



A STUDY ON EMERGENCY RETROFIT USING PRESTRESSING BARS AND STEEL PLATES FOR DAMAGED COLUMNS

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SUMMARY

Emergency seismic retrofit technique is indispensable for the rehabilitation of damaged RC buildings immediately after earthquake attack. Therefore, it is important to develop quick and convenient emergency retrofit technique to recover the seismic performance of the structures damaged by earthquakes. It is well known fact that the strength and ductility of RC columns can be extremely enhanced by transverse confinement. Considering this fact, an emergency retrofit technique utilizing pre-tensioned high steel bars (PC bars) and steel plates is proposed in this paper. The effectiveness of the proposed retrofit technique is experimentally investigated and analytically evaluated.

In the experiment, specimens with poor hoop arrangement were subjected to cyclic loading tests under constant axial force to cause shear failure first. Then, after providing an emergency retrofit for the damaged specimens, the cyclic loading tests were carried out again. At the same time, to confirm the residual axial compression capacity of the damaged column specimens and of the column specimens reinforced by the emergency retrofit, axial compression tests were carried out as well. In addition, the various damage levels recoverable by emergency retrofit were confirmed. Experimentally, it was verified that the emergency retrofit on damaged column was effective not only for the improvement of ductility but also for the recovery of axial and lateral capacity. By introducing the pretension force into the PC bars, the emergency retrofit makes the cracks close with the help of active confinement in addition to passive confinement and shear strengthening by PC bars.

INTRODUCTION

The collapse of the buildings can be prevented by the emergency retrofit, if the vertical load for long term can be sustained by the column after the damaged caused in the earthquake. Therefore, the emergency retrofit for post-earthquake damaged RC columns is important. Based upon the investigation of previously proposed retrofit technique for independent RC column which introduced prestress using the high strength

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steel rod by T. Yamakawa and M. Kurashige et al. [1], the emergency retrofit technique for ensuring seismic performance and safety on the building damaged in the earthquake is proposed in this paper. The emergency retrofit can be applied without heavy machinery as a dry construction method. From the past investigation [2], it was observed that the recovery of the seismic performance was small when the pretension force was not introduced into the PC bars but when the pretension force was introduced, the seismic performance was improved a little. However, it did not return to the level equal to that of the sound concrete. For this reason, in the proposed retrofit technique the steel plates were considered to attach on the four faces of column in addition to the pre-tensioned high strength steel bars and the earthquake performance of this retrofit technique was evaluated through cyclic loading test. Moreover, the recovery level is also affected by the degree of damage. So, in the cyclic loading test, the seismic performance at different damage levels were also verified. Furthermore, the residual axial compression capacity was measured by axial compression tests on shear damage specimens before and after retrofit. In order to evaluate the experimental results, the flexural strength is calculated by fiber model and by the AIJ simplified equation. The shear strength is also calculated by AIJ guideline equations [3]. In both the cases, the active confinement effect due to pretension force into PC bars and the passive confinement effect by PC bars and steel plates are considered.

TEST PLAN

The specimens used in this experiment were shear sensitive columns with square cross sectional dimension of 250x250mm, height of 500mm, shear span to depth ratio of 1.0, shear reinforcement ratio of $p_w=0.08\%$, and longitudinal reinforcement ratio of $p_g=1.36\%$. Mechanical properties of longitudinal reinforcement, hoop steel, PC bars and steel plates are given in Table 1. The symbol “ σ_B ” in this table is the cylinder strength of concrete. The cyclic loading tests were carried out in two steps. At first step, the cyclic loading test was continued until the shear failure happened in non-retrofitted specimens. At second step, the cyclic loading test was again carried out in the same specimens after emergency retrofitting. The axial force was always existed in the shear damaged specimens with low to medium crack width. Fig. 1 represents the shear damaged specimens before and after retrofit. Three specimens ER02S-P41S1, ER03S-P41S2 and ER02S-P41S3 were prestressed but the last specimen ER03S-P41SN was not prestressed.

