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STRATEGY OF WATER SUPPLY NETWORK RESTORATION WITH REDUNDANCY INDEX

Shigeru NAGATA [1] Kinya YAMAMOTO [2]

SUMMARY

Various factors such as damage characteristics, network formation, restoration materials and equipment, and the number of workers will affect the restoration of a water supply system. Moreover, since many of the conduits constituting a water supply network are buried in the ground and hence have more than one uncertain element, damage to a buried pipe conduit network should be treated as a random event in a network restoration strategy immediately after an earthquake.

In the present study, we not only study the form of the network and the probability of the occurrence of pipe conduit failure, but we also propose a method to evaluate a network restoration strategy that uses the redundancy index as an indicator. This index is defined by an information entropy.

As examples, we evaluate strategies for the restoration of the major water-pipe conduit network of the water distribution area in the Kobe water supply system.

INTRODUCTION

The 1995 Hyogoken-Nanbu earthquake caused such serious damage to the waterworks that it took about 2.5 months to completely restore it [1], thus having a great effect on the residents. Minimizing earthquake damage to lifelines, including water supply systems, by providing them with aseismic reinforcement and by other means is an important measure against earthquakes. However, in reality, it is impossible to make any structure perfectly immune from failure. Accordingly, to reduce the possible negative effects on the residents, it is also important to establish measures to quickly fix interrupted water supplies. Although it is difficult to predict all damage that will be caused by an earthquake to a water supply system consisting of a buried pipe conduit network [2], a prior examination of the optimum system restoration strategy should be done.

Various factors influence the restoration of a water supply system. These include the form of the network, the damage characteristics, the regional characteristics, the restoration materials and equipment, and the

[1] Chief Research Engineer, Kajima Technical Research Institute, Tokyo, Japan. Email: nagata-shigeru@kajima.com
[2] Professor, Kogyokusha College of Technology, Tokyo, Japan. E-mail: yamamoto@kogyokusha.ac.jp

number of workers. Moreover, every earthquake is governed by some uncertain elements, and its occurrence and the damage caused by it have to be treated as random events. On the other hand, this restoration problem cannot be solved without prior examination of water supply system restoration strategies.

In this study, we investigate the network and damage characteristics, and then examine a network restoration strategy that involves the redundancy index [3][4].

NETWORK RESTORATION STRATEGY WITH REDUNDANCY INDEX

Modeling the water supply system

Various factors such as the configuration of the network, the damage characteristics, the regional characteristics, restoration materials and equipment, and the number of workers influence the restoration of a water supply system. Moreover, because it is governed by some uncertain elements, an earthquake has to be treated as a random event. Therefore, we examine a system restoration strategy that involves the redundancy index [3][4] in which consideration is given to the form of the network and the damage characteristics.
An analysis network model of a real water supply system is developed to examine the system restoration strategies.

Major water pipes in the real water supply system are regarded here as links, sources of water such as water purification plants and service reservoirs are regarded as supply nodes, and simplified networks of small-diameter water pipes in the lower stratum of the water supply system are regarded as demand nodes that have water consumption or water consumers. Moreover, we assumed that an earthquake causes damage to links alone in the direction of their extension according to a Poisson stochastic process [5].
With the above analysis network model, we calculated the mode of water supply system failure using a Monte Carlo simulation method and determined the redundancy index for each demand node.

Redundancy index

Figure 1 is a Venn's diagram of the mode of failure caused to a water supply network system with *m* links. This diagram is based on the assumption that an earthquake causes damage to links alone.
The entire sample space *U* is given by equations (1), (2) and (3).

$$U = \{D_0, D_1, D_2, \cdots, D_{m-1}, D_F\} \quad (1)$$

$$U = O \cup NO \quad (2)$$

$$U = D \cup ND \quad (3)$$

Here, *O* in eq. (2) is the subset of operational modes in which the function of the system is maintained, whereas *NO* in the same equation is the subset of non-operational modes in which the function of the system fails. Maintenance of the system function means a state of the system in which some demand node is connected to some supply node through some link. Failure of the system function means a state of the system in which all demand nodes become disconnected from supply nodes. Whether or not a connection exists between the supply and demand nodes is a state that concerns each demand node in the water supply network, and, therefore, there are as many Venn's diagrams as there are demand nodes.
O and *NO* are given by the equations as follows.

