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DEFINITION OF SUITABLE BILINEAR PUSHOVER CURVES IN NONLINEAR STATIC ANALYSES

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SUMMARY

Reliability of results obtained through nonlinear static analyses is strongly dependent on the assessment of the performance point, usually computed by means of the capacity spectrum method. The use of constant ductility spectra as demand spectra, in place of reduced spectra for assigned equivalent viscous damping, can provide better results and a larger stability in the evaluation method of the performance point. In this case, the capacity curve of the SDOF equivalent system has to be transformed in a bilinear curve for computing the available ductility. Such a conversion can be performed according to several criteria that significantly influence results. In this paper, results of analyses carried out in order to assess the dependence of the performance point value on parameters controlling the bilinear relationship and on conversion procedure are shown. Reliability and accuracy of procedures proposed by ATC 40, Eurocode 8 and italian seismic code PCM 3274 as well as of procedures based on the use of constant ductility spectra is assessed by comparing results with the ones from nonlinear dynamic analyses.

INTRODUCTION

The non-linear static analysis (NLSA), originally introduced by ATC-40 [1] and FEMA 273 [2] documents for existing buildings, is more and more spreading as analysis procedure for structural design in seismic areas, and represents a simple yet effective method for verifying the achievement of selected performance requirements.

The procedure outlined in the above-mentioned documents is based on the "Capacity Spectrum Method", developed by Freeman et Al. [3, 4] and can be summarized as follows.

1.- The so-called pushover curve, expressed in terms of base shear V_b versus displacement d_c of a control point, generally located at top floor level, is computed or estimated assuming a pre-determined horizontal forces distribution.

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2.- The pushover curve is converted in a capacity spectrum using the dynamic characteristics of the first vibration mode, dividing V_b and d_c respectively by:

$$M_1^{eff} = \frac{\left(\sum_{j=1}^{n} m_j \phi_j^1\right)^2}{\sum_{j=1}^{n} m_j {\phi_j^1}^2} \qquad \text{and} \qquad \Gamma_1 \phi_n^1 = \frac{\sum_{j=1}^{n} m_j \phi_j^1}{\sum_{j=1}^{n} m_j {\phi_j^1}^2} \phi_n^1$$

with obvious and well known meaning of the symbols.

3.- The elastic or design spectrum is represented in the ADRS format, i.e. with spectral pseudo-accelerations plotted as a function of spectral displacements (demand spectrum).

4.- Capacity and demand spectra are plotted together and the demand displacement, i.e. the “performance point” coordinates, is determined. To do this, a series of equivalent linear systems has to be analyzed, with updated values of period and equivalent viscous damping, in order to account for energy dissipation and period shift triggered by deformation in the non-linear range.

5.- The displacement demand connected with the performance point is then re-converted in terms of control point displacement. From this last one, the structural deformations to be compared with the specific requirements can be calculated.

It is self-evident that the reliability of the procedure is substantially dependent on the correct determination of the performance point. Besides, the application of the procedure itself carries some significant approximations. In particular, the pushover curve is usually determined using a fixed force distribution depending on the first modal shape of the elastic system, which renders the analysis representative only of systems with negligible higher modes effects. In this regards, some extensions have been proposed in order to account both for higher modes influence and modification of the vibration modes due to inelastic strains (Reinhorn et Al. [5]). Even more significant is the implicit assumption that, in order to avoid the computational effort needed to perform non-linear dynamic analysis of the equivalent inelastic SDOF system, the response can be reliably determined through an iterative procedure which uses equivalent linear systems with variable period and damping.

To do this, ATC-40 document proposes three procedures (A, B and C), based on the same principles, but different as far as the implementation is regarded. Procedures A and B are analytical, with bilinear capacity curve fixed in procedure B and iteratively determined in procedure A. Procedure C is fully graphical.