In order to verify the effectiveness of the retrofit at different damage levels, three damage levels were considered in the cyclic loading program. Level 1 indicated a situation where the damage level was small and the maximum crack width was not greater than 5mm. Level 2 indicated a situation where the damage level was medium and the maximum crack width was about 5 to 10mm. Levels 1 and 2 indicated the situations where the axial force (axial force ratio 0.2) could be maintained even after the damage caused in shear. Level 3 indicated a situation where the crack width was greater than 10mm and the axial force ratio 0.2 corresponding to live and self-weight loads working as long term vertical load could not be maintained.

Next, after providing an emergency retrofit for each damaged specimen as shown in Table 2, cyclic loading tests were carried out. The emergency retrofit as shown in Fig. 2 was a method of installation in which corner blocks were located at four corners of a damaged column, PC bars with diameter of 5.4mm were installed to connect them, and specified prestress was introduced with a torque wrench. Four steel plates (240x470x3.2 mm) were also simply attached to the four faces of a damaged column and no welding was necessary. This retrofit method is very convenient. The pretension strain level was about 2450μ , which was equal to approximately 1/3 of the yield strain of PC bar. The corner blocks to be used as shown in Fig. 3, are usually Type 1. The Type 2 blocks are to be used when damage levels are so large that PC bars cannot be placed with Type 1 blocks.

Table 1 Mechanical properties of reinforcements

Reinforcement		a(cm ²)	f _y (MPa)	ε _y (%)	E _s (GPa)
Rebar	D10	0.71	372	0.20	186
Hoop	3.7φ	0.11	390	0.19	205
PC bar	5.4φ	0.23	1220	0.61	200
Steel plate	3.2mm	0.76	276	0.13	212

Notes: a=cross section area, f_y=yield strength of steel,
ε_y=yield strain of steel, E_s=modulus of elasticity.

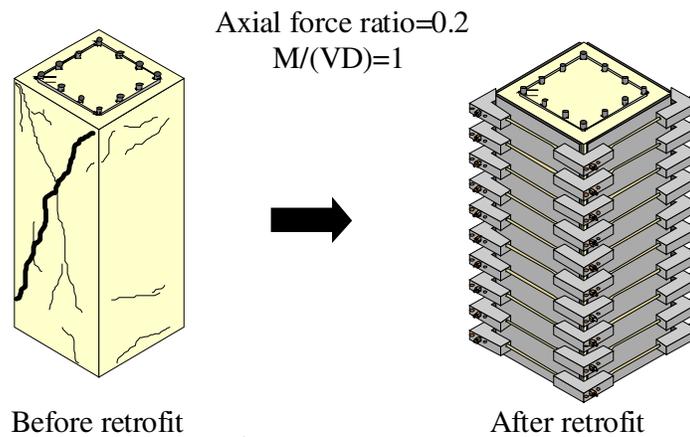


Fig. 1 Emergency retrofit for extreme short RC columns by PC bars and steel plates

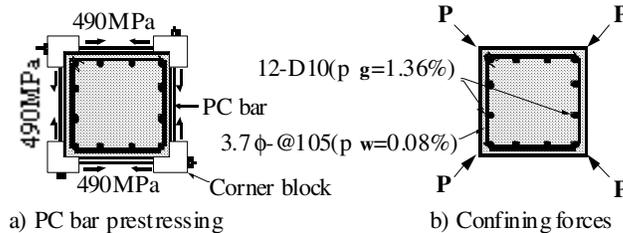


Fig. 2 Details of retrofit by PC bar prestressing and confining forces

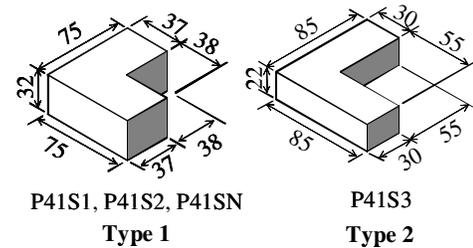
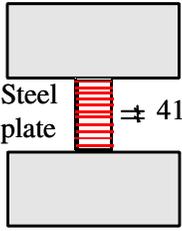
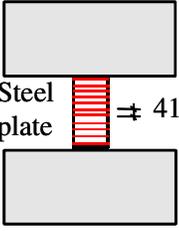
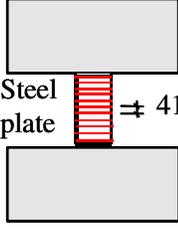
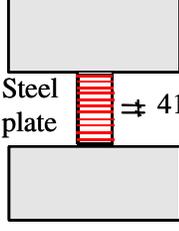


