



MODIFIED EQUIVALENT LINEAR SYSTEM METHOD BASED ON CHARACTERISTICS OF STRONG GROUND MOTIONS

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SUMMARY

The equivalent period for equivalent linear systems is proposed using the ratio of the peak displacement in the previous one-cycle to maximum displacement for considering duration of earthquakes. A estimation method using this equivalent period and other estimation methods proposed in past studies for inelastic displacement demands from design earthquake spectrum are compared. The strength reduction factors for nonlinear single-degree-of-freedom system with degrading bilinear Takeda hysteresis model are defined based on the estimation procedures and simulated earthquake motions with three types of duration. Comprehensive time-history analyses were carried out under simulated earthquake motions with different durations. Calculated maximum displacement responses are compared with target ductility levels estimated from the procedures. Most of the procedures are found to underestimate the responses generally under the motions of the short duration type, which simulates response to near field earthquakes.

INTRODUCTION

In case that the design earthquake is given by an elastic response spectrum or Fourier spectrum as commonly adopted in recent performance-based design codes, the maximum responses of any elastic system can be estimated by "modal analysis." The maximum response can definitely be determined for each fundamental mode and the maximum responses can be estimated by the superposition of all dominant modes, for example, by means of square root of sum of squares. However, if the response of a structure exceeds the elastic limit, nonlinear response must be estimated from the elastic response spectrum. This is nothing but so-called "equivalent linearization," which correlates nonlinear responses with linear response spectrum. If the design motion is given by the elastic response spectrum and the nonlinear displacement response is used as design criteria, a rational theoretical background on the linearization is essential in the development of design code. A rational theory is still needed to explain the nonlinear response based on the expected characteristics of the earthquake motion at the site.

Even though the earthquake is specified by the elastic response spectrum, another way of calculating nonlinear response is to perform time-history response analysis under an artificial motion, which is synthesized so that its response spectrum are fitted to the specified target spectrum. However, the response spectrum is only smoothed from past earthquakes and not general for the future earthquake on the site,

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and the essential time-history characteristics are assumed in the synthesis of the artificial motion, such as, in terms of "envelope curve" or "phase spectrum." Therefore, nonlinear time-history response could be different under different motions with the same elastic response spectrum. Therefore, another approach is to calculate nonlinear responses from the past records directly and correlate the responses with structural parameters, such as initial fundamental period, strength reduction factors, and hysteresis types. This type of estimation methods may be classified as inelastic spectrum-based procedures.

Here, estimation methods from a proposed procedure and four procedures proposed in the past studies or codes, three equivalent linearizations and two inelastic spectrum-based procedures, are compared with numerical results from time-history response analyses.

METHOD OF TIME-HISTORY ANALYSES

Simulated Earthquake Motions

The velocity and acceleration response spectra used in the analysis are shown in Figure 1. The target demand linear response spectrum for 5 percent damping coefficient was selected as shown with thick solid line in Figure 1. These acceleration and velocity response spectra are calculated for a site in Tokyo with return period of 500 years according to AIJ Recommendations for Loads on Buildings[1]. Input ground motions were generated by phase difference method. The reference duration of the ground motions are 40.96 seconds and the sampling rate is 0.01 second. For the phase difference distributions, normal deviations whose normal distribution is 0.03π , 0.10π and 0.20π are used, as illustrated in Figure 2. The three kinds of phase distributions are based on three cumulative probability density functions as shown in Figure 3, for each of which five different ground motions are generated with five different random seeds. Therefore, fifteen motions were used as input record with three types of phase characteristics, or durations of motions. The calculated acceleration and velocity response spectra from these motions are also shown in Figure 1. Typical acceleration time-histories of the three types are shown in Figure 4.

Structural Models

For simplicity of the theoretical interpretation on the calculated results, the hysteresis model for the system is selected as a degrading bilinear model(bilinear Takeda model), which is shown in Figure 5. The model is can represent the hysteretic characteristics of reinforced concrete structures by neglecting cracking behavior and regarding the elastic stiffness of the model as the yielding stiffness of the structure. In past studies on inelastic system of reinforced concrete structures, the yielding stiffness was found to be essential than the initial stiffness before cracking. The constant parameters for the systems are as follows: the stiffness after yielding is 0.01 times the initial stiffness and the unloading stiffness degradation parameter is 0.4, the elastic damping coefficient is 0.05 which is assumed to be proportional to the inelastic stiffness. The initial fundamental period were varied systematically. The strength reduction factors are selected in two cases to attain two levels of ductility μ as $\mu=2$ and $\mu=4$.

Estimation of Maximum deformations

Estimated maximum deformations were compared with response deformations using the required strength spectrum for constant maximum ductility factor are calculated from design spectrum (Figure 1).

