

PROPOSAL OF ONE-DIMENSIONAL DYNAMIC RESPONSE ANALYSIS METHOD OF EMBANKMENT AND SUPPORT GROUND SYSTEM CONSIDERING DYNAMIC INTERACTION WITH THE SIDE GROUND

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SUMMARY

This paper presents a one dimensional dynamic response analysis method to evaluate the dynamic response of an embankment and a support ground system based on the stiffness matrix method to consider the dynamic interaction between an embankment and a support ground. The motion of equation for a embankment is formulated under the assumption that the shape of embankment is a arbitrary trapezoid and the embankment is divided into some horizontally layered thin elements. The whole stiffness matrix for an embankment and a support ground system is obtained by considering not only the dynamic interaction between an embankment and a support ground but also that between the embankment-support ground system and the side layered ground. In order to verify an accuracy of the proposed method, seismic response analysis for two-dimensional FE model of embankment and support ground is carried out. Furthermore, this method is applied to estimate the dynamic response of the river dike where has been suffered by the northern Miyagi earthquake, July 26, 2003 Japan. It is found that the dynamic interaction between embankment-support ground system and side layered ground is very important role for evaluating the dynamic response of embankment and that the natural period at 1st mode for the river dike and support ground system calculated by the proposed method is good agreement with that obtained by the analysis of measured micro-tremor on the top of the river dike.

INTRODUCTION

A great deal of earth structure such as a embankment and a river dike has repeatedly suffered by the earthquake. However, it is difficult to design the structure which will not have any damage for the strong earthquake ground motion. Because, if the structure which satisfied the requirement mentioned above could be designed, a scale of the designed structure is too big to exist in this real space and much expenses are required to construct the structure. Therefore, it is necessary for the design concept that the damage of a structure is considered to be under a allowable limit. In order to establish such a design procedure, the deformation caused by a failure of ground has to be evaluated quantitatively. FEM [1] and sliding block method proposed by Newmark[2] have been used to evaluate the deformation. The former has an advantage to be able to consider the nonlinear behavior

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Figure.1 Schematic illustration of Layered embankment element

of soil directly under the complex soil and constructing condition. However, an accuracy of the analysis is remarkably depend on a quality of information and a performance of a engineer for modeling a material parameter and a ground structure. Newmark's method has been commonly used because the same slip circle with the ordinary design is possible to be used for the stability analysis during earthquake in the existing design code in Japan. However, a dynamic response of a sliding block in a earth structure is not considered in the method. Then, it is difficult to evaluate a large deformation accurately for a strong earthquake motion. Recently, modified Newmark's method which take into account of the effect by modeling a embankment as single degree of freedom system has been proposed [3],[4]. Furthermore, it has been pointed out that many damages for the river dike have observed at the embankment constructed on the soft ground [5.] This evidence shows it is important to consider the dynamic interaction between the embankment and the support ground. This report describes two subjects. One is to propose one-dimensional dynamic response analysis method for a embankment considering the dynamic interaction with not only the support ground just below a embankment but also the side layered ground. Second is the application of the method for evaluating the dynamic behavior of embankment suffered by the northern Miyagi earthquake, July 26,2003. Hence, the method is formulated by use of the stiffness matrix method proposed by kausel [6]. The accuracy of the method is verified by the comparison with the analytical result obtained by two-dimensional FE method.