Such procedures are conceptually simple, but the iterations can be time consuming. In this regard some improvements have been proposed. In Albanesi et Al. [6] a single variable damping spectrum is used instead of a series of constant damping spectra, so that the procedure becomes non iterative. Chopra and Goel [7] have shown that the ATC procedures do not converge in numerous realistic cases. Besides the maximum deformation demand can be significantly underestimated (50%), with respect to the “exact” values determined through dynamic analysis, over a wide range of periods.

Starting from a concept originally proposed by Bertero [8], Reinhorn [9] and Fajfar [10], Chopra and Goel proposed an improved procedure that uses constant ductility spectra. In this method, the demand is computed by analyzing an inelastic system rather than a series of equivalent linear systems with updated period and damping, that tend to become unrealistically high. In the opinion of such authors, also confirmed by Faella [11], the procedure based on constant ductility spectra provides more reliable results in most cases, while maintaining the same graphical appeal of the ATC procedures. On the other hand, the Capacity Spectrum Method with constant ductility spectra contextually introduces the need of reducing the capacity curve in a bilateral shape.

Similarly, the procedures proposed by European codes, Eurocode 8 [12], and more recently by the Italian seismic code [13], use bilinear curves for capacity spectra, while iterations are optional in the Eurocode and eliminated in the Italian code.

THE CAPACITY CURVE CONVERSION ISSUE

Both in ATC 40 procedure B and European/Italian codes, as well as the procedures using constant ductility spectra, the capacity curve derived form pushover analyses has to be converted into a bilinear curve. Criteria to be used for such conversion can be various and may significantly influence the results, especially taking into account that pushover curves of structures with different typology, material and geometrical configuration can show different shape with horizontal, hardening or even softening post-elastic slopes. Such influence might be critical when inelastic spectra are used, since the iterations converge when the ductility demand matches the capacity, this last one being strongly dependent on the bilinear curve shape.

Moreover, conversion issues are amplified if real earthquake spectra are used. As shown in figure 1, the demand curves plotted in the ADRS format are characterized by non-monotonic shape, so that the intersection with the capacity curve may not be univocally defined.

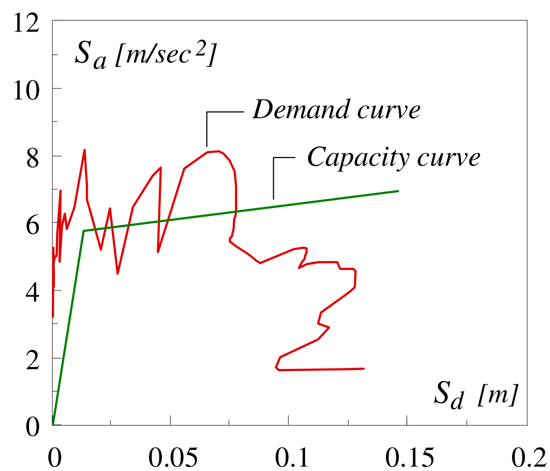

Figure 1. Use of real earthquake spectra

This aspect can influence to some extent the determination of the performance point on the capacity curve, on whose correct identification the reliability of the procedure is based, whatever its implementation might be.

In this paper a series of parametric analyses is performed, with the aim of assessing sensibility of the performance point with respect to the parameters that define the capacity curve conversion. Besides, reliability of American (ATC-40 [1]) and European (Eurocode 8 [12] and Italian seismic code [13]) code procedures, as well as of procedures using constant ductility spectra, is investigated. Comparing the results obtained through the different procedures with "exact" values derived from nonlinear dynamic analyses assesses such accuracy.

ANALYSES

The analyses have been carried out for a 6 floors r.c. building with L-shape plan as schematically shown in figure 2. Structural members have been designed following the provisions of Eurocode 8 draft n.3 [14], assuming ductility class "E" (Enhanced ductility) and behavior factor equal to 4.