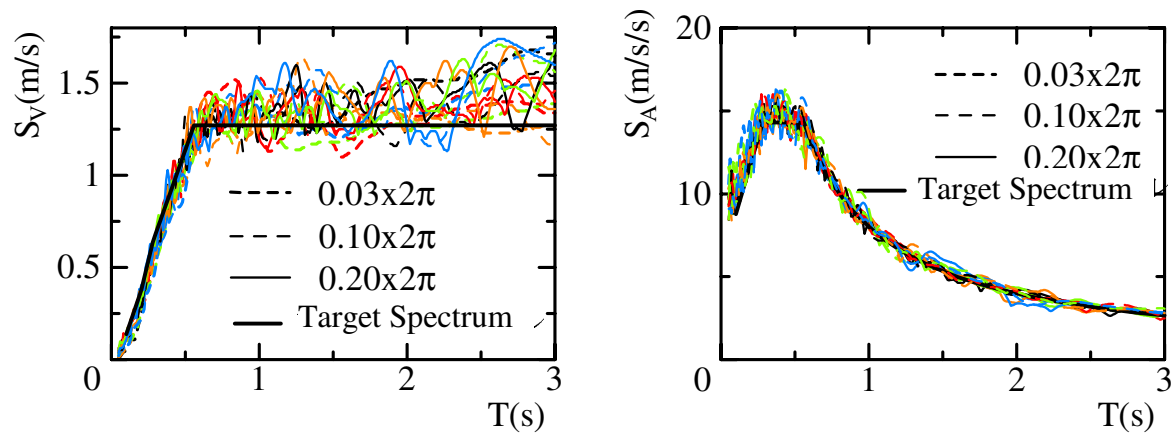


Figure 1 Target and calculated response spectra

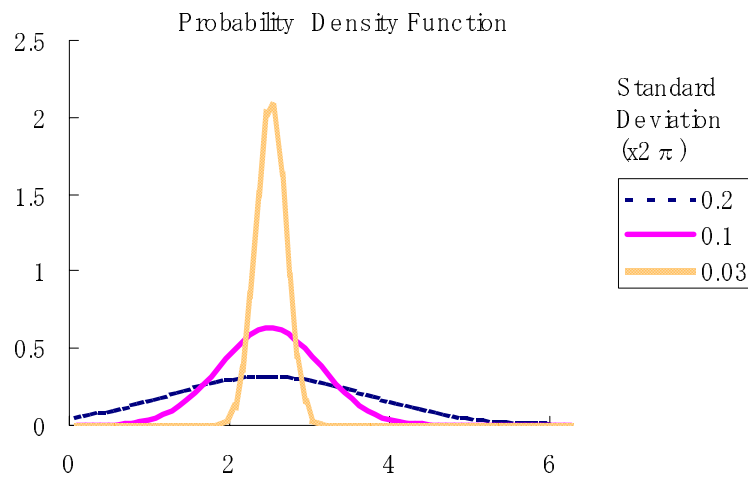


Figure 2 Probability density functions of the phase difference spectrum

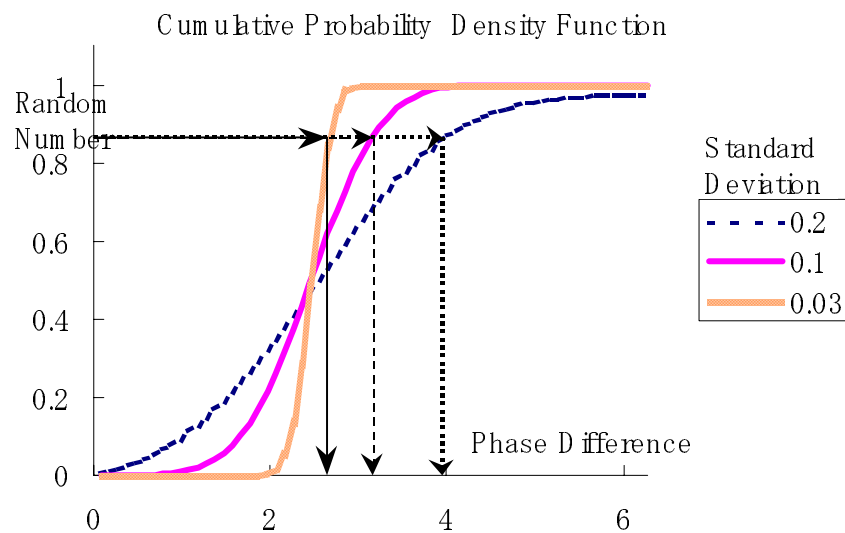


Figure 3 Cumulative probability density functions of the phase difference spectrum

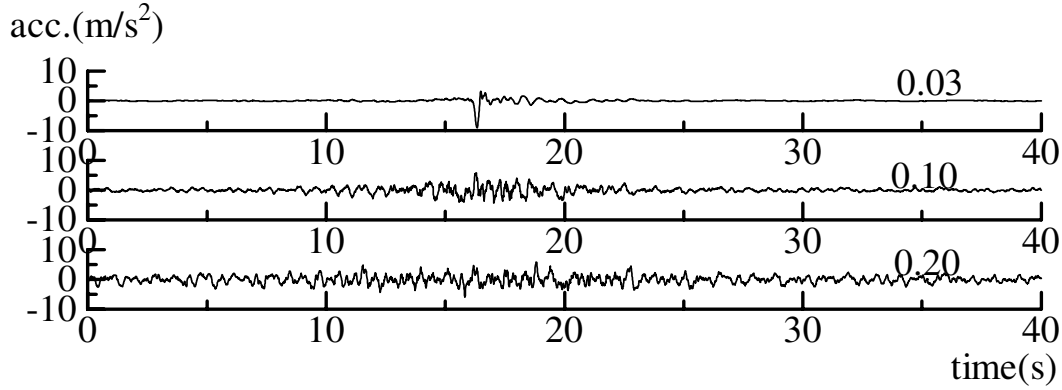


Figure 4 Time-history of simulated earthquake motions

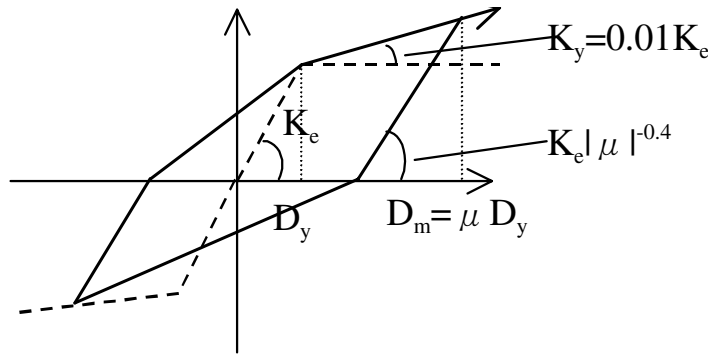


Figure 5 Degrading bilinear hysteresis model

ESTIMATION OF MAXIMUM DEFORMATIONS

Numerous studies have been carried out to formulate inelastic demands as design spectra. The following five methods are selected for comparison in this study.