Fig. 3 Types of corner block

In specimens ER03S-P41S2 and ER03S-P41SN, the axial compression tests were carried out to confirm the residual axial compression capacity by returning the horizontal force to zero right after the shear failure. However, when axial force was increased to the compressive strength, compression failure might take place in a column with shear failure and thus, there seemed to be certain occasions where emergency retrofitting could not be implemented. Therefore, values of the residual axial compression capacity shown in this paper are assumed to be somewhat conservative before reaching to the compressive strength. After that, the axial force was returned to initial axial force (axial force ratio 0.2), and emergency retrofitting was implemented. In the same way, restorable axial strength of a damaged column after emergency retrofitting was measured. Afterwards, the cyclic loading tests were carried out again up to the drift angle

Table 2 Column specimens

	ER02S-P41S1	ER03S-P41S2	ER02S-P41S3	ER03S-P41SN
Specimen Unit : mm				
PC bar	5.4 ϕ @41			
Prestress(MPa)	490			Non
Steel plate	480 \times 240 \times 3.2			
σ_B (MPa)	23.9	28.3	25.0	28.3
Max. crack width	1.8	6.0	20	0.8
Damaged level	1	2	3	1
Compression test	×	○	×	○
Common details	Specimen: $M/(VD)=1.0$, $N/(bD \square \sigma_B)=0.2$, Rebar : 12-D10 (SD295) ($p_g=1.36\%$), Hoop : 3.7 ϕ -@105 ($p_w=0.08\%$)			

of 5% at a constant axial force ratio of 0.2. After that, the residual axial compression capacity was confirmed after returning the horizontal force to zero. Therefore, the axial compression tests were carried out at three times: first, for confirmation of residual axial compression capacity immediately after damage and before emergency retrofitting; second, for confirmation of restorable axial strength immediately after emergency retrofitting; and third, for confirmation of residual axial compression capacity after final cyclic loading test. Cyclic loading tests were carried out twice, before and after emergency retrofitting.

EXPERIMENTAL RESULTS AND DISCUSSIONS

Lateral shear force versus drift angle relationship ($V - R$ curve) before and after retrofitting in cyclic loading tests and average axial compressive strain versus drift angle relationship ($\epsilon_v - R$ curve) are shown in Fig. 4. A dashed line indicates the flexural strength of sound RC column without shear failure calculated by AIJ simplified equation. In addition, the vertical load versus average axial compressive strain relationship ($N - \epsilon_v$ curve) by axial compression test is shown in Fig. 5.

The specimen ER02S-P41S1 is the specimen for which damage level was assumed to be small damage, namely, Level 1 and the emergency retrofitting was provided. In the cyclic loading test before retrofitting, clear shear cracks were generated at $R=0.2\%$ of first cycle of loading and shear failure occurred at approximately $R=0.5\%$. As shown in Fig. 4, lateral shear force of the specimen before retrofitting decreased rapidly at approximately $R=0.5\%$; and maximum crack width after cyclic loading tests of $R=\pm 0.5\%$ with three cycles of loading became 1.8mm. On the other hand, it can be confirmed that the $V - R$ curve by cyclic loading test after retrofitting indicates typical elastoplastic flexural behavior and at $R=1.0\%$, lateral shear force is greater than the flexural strength calculated by AIJ simplified equation. For this reason, an emergency retrofit on damaged columns by utilizing pre-tensioned high-strength steel bars (PC bars) and steel plates is considered effective to recover flexural strength. Furthermore, it could be

