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IDENTIFICATION OF STRUCTURAL DAMAGE BASED ON VIBRATION RESPONSES

Aiko FURUKAWA[1] and Junji KIYONO[2]

SUMMARY

A technique for detecting damage to structures is proposed that uses FRF data generated by the harmonic excitation force. It is based on the fact that structural damage usually causes a decrease in structural stiffness and an increase in structural damping, thereby producing change in vibration characteristics. This change in vibration responses provides useful information about the location and severity of the damage done. Damage identification equations were established by comparing the equations of motion of the structure before and after damage. The proposed technique is advantageous in that data are easily accumulated solely by changing the excitation and measurement points and the excitation frequency. The optimal arrangement of the apparatus for obtaining good identification results was determined by numerical simulations. When the number of measurements was insufficient, a grouping method, which sets up a well-posed problem for a small number of data, was introduced. Numerical results for a cantilever beam are presented to assess the validity of the proposed technique.

INTRODUCTION

During the 1995 Hyogo-ken Nanbu Earthquake, a great number of structures suffered various levels of severe damage. When a large earthquake occurs, the damage done to important infrastructures must be assessed and repairs made immediately to prevent the expansion of secondary damage. Damage to a structure accumulates when it undergoes natural disasters, such as typhoons, heavy rains, and strong earthquakes, as well as because of daily loading, corrosion, fatigue, and its inevitable aging. Damage assessment as to whether a structure needs repair or reinforcement therefore must be made and adequate measures taken to avoid catastrophic situations. To meet these needs, damage detection techniques have been developed.

The conventional way of detecting damage is by visual inspection, considered an efficient approach because the structure's condition and damage can be easily and simply checked. But the increased size and complexity of recently erected structures have lessened the efficacy of visual inspection. Moreover, when damage is inevitable inside the finish materials and fire proofing, removal of the facing material is

[1] Department of Urban Management, Kyoto University, Japan Email: furukawa@quake2.kuciv.kyoto-u.ac.jp
[2] Department of Urban Management, Kyoto University, Japan Email: kiyono@quake.kuciv.kyoto-u.ac.jp

necessary and that may be costly and time consuming. With improvements in measuring apparatuses and technology, various new damage detection techniques have been developed through experiments and measurements. Such nondestructive damage evaluation methods as ultrasonic, acoustic emission and x-ray inspections are techniques which indirectly investigate structural defects without tempering with the structure [1]. Slight damage which cannot be seen by the human eye can be detected with these techniques, but, only the local damage not the global structural damage condition.

Vibration-based damage detection is a technique that evaluates the global structural condition. It is based on the fact that structural damage usually causes a decrease in structural stiffness and an increase in structural damping, thereby producing change in vibration characteristics. Vibration data used to detect structural damage fall into two categories; modal data and the frequency response function (FRF) [2], [3], [4]. Modal data include the natural frequency [5], [6], mode shape [7], [8], mode shape curvature [9], modal flexibility [10], and modal strain energy [11], [12].

As pointed out by Banks [13], Wang [2], and Lee [4], the use of modal data has disadvantages. As they are indirectly measured test data, they may be contaminated by measurement as well as modal extraction errors. In addition, the majority of methods require complete modal data, which cannot be obtained in most cases because a large number of sensors usually are required. In contrast, the use of FRF data has advantages: FRF data are less contaminated because they are measured directly from structures. Also, they provide much more information about damage in a desired frequency range than do modal data because the latter mainly are extracted from very limited FRF data related to resonance [2]. For these reasons, the use of FRF data has greater potential than the use of modal data.

A damage detection technique by which to identify both the location and magnitude of damage, is proposed which uses FRF data obtained by harmonic excitation force. In that procedure, the structure first is excited at one excitation point by the harmonic excitation force. Acceleration responses at one measurement point then are measured and the acceleration responses converted to FRF data. Damage is assumed to be accompanied by changes in structural parameters, a decrease in stiffness and an increase in damping, and that such change alters the FRF data. Comparison of the equations of motion of the structure before and after damage provides damage identification equations, that relate local changes in structural parameters to change in the FRF data. If FRF data can be obtained, then changes in the structural parameters can be determined by solving the damage identification equations. Change in a structural parameter directly pinpoints the location and magnitude of the damage done. Our proposed method that uses FRF data has several advantages. The major one is that data are easily accumulated, solely by changing the excitation and measurement points and the excitation frequency. When as many measurements as there are unknown parameters are made, the damage identification equations present a determined problem. Also, by conducting more measurements than there are unknown parameters, the measurement noise effects are reduced. Owing to these advantages, more realistic modeling is feasible which reduces modeling errors.