- (1) Procedure A[2]: Equivalent linearization using substitute damping .
- (2) Procedure B[3]: Equivalent linearization using damping modification factor
- (3) Procedure C[4]: Equivalent linearization with modified damping and equivalent period considering duration of earthquakes.
- (4) Procedure D[5]: Capacity-diagram method using inelastic spectrum based on Newmark-Hall formula
- (5) Procedure E[6]: Modified inelastic spectrum considering branch of elastic spectrum

The first three procedures A, B, C may be classified as equivalent linearization. The last two may be as procedures based on inelastic spectrum. The key equations of the methods are briefly quoted below.

(i) The Equivalent Linear System Procedures

Procedure A[2]:

The equivalent linear systems are required for the SA-SD diagram method based on elastic response spectrum. The preciseness of this method depends on the reduction factors for elastic response spectrum due to viscous damping. The damping effect of response spectrum is estimated using the following equation.

$$\frac{S(\zeta)}{S(5\%)} = \frac{1.5}{1+10\zeta} \quad (1)$$

In the capacity diagram method, the equivalent periods were determined implicitly as the secant stiffness to the maximum response displacement point. Equivalent viscous damping factors named as substitute damping ζ_e are proposed as follows:

$$\hat{\zeta}_{eq} = 0.05 + \zeta_{eq} \quad , \quad \zeta_e = 0.2 \left(1 - \frac{1}{\sqrt{\mu}} \right) \quad (2)$$

which were reduced from the equivalent inelastic energy dissipation at stationary cyclic response at maximum displacement response. The reduction factor is based empirically on averaged energy throughout the responses of nonlinear systems under recorded motions (Figure 7 and equation (4) below).

Procedure B[3]:

The damping modification factor, κ which depends on the hysteretic behavior of the system as below, shown in Figure 6, are defined and used for equivalent linear system.

Type A: Stable hysteresis loops

Type B: Between type A and type C

Type C: Severely pinched and/or degraded loops

$$\hat{\zeta}_{eq} = 0.05 + \kappa \zeta_{eq} \quad (3)$$

$$\zeta_{eq} = \frac{1}{4\pi} \frac{\Delta W}{W_e} \quad (4)$$

ΔW : Hysteresis dissipation energy, W : Equivalent potential energy, κ : The damping modification factor (Figure 7).

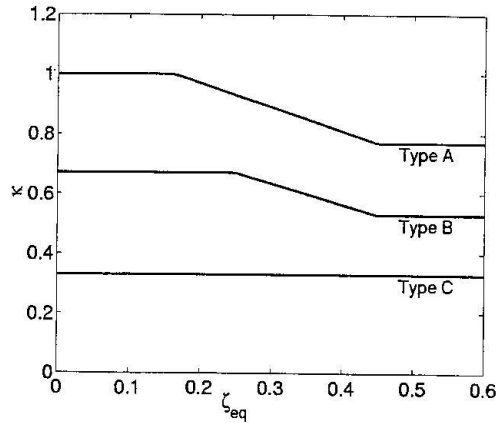


Fig. 6 κ The damping modification factor

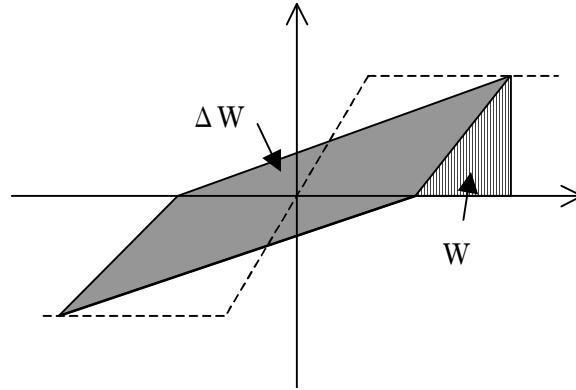


Fig. 7 Equivalent viscous damping ζ_{eq}

Procedure C[4]:

This is proposed method which is using modified equivalent periods. The equivalent period defined peak displacement ratios. The peak displacement ratio γ is defined as the ratio of the peak displacement in the previous one-cycle to the maximum displacement for considering duration of earthquakes. The peak displacement ratio γ is approximated using the duration of motion as:

$$\gamma = \frac{t_0/2 - T_e}{t_0/2 + T_e} = \frac{t_0 - 2T_e}{t_0 + 2T_e} \quad (5)$$

t_0 : duration of earthquake. T_e : equivalent period.

Equivalent viscous damping is defined using reduced energy dissipation as shown in Figure 8.

$$\zeta_{eq} = \frac{1}{4\pi} \left(\frac{\Delta W}{W} \right) \quad (6)$$

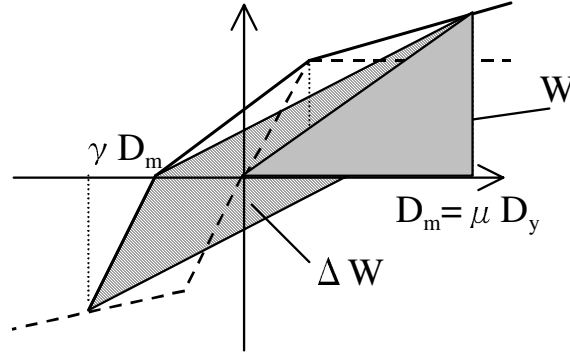


Figure 8 Hysteretic Absorption Energy

The effect of damping to the reduction of response spectrum is derived as: [7]:

$$\frac{S_v(\zeta)}{S_v(0)} = \sqrt{\frac{1 - e^{-4\pi\zeta t_0/T}}{4\pi\zeta t_0/T} \{0.424 + \ln(4\pi\zeta t_0/T + 1.78)\}} \quad (7)$$

where, t_0 : duration of earthquake, T : initial period. ζ : damping coefficient.

(ii) Inelastic Spectrum-based Procedures

The inelastic design spectra under constant ductility factors were calculated from elastic spectra by the yield strength reduction factors.