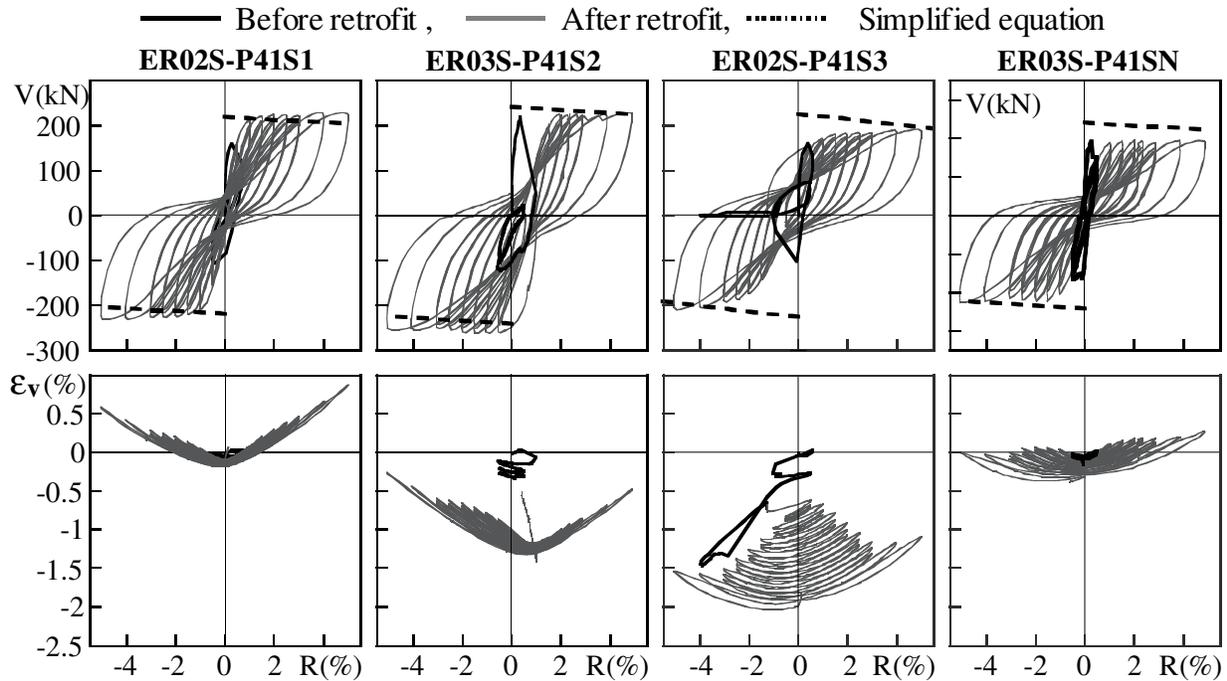


Fig. 4 Shear capacity V and average axial strain ϵ_v versus drift angle $R(\%)$ relationships

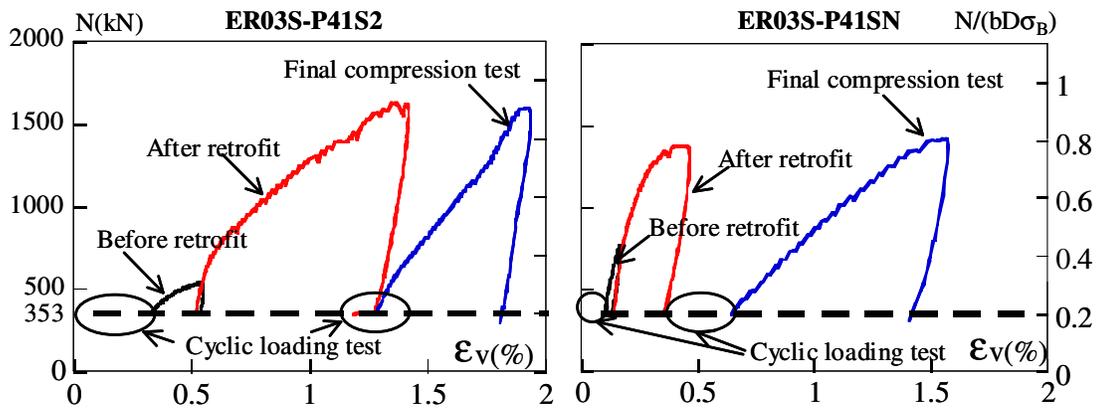


Fig. 5 Measured axial force vs. axial compression strain

confirmed that the maximum crack width after cyclic loading test was 1.8mm, which was the same as the crack width after the same tests before retrofitting; so the crack did not become wider, especially when drift angle ranges were greater.