In the next section, the theoretical algorithm of the proposed damage detection technique first is modeled. The optimal apparatus arrangement then is investigated through numerical simulations to obtain good identification results. When the number of measurements is insufficient, the damage identification equations give an underdetermined problem. In that case, a grouping method, which converts an ill-posed problem to a well-posed determined one with insufficient data, is introduced. The validity and accuracy of this proposed damage detection technique are shown through numerical studies of a cantilever beam.

PROPOSED DAMAGE DETECTION METHOD

Vibration response of an intact structure

The equation of motion for an intact structure is

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{f\}\cos\omega t \tag{1}$$

where $[M]$, $[C]$, and $[K]$ respectively are the mass, damping, and stiffness matrices of the structure. $\{x\}$ is the displacement response vector, $\{f\}$ the amplitude vector of the harmonic external force, and ω the excitation circular frequency.

The forced vibration response is obtained by solving the equation of motion;

$$\{x\}=[R(\omega)]\{f\}\cos\omega t-[I(\omega)]\{f\}\sin\omega t \tag{2}$$

where $[R(\omega)]$ and $[I(\omega)]$ are the real and imaginary parts of the transfer function for an intact structure;

$$[R(\omega)]=\mathrm{Re}\left(\left[-\omega^2[M]+i\omega[C]+[K]\right]^{-1}\right) \tag{3}$$

$$[I(\omega)]=\mathrm{Im}\left(\left[-\omega^2[M]+i\omega[C]+[K]\right]^{-1}\right) \tag{4}$$

Modeling of damage

The total stiffness $[K]$ and damping $[C]$ matrices of a structure in the intact state are the summation of the element matrices;

$$[K]=\sum_{e=1}^{n}[K^e] \tag{5}$$

$$[C]=\sum_{e=1}^{n}[C^e] \tag{6}$$

where n is the number of elements, and $[K^e]$ and $[C^e]$ (e=1, …, n) are the contribution of the e-th element to the total stiffness and damping matrices of the intact structure.

Damage to the structure is assumed to cause a decrease of $[\delta K]$ in the stiffness matrix and an increase of $[\delta C]$ in the damping matrix. Further, damage is assumed not to be accompanied by a change in the mass matrix. Changes in the e-th element stiffness $[\delta K^e]$ and damping $[\delta C^e]$ matrices are assumed to be proportional to the element matrices;

$$[\delta K^e]=\delta k_e[K^e] \tag{7}$$

$$[\delta C^e]=\delta c_e[C^e] \tag{8}$$

where δk_e and δc_e respectively are proportional changes in the stiffness and damping of e-th element.

Variations in the total stiffness and damping matrices therefore are expressed as the summation of the changes in element stiffness and the damping matrices;

$$[\delta K]=\sum_{e=1}^{n}\delta k_e[K^e] \tag{9}$$

$$[\delta C]=\sum_{e=1}^{n}\delta c_e[C^e] \tag{10}$$

where δk_e and δc_e the parameters to be identified, indicate the location and magnitude of damage. If δk_e and δc_e equal 0.0, the e-th element is not damaged, if δk_e or δc_e is larger than 0.0, the e-th element is damaged and the value of the parameter indicates the magnitude of damage. In contrast, when structures are reinforced, δk_e or δc_e may become smaller than 0.0. Such values indicate the efficiency of reinforcement of the e-th element.

Vibration response of a damaged structure

The equation of motion for a damaged structure is

$$[M](\{\ddot{x}\}+\{\delta\ddot{x}\})+([C]+[\delta C])(\{\dot{x}\}+\{\delta\dot{x}\})+([K]-[\delta K])(\{x\}+\{\delta x\})=\{f\}\cos\omega t \tag{11}$$

where $[\delta K]$ and $[\delta C]$ are respectively variations in the stiffness and damping matrices, and $\{\delta x\}$ is the increase of displacement response.

Substituting Eq. (1) into Eq. (11) and applying Eqs. (9) and (10) gives the equation for $\{\delta x\}$;

$$[M]\{\delta\ddot{x}\}+([C]+[\delta C])\{\delta\dot{x}\}+([K]-[\delta K])\{\delta x\}=\sum_{e=1}^{n}\delta k_e[K^e]\{x\}-\sum_{e=1}^{n}\delta c_e[C^e]\{\dot{x}\} \tag{12}$$

Substituting Eq. (2) into Eq. (12) gives

$$\begin{aligned}&[M]\{\delta\ddot{x}\}+([C]+[\delta C])\{\delta\dot{x}\}+([K]-[\delta K])\{\delta x\}\\&=\sum_{e=1}^{n}\delta k_e[K^e]([R(\omega)]\cos\omega t-[I(\omega)]\sin\omega t)\{f\}+\omega\sum_{e=1}^{n}\delta c_e[C^e]([R(\omega)]\sin\omega t+[I(\omega)]\cos\omega t)\{f\}\end{aligned} \tag{13}$$