Procedure D[5]:

The yield strength reduction factor is based on Newmark-Hall[8]

$$R_y = \begin{cases} 1 & T_n < T_a \\ (2\mu - 1)^{\beta/2} & T_a < T_n < T_b \\ \sqrt{2\mu - 1} & T_a < T_n < T_{c'} \\ \frac{T_n}{T_c} \mu & T_{c'} < T_n < T_c \\ \mu & T_c < T_n \end{cases} \quad (8)$$

where : T_a , T_b and T_c are 1/33 s, 0.125s and 0.56s.

Procedure E[6]:

The yield strength reduction factor is given in rather sophisticated formula as:

$$R_u = \begin{cases} C_1(\mu - 1)^{C_R} \frac{T}{T_0} + 1 & T \leq T_0 \\ C_1(\mu - 1)^{C_R} + 1 & T_0 \geq T \end{cases}, \quad T_0 = C_2 \mu^{C_T} T_c \quad (9)$$

C_1 , C_2 , C_R , C_T are 0.75, 0.65, 1.0, 0.30

T_c is the constant S_A and constant S_D branches(0.56s).

COMPARISON OF MAXIMUM RESPONSE DEFORMATIONS

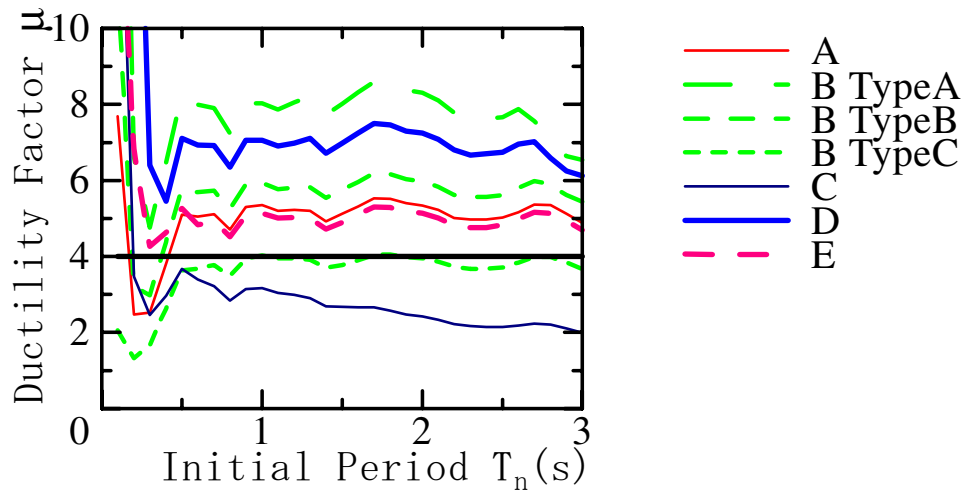
Strength reduction factor

To compare above methods generally, strength reduction factors are calculated for the ductility levels of $\mu=2$ and $\mu=4$ from the original target spectrum. For Procedure B, the reduction factors are calculated for the three types of hysteresis loops. Procedure C, which takes into account the duration of motion, gives three types of reduction factors corresponding to the given three types of earthquake motions. The durations of 4.92, 16.4, 32.8 seconds are used to derive the strength reduction factors in the Procedure C.

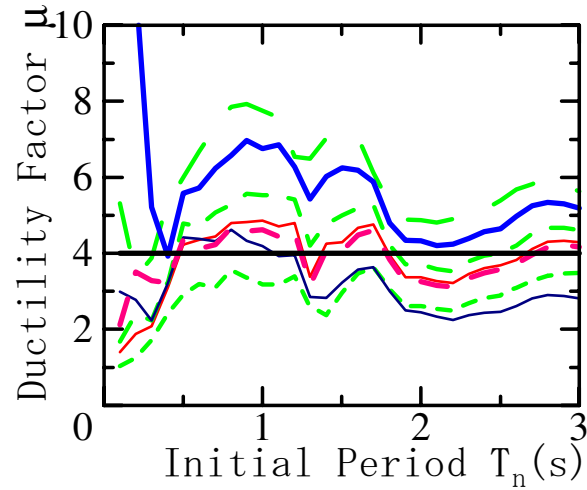
For each period of the structure, the strength of the structures were reduced by the strength reduction factor as above, for which time-history analyses were carried out under the fifteen simulated earthquake motions. If the calculated ductility factors of maximum response displacement are close to $\mu=2$ or $\mu=4$, then the method can be regarded as accurate.

Comparison with time-history response analysis

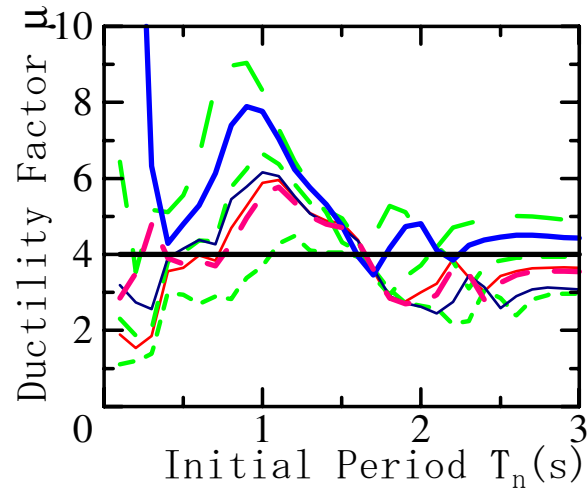
Typical results are compared for five methods in Figure 9 in case of target ductility factor of 4. Maximum displacement responses in terms of ductility factors calculated from time-history analysis are shown for each estimation method in a figure, which are averaged separately for the three types of ground motions in the figure (a) through (c). For example, notation A in the legend in the figure (a), the maximum response ductility factors are calculated based on the estimation of Procedure A and averaged for five ground motions of short duration type. That is, the yield strength of the systems are determined from the reduction factor calculated from the required strength reduction factor derived from the Procedure A[2], which is shown in the figure (a) of Figure A for the maximum ductility factor of 4, and then the maximum ductility factors calculated from time-history analysis under five motions of short duration type are averaged and shown with the thin solid line. Therefore, if the calculated responses are larger than 4, the method underestimate the responses. As for the Procedure B[3], the hysteresis used here, which is degrading bilinear model, may be regarded as Type B, although the cases for the other types, Type A(fat and stable) and Type C(pinching), are also included and shown for comparison.