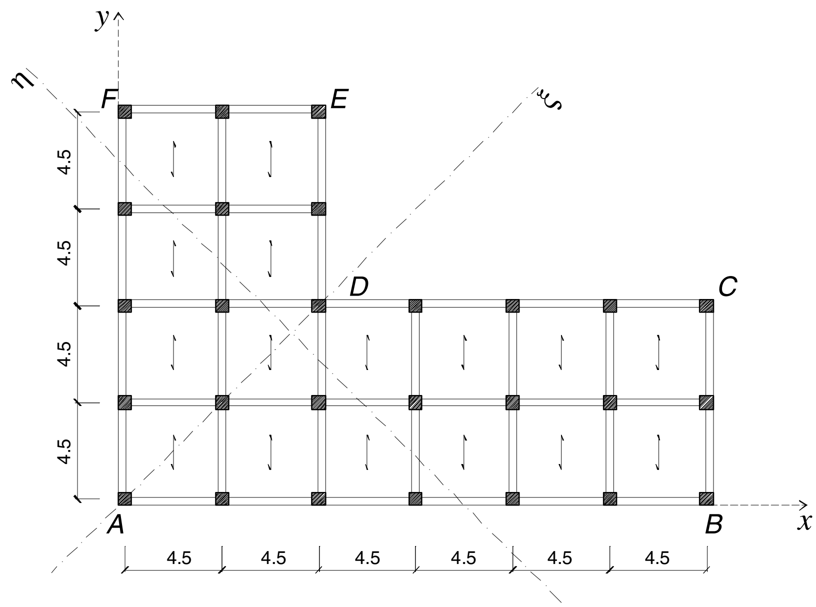

Figure 2. Geometry of the analyzed building

Columns have square cross section with 600x600 mm side dimension for the first two floors, with 100 mm taper every two floors. Beams have 350x500mm cross section. Different design strengths are obtained by varying the longitudinal and transversal reinforcement; this last one has been calculated also to take into account, in the non linear modeling, the confinement of the concrete core. In columns the reinforcement has been designed considering biaxial bending.

In the design, category B and B-type soil have been assumed, with 0.35g ground acceleration and importance factor equal to 1. Dead and live loads values are respectively 6 kN/m^2 and 3 kN/m^2. At floor perimeter, a 8.13 kN/m dead load has been considered for the presence of walls. C25/30 strength class and 430 MPa yield stress have been assumed respectively for concrete and reinforcement steel.

Modal analysis has been carried out, combining the effects of the two horizontal components, in compliance with EC8 requirements. First three vibration modes have periods T_1 = 0.58, T_2 = 0.56, T_3 = 0.48 seconds, the first two being mainly translational in the ξ and η diagonal directions, and the latter being torsional.

The nonlinear dynamic analyses, as well as the nonlinear static analyses, have been carried out using the software code CANNY (Li [15]) which accounts for the main aspect of the inelastic response of r.c.

structures, such as confined and unconfined concrete behavior, biaxial bending – axial force interaction, pinching, strength and stiffness decay, etc.

Giberson model has been used for beams, with inelastic deformation concentrated at element ends by means of nonlinear springs with non-symmetric bilinear moment-rotation relationship. This accounts for the difference in the behavior of the section before and after the yielding of steel reinforcement. In the dynamic analyses, cyclic response has been simulated using the Takeda hysteretic model, modified in order to introduce strength degradation.

As far as columns are regarded, "multisprings" elements have been used for modeling the interaction of bending actions with axial forces. Figure 3 shows the disposition of reinforcement bars and the discretization used for concrete. Modified Kent & Park model has been attributed to concrete fiber springs, to account for both the difference between confined and non-confined regions and the strength decay due to cyclic loading. Steel fiber springs use simple bilinear stress-strain low with hardening.