ANALYTICAL METHOD FOR A EMBANKMENT AND A SUPPORT GROUND SYSTEM

Fundamental solution for layered embankment element

The following general assumptions for a one-dimensional analysis are used to model an embankment as a one-dimensional system: (1) vertical incident wave is considered as an input motion. (2) a deformation at each element is only a horizontal direction. Furthermore, the shape of embankment is assumed to be a trapezoid. A layered embankment element as shown in Fig.1 is defined to be horizontally sliced layer whose thickness and area of a horizontal close section at an arbitrary depth z is dx, A(z) respectively. Equilibrium equation (1) is introduced based on the balance between inertia force and shear force applied to boundary surface at the element. Here, G, ρ represent shear modulus and mass density respectively. Equation (2) expresses an area of a close section at an arbitrary depth z considering the slope angle of embankment. A variable z is changed to Z by use of equation (3) after substituting A(z) shown in equation (2) to equation (1). Then, the partial differential equation (4) is obtained. Next, a horizontal displacement u represents as the function of space variable and time variable based on separating variable technique. After substituting the new displacement into equation (4), a variable z is changed to x using equation (6). Then, equation (1) is expressed as Bessel equation (5) with respect to a spatial function $\phi(x)$.



embankment element



$$\frac{\gamma \cdot A(z)}{g} \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial z} \left(G \cdot A(z) \frac{\partial u}{\partial z} \right) \tag{1}$$

$$A(z) = B(z) \cdot 1 = 2a' \cdot z + B_0 = a \cdot z + B_0 \qquad (2), \qquad Z = a \cdot z + B_0 \qquad (3)$$

$$Z\frac{\partial^2 u}{\partial t^2} = a^2 V_s^2 \left(Z\frac{\partial^2 u}{\partial Z^2} + \frac{\partial u}{\partial Z}\right) \qquad (4), \quad \frac{d^2 \phi(z)}{dZ^2} + \frac{1}{Z}\frac{d\phi(z)}{dZ} + \frac{\omega^2}{a^2 V_s^2}\phi(z) = 0 \qquad (5), \quad x = \frac{\omega}{a \cdot V_s}Z \qquad (6)$$

$$\frac{\varphi(x)}{dx^2} + \frac{1}{x}\frac{d\varphi(x)}{dx} + \phi(x) = 0 \tag{7}$$

It is well known that s general solution of Bessel equation is expressed by use of Bessel function and Neumann function at 0th order. Then, a displacement and shear stress in a layered embankment element as a stationary solution with respect to a circular frequency ω is expressed as the following equation (8) with two unknown coefficient C, D. Here, k represents a wave number ω/V_s .

$$u(z,\omega) = [C \cdot J_0 \{k(z + \frac{B_0}{a})\} + D \cdot Y_0 \{k(z + \frac{B_0}{a})\}] \exp(i\omega t)$$

$$\tau(z,\omega) = G \frac{\partial u(z,\omega)}{\partial z} = -Gk[C \cdot J_1 \{k(z + \frac{B_0}{a})\} + D \cdot Y_1 \{k(z + \frac{B_0}{a})\}] \exp(i\omega t)$$
(8)

Stiffness matrix of a layered embankment element and a layered support ground

Lavered embankment element

The displacement and the shear stress at upper and lower boundary of m th element in a embankment as shown in Fig.2 are obtained by use of equation (8). By combining the four equations at both boundaries, equation (9) is obtained by eliminating the unknown coefficients. The equation has a important relationship to connect the displacement and the shear stress at between m th layer with those at m+1 th layer. By use of the method proposed by kausel et al, the equation (9) is changed to the relationship between the displacement at m and m+1 the boundary and the shear force at those boundary. The obtained equation (10) is the same with a static governing equation by use of FE

method. $\begin{bmatrix} {}_{e}K^{m}\end{bmatrix}$ represents stiffness matrix. ${}_{e}P_{m}$, ${}_{e}P_{m+1}$ are calculated shear forces by multiplying unit area to shear stress and those sign is noticed to be determined based on a sign with respect to a force. Subscripts for a displacement and a shear force represents location number of boundary under the top of embankment and is correspond to a nodal point which has been used in FE method. The components of the stiffness matrix are shown in equation (11).