(a) (0.03π) : motions of short duration



(b) (0.10π) : motions of medium duration



(c) (0.20π) : motions of long duration

Figure 9 Calculated ductility factors of maximum displacement response from time-history analysis averaged five motions with the same duration

Generally, in case of medium or long duration of motions, which simulate far field earthquakes, most of the estimation procedures gives a fair correlation with the time-history analysis, although in the method D underestimate with relatively larger discrepancy. However, in case of the figure (a) under short duration motions, which simulate near field earthquakes, most procedures generally, especially Procedure D, underestimates the responses, except for the Procedure C. This due to the effect of damping on the reduction of response, both in cases of viscous damping in elastic response and hysteretic damping in inelastic response, is generally and theoretically smaller under the motions of short duration, or pulse-type motions[4]. A large discrepancy is found also in the short period region less than 0.3 seconds. This is due to another reason because the acceleration response spectrum increases steeply in the region, where the response could vary by a small difference of yield strength.

CONCLUSIONS

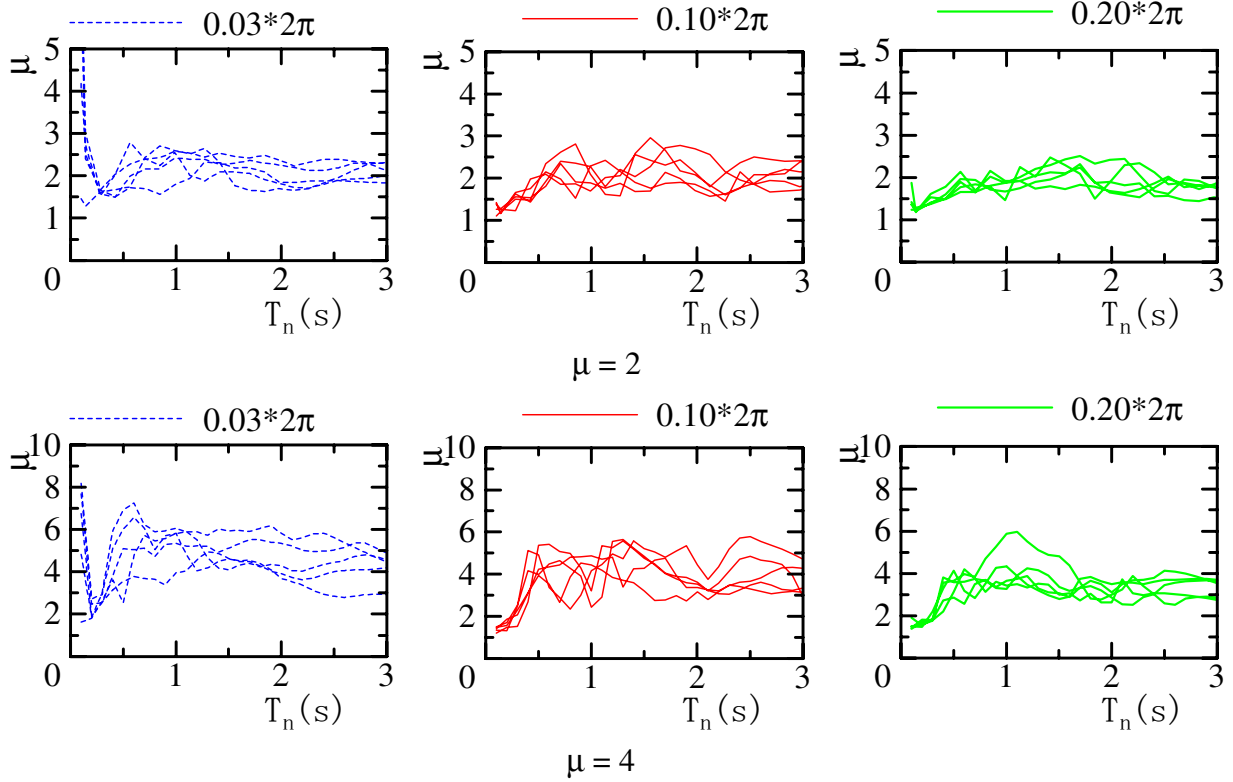
Procedures proposed in past studies for estimating inelastic displacement demands from elastic design earthquake spectrum are compared and verified through comprehensive time-history analyses. Most of the procedures are found to underestimate the responses generally under the motions of the short duration type simulating near field earthquakes, which might be due to overestimation of the effect of damping. The estimation improved using the modified equivalent period by the peak displacement ratios that calculated from the duration of motion.