The increase in response, $\{\delta x\}$, is obtained by solving Eq. (13);

$$\begin{aligned}\{\delta x\}=&\left\{\sum_{e=1}^{n}\delta k_e\begin{pmatrix}[R'(\omega,\mathbf{\delta c},\mathbf{\delta k})][K^e][R_H(\omega)]\\-[I'(\omega,\mathbf{\delta c},\mathbf{\delta k})][K^e][I_H(\omega)]\end{pmatrix}+\omega\sum_{e=1}^{n}\delta c_e\begin{pmatrix}[R'(\omega,\mathbf{\delta c},\mathbf{\delta k})][C^e][I_H(\omega)]\\+[I'(\omega,\mathbf{\delta c},\mathbf{\delta k})][C^e][R_H(\omega)]\end{pmatrix}\right\}\{f\}\cos\omega t\\&+\left\{-\sum_{e=1}^{n}\delta k_e\begin{pmatrix}[R'(\omega,\mathbf{\delta c},\mathbf{\delta k})][K^e][I_H(\omega)]\\+[I'(\omega,\mathbf{\delta c},\mathbf{\delta k})][K^e][R_H(\omega)]\end{pmatrix}+\omega\sum_{e=1}^{n}\delta c_e\begin{pmatrix}[R'(\omega,\mathbf{\delta c},\mathbf{\delta k})][C^e][R_H(\omega)]\\-[I'(\omega,\mathbf{\delta c},\mathbf{\delta k})][C^e][I_H(\omega)]\end{pmatrix}\right\}\{f\}\sin\omega t\end{aligned} \tag{14}$$

where $[R'(\omega,\mathbf{\delta c},\mathbf{\delta k})]$ and $[I'(\omega,\mathbf{\delta c},\mathbf{\delta k})]$ are the real an imaginary parts of the transfer function for the damaged structure, and both are functions with respect to, $\omega,\delta c_e$, and δk_e (e=1, c,n) expressed as

$$[R'(\omega,\mathbf{\delta c},\mathbf{\delta k})]=\mathrm{Re}\left(\left[-\omega^2[M]+i\omega([C]+\sum_{e=1}^{n}\delta c_e[C^e])+([K]-\sum_{e=1}^{n}\delta k_e[K^e])\right]^{-1}\right) \tag{15}$$

$$[I'(\omega,\mathbf{\delta c},\mathbf{\delta k})]=\mathrm{Im}\left(\left[-\omega^2[M]+i\omega([C]+\sum_{e=1}^{n}\delta c_e[C^e])+([K]-\sum_{e=1}^{n}\delta k_e[K^e])\right]^{-1}\right) \tag{16}$$

Displacement response after damage, $\{x'\}$, is obtained by adding $\{x\}$ in Eq. (2) to $\{\delta x\}$ in Eq. (14).

$$\begin{aligned}\{x'\}=\{x\}+\{\delta x\}=&\left([R(\omega)]+\sum_{e=1}^{n}\delta k_e[S^e(\omega,\mathbf{\delta c},\mathbf{\delta k})]+\omega\sum_{e=1}^{n}\delta c_e[T^e(\omega,\mathbf{\delta c},\mathbf{\delta k})]\right)\{f\}\cos\omega t\\&-\left([I(\omega)]+\sum_{e=1}^{n}\delta k_e[U^e(\omega,\mathbf{\delta c},\mathbf{\delta k})]+\omega\sum_{e=1}^{n}\delta c_e[V^e(\omega,\mathbf{\delta c},\mathbf{\delta k})]\right)\{f\}\sin\omega t\end{aligned} \tag{17}$$

where $[S^e(\omega,\mathbf{\delta c},\mathbf{\delta k})]$, $[T^e(\omega,\mathbf{\delta c},\mathbf{\delta k})]$, $[U^e(\omega,\mathbf{\delta c},\mathbf{\delta k})]$, and $[V^e(\omega,\mathbf{\delta c},\mathbf{\delta k})]$ are

$$[S^e(\omega,\mathbf{\delta c},\mathbf{\delta k})]=[R'(\omega,\mathbf{\delta c},\mathbf{\delta k})][K^e][R(\omega)]-[I'(\omega,\mathbf{\delta c},\mathbf{\delta k})][K^e][I(\omega)] \tag{18}$$

$$[T^e(\omega,\mathbf{\delta c},\mathbf{\delta k})]=[R'(\omega,\mathbf{\delta c},\mathbf{\delta k})][C^e][I(\omega)]+[I'(\omega,\mathbf{\delta c},\mathbf{\delta k})][C^e][R(\omega)] \tag{19}$$

$$[U^e(\omega,\mathbf{\delta c},\mathbf{\delta k})]=[R'(\omega,\mathbf{\delta c},\mathbf{\delta k})][K^e][I(\omega)]+[I'(\omega,\mathbf{\delta c},\mathbf{\delta k})][K^e][R(\omega)] \tag{20}$$

$$[V^e(\omega,\mathbf{\delta c},\mathbf{\delta k})]=-[R'(\omega,\mathbf{\delta c},\mathbf{\delta k})][C^e][R(\omega)]+[I'(\omega,\mathbf{\delta c},\mathbf{\delta k})][C^e][I(\omega)] \tag{21}$$