The specimen ER03S-P41S2 is the specimen for which damage level was assumed to be moderate damage, namely, Level 2 and emergency retrofitting was provided. In the cyclic loading test before retrofitting, large shear cracks were generated at $R=0.4\%$ and at $R=1\%$ with one cycle of loading, the column failed in a brittle shear manner with the formation of large diagonal crack. The shear crack width was about 4-5mm. Then, in order to increase the damage level, the cyclic loading tests were carried out at the same drift angle with three cycles of loading. As a result, the crack width widened to about 6mm. The residual lateral displacement at that time was 0.09% ($R=0.09\%$). When the residual axial compression

capacity was confirmed in this situation, it was approximately $0.3 \sigma_B$ of an RC column (see Fig. 5). The crack width widened to 6.6mm in this axial load test. In the same situation after returning the axial force to an axial force ratio of 0.2 and implementing emergency retrofitting the axial compression tests were carried out again and restorable axial strength ($0.9 \sigma_B$ of an RC column) was confirmed. Then, cyclic loading tests were carried out with maintaining the axial force ratio of 0.2. As a result, although shear failure was prevented and shifted to elastoplastic flexural behavior as shown in Fig. 4, lateral capacity were slightly different in positive and negative sides and axial compression strain increased. In addition, values of lateral force are almost equal to those of flexural strength by AIJ simplified equation.

The specimen ER03S-P41S3 is the specimen for which damage level was assumed to be large damage, namely, Level 3 and emergency retrofitting was provided. In cyclic loading tests before retrofitting, initial shear cracks were generated at $R=0.2\%$ and shear failure occurred before $R=0.5\%$. Then, at the first cycle of $R=-0.5\%$, shear crack width widened to about 4-5mm. After the second cycle at $R=0.5\%$ and on the way to the second cycle at $R=-0.5\%$, the shear cracks widened and it was apprehended that cyclic loading tests could not be carried out. Thus, an axial force of 300kN was reduced to 60kN in a test setup. The shear crack width at that time reached 20mm and if the axial force ratio of 0.2 had been maintained, emergency retrofitting could not have been implemented due to the destruction of the column itself. For this reason, it was considered that the specimen was at a level equivalent to the critical damage level for emergency retrofitting. During the process to implement emergency retrofitting to the specimen, other longer corner blocks with the same bearing areas were used because the shear crack width was so wide that the first type of corner blocks were not usable (see Fig. 3). The axial force ratio was returned to 0.2 after emergency retrofitting. The residual drift angle ranges at that time decreased from $R=-3.9\%$ to $R=-1.3\%$. This meant that the crack width narrowed due to the introduction of prestress. On the other hand, during reversed cyclic loading tests after retrofitting the shear capacity recovered gradually and indicated desirable elastoplastic behavior. Even though the lateral force did not reach the flexural strength of sound column calculated by AIJ simplified equation, the lateral capacity almost recovered and ϵ_v was small. Furthermore, the maximum crack width after the tests was 2.5mm, which was smaller than the value before retrofitting.

Based upon the results of these tests, an emergency retrofit on damaged columns by utilizing pre-tensioned high strength steel bars (PC bars) and steel plates, is confirmed to prevent shear failure, to recover lateral capacity and to improve ductility. This means that if columns can still maintain axial force after an earthquake, it is possible that the lateral capacity can be recovered and also the ductility can be improved.

Next, the specimen ER03S-P41SN is the specimen for which damage level was assumed to be Level 1 and emergency retrofitting was provided. However, this is a specimen in which no prestress had been introduced. During cyclic loading tests before retrofitting, initial shear cracks occurred at $R=0.3\%$ and widened to approximately 1.0mm after three cycles of loading at $R=0.5\%$. Afterwards, by axial compression tests, it was confirmed that the residual axial compression capacity was approximately $0.4 \sigma_B$ of RC column. After confirmation, emergency retrofitting was implemented by using PC bars; and without introducing prestress, the second axial compression test was carried out to have the restorable axial compression capacity of RC column approximately $0.8 \sigma_B$ (see Fig. 5). Then cyclic loading test was carried out again. In the specimen ER02S-P41Sh with the same damage level, in which prestress had been introduced, the lateral force was greater than flexural strength calculated by AIJ simplified equation. However, on the other hand, in the same specimen without prestressing, the lateral capacity did not reach the flexural strength by AIJ simplified equation. From this fact it can be considered that emergency retrofitting without prestress would not allow the lateral capacity to recover to the level of sound columns, even though damage level of the damaged columns was small.