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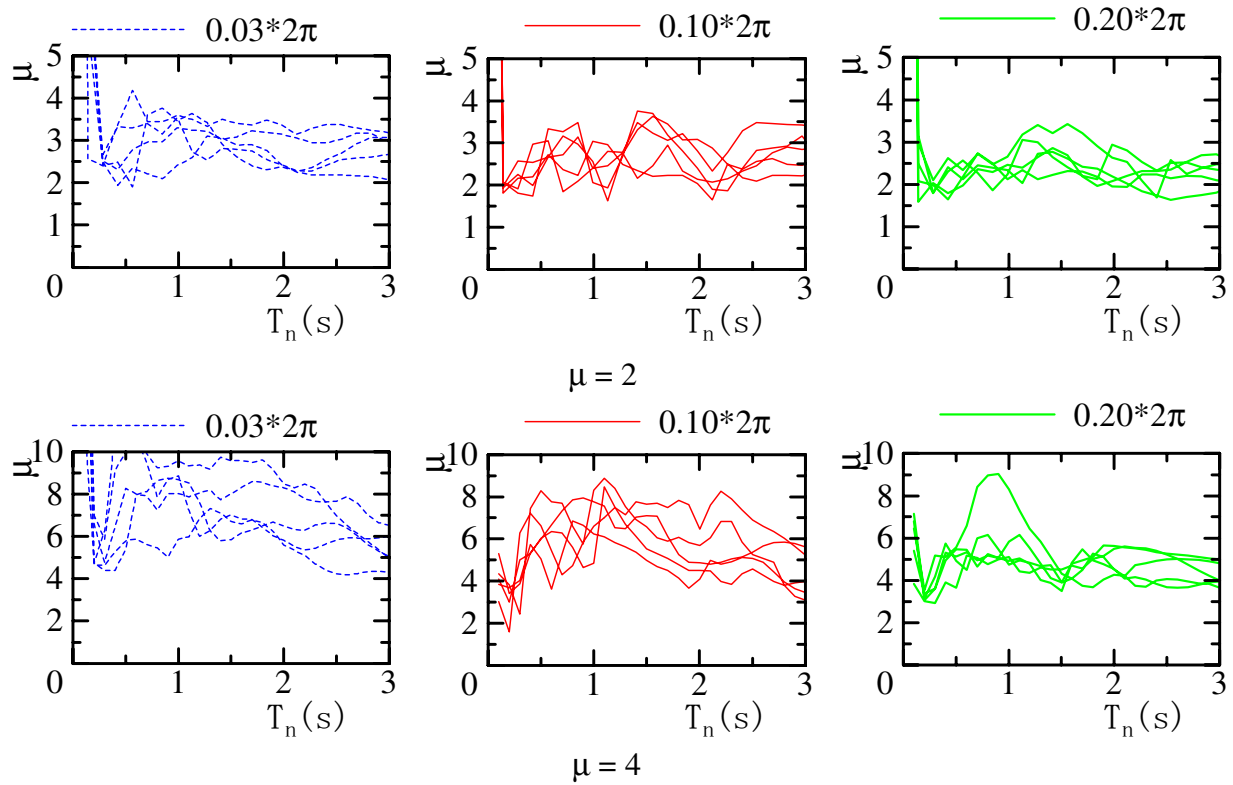
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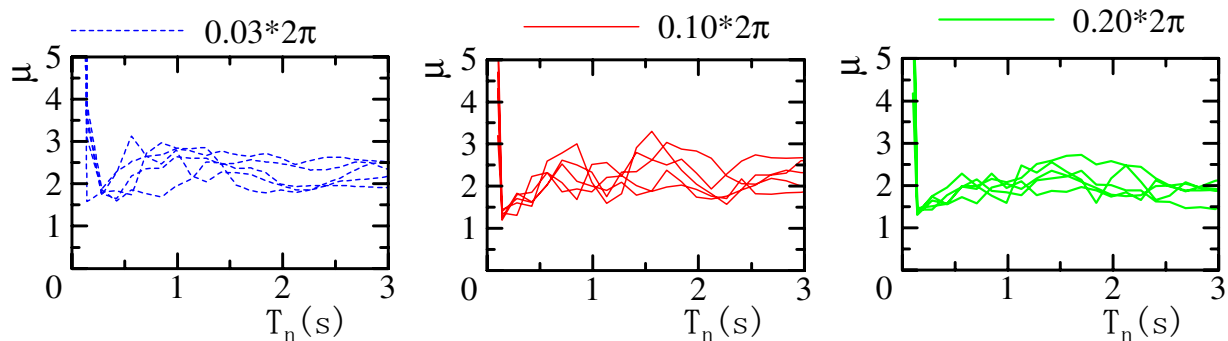


(a) Procedure A (Shibata and Sozen)

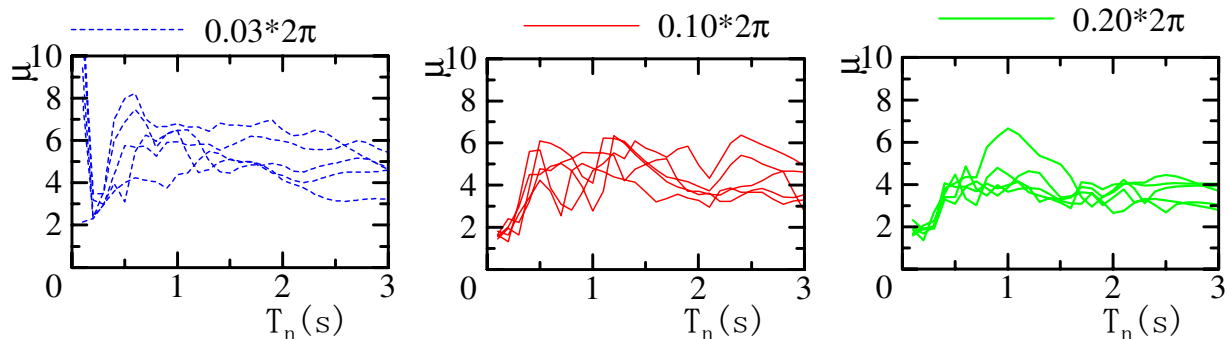


(b1) Procedure B (ATC40) Type A

Figure A Calculated ductility factors of displacement response from time-history analysis