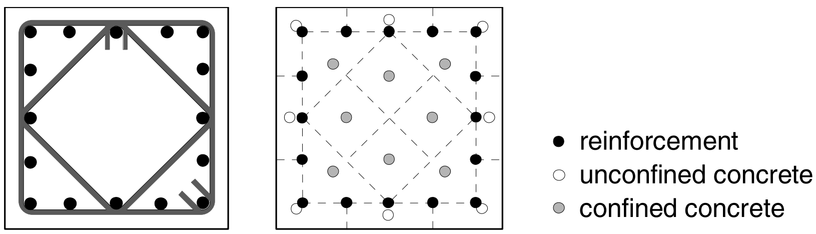

Figure 3. Column reinforcement and "multisprings" discretization

In order to apply the capacity spectrum method, the pushover curves of the structure, in terms of base shear V_b and top displacement d_{top}, are converted into capacity spectrum dividing the base shear by the first modal mass M_1^* and the top displacement by the product $\Gamma_1\phi_n^1$, where Γ_1 is the participation factor of the first mode and ϕ_n^1 the modal top displacement in the first mode.

Performance point coordinates are determined calculating the spectral displacement d_o, given by the intersection of the capacity curve with the demand curve of the selected earthquake. The demand curve is a constant ductility spectrum derived through Newmark-Hall method and plotted in the ADRS format.

Conversion of the capacity curve into bilinear form is performed using an equal energy criterion, in order to estimate a "yielding" value d_y of the top displacement and the corresponding acceleration value a_y, as well as the post-elastic stiffness. In this manner a ductility value d_o/d_y can be attributed to the target condition. Such value must equal, in the performance point, the ductility of the inelastic spectrum.

Equal energy criterion is applicable in the following cases. (i) Elastic-perfectly plastic behavior, with plastic threshold defined by the maximum strength, to be assumed. In this case the only parameter governing the bilinear curve is the initial stiffness, and can be calculated through the equal energy criterion. (ii) Elastic-plastic behavior, with hardening and maximum strength values not necessarily coincident with the ones of the original curve. Such hypotheses, can lead to more realistic bilinear curves, though they cannot be univocally defined by the equal energy criterion. Actually, they depend on the slopes of the elastic and post elastic branches.

The influence of the bilinear conversion on the structural response has been studied through parametric analyses that take into account the variation of the following parameters (figure 4): α, ratio of the maximum acceleration (final point) of the bilinear relationship with respect to the actual curve acceleration; β, ratio of the elastic stiffness of the bilinear relationship to the initial stiffness of the capacity curve.

In the parametric analyses, aimed to define the equivalent bilinear curve from the capacity spectrum, α is ranging between 0.9 and 1.0 with 0.025 increments, while β ranges from 0.6 to 1.0 with 0.05 increments.

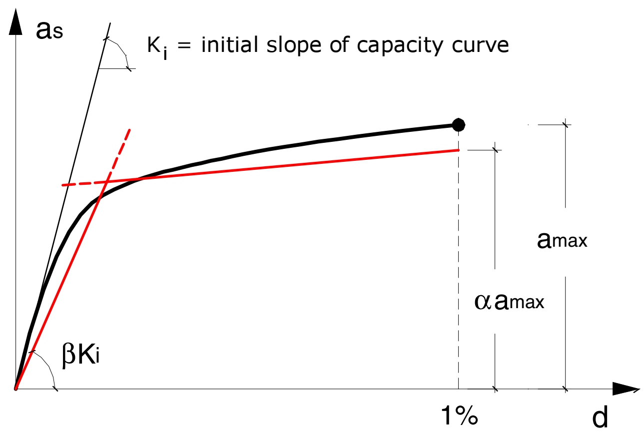

Figure 4. Parametric bilinear curve

Additional parameter that influences the equivalent bilinear curve is the maximum assumed value of the displacement on the original curve. In this study, the analyses have been carried out with 1% of the building height and repeated with 1.33%.

Non linear static analyses have been performed in the ξ and η diagonal directions (see figure 2). Then, "exact" response values have been determined using the S00E component of El Centro earthquake (PGA=0.348 g).

Additional analyses, aimed at assessing the accuracy of the different procedures herein recalled, have been performed loading the building in the y direction. For these cases, besides the El Centro accelerogram, also El Almendral record (component N50E, Valparaiso earthquake dated March 3, 1985, PGA=0.284 g) has been used, along with 8 generated signals, compatible with the soil B spectrum shape provided by the Italian seismic code [13]. ATC-40 procedure A (hysteretic behavior type A) has been applied, together with Eurocode 8, Italian seismic code and constant ductility spectra procedures.