The Fourier amplitude $\{X'(\omega)\}$ of the displacement response in the damaged state $\{x'\}$ is

$$\{X'(\omega)\} = \left([R(\omega)] + \sum_{e=1}^{n} \delta k_e [S^e(\omega, \boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k})] + \omega \sum_{e=1}^{n} \delta c_e [T^e(\omega, \boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k})]\right)\{f\}$$
$$+ i\left([I(\omega)] + \sum_{e=1}^{n} \delta k_e [U^e(\omega, \boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k})] + \omega \sum_{e=1}^{n} \delta c_e [V^e(\omega, \boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k})]\right)\{f\} \tag{22}$$

Development of damage identification equations

In this study, the frequency response function (FRF), obtained by dividing the Fourier amplitude of the acceleration response by the amplitude of the harmonic excitation force, is used. Assuming that the excitation force is applied at point *j* and acceleration is measured at point *i*, the real and imaginary parts of the FRF, $a_R(i, j, \omega)$ and $a_I(i, j, \omega)$, are

$$a_R(i, j, \omega) = -\omega^2 \left(R_{ij}(\omega) + \sum_{e=1}^{n} \delta k_e S_{ij}^e(\omega, \boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k}) + \omega \sum_{e=1}^{n} \delta c_e T_{ij}^e(\omega, \boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k}) \right) \tag{23}$$

$$a_I(i, j, \omega) = -\omega^2 \left(I_{ij}(\omega) + \sum_{e=1}^{n} \delta k_e U_{ij}^w(\omega, \boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k}) + \omega \sum_{e=1}^{n} \delta c_e V_{ij}^e(\omega, \boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k}) \right) \tag{24}$$

In Eqs. (23) and (24), both $R_{ij}(\omega)$ and $I_{ij}(\omega)$ are known values obtained from parameters [*M*], [*C*] and [*K*] of the intact structure and the excitation circular frequency, ω. The real and imaginary parts of the FRF, $a_R(i,j,\omega)$ and $a_I(i,j,\omega)$, are obtained from measurements of the damaged structure.

$[S^e(\omega,\boldsymbol{\delta}\mathbf{c},\boldsymbol{\delta}\mathbf{k})]$, $[T^e(\omega,\boldsymbol{\delta}\mathbf{c},\boldsymbol{\delta}\mathbf{k})]$, $[U^e(\omega,\boldsymbol{\delta}\mathbf{c},\boldsymbol{\delta}\mathbf{k})]$, and $[V^e(\omega,\boldsymbol{\delta}\mathbf{c},\boldsymbol{\delta}\mathbf{k})]$, however, are comprised of the known values [*M*], [*C*], [*K*], and ω, and the unknown parameters δc_e and δk_e.

Transposing the unknown terms to the left side and the known ones to the right side, Eqs.(23) and (24) become

$$-\omega^2 \sum_{e=1}^{n} \delta k_e S_{ij}^e(\omega, \boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k}) - \omega^3 \sum_{e=1}^{n} \delta c_e T_{ij}^e(\omega, \boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k}) = a_R(i, j, \omega) + \omega^2 R_{ij}(\omega) \tag{25}$$

$$-\omega^2 \sum_{e=1}^{n} \delta k_e U_{ij}^e(\omega, \boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k}) - \omega^3 \sum_{e=1}^{n} \delta c_e V_{ij}^e(\omega, \boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k}) = a_I(i, j, \omega) + \omega^2 I_{ij}(\omega) \tag{26}$$

In Eqs.(25) and (26), the measurement point *i*, excitation point *j*, and excitation frequency ω are arbitrary values. Thus, choosing *m* different sets of *i*, *j*, ω, may yield 2*m* equations for 2*n* unknowns, herein called the damage identification equations.

$$[X(\boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k})]\{\alpha\} = \{y\} \tag{27}$$

where

$$X_{l,e}(\boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k}) = -\omega^2 S_{ij}^e(\omega, \boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k}) \tag{28}$$

$$X_{l,n+e}(\boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k}) = -\omega^2 T_{ij}^e(\omega, \boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k}) \tag{29}$$

$$X_{m+l,e}(\boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k}) = -\omega^2 U_{ij}^e(\omega, \boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k}) \tag{30}$$

$$X_{m+l,n+e}(\boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k}) = -\omega^2 V_{ij}^e(\omega, \boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k}) \tag{31}$$

$$\alpha_e = \delta k_e \tag{32}$$

$$\alpha_{n+e} = \delta c_e \tag{33}$$

$$y_l = a_R(i, j, \omega) + \omega^2 R_{ij}(\omega) \tag{34}$$

$$y_{m+l} = a_I(i, j, \omega) + \omega^2 I_{ij}(\omega) \tag{35}$$

where *m* is the number of measurements and *l* the measurement number (l=1,…, *m*) corresponding to a different combinations of *i*, *j* and ω.