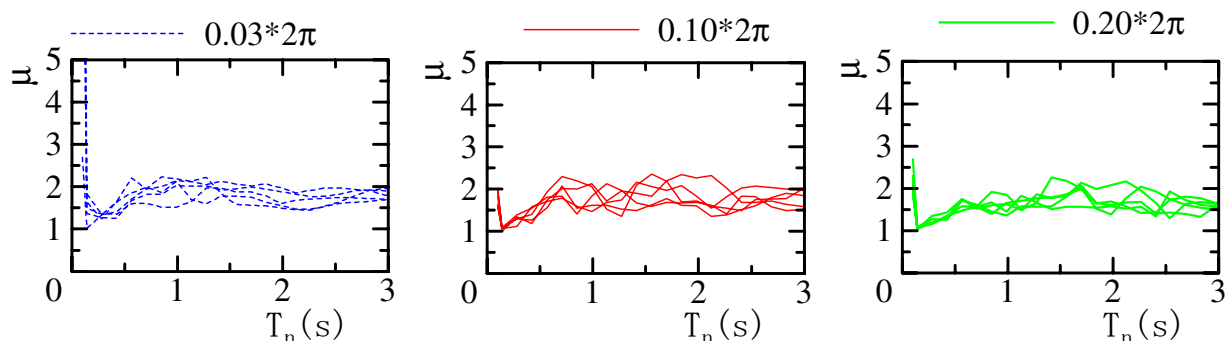


$\mu = 2$

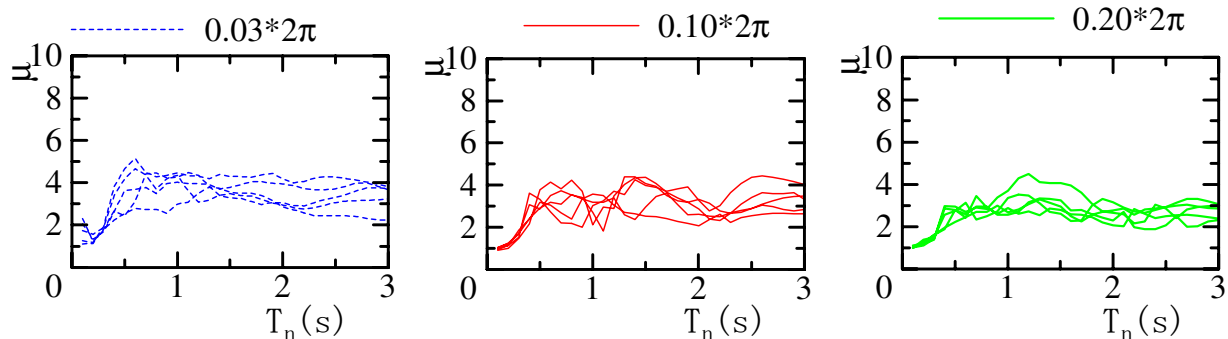


$\mu = 4$

(b2) Procedure B (ATC40) TypeB



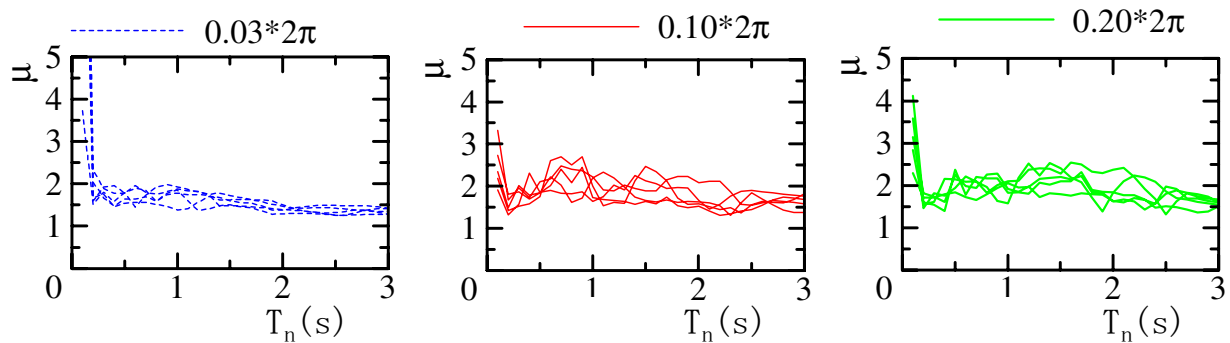
$\mu = 2$



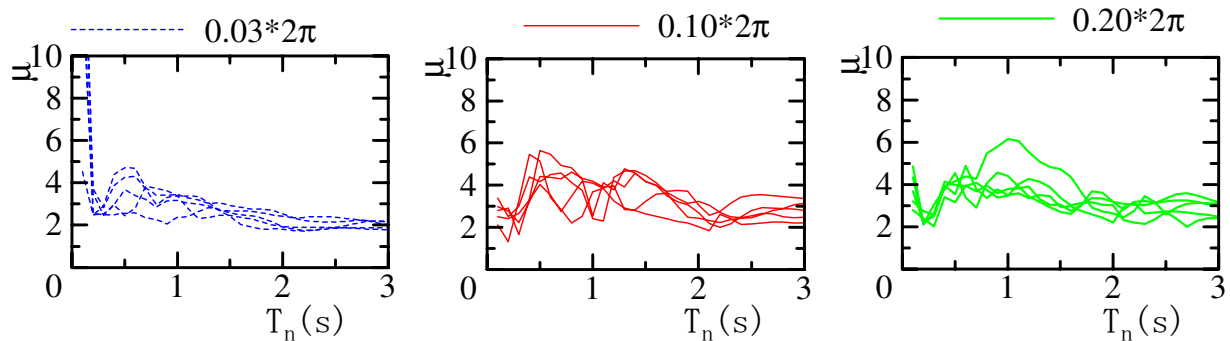
$\mu = 4$

(b3) Procedure B ATC40 TypeC

Figure A Calculated ductility factors of displacement response from time-history analysis

