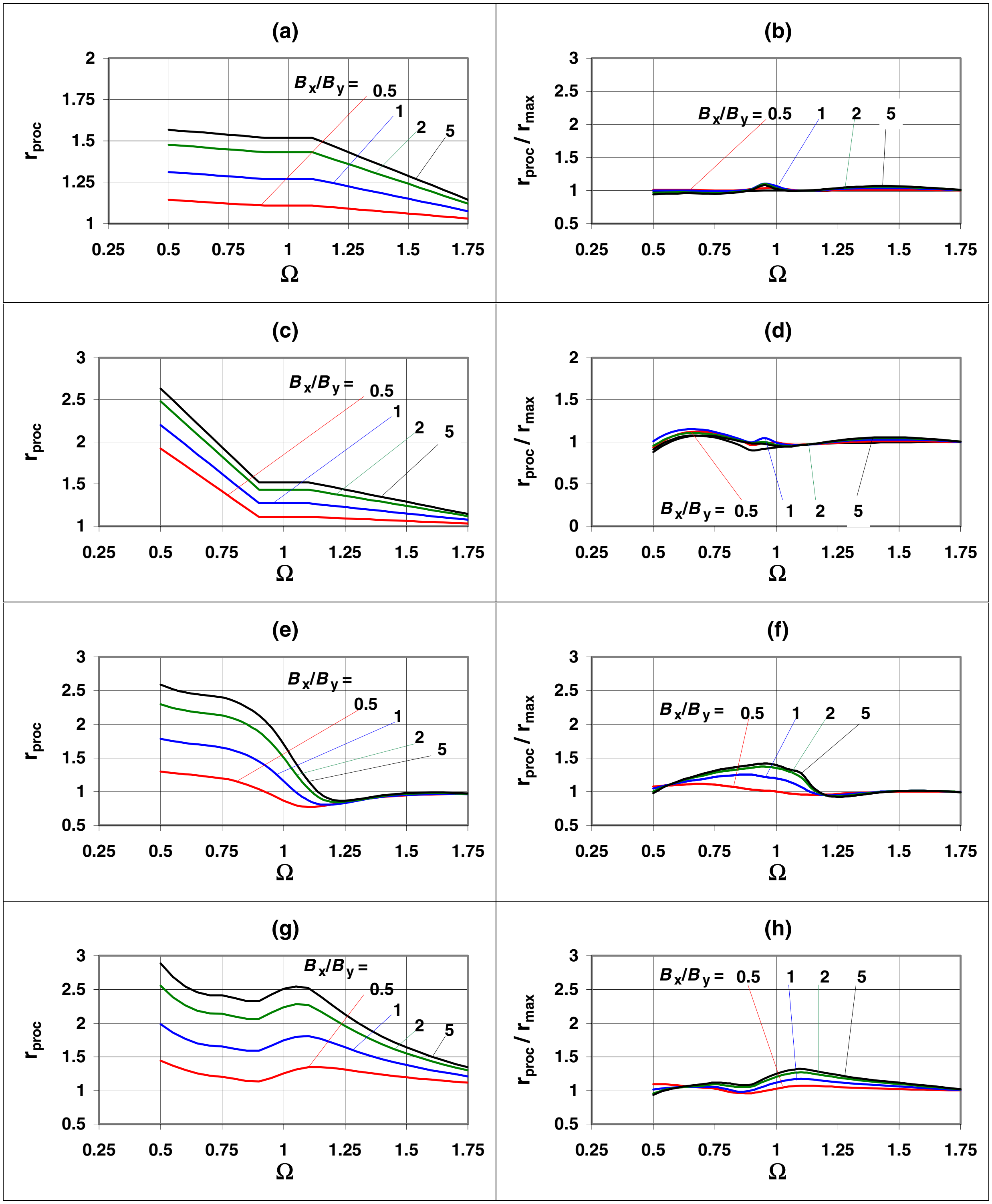

Figure 10. Values of r_{proc} and r_{proc}/r_{max} for the proposed procedure; $B_x = 50$ m; $\Delta e/B = 0.05$; (a) and (b): edges of symmetric systems, $\varepsilon_x = \varepsilon_y = 0$; $T_x = T_y = 0.5$ sec.; (c) and (d): edges of symmetric systems, $\varepsilon_x = \varepsilon_y = 0$; $T_x = T_y = 0.1$ sec.; (e) and (f): stiff edge of asymmetric systems, $\varepsilon_x = 0.2$, $\varepsilon_y = 0$; $T_x = T_y = 0.5$ sec.; (g) and (h): flexible edge of asymmetric systems, $\varepsilon_x = 0.2$, $\varepsilon_y = 0$; $T_x = T_y = 0.5$ sec.