For a damped structure, [X ($\delta\mathbf{c}$,$\delta\mathbf{k}$)] is the $2m \times 2n$ matrix, {α} the $2n$ unknown vector and {y} the $2m$ known vector. By solving Eq.(27), the location and magnitude of damage are obtained. In solving Eq.(27), however, iteration calculation is required because matrix [X ($\delta\mathbf{c}$,$\delta\mathbf{k}$)] includes the unknown parameters δc_e and δk_e.

The present damage detection method requires structural parameters of the intact state, [M], [C], and [K], and FRFs of the damaged state, $a_R(i,j,\omega)$ and $a_I(i,j,\omega)$. By changing the measurement point, i, the excitation point, j, and the excitation frequency, ω, various damage identification equations are obtained. Conducting as many measurements as there are elements, the damage identification equations give a determined problem, and its unique solution can be calculated.

NUMERICAL TESTS OF DAMAGE DETECTION

The feasibility of the proposed damage detection technique was tested through numerical simulations done with a cantilever beam. The beam was the 50.0cm long, 2.0cm wide, and 0.5cm thick (Fig.1) and was made of aluminum. It had a Young's modulus of 7200kg/mm^2, poisson's ratio of 0.34, and mass density of 2.7kg/m^3, and was divided into 20 elements. The numbering of the nodes and elements is shown in Fig.1. Assumed damage models for a cantilever beam are shown in Fig.2. The horizontal axis gives the element number, the vertical one the damage ratio; i.e., δk_e and δc_e. The natural frequencies before and after damage are shown in Table 1.

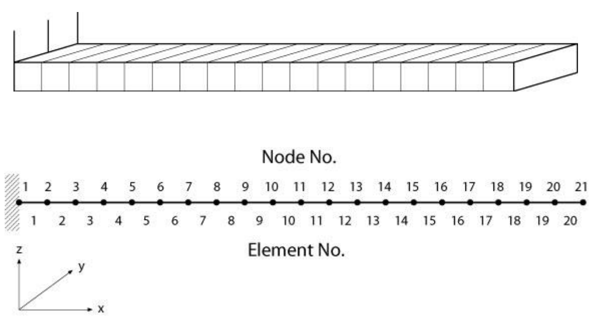
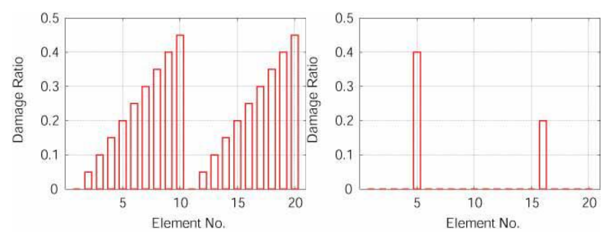


Fig. 1. Analytical model for a cantilever beam



(a) damage model A **(b) damage model B**

Fig 2. Assumed damage models

Table 1. Natural frequencies of the cantilever beam (Hz)

	1st	2nd	3rd
intact state	16.53	102.9	285.9
damage model A	15.09	92.81	255.0
damage model B	16.02	102.5	278.5

OPTIMAL APPARATUS ARRANGEMENT

Analytical case

If as many measurements as there are elements are conducted, the damage identification equations should provide a determined problem with only one solution. The question is which measurement patterns should be chosen? Optimal apparatus arrangement therefore was investigated. To simplify the problem, we assumed an undamped cantilever beam; [C]=0 and δc_e=0.0. Therefore, there are n unknowns, δk_e, for n elements, and m data, $a_R(i,j,\omega)$, is obtained for m measurements.

The analytical conditions are shown in Table 2. Because there are 20 elements, 20 different measurements were made by changing the excitation and measurement points and the frequency (Table 2). In Case A1, the excitation point was fixed at node 21 and the frequency at 1Hz, and the measurement point changed from node 2 to 21. In Case A2, the measurement point was fixed at node 21, the frequency at 1Hz, the excitation point being changed from node 2 to 21. In Case A3, the frequency was fixed at 1Hz, the excitation point changed from node 2 to 21 and the response measured at the latter nodes. In Case A4, nodes 11 and 21 were excited separately at 1Hz, and the measurement point changed at odd nodes from 3 to 21. In Case A5, the excitation point was fixed at node 21 and excited separately at 1 and 5Hz. Next, the measurement point was changed at odd nodes from 3 to 21. The damage model used in the analysis is model 1, shown in Fig.2(a).