ANALYTICAL INVESTIGATION

The flexural strength is calculated by fiber model and AIJ simplified equation considering the active confinement effect by prestress induced by PC bars in addition to passive confinement effect by PC bars and steel plates. The enhancement of concrete strength due to active confinement is calculated based on the Richart's formula [4] as shown in the following. The constitutive law of concrete (stress-strain relationship) is considered according to Mander's model [5]. In order to calculate the shear strength enhancement by PC bars, the limiting strength of PC bar is considered 800 MPa instead of yield strength when no prestress is introduced, but when prestress is introduced then the difference between the yield strength and the applied prestress is considered not to exceed 800 MPa.

$$\sigma_{ac} = 4.1ke\sigma_r \quad ; \quad ke = \left(1 - \sum_{i=1}^4 \frac{(w'_i)^2}{6b^2}\right) \left(1 - \frac{s'}{2b}\right)^2 \quad ; \quad \sigma_r = \frac{2\sigma_p a_p}{sD} \quad (1)$$

where σ_{ac} = concrete strength increased by introducing prestress (active confinement effects), ke = effective confinement coefficient (the ratio of effectively confined concrete area to the total area of cross section) (see Fig. 6), σ_r = lateral pressure by introduction of prestress, w'_i = clear distance between corner blocks placed in column, b = width of column section, s' = clear distance between adjacent corner blocks in the vertical direction, σ_p = pre-tensioned stress of PC bar, a_p = section area of PC bar, s = interval of PC bar, and D = depth of column section.

The concrete strength of f'_{cc} used for flexural analysis is calculated by equation (2). Especially in this analysis, taking into account the damage level of specimens, the decreased concrete strength of σ_B' is used instead of the initial concrete strength of σ_B . Moreover, f'_{cc} is used to calculate shear strength:

: Effective area of confined concrete

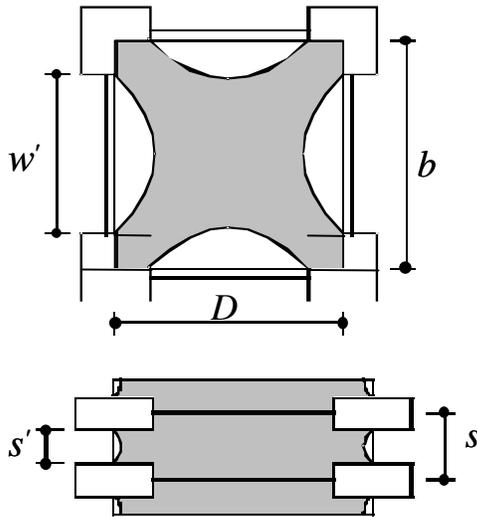


Fig. 6 Effectively confined concrete

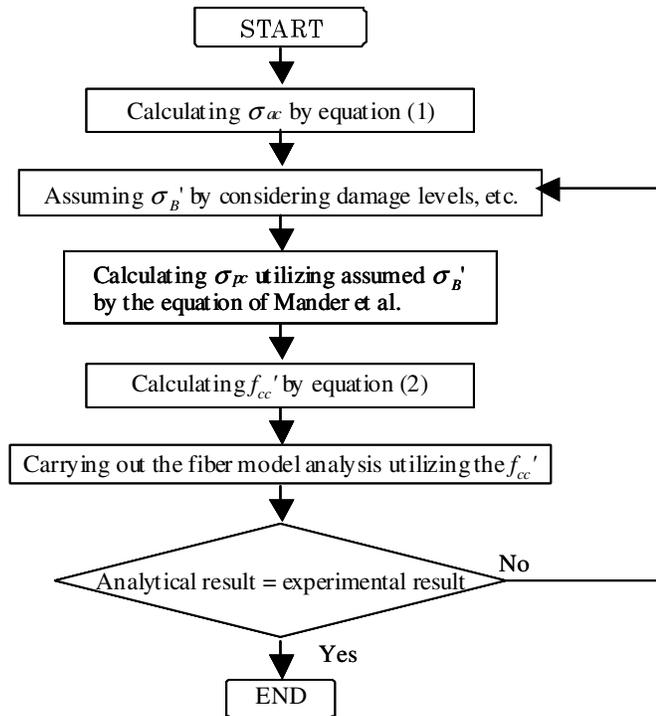


Fig. 7 Flowchart for calculation of lateral capacity

$$f'_{cc} = \sigma_B' + \sigma_{ac} + \sigma_{pc} \quad (2)$$

where f'_{cc} = concrete strength used for flexural and shear analysis, σ_B' = concrete strength decreased by damage, σ_{ac} = incremental concrete strength by active confinement effect, σ_{pc} = incremental concrete strength by passive confinement effect.