$$O = \{D_0, D_1, D_2, \cdots, D_{m-1}\} \quad (4)$$

$$NO = \{D_F\} \quad (5)$$

D in eq. (3) is the subset of damage modes in which some damage is caused to some link and ND in the same formula is the subset of non-damage modes in which no link fails. They are given by the equations as follows.

$$D = \{D_1, D_2, \cdots, D_{m-1}, D_F\} \quad (6)$$

$$ND = \{D_0\} \quad (7)$$

Here, O in eq. (2) is the subset of operational modes in which the function of the system is maintained, whereas NO in the same formula is the subset of non-operational modes in which the function of the system fails. Moreover, D_0 is the mode in which none of the links fails. D_1 is the mode in which only one link fails and the function of the system is maintained. Also, D_{m-1} is the mode in which, while all the elements but one link fail, the function of the system is maintained. Finally, D_F is the mode in which the function of the system fails.

Modes D_0 to D_F are mutually exclusive and collectively exhaustive. Moreover, D_F includes modes ranging from d_{F1} to d_{Fm} as shown in eq. (8).

$$D_F = \{d_{F1}, d_{F2}, \cdots, d_{Fm-1}, d_{Fm}\} \quad (8)$$

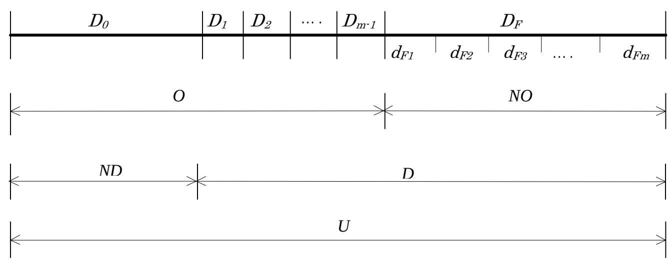


Figure 1 Venn's Diagram of the Mode of Failure Caused to Water Supply System

Here, d_{F1} is the mode in which only one link fails and the function of the entire system also fails. Similarly, d_{F2} is the mode in which two links fail and the function of the entire system also fails. d_{Fm-1} is the mode in which all links except one fail and the function of the system also fails, and d_{Fm} is the mode in which all m links fail and the function of the system also fails.

The redundancy index [3][4] is defined by the following equation with the information entropy [5][6][7]:

$$\begin{aligned} R_E &= H_D|_D / \log_2(m) \\ &= \frac{\{-\sum_{i=1}^{m-1} P_{D_i}|_D \log_2 P_{D_i}|_D - P_{D_F}|_D \log_2 P_{D_F}|_D\}}{\log_2(m)} \end{aligned} \quad (9)$$

Here, the numerator $H_D|_D$ of eq. (9) is the conditional information entropy for the subset of the mode in which one or more links fail under the conditions imposed by the damage occurrence mode D. $P_{D_i}|_D$ and $P_{D_F}|_D$ are the conditional probabilities of modes D_i and D_F, given that some damage D occurs to the system, and they are given by the following equations when the probability of occurrence of the mode D is represented by P_D.