Table 2. Analytical conditions, Case A

	_excitation point	measurement point	excitation frequency	direction
A1	No.21	all nodes in No.2-21	1Hz	Z
A2	all nodes in No.2-21	No.21	1Hz	Z
A3	all nodes in No.2-21	the same as excitation point	1Hz	Z
A4	No.11, 21	odd nodes in No.3-21	1Hz	Z
A5	No.21	odd nodes in No.3-21	1, 5Hz	Z

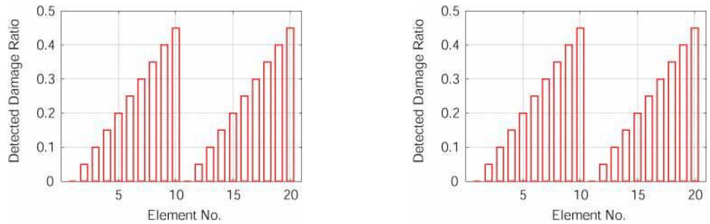
Results

On solving the damage identification equation, Eq.(23), the solutions for Cases A1 and A2 converged, and the damage to each element was identified (Fig.3). In contrast, the solutions for Cases A3, A4, and A5 diverged, and damage identification failed. To obtain convergence of the solutions for Cases A3, A4, and A5, truncation singular value decomposition (TSVD) was introduced. TSVD is intended to reduce the original parameter space to a smaller subspace such that damage equations not only become much more numerically stable but provide a sufficiently accurate solution. Singular values whose ratio to the maximum were less than 10^6 are regarded as 0, and the matrix [X] rank is reduced. After TSVD was used, the damage for Cases A3, A4 and A5 could be identified (Fig.4).

In Case A3, the first 19 singular values decreased smoothly, but the 20th decreased erratically. The matrix [X] rank was decreased to 19 after TSVD. This means only 19 data were available even though 20 measurements were conducted, and that damage ratios for the 20 elements were identified by the use of 19 data. Identification accuracy therefore is slightly less than for Cases A1 and A2, for which 20 data were available.

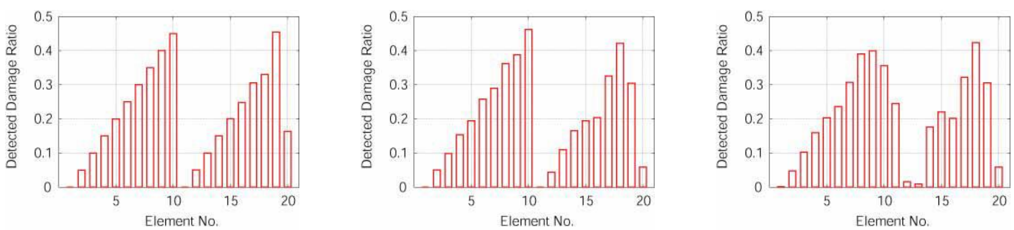
Case A4 gave the second worst identification results. The first 15 singular values decreased smoothly, whereas the 16th decreased erratically. The matrix [X] rank decreased to 15 after TSVD. This means that only 15 data were available, and that damage was identified using all 15. Measurements with other excitation points gave half-independent information about the damage done. Case A5 gave the worst accuracy. The 11th singular value decreased markedly and the matrix [X] rank had decreased to 10 after TSVD. Case A5 had 20 different measurement data, but only 10 were independent, the rest provided no useful information about damage.

When the rank of [X] has the same value as the number of elements, the error is very small, and accuracy decreases with the rank of [X]. The rank of [X] represents the number of measurements that give independent information about damage. The conclusion is that as many points as possible should be measured or excited to obtain a good identification. When complete independent data are not obtainable, TSVD gives a stable solution. When all nodes cannot be measured, the excitation point, rather than the excitation frequency, should be changed.



(a) Case A1 **(b) Case A2**

Fig. 3 Damage identification results, Case A



(a) Case A3 **(b) Case A4** **(c) Case A5**

Fig. 4 Damage identification results, Case A after TSVD

GROUPING METHOD

Damage identification is a kind of inverse analysis. As stated, the newly proposed method identifies structural damage by solving the damage identification equation Eq.(23). As for the damped structure, matrix $[X(\boldsymbol{\delta}\mathbf{c},\boldsymbol{\delta}\mathbf{k})]$ is the $2m \times 2n$ matrix, in which m is the number of measurements and n the number of elements. There are 2m known data; the m real parts and m imaginary parts of the FRFs. Moreover, there are 2n unknowns, n δc_e and n δk_e.