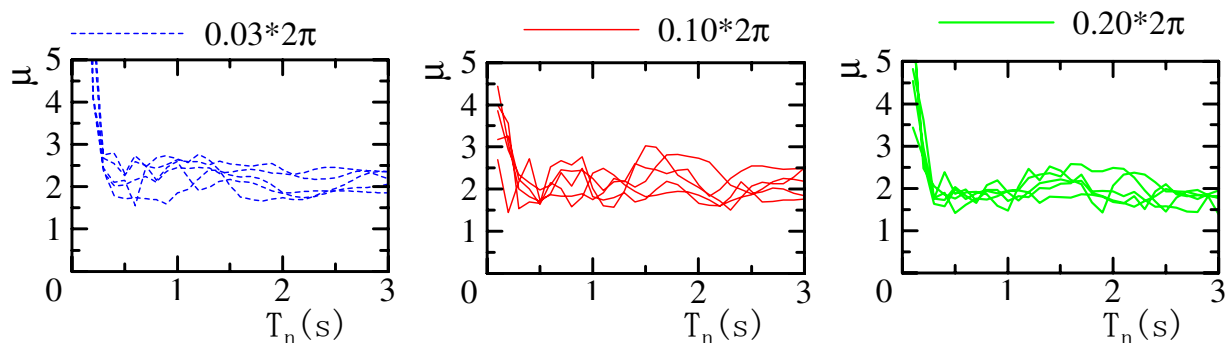


$\mu = 2$

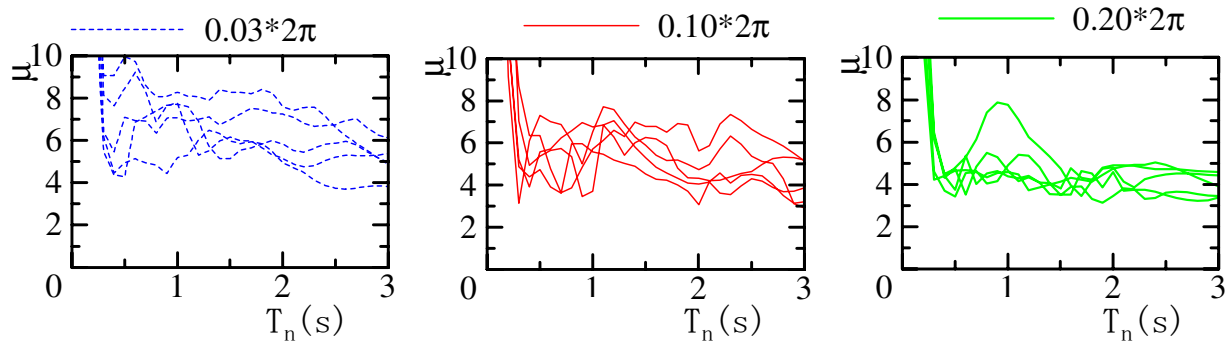


$\mu = 4$

(c) Procedure C(Kabeyasawa and Nakamura)



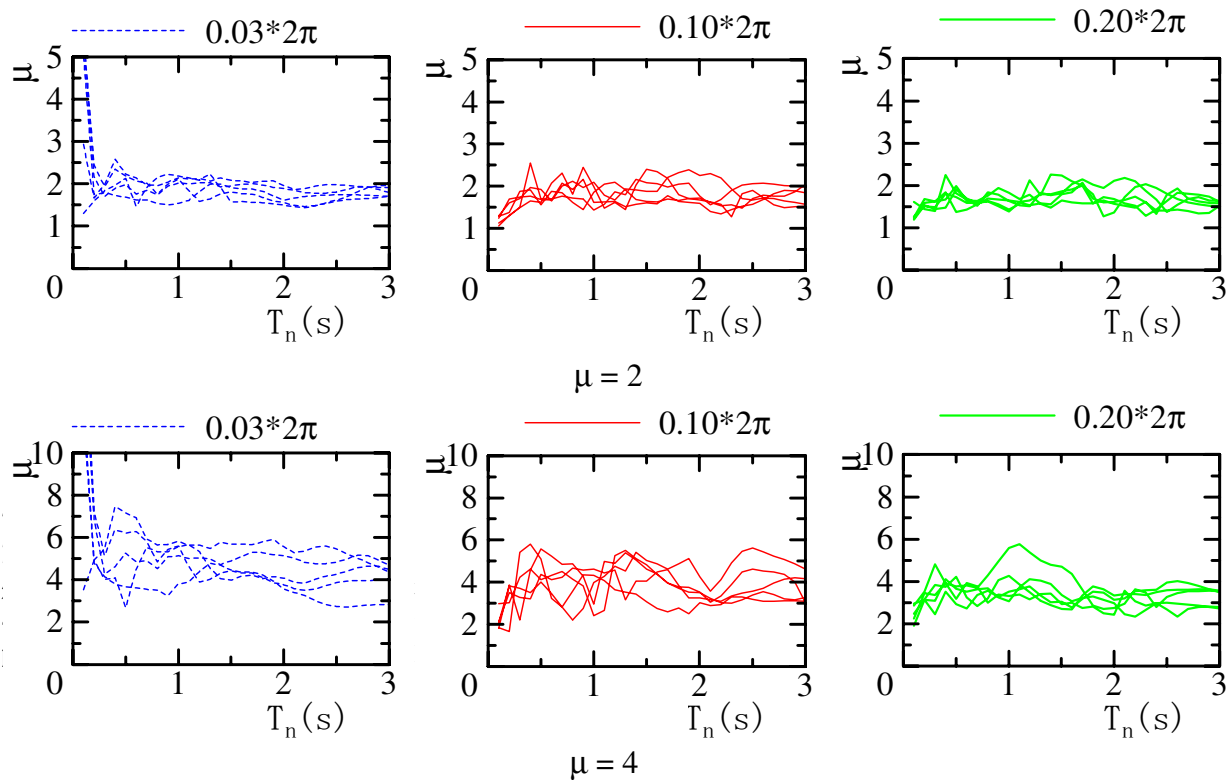
$\mu = 2$



$\mu = 4$

(d) Procedure D(Chopra and Goel)

Figure A Calculated ductility factors of displacement response from time-history analysis



(e) Procedure E(Vidic, Fajfar and Fischinger)

Figure A Calculated ductility factors of displacement response from time-history analysis