$$P_{D_i}|_D = \frac{P(D_i)}{P_D} \quad (10)$$

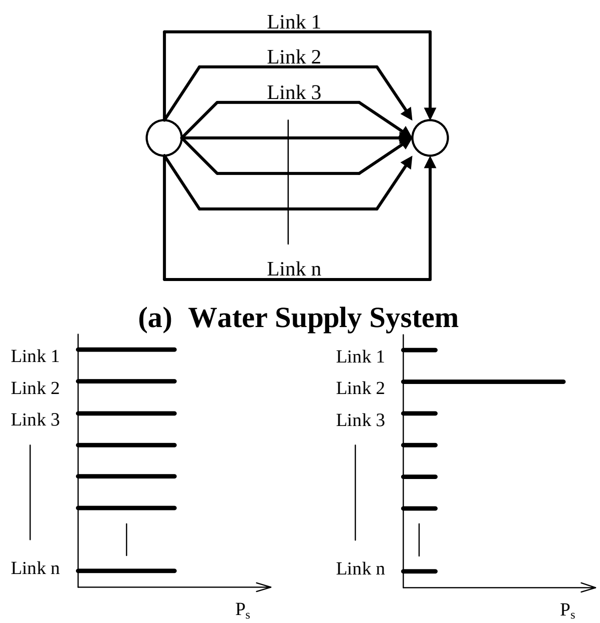
$$P_{D_F}|_D = \frac{P(D_F)}{P_D} \tag{11}$$

Between the mode D_1, in which one element fails, and the mode D_F, in which all the functions of the system are interrupted, there exist modes D_2, D_3, D_4 …… D_{m-1}. An earthquake causes the system to have one of these modes. Since the mode of the system is constant, the existence of many modes between the mode D_1 and the mode D_F ensures that no spare time exists. However, when compared with a system that is made to undergo a definite progress of modes from the mode D_1 to the mode D_F, this system is thought to have more time. The more time this system has is expressed as notional reserve distance.

The greater the reserve distance is, the more time and, therefore, the greater redundancy the system has. That is, the larger the $H_D|_D$, the greater both the reserve distance and the redundancy of the system. R_E defined by eq. (9) is normalized such as to range between 0 and 1 by dividing $H_D|_D$ by the maximum value, $log_2(m)$. Therefore, when R_E is 0, the redundancy of the system is minimum, and when it is 1, the redundancy is maximum.

Redundancy index and system restoration strategy

Figure 2 shows a diagram of the model of water supply system. Here, it is assumed that the system has n links that are arranged in parallel as shown in figure 2 (a). Also, we assume that water is supplied from the node on the left to the node (demand node) on the right in the direction of the arrows of links. If the probability of non-failure of each link is given by P_{Si}; i=1-n, the probability connectivity of two nodes is defined by the expression $1-\prod_{i=1}^{n}(1-\mathrm{P_{Si}})$. This probability of connectivity is used to ensure reliability of the lifeline network system in many cases [8][9][10].



(b) System 1 (c) System 2

Figure 2 Model of Water Supply System

The distributions of probabilities P_s of non-failure of links for two network systems that have the same reliability defined by the probability of connectivity are shown in Figs. 2 (b) and 2 (c). P_s of each link in System 1 is not so large and similar in value to those of other links. On the other hand, in System 2, the value of P_s for only one link is very high while those for other links are low. The following facts are made clear by a close look at the figures.

In System 1, because the probabilities of non-failure of all the links are almost equal, it is possible that at least one link may survive an earthquake without failure. On the other hand, in System 2, all the links except link 2 may almost definitely fail in an earthquake. Therefore, the reliability of System 2 is maintained by link 2 alone. The difference between these two systems cannot be made clear by the reliability defined by a probability of connectivity. However, that the redundancy of System 1 is greater than that of System 2 is revealed by the use of the redundancy index defined in eq. (9).

The above fact is examined by a simple calculation example below.

Figure 3 shows a simple water supply system consisting of two links. The probabilities of non-failure of the two links in such a system, which corresponds to System 1 in Figure 2 (b), are set to $P_1 = 0.9$ and $P_2 = 0.9$, whereas those for System 2 in Figure 2 (c) are set to $P_1 = 0.95$ and $P_2 = 0.8$. In this case, Systems 1 and 2 have an equal reliability of *0.99*, which is defined by the probability of connectivity of nodes A and B. However, the redundancy indices R_E of Systems 1 and 2 are *0.785* and *0.561*, respectively. This means that System 1 has a greater redundancy than System 2.