$$\begin{cases} e^{u}_{m+1} \\ e^{\tau}_{m+1} \end{cases} = \begin{bmatrix} E^{m} \\ e^{\tau}_{m} \\ e^{\tau}_{m} \end{cases} = \begin{bmatrix} E^{m}_{11} & E^{m}_{12} \\ E^{m}_{21} & E^{m}_{22} \end{bmatrix} \begin{cases} e^{u}_{m} \\ e^{\tau}_{m} \\ e^{\tau}_{m} \end{cases}$$

$$\begin{cases} e^{P}_{m} \\ -e^{P}_{m+1} \\ e^{\tau}_{m+1} \end{cases} = \begin{bmatrix} e^{K^{m}} \\ e^{\tau}_{m} \\ e^{u}_{m+1} \\ e^{u}_{m+1} \\ e^{u}_{m+1} \end{cases}$$

$$= \begin{bmatrix} e^{k} K^{m}_{11} & e^{k} K^{m}_{12} \\ e^{k} K^{m}_{21} & e^{k} K^{m}_{22} \\ e^{k} K^{m}_{21} & e^{k} K^{m}_{22} \\ e^{u}_{m+1} \\ e^{u}_$$

$${}_{e}k_{11}^{m} = -G_{m}k_{m} \frac{J_{1}(kh_{m}^{0})Y_{0}(kh_{m}^{1}) - J_{0}(kh_{m}^{1})Y_{1}(kh_{m}^{0})}{J_{0}(kh_{m}^{0})Y_{0}(kh_{m}^{1}) - J_{0}(kh_{m}^{1})Y_{0}(kh_{m}^{0})}$$

$${}_{e}k_{12}^{m} = G_{m}k_{m} \frac{J_{1}(kh_{m}^{0})Y_{0}(kh_{m}^{0}) - J_{0}(kh_{m}^{0})Y_{1}(kh_{m}^{0})}{J_{0}(kh_{m}^{0})Y_{0}(kh_{m}^{1}) - J_{0}(kh_{m}^{1})Y_{0}(kh_{m}^{0})}$$

$${}_{e}k_{21}^{m} = G_{m}k_{m} \frac{J_{1}(kh_{m}^{1})Y_{0}(kh_{m}^{1}) - J_{0}(kh_{m}^{1})Y_{1}(kh_{m}^{1})}{J_{0}(kh_{m}^{0})Y_{0}(kh_{m}^{1}) - J_{0}(kh_{m}^{1})Y_{1}(kh_{m}^{1})}$$

$${}_{e}k_{22}^{m} = -G_{m}k_{m} \frac{J_{1}(kh_{m}^{1})Y_{0}(kh_{m}^{0}) - J_{0}(kh_{m}^{0})Y_{1}(kh_{m}^{1})}{J_{0}(kh_{m}^{0})Y_{0}(kh_{m}^{1}) - J_{0}(kh_{m}^{0})Y_{1}(kh_{m}^{1})}$$

$$(11)$$

Layered support ground

In a layered support ground just below a embankment mentioned above, stiffness matrix proposed for SH wave field by Kausel et al is used for expressing the relationship between displacement and shear force. The relationship and each component of stiffness matrix are shown in equations (12) and (13) respectively. ${}_{s}G_{n}$, ${}_{s}k_{n}$ and ${}_{s}h_{n}$ represent shear modulus, wave number and thickness in the n th layer respectively.

Assembling whole stiffness matrix and dynamic response analysis method

Considering force balance between the bottom of embankment and the top of the support ground just below the embankment, a the whole stiffness matrix [K] for embankment and support ground system as shown in fig.3 is assembled by use of the stiffness matrix for a embankment element and layered support ground element. Then, the displacement and shear force relationship is expressed as equation (13). The model as shown in fig.3 consists of a layered embankment element and three layered element in support ground including base layer. Here, U_e, U_s, P_e and P_s represent the displacement vector in the embankment, that in the support ground, the shear force vector in the embankment and that in the support ground respectively. The values of each vector component as the response are expressed by equation (14). Except the nodal force on the base layer, the all shear forces are zero. Each component of the whole stiffness matrix [K] is expressed as equation (15). Here,