IMPROVEMENT OF TORSIONAL PROCEDURE

A modification of the De la Llera and Chopra Procedure is proposed next in order to improve the response estimation of the accidental effects for a wider range of system parameters and to ensure torsional stability. This procedure is used since it achieves very good estimates for symmetric systems with relatively large periods ($T \geq 0.5$ sec) for all the range of torsional stiffnesses and aspect ratios. It seems feasible to set out, as a first approach, some simple modifications in order to improve the estimation for asymmetric systems and low periods. On the other hand, an improvement of the code-specified provisions would require rather complex modifications that would involve establishing a variable accidental eccentricity coefficient (β) depending on the system properties.

The proposed torsional procedure takes into account the variations of r_{max} / r_{nom} with system properties observed in Figures 4 and 5. These results and suitable measures for incorporating them are described next:

1) In order to avoid a possible torsional instability it is necessary to bound the inferior value of the torsional stiffness, for preventing excessive displacements at the plan edges, as done in the Venezuelan Seismic Code. On this matter we have to take into account the accidental variations of the center of mass and rigidity. Under nominal conditions, for $\Omega = \varepsilon' = (\varepsilon_o^2\Omega_o^2 + \varepsilon^2)^{1/2}$ there is torsional instability; to get away from that it is advisable that $\Omega \geq 1.1\varepsilon'$. However, considering an accidental variation of eccentricities $\Delta e/B = 0.05$ and all possible plan aspect ratios (since $\max\{B/\rho\} \cong 3.46$; $1.1*3.46*0.05 \cong 0.2$) we propose that $\Omega \geq 1.1\varepsilon' + 0.2$. In addition, for very small torsional stiffnesses, $\Omega < 0.5$, the displacements may increase too much for some values of periods or eccentricities. Given the typical uncertainties of system properties this condition should be avoided, because a small variation in those may lead to a large increase in the response. Therefore, it is proposed that for applying the procedure the additional requirement $\Omega \geq 0.5$ is included; then, $\Omega_{min} = \max\{1.1\varepsilon' + 0.2\ ;\ 0.5\}$.

2) For the region of medium torsional stiffness, $0.9 \leq \Omega \leq 1.1$, the amplification corresponding to symmetric systems with a given value of B_x/B_y is almost constant, regardless of period values. For this amplification either the expression proposed in De la Llera [5] for $T=1$ sec. or one slightly modified, $A' = 1 + 0.045(B/\rho)^2$, can be used for all period values.

3) In order to use the bounds indicated in the preceding item we must have $\Omega_{min} < 0.9$. Therefore, the method has to be limited to eccentricities so that $\varepsilon' \leq 0.6$.

4) For symmetric systems, the increment of amplification for small periods ($T<0.7$ sec.) grows in inverse proportion to Ω, from $\Omega \cong 0.9$ until $\Omega \cong \Omega_{min}$. Let A_{max} be the amplification for $\Omega = \Omega_{min}$; an approximate fit of ratio A_{max} / A' can be accomplished, such as $A_{max} / A' = \exp\{2.1(0.7-T)^{2.6}\}$ for $T \leq 0.7$ sec. and $A_{max} / A' = 1$ for $T>0.7$ sec., which increases continuously as long as T decreases.

5) At the stiff edge of asymmetric systems is not necessary to increase the amplifications corresponding to symmetric ones; in this proposal the possible reductions of them are not included. At the flexible edge of asymmetric systems the values of A_{max} for $\Omega = \Omega_{min}$ increase by a factor of $(1+\varepsilon')$ relative to those for symmetric ones, for moderate eccentricities; this factor is somehow conservative for large eccentricities.

6) For very large torsional stiffnesses the amplification approaches 1, without reaching it for $\Omega = 1.75$. It is proposed to replace the De la Llera and Chopra interpolation (between A and 1 for $1 \leq \Omega \leq 1.8$) with an interpolation between A' and 1 for $1.1 \leq \Omega \leq 2$.

7) When the frequency ratio $\Omega_o < 1$ and $\Omega_o \cong \Omega$, the accidental effects are not adequately provided by the proposed procedure and a more refined analysis becomes necessary.

According to the above criteria, we propose a procedure broke down as follows. The method is appropriate if the verification given in step 3 is fulfilled; otherwise, it is recommended to perform a dynamic analysis with accidental effects. For each direction of analysis the steps of the improved procedure are:

Step 1. Compute the fundamental periods of the building (translational ones and rotational one) by means of dynamic analysis or Rayleigh method. Obtain the global values of Ω and Ω_o.

Step 2. Estimate the normalized eccentricities between the C.M. and the C.R. in each floor: ε, ε_o, $\varepsilon' = (\varepsilon_o^2\Omega_o^2 + \varepsilon^2)^{1/2}$. Then, assign a global value of ε' to the building either conservative or representative of the building plans.