When data numbers equal the number of unknown parameters, 2m = 2n, damage identification equations present a determined problem whose solution is uniquely determined by

$$\{\alpha\} = \left[X(\boldsymbol{\delta}\mathbf{c},\boldsymbol{\delta}\mathbf{k})\right]^{-1}\{y\} \tag{37}$$

When the data exceed the unknowns, 2m > 2n, damage identification equations give an overdetermined problem which may have no unique solution. In that case, the least-squares method provides the best-fit solution;

$$\{\alpha\} = [X(\boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k})]^{+}\{y\} \tag{38}$$

where $[X(\boldsymbol{\delta}\mathbf{c},\boldsymbol{\delta}\mathbf{k})]^{+}$ is a pseudo-inverse of $[X(\boldsymbol{\delta}\mathbf{c},\boldsymbol{\delta}\mathbf{k})]$ expressed by

$$[X(\boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k})]^{+} = \left([X(\boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k})]^{T}[X(\boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k})]\right)^{-1}[X(\boldsymbol{\delta}\mathbf{c}, \boldsymbol{\delta}\mathbf{k})]T \tag{39}$$

When the number of measurements is insufficient, 2m < 2n, the damage identification equations provide an underdetermined problem which has an infinite number of solutions. This may occur even though data are easily accumulated by changing the measurement conditions, if the size and complexity of the structure are great. An underdetermined problem can be solved uniquely only by introducing an optimality criterion. For example, Hassiotis [6] solved an underdetermined problem by regarding it as an optimization problem in order to minimize the norm of the changes in the structural parameters. Xia [14] used the Moore-Penrose pseudo-inverse which minimizes the norm and error. Minimization of changes in the structural parameters caused by damage, however, tends to spread localized damage throughout all the elements. Application of the optimization problem to a damage detection problem, which is common in real situations, is not always a good choice.

To surmount this, the grouping method was introduced. It is intended to reduce the number of unknowns by grouping those elements assumed to have the same magnitude of damage. The algorithm is

- Step 1. Assume that all n elements are damaged element candidates.
- Step 2. Divide all elements suspected of damage into m groups.
- Step 3. Assume that elements which belong to the same group have the same magnitude of damage. In that case, changes in the damping and stiffness of the g-th group, δc_g and δk_g (g=1,…,m), are the parameters to identify.
- Step 4. Reconstruct the damage identification equations with respect to δc_g and δk_g.
- Step 5. Solve the damage identification equations for δc_g and δk_g.
- Step 6. Check if both δc_g and δk_g are less than the threshold value assumed in advance. If yes, the elements belonging to the group are identified as undamaged; otherwise damage still is suspected.
- Step 7. Check the number of elements suspected of damage. If less than n, stop here; otherwise go back to Step 2.

This grouping method is based on the fact that if the number of groups coincides with the number of measurements, the damage identification equations give a determined problem. By solving damage identification equations for grouped elements, the estimated unique magnitude of damage for each group is obtained. Any group for which the estimated magnitude of damage is smaller than the threshold is excluded as having damage candidates. The elements with suspected damage next are divided into new groups. By using the grouping method iteratively, all undamaged elements are excluded as damage candidates; only damaged elements remain.

VERIFICATION OF THE GROUPING METHOD

Analytical case

This method was used for a damped cantilever beam and a small number of measurements. The analytical condition is shown in Table 3. The number of measurements is 5 only for 20 elements. The damping matrix for the intact state, [C], was considered Rayleigh damping on the assumption that the damping constant of both the 1st and 2nd modes is 2%. Damage models for δc_e and δk_e are shown in Fig.2(b). Change in both the stiffness and damping for element No.5 is 40%, and for element No.16 20%. We identified these parameters independently.

Table 3. Analytical conditions for Case C

Excitation point	Measurement point	Excitation frequency	direction
No.21	No.5, 9, 13, 17, 21	1Hz	Z

Results

As the number of measurements is less than that of elements, the damage identification equations give an underdetermined problem, for which an infinite number of solutions exist. For one solution damage identified by use of Moore-Penrose pseudo-inverse is shown in Fig.5. The Moore-Penrose pseudo-inverse spreads localized damage throughout all the elements.

The process of damage detection by the grouping method is shown in Fig.6. As there are five measurements (10 data), five groups (10 unknowns) are necessary to obtain a determined problem. In step 1, five groups are made by combining four elements per group. The threshold value is assumed to be 0.02. Elements with estimated damage less than 0.02 are excluded as damage candidates. In step 1, 8 elements, Nos.9, 10, 11, 12, 17, 18, 19, and 20, are excluded (Fig. 6(a)). In step 2, five groups are made from the remaining 12 elements. Element Nos.1, 2, 13, and 14 are excluded in this step (Fig. 6(b)). In step 3, five groups are made by combining the remaining 8 elements, leaving 4 elements, Nos.5, 6, 15, and 16 (Fig.6(c)). In step 4, there are only 4 suspected elements for 5 measurements. The damage identification equations therefore were solved by the least-squares method. Element No.6 is excluded as a damage candidate leaving 3 elements, Nos.5, 15, and 16. In step 5, the solution for 3 elements obtained by the least-squares method, leaves element Nos.5 and 16 (Fig. 6(a)). Because they gave a high damage ratio for both stiffness and damping, iteration was stopped.