Figure.4 Schematic illustration for the dynamic interaction between the embankment - support Ground system and the side layered ground

subscripts e, s for an each component represents embankment and support ground respectively and subscripts ij represent a raw and a column for a stiffness matrix of each layer element. Finally, superscripts i represents a number of element. The component k^3 in equation (15) represents the stiffness for considering the base layer as the elastic layer and is expressed in equation (16). So far, the formulation for the embankment and support ground system considering the dynamic interaction between the embankment and the support ground just below the embankment is described. The onedimensional system is named as a soil column A as shown in fig.4. Next, Another dynamic interaction between the soil column A and the side layered ground shown in fig.4 also has to be considered. The side layered ground which is named as the soil column B as shown in fig.4 don't have a embankment. Then, the displacement and shear force relationship is easy to be expressed as equation (17). The additional force vector P_s^* for the layer of both soil columns under the embankment is applied to satisfy with the condition that each component of the response displacement vector U_s^* in the soil column A caused by the seismic force P_B on the base layer and the additional force is equal to each component of that in the soil column B. Under the condition mentioned above, equilibrium matrix equation for each soil column is expressed as equations (18) and (19). By adding both equations, the displacement and shear force relationship for a whole system considering two kind of the dynamic interaction is obtained as the equation (20)

As a seismic response analysis for the embankment and support ground system, input motion u_B^s on the elastic base layer is considered as an incident component (2E). The shear force P_B^s on the base layer is expressed as equation (21) by use of the input motion and represents the applied seismic force for the whole system. Then, a response displacement is calculated by multiplying the inverse whole stiffness matrix $[K^*]^{-1}$ to the nodal force vector as shown in equation (22).

$$\begin{bmatrix} K \end{bmatrix} \begin{bmatrix} U_e \\ U_s \end{bmatrix} = \begin{bmatrix} P_e \\ P_s \end{bmatrix}$$
(13),
$$\begin{bmatrix} U_e \\ P_s \end{bmatrix} = (u_1^e), \quad \begin{bmatrix} U_e \\ P_s \end{bmatrix} = (u_1^s, u_2^s, u_B^s)^T \\ \begin{bmatrix} P_s \\ P_1 \end{bmatrix} = (P_1^e) = (0), \quad (P_1^s, P_2^s, P_B^s)^T = (0, 0, P_B^s)^T \end{bmatrix}$$
(14)



Figure.5 FE model for the embankment and the support ground system

$$[K] = \begin{bmatrix} {}_{e}k_{11}^{1} & {}_{e}k_{12}^{1} & 0 & 0 \\ {}_{e}k_{21}^{1} & {}_{e}k_{22}^{1} + {}_{s}k_{11}^{1} & {}_{s}k_{12}^{1} & 0 \\ 0 & {}_{s}k_{21}^{1} & {}_{s}k_{22}^{1} + {}_{s}k_{11}^{2} & {}_{s}k_{12}^{2} \\ 0 & 0 & {}_{s}k_{21}^{2} & {}_{s}k_{22}^{2} + {}_{k}k_{1}^{3} \end{bmatrix}$$
(15), $k^{3} = ik_{B}G_{B}$ (16)

$$\begin{bmatrix} {}_{f}K \end{bmatrix} \{ U_{f} \} = \begin{cases} 0(=P_{f}) \\ P_{B} \end{cases} \quad (18), \quad \begin{bmatrix} {}_{e}k_{s} \end{bmatrix} \{ U_{e}^{*} \} + \begin{bmatrix} {}_{s}k \end{bmatrix} \{ U_{s}^{*} \} = \begin{cases} -P_{s}^{*} \\ P_{B} \end{cases} \quad (19), \quad \begin{bmatrix} {}_{f}K \end{bmatrix} \{ U_{s}^{*} \} = \begin{cases} P_{s}^{*} \\ P_{B} \end{cases} \quad (20)$$