From the above, it is clear that, in System 2 of which the redundancy is small, the possibility that links other than link 2 can escape failure under the conditions which makes link 2 fail is small, and thus the failure of link 2 directly leads to the interruption of the system functions. On the other hand, in System 1, which has a greater redundancy, the failure of any one link does not mean the failure of the other links and the possibility that a few links may escape failure is large. Therefore, even when the system functions are interrupted, it is possible that each demand node with a greater redundancy index in System 1, unlike one in System 2, may have more than one slightly damaged link between it and its supply node.

Thus, when it is assumed that the function of the system can be maintained if its demand nodes are connected with its supply nodes via links, giving restoration priority to links that lead to demand nodes with greater redundancy indices should make the restoration of the system function more efficient.

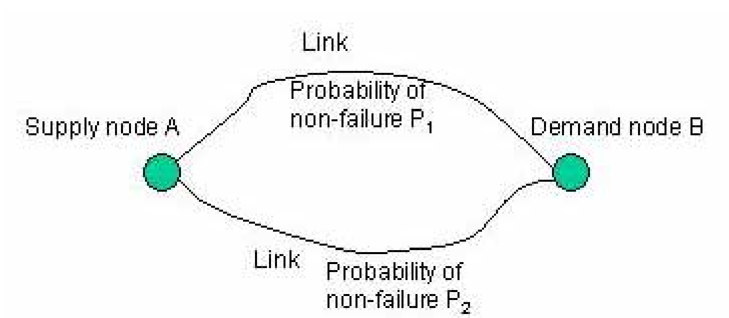


Figure 3 Simple Water Supply System

NUMERICAL EXAMPLES

As the numerical examples we analyze one of the urban water distribution areas in the Kobe water supply network [11][12]. To estimate the damage rate (number of damages / km), we use the equation for the water supply conduit damage estimation that is based on the peak ground acceleration (PGA) that was proposed by Isoyama, et al. [2].

Figure 4 and Table 1 show the model of the major water pipe conduit network.

The network restoration is simulated under the conditions shown below.

(a) Simultaneous restoration of more than one link is not attempted.

(b) The direction of water supply is shown by arrows attached to links and it is not changed even after earthquake occurrence.

(c) Restoration of links starts with the link that is nearest to the service reservoir or upstream and is conducted one by one in the direction of the arrows.

(d) The connection of a demand node with a supply node enables normal water consumption.

(e) The time required for the restoration is proportional to the number of damaged places in the links, regardless of the failure mode.

(f) In the restoration work, no consideration is given to the time for travel between work areas.

(g) The point of time when all the demand nodes are connected with supply nodes is regarded as the restoration completion time.

Three types of network restoration strategy (for cases I to III of numerical example) are established and the calculation is done under the above conditions.