Step 3. Verify that $\varepsilon' \leq 0.6$; $\Omega \geq 0.5$; $\Omega \geq 1.1\varepsilon' + 0.2$ and that $\{\Omega_o$ is far enough from Ω , if $\Omega_o < 1\}$.

Step 4. Determine the amplifications, A, at each edge of building plan:

4.1 $A = A' = 1+0.045(B/\rho)^2$, if $0.9 \leq \Omega \leq 1.1$

4.2 $A = A_{max} = \eta\alpha A'$, if $\Omega = \Omega_{min} = \max\{1.1\varepsilon' + 0.2; 0.5\}$. Where: $\{\alpha = \exp\{2.1(0.7-T)^{2.6}\}$ if $T \leq 0.7$ sec.; $\alpha = 1$ if $T \geq 0.7$ sec.$\}$, $\{\eta = 1+\varepsilon'$ at flexible edge; $\eta = 1$ at stiff edge $\}$.

4.3 $A = 1$, if $\Omega \geq 2$.

4.4 Interpolate linearly between A_{max} and A' for $\Omega_{min} < \Omega < 0.9$, and between A' and 1 for $1.1 < \Omega < 2$.

Step 5. Compute the amplifications at the resisting elements of building by interpolating linearly between the amplification at the edge and the value of 1 assigned either to the C.M. or to the C.R.

Step 6. Compute the inherent responses (displacements and forces) at the resisting elements for all storeys of the building.

Step 7. Compute the design responses (displacements and forces) by multiplying the inherent responses by the corresponding amplifications.

Presented in Figures 10(a), 10(c), 10(e) and 10(g) are the design displacements derived from applying the proposed procedure for the four systems previously evaluated. Figures 10(a) and 10(c) correspond to symmetric systems for which $r_{nom} = 1$; hence, we can observe the graphical expression of the procedure amplifications, for $T = 0.5$ sec. and 0.1 sec., respectively. Shown in Figures 10(e) and 10(g) are the design displacements at the stiff and flexible edges of a one-way asymmetric system with $\varepsilon = 0.2$. Figures 10(b), 10(d), 10(f) and 10(h) present the values of r_{proc}/r_{max}; the proposed procedure provides a good estimate of the accidental effects for most of the system parameters. It can be seen a very good estimation for the symmetric systems, reducing the errors for $T = 0.1$ sec. of the original procedure. In the same way, for asymmetric systems a better estimation is now obtained. Some overestimations for systems of moderate torsional stiffness still remain, because reductions of the amplification for asymmetric systems have not been included.

CONCLUSIONS

Investigated were the maximum elastic displacements at edges of single storey systems subjected to seismic excitation, considering two sources of accidental effects: rotational seismic component and variations of the center of mass. The position of the center of mass was varied along two orthogonal directions spanning nine positions on the plane. The critical response to the rotational and two horizontal seismic components considering all possible incidence angles was determined. The influence of system properties on the accidental effects was examined, several torsional procedures that incorporate these effects were evaluated, and an improved procedure was proposed. The main conclusions are:

a) The relevant parameter is the ratio, named amplification, of the maximum response and the nominal inherent response. Significant results are: a.1) a quasi-proportional relationship between the amplification and the degree of accidental variation of the center of mass position; a.2) an uniform amplification for symmetric systems with medium torsional stiffness, irrespective of system periods; a.3) a tendency of the amplification to increase for very torsionally flexible systems, exceeding the aforementioned uniform value, for systems with moderate and large periods; a.4) a tendency to large increments of amplification for symmetric systems of low periods and small torsional stiffness; a.5) a large local increment of amplification for systems with a larger period in the direction orthogonal to the examined displacement, when that matches the torsional period; a.6) an increment of amplification at the flexible edge of asymmetric systems with small torsional stiffness; a.7) a very large increment of amplification when torsional stiffness is close to the torsional instability condition.

b) The torsional provisions in the Venezuelan, Mexican and International Building Codes and the De la Llera and Chopra Procedure do not provide a uniform estimate of the theoretical values. All procedures lead to good estimates for torsionally stiff systems, but they can be either conservative or unconservative for the other systems. De la Llera and Chopra Procedure gives accurate results for symmetric systems with moderate or large periods, irrespective of torsional stiffness and aspect ratio, but its accuracy diminishes for short periods and asymmetric systems.

c) The proposed torsional procedure, which improves that of De la Llera and Chopra, takes into account the influence of structural system properties indicated above. It includes restrictions to ensure torsional stability, expressed as minimum values for the ratios of torsional and translational frequencies. The proposed procedure shows a significant improvement in the estimate responses for short period systems and to a lesser extent for asymmetric systems.

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