RESULTS

With reference to the analyses performed in the ξ and η directions, figure 5 shows the values of ductility demand corresponding to the performance point as a function of β, for different values of α. Strong sensibility of the computed ductility demand can be noted, with values ranging from 1.35 to 2.76. The variability, thus, exceeds 100%.

Values moderately increase with α, while the dependence on β is significantly more evident, as expected due to the meaning of the two parameters.

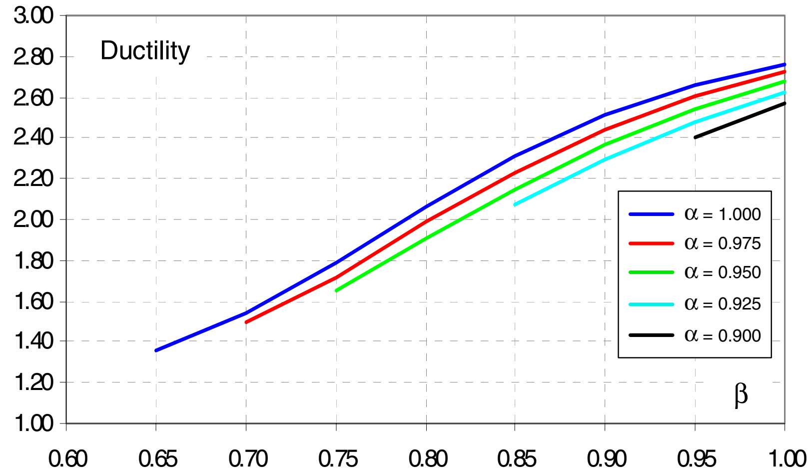

Figure 5. Ductility demand as a function of displacement

Figure 6 shows the displacement of the control point at the top of the building corresponding to the performance point. In the following such displacement value is regarded as “performance displacement”. As in the previous case, the curves correspond to different α values, while the horizontal line denotes the displacement value calculated via dynamic analysis. For any α value, the performance displacement increase with β up to a maximum achieved at β values ranging between 0.85 and 0.95. Furthermore, the values of the performance displacement can turn out smaller or greater than the dynamic displacement.

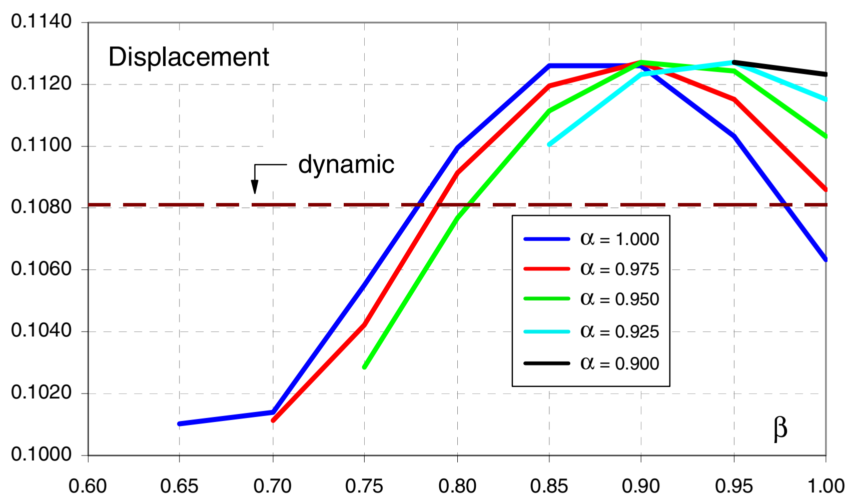

Figure 6. Performance displacement as a function of the conversion

The difference between the performance displacement and the dynamically calculated displacement is then shown in figure 7. The percentual variation is always bounded in a -7% to +5% band. It can be derived that, though the ductility demands can be significantly different, the performance displacement is almost stationary with respect to the parameters that define the curve conversion.