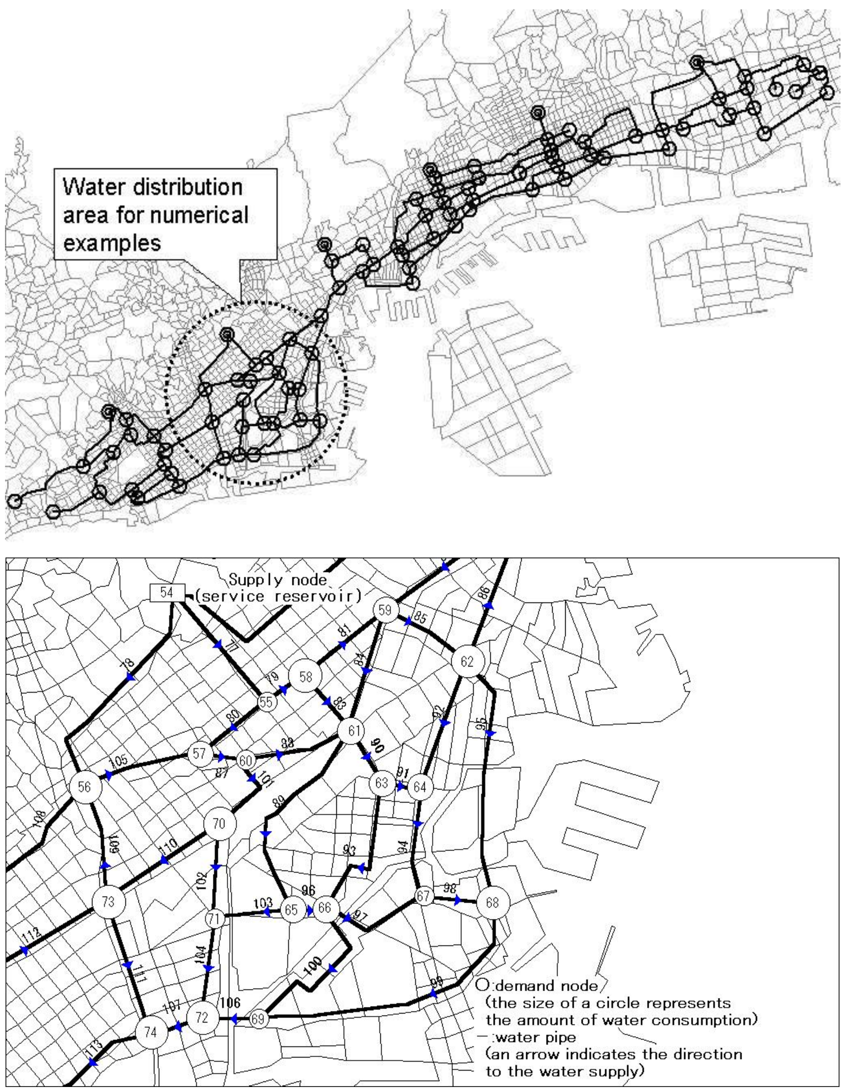


Figure 4 Water Supply Network for Numerical Example

Table 1 Data on Nodes and Links of Water Supply Network for Numerical Example

Link No.	Start node	End node	Average of damaged places	Distance of link (km)
77	54	55	0.362	0.780
78	54	56	1.088	1.358
79	55	58	0.104	0.269
80	55	57	0.179	0.508
81	58	59	0.228	0.610
83	58	61	0.937	0.411
84	59	61	0.433	0.728
85	59	62	0.213	0.557
87	57	60	0.162	0.270
88	60	61	0.428	0.607
89	61	65	1.157	1.246
90	61	63	1.083	0.359
91	63	64	0.186	0.223
92	62	64	0.675	0.788
93	63	66	0.785	0.911
94	64	67	0.687	0.652
95	62	68	2.113	1.473
96	65	66	0.158	0.180
97	66	67	0.540	0.613
98	67	68	0.627	0.403
99	68	69	1.809	1.744
100	66	69	0.839	1.014
101	60	70	0.330	0.480
102	70	71	0.469	0.532
103	65	71	0.410	0.443
104	71	72	0.510	0.560
105	56	57	0.902	0.701
106	69	72	0.258	0.311
107	72	74	0.298	0.311
109	56	73	0.393	0.682
110	70	73	0.452	0.789
111	73	74	0.617	0.810
			Total distance (km)	21.323

Node No.	Water consumption (m^3/day)
54	
55	171
56	599
57	314
58	456
59	399
60	200
61	285
62	570
63	257
64	314
65	257
66	285
67	200
68	570
69	228
70	570
71	228
72	599
73	599
74	570
Total water consumption (m^3/day)	7,670