In deciding the value of σ_B' an arbitrary value of σ_B' is assumed first with respect to damage levels, as shown in Fig. 7. Using this value of σ_B' , σ_{pc} by passive confinement effect is calculated according to Mander's constitutive law. Then using f'_{cc} calculated by equation (2), the flexural strength is calculated. The calculations are repeated until the calculated and experimental results are equal. The calculated flexural and shear strengths in comparison to experimental skeleton curves are shown in Fig. 8. In case of specimen ER03S-P41SN, active confinement effect was not considered because prestress was not introduced. In Fig. 8, it is observed that the calculated results agree well with the experimental results. This means that the flexural capacity of shear damaged columns can be predicted by fiber model analysis and simplified equations with adopting the Mander's constitutive law of concrete that involves the recovery of concrete strength by active and passive confinement effects.

For all the specimens, at a drift angle of 1 to 1.5%, the shear strength calculated by AIJ simplified equation is larger than the experimental results, but after that, calculated values are smaller than experimental values and thus, calculated values are considered conservative. So, more investigations will be necessary.

The enhancement of concrete strength after retrofitting of different damaged specimens is shown in Fig. 9. For the specimen ER02S-P41S1, the damaged concrete strength was 7.2MPa (approximately $0.3\sigma_B$), which then recovered to 24.4MPa (little greater than σ_B) by retrofitting. In the same manner, the concrete strength values of the specimens ER03S-P41S2 and ER02S-P41S3 recovered from approximately 0.2 to $0.8\sigma_B$ and 0 to $0.7\sigma_B$ respectively. Though the values of concrete strength σ_B of the damaged columns based upon damage levels were 7.2MPa, 6.0MPa and 0MPa, the values of concrete strength with the addition of active and passive confinement effects ($\sigma_{ac} + \sigma_{pc}$) were 17.3MPa, 17.8MPa and 17.3MPa respectively, which were almost the same values regardless of the damage levels. On the other hand, the recovery of concrete strength of the specimen ER03S-P41SN in which prestress was not introduced was approximately $0.6\sigma_B$ despite the small damage level.

Therefore, an emergency retrofit on damaged columns by utilizing pre-tensioned PC bars and steel plates, is considered very effective to recover concrete strength due to the lateral confinement effect.

The maximum compressive stress resisted by shear damaged specimens of ER03-P41S2 and ER03-P41SN during axial compression test before and after retrofit are shown in Table 3. The axial compression force carried by the concrete is given by deducting the axial force carried by the longitudinal reinforcement. It is assumed that the longitudinal reinforcement strain is equal to the average axial strain of the column.

Experimentally, it was observed that in case of specimen ER03S-P41S2, the concrete strength before retrofitting was 3.7MPa and after retrofitting, that recovered to 21.2MPa. On the other hand, the strength of damaged concrete calculated by analysis was 6.0MPa, which recovered to 23.0MPa after retrofitting. In the damaged test specimens, axial compression capacity obtained in each compression test was a moderate value, because axial force was not applied until it was destroyed by compression. The concrete strength ($f'_{cc} - \sigma_B'$) recovered by retrofitting showed almost the same for experimental values and analysis results. Similarly, in case of ER03S-P41SN, the experimental results agree well with the analytical results.

Therefore, based upon this analysis method with concern for the decrease in concrete strength of damaged columns and the recovery of concrete strength by emergency retrofitting, active and passive confinement effects after the recovery can be evaluated; and thus, it is considered effective for the investigation of emergency retrofitting.