$$\begin{bmatrix} {}_{e}k & {}_{s}k_{e} \\ {}_{e}k_{s} & {}_{s}k \end{bmatrix} \begin{bmatrix} U_{e}^{*} \\ U_{s}^{*} \end{bmatrix} = \begin{bmatrix} k^{*} \end{bmatrix} \begin{bmatrix} U_{e}^{*} \\ U_{s}^{*} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 2P_{B} \end{bmatrix}$$
(21)

$$P_{B}^{s} = k^{3} u_{3,0}$$
(22),
$$\begin{cases} {}_{e} u_{1}^{*} & {}_{s} u_{1}^{*} & {}_{s} u_{2}^{*} & {}_{s} u_{3}^{*} \end{cases}^{T} \\ = [K^{*}]^{-1} \{ 0 \quad 0 \quad 0 \quad P_{B}^{s} \}^{T} \end{cases}$$
(23)

VERIFICATION OF THE PROPOSED METHOD WITH THE COMPARISON OF THE RESULTS OBTAINED BY TWO-DIMENSIONAL FE ANALYSIS

In order to verify the proposed one-dimensional seismic response analysis method for the embankment and the support ground system, an analytical result obtained by the proposed method is compared with an analytical result calculated by two-dimensional FE method. Stadas [7] is used for the analytical code for two-dimensional FE analysis. The analytical FE model is shown in Fig.5. The number of nodal point and that of element are 2930 and 2882 respectively. As the shape characteristics of embankment, the shape is a trapezoid. The slope angle is 45 degree. The height and the width of the top are 20m and 5m respectively. Furthermore, the shear wave velocity is 150m/s. The thickness and the shear wave velocity of the support ground are 30m and 200m/s respectively. In order to calculate the viscous damping for considering the base layer as the elastic layer, the shear wave velocity is 500m/s. The mass density and the poison ratio for all element are 1.8t/m³ and 0.49 respectively. The seismic record observed at Fukiai due to the Hyogoken Nanbu earthquake is used as the input motion.

Fig.6 shows the comparison of the response acceleration time histories at the top of the embankment obtained by 2D FE analysis to that obtaind by the proposed 1D analysis. Furthermore, Fig.7 shows the comparison of the frequency transfer function obtained by 2-dimensional FE analysis and that obtained by the proposed one-dimensional analysis method.



Figure.6 Comparison of the acceleration time history at the top of the embankment obtained by 2D FE analysis to that obtained by the proposed 1D analysis

The frequency transfer function between the top of the embankment and the base layer is calculated as the spectral ratio. It is found that both the response characteristics for the proposed 1D analysis with respect to an acceleration and a frequency transfer function are good agreement with those for 2D FE analysis. Therefore, the proposed 1D analysis has a good accuracy to apply the dynamic response of the embankment considering the dynamic interaction with the support ground.



Figure.7 Comparison of the frequency transfer Function by 2D FE analysis to that by the proposed 1D analysis

DYNAMIC BEHAVIOR OF THE SUFFERED RIVER DIKE DUE TO THE NORTHERN MIYAGI EARTHQUAKE, JULY 26,2003

Overview of the earthquake and the damages of river dike

The northern Miyagi earthquake which had 3 events happened within a day at July 26, 2003 and caused a serious damages. Main shock has a dip-slip type fault and the JMA magnitude is 6.2. Epicenters of each earthquake are located around Asahi Mountain. The focal depth of main shock is

about 10 km. Among the structural damages, many damages of the river dikes were caused by the main shock. The most serious damage of river dike at Kimazuka along Naruse river is shown in photo.1 The location of the damaged river dike around Kimazuka along Naruse river and the close section of the ground structure along Naruse river is shown in fig.8. It is found that the depth from the bottom of embankment to the base rock varies thinner from the lower stream of the river to the upper stream. The variance of the thickness is caused by the change of the soft clayey soil which exists under the embankment. The shear wave velocity the soil is about 110m/s. The damage has been supposed to be caused by the failure of the support ground due to



Photo.1 Damage of river dike at Kimazuka (Office of lower stream Kitakami River construction, Ministry of transportation)



a) Comparison of the natural period at 1st mode by microtremor with that obtained by the proposed method





Figure.9 Comparison of the natural period at 1st mode obtained by micro-tremor



dike(\times) and the ground structure along Naruse river Figure.8 The comparison of natural period for the river dike and the support ground system and the ground structure along Naruse river

b) The relationship between the damaged location of river

Figure.10 Comparison of the frequency transfer functions obtained by the proposed method

liquefaction and the failure of the embankment due to the strong earthquake motion. Especially, the damage at the upper stream around Furudake has supposed to be caused by the liquefaction.