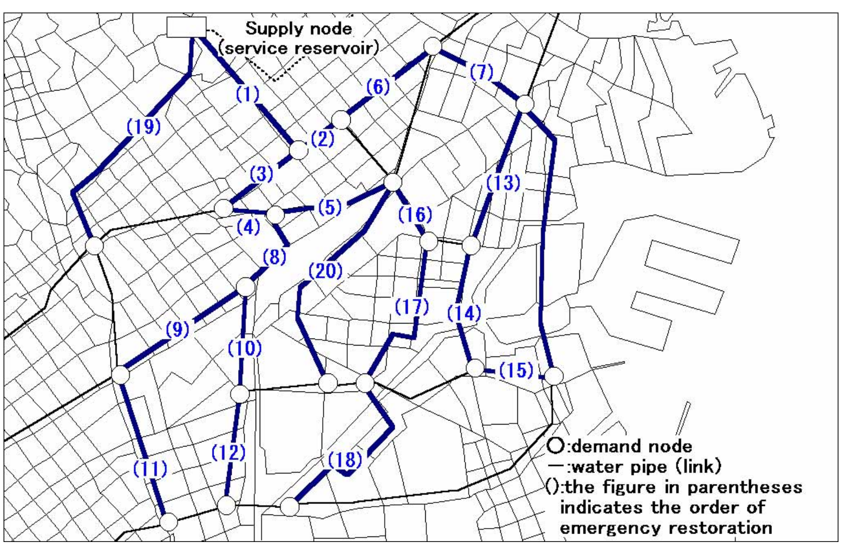


Figure 5 Calculation Result of Case I

Calculation result of case I shown in Figure 5 deals with a strategy in which priority for restoration is given to the node with the greater redundancy index. First of all, from among the demand nodes that are directly connected with a supply node without passing through other demand nodes, the one (node 55 in Figure 4) with the largest redundancy index is chosen and the link (link 77 in Figure 4) that connects it to the supply node is restored. Next, the probability P_f of failure in the restored link 77 is set at 0, and the redundancy index of each demand node is recalculated. Next, from among the demand nodes that are directly connected to either the supply node or the demand node 55, the one with the largest redundancy index is chosen and one link leading to it, the demand node, is restored. Then, the probability P_f that failure occurs to the restored link is set at 0, and the redundancy index of each demand node is recalculated to choose the next link to be restored. The above procedure is repeated until the end of the network restoration.

Here, five demand nodes are chosen, and the variations of their redundancy index that are effected by the time they are connected to the supply node are shown in Figure 6. There are 20 links needed to complete the network restoration by connecting through them all the demand nodes to the supply node in the network. After a demand node has been connected to the supply node, its redundancy index is no longer recalculated. The demand node 61 is connected to the supply node in the initial stage of the network restoration work. Demand nodes 71, 72, and 73 are connected to the supply node in the middle stage of the network restoration work. Also, demand node 56 is not connected to the supply node until the final stage of the network restoration work. Demand nodes that are restored early show a sharp rise in their redundancy indices immediately before the start of their restoration.

Calculation result of case II shown in Figure 7 deals with a strategy in which, for each demand node, we determine in every link, the route from the supply node to itself that has the smallest cumulative sum of the average number of damaged places. Then the link with the smallest average of the number of damaged places in that route is given priority for restoration.

Calculation result of case III shown in Figure 8 deals with a strategy in which for links needed to connect to the supply node, we choose and restore the demand node that supplies the most water to residents. When there are demand nodes that supply the same amount of water, the link that has the smallest average number of damaged places is restored. The intention of this strategy, however, is not to restore the demand node that supplies the greatest amount of water to residents in the system earlier than others, but to compare the amounts of water that demand nodes supply in the selection of the direction of restoration at a demand node and then to do restoration work towards the one that supplies more water.

The number in parentheses assigned to each link in Figures 5, 7 and 8 indicates the restoration order of the link.

Figure 9 shows the restoration curve based on the results of the calculation. On the horizontal axis is the restoration rate that is given by dividing the number of days elapsed in the restoration work by the number of days needed to complete the restoration work from calculation result of case II. On the vertical axis is the water supply rate that is given by dividing the cumulative water consumption that occurs because the water supply is resumed due to restoration work by the water consumption of the entire water supply system. The broken line that connects two points of coordinates (0, 0) and (1, 1) in Figure 9 has been drawn as a guide for the evaluation of network restoration.