ANALYTICAL INVESTIGATION

The flexural strength is calculated by fiber model and AIJ simplified equation considering the active confinement effect by prestress induced by PC bars in addition to passive confinement effect by PC bars and steel plates. The enhancement of concrete strength due to active confinement is calculated based on the Richart's formula [4] as shown in the following. The constitutive law of concrete (stress-strain relationship) is considered according to Mander's model [5]. In order to calculate the shear strength enhancement by PC bars, the limiting strength of PC bar is considered 800 MPa instead of yield strength when no prestress is introduced, but when prestress is introduced then the difference between the yield strength and the applied prestress is considered not to exceed 800 MPa.

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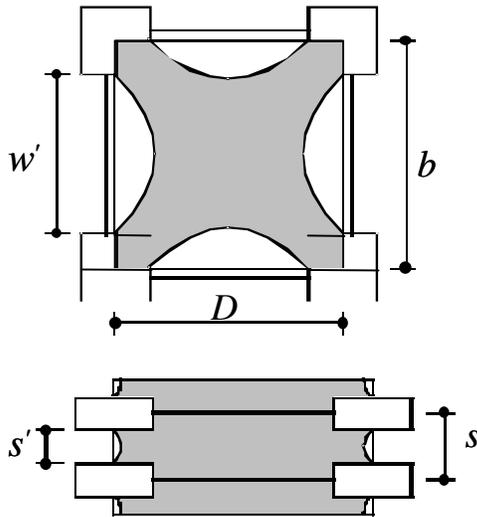


Fig. 6 Effectively confined concrete

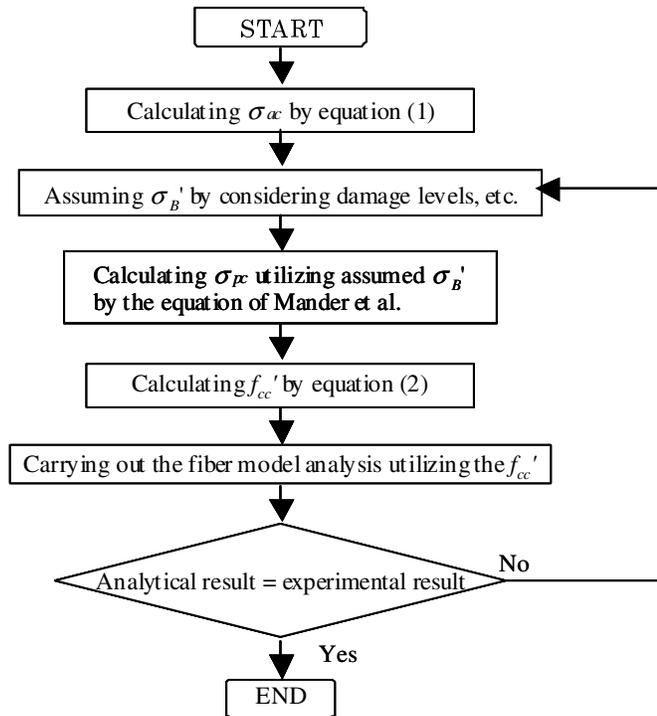


Fig. 7 Flowchart for calculation of lateral capacity

CONCLUSIONS

- (1) The proposed emergency retrofit technique utilizing high strength steel bar prestressing provides an effective tool for restoring the lateral capacity in addition to axial capacity of RC column damaged immediately after earthquake attack. Moreover, it is possible to drastically recover the earthquake performance to original sound condition, when the steel plates are also utilized. However, the recovery level is affected by the degree of damage.
- (2) From the experimental results of axial compression capacity immediately after applying the emergency retrofit on damaged columns it is proven that the concrete strength of damaged columns recover significantly.
- (3) The prestressing as an outstanding characteristic of this technique is a beneficial procedure that takes advantages of active confinement as well as passive confinement and shear strengthening.
- (4) The emergency retrofit can be applied quickly as a dry construction method without heavy machinery.
- (5) Moment curvature analysis considering effective compressive strength of cracked concrete was conducted in order to evaluate the retrofit effects. Active confinement effect from prestressing bars and additional confinement effects from the steel plates were also taken into account. The simulation gave good agreement with the test results, showing the design criteria for the retrofitting.
- (6) Based upon the comparative investigation of analytical results and axial compression tests carried out to confirm residual axial compression capacity, the analysis method concerning concrete strength is considered useful for the investigation of emergency retrofitting.

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