By iteration in the grouping method, final results were obtained in step 5 (Fig. 6(e)). The damage detected by this method obviously has higher accuracy than that found by the Moore-Penrose generalized inverse. The grouping method improves the accuracy of identification by reducing the number of parameters. The grouping method is a useful algorithm for coping with an inverse problem with a small number of data. It reduces the number of unknowns by assuming equality of the damage ratios for several elements and reduces the number of parameters stepwise by exclusion of unsuspected damage.

CONCLUSIONS

An FRF-based damage detection technique is proposed that is derived from the fact that changes in vibration responses provide information about the location of damage and its severity because damage to structures causes changes in their structural parameters. Damage identification equations were derived from the equations of motion of a structure before and after damage. This detection technique has the advantage that data accumulated simply by changing the excitation point, measurement point, or excitation frequency. Optimal arrangement of the apparatus was found by numerical testing of a cantilever beam. Measurement or excitation should be made at as many points as possible to obtain good identification. When complete independent data are not obtainable, TSVD gives a stable solution. When measurement at all nodes is not possible, the excitation point rather than the excitation frequency should be changed.

When the number of measurements is insufficient, introduction of the grouping technique enables damage identification. This technique was verified for simulated data from a numerical vibration test on a cantilever beam.

The proposed method identifies not only the location of damage but the amount of damage with high accuracy when data are insufficient, and instruments are properly set. In future analysis, this technique will be used to assess various damage types, the system made more robust by introducing additional data and mathematical operations of high order, as well as the verification of damage by experiments.

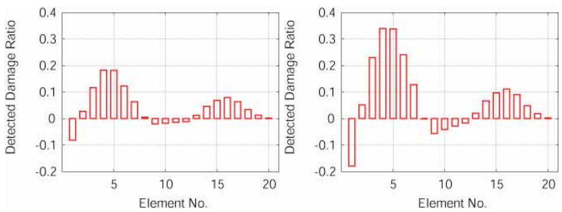
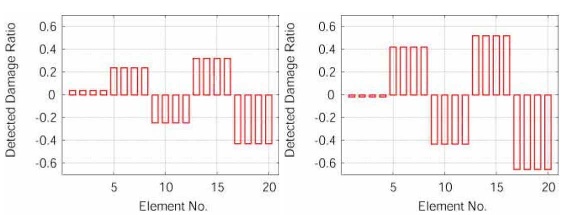
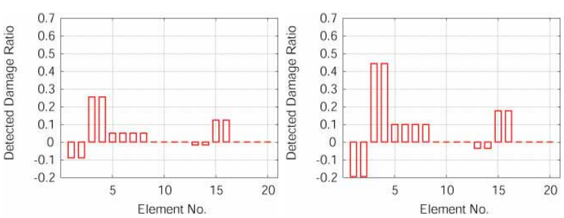


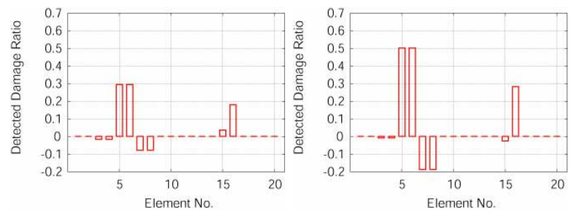
Fig. 5. Damage identification results with the Moore-Penrose inverse (left: stiffness, right: damping)



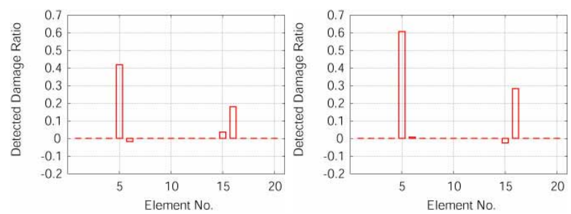
(a) Step 1



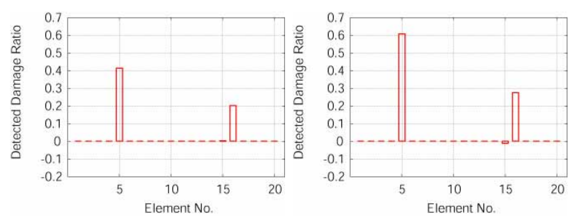
(b) Step 2



(c) Step 3



(d) Step 4



(e) Step 5

Fig. 6. Damage identification results with the grouping method (left: stiffness, right: damping)

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