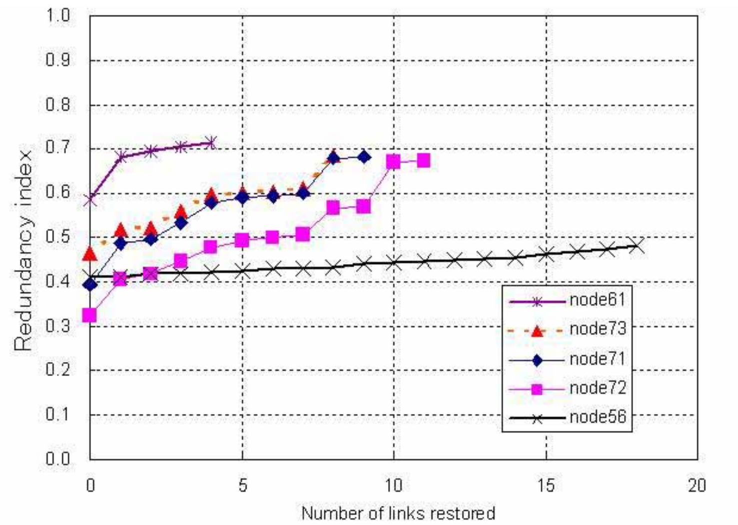


Figure 6 Variation of the Redundancy Index

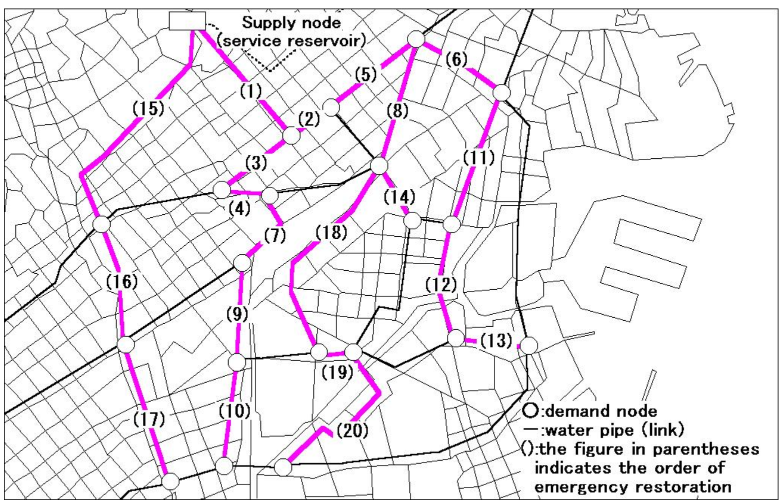


Figure 7 Calculation Result of Case II

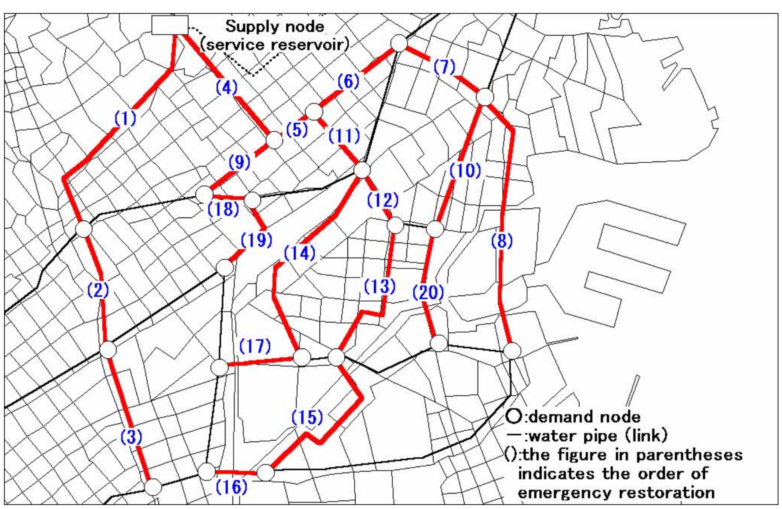


Figure 8 Calculation Result of Case III

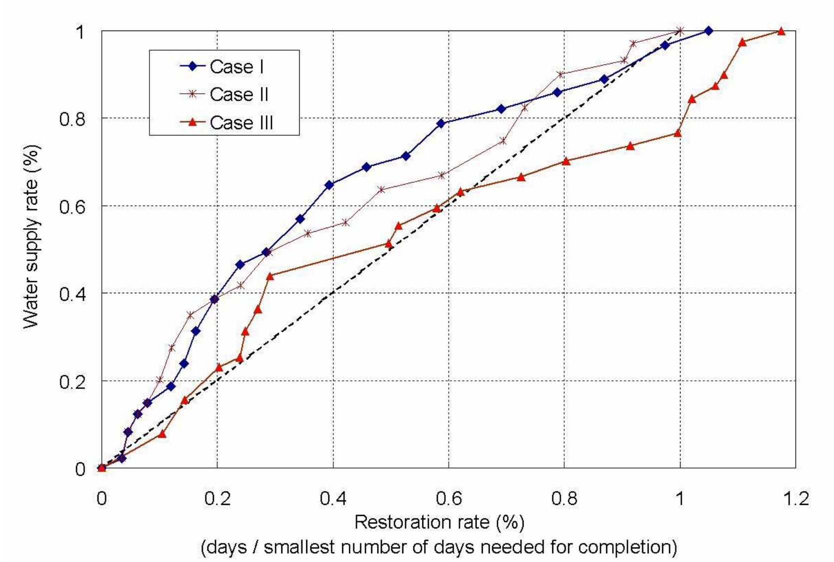


Figure 9 Restoration Curves of Numerical Analysis

The restoration curve obtained from calculation result of case II has a sharp rise in the initial stage of network restoration work and shows that, when the restoration rate is 0.3, the water supply rate is about 0.5. Thereafter, however, the curve has a more gentle rise such that a relatively long time is needed to raise the water supply rate from 0.5 to 0.8. For example, even when the restoration rate reaches 0.7, the water supply rate is still below 0.8.

The restoration curve obtained in calculation example III has the shape of a reversed letter S. Moreover, the curve, showing a slackening of the pace of network restoration, goes down below the broken line at the point where the water supply rate exceeds 0.6. Of the three strategies, the strategy for this example consumes the most time to complete network restoration.

On the other hand, the restoration curve obtained in calculation example I, in which the redundancy index is used as an indicator, is convex and shows that the water supply rate rises quickly at the middle stage of network restoration. But the curve is overtaken by the restoration curve of calculation example II at the point where the water supply rate exceeds 0.75, and shows that, beyond that point, much time is used by the time network restoration is completed (i.e., the time the water supply rate reaches 100%).