Dynamic behavior of the suffered river dike

In order to make clear the vibrating characteristics of the dike-support ground system, micro-tremor was measured at 9 sites on the top of river dike. The observed sites was located at almost ever 0.5km along the river. The natural period of the dike-support ground system was obtained as the predominant period of H/V spectral ratio as shown in Fig.9 and the natural period at each observed site is shown in Fig.8a). Here, the component with respect to the longitudinal direction of the river dike is used as the horizontal component. It is found the natural period changes from 1.3 seconds to 0.65 seconds with the change of the depth between the bottom of the embankment and the base layer. Furthermore, the natural periods at 1st mode for the three sites where are located 13.5km, 15.5km and 16.5km from a front of the river was evaluated based on the frequency transfer function shown in Fig.10 calculated by the proposed 1D method and were shown in Fig.8a) as solid circle. The ground structure model on the base rock shown in Fig. 8b) is considered as the analytical models for calculating the frequency transfer functions. It is found that the predominant period obtained by the



b)Comparison of fourier spectrum Figure.11 Characteristics of the observed records around the front of Naruse river for the offshore Miyagi earthquake, May 26 and the northern Miyagi Earthquake, July 26



micro-tremor is good agreement with the natural period at 1st mode calculated for the ground structure model over the base rock by the proposed 1D method. It is noticed that the dynamic interaction between the embankment and the support ground including the side layered ground is important to evaluate the vibrating characteristics of the river dike.

Finally, the seismic responses at two sites where was located 13.5km and 16.5km from the front of river

were compared each other by use of the proposed 1D method. The seismic records observed in the ground at Nakasita around the front of Naruse river were used as the input motion. Not only the NS component observed at the main shock but also that observed at the offshore Miyagi earthquake were use for the analysis. Those acceleration time history and fourier spectrum are shown in Fig.11. The maximum response of shear stress and acceleration with depth are shown in Fig.12. Here, the converged shear modulus and damping constant obtained by a nonlinear dynamic response analysis in frequency domain for the layered ground under the river dike are used as the material properties of soil for considering the non-linearity of soil. It is found that each dynamic responses for the record observed at the earthquake, May 26. At the northern Miyagi earthquake, the response under the depth of 3.0m at the site located 13.5km is larger than that at the site located 16.5km. It is found that the difference of the embankment response is caused by the difference of failure mode at each site is occurred.

CONCLUDING REMARKS

One of the subjects on this report is to propose one-dimensional dynamic response analysis method for a embankment considering the dynamic interaction with the support ground just below a embankment and with the side layered ground. Here, a few features of this method is that the stiffness matrix method proposed by kausel [6] is used to formulate the governing equation of the motion and that the stiffness matrix for the layered embankment element is introduced. Twodimensional FE analysis is used to make sure the accuracy of the method. Second subject is that this proposed method is applied to evaluate the dynamic behavior of embankment suffered by the northern Miyagi earthquake, July 26,2003. Major results are as follows:

- (1) The proposed 1D analysis is applicable to evaluate the dynamic response of embankment accurately because the analytical results by 2D FE analysis is good agreement with those by the proposed 1D analysis.
- (2) The predominant period of the vibration for the river dike is correspond to the natural period at 1st mode for the river dike and support ground system.
- (3) The thickness of soft clayey soil under the embankment and the characteristics of the earthquake motion is a important role to evaluate the damage degree of the river dike.

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