In many cases, the restoration of some areas do not occur until the final stage of network restoration because its damage has been especially serious. In the restoration work at the final stage, there are many cases where it is impossible to restore an area to the original state because houses in the area have collapsed or burnt down. For this reason, there is a view that a water supply rate of about 0.9. should be the upper limit of the network restoration evaluation. When restoration curves drawn up to the water supply rate of 0.9 are evaluated according to the above view, the one obtained from calculation example I is found to be relatively good.

In the future, efforts will be made to include a quantitative indicator that can show the relative merits of the restoration curves.

CONCLUSIONS

As a tentative procedure to examine water supply system restoration strategies and the strategy evaluation method, we did three numerical examples. The calculation results showed that the network restoration work that was put forward simply along the shortest route did not necessarily have the best result. Moreover, the strategy in which restoration was directed, successively and without a long perspective, at the demand node with the larger water consumption was also not the most effective. Among the three network restoration strategies in the examples, the one in which the redundancy index was used as an indicator produced relatively good results. This is the recommended strategy.

In the future, we will study strategies that include such factors as the restoration rate of conduits, the number of workers, the optimum placement of materials and equipment, transport of man and material between restoration sites, and other factors. In addition, we will develop a strategy evaluation method that includes the degree of annoyance that a given strategy gives to residents.

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