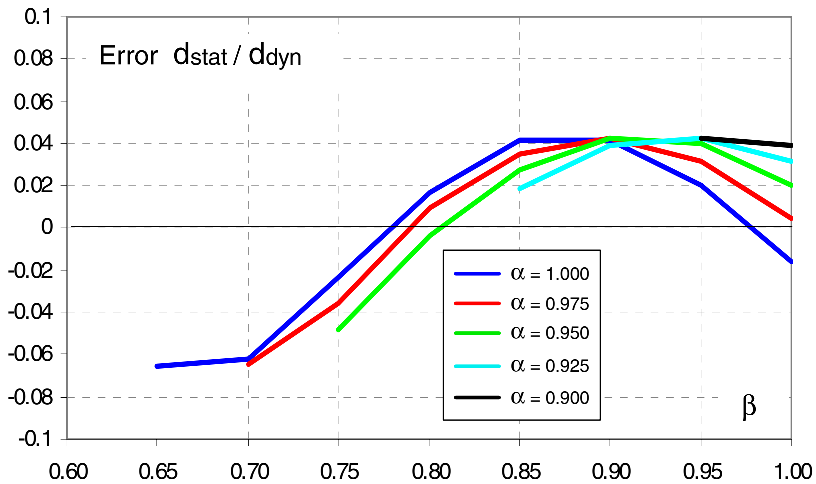

Figure 7. Error in the performance displacement

As far as the accuracy assessment is regarded, the following tables 1, 2 and 3 respectively report results corresponding to El Centro and El Almendral records, and the mean values of the ones obtained with the 8 generated accelerograms. Bold characters denote in each table the minimum error.

In the examined cases, the procedure using constant ductility spectra appears to be the most accurate, except for El Centro record. In such case the errors in the procedures are yet very similar, scoring a 5% value. Further analyses are now being carried out by the authors, in order to assess this issue in a wider variety of records.

Table 1. Results from El Centro record

Procedure	Intermediate data	Spectral displacement	Top displacement	Error (%)
ATC 40 - Procedure A *Hysteretic behavior A*		76.5	102.5	**-4.12**
Constant ductility spectra *Equivalent EP curve up to d_{max} = 1% height*	μ = 2.845	84.1	112.4	+5.14
Constant ductility spectra *Equivalent EP curve up to $d_{max} \approx$ solution*	μ = 2.915	69.7	93.1	-12.9
Italian Seismic Code [13] *Equivalent EP curve up to horizontal slope*	M* = 1333 K* = 87500 T* = 0.7755	84.4	111.8	+4.58
Eurocode 8 *Equivalent EP curve up to $d_{max} \approx$ solution*	M* = 1333 K* = 108375 T* = 0.6968	75.5	101.0	-5.52
Dynamic analysis			106.9	

Table 2. Results from El Almendral record

Procedure	Intermediate data	Spectral displacement	Top displacement	Error (%)
ATC 40 - Procedure A *Hysteretic behavior A*		81.57	109.0	+13.68
Constant ductility spectra *Equivalent EP curve up to* d_{max} *= 1% height*	μ = 2.52	74.4	99.46	**+3.73**
Constant ductility spectra *Equivalent EP curve up to* $d_{max} \approx$ *solution*	-----	no solution	-----	----
Italian Seismic Code [13] *Equivalent EP curve up to horizontal slope*	M* = 1333 K* = 87500 T* = 0.7755	75.8	100.43	+4.75
Eurocode 8 *Equivalent EP curve up to* $d_{max} \approx$ *solution*	M*=1333 K*=108600 T* = 0.699	81.9	108.5	+13.16
Dynamic analysis			95.88	

Table 3. Mean values of results from the 8 generated signals

Procedure	Intermediate data	Spectral displacement	Top displacement	Error (%)
ATC 40 - Procedure A *Hysteretic behavior A*		98.8	132.0	+16.1
Constant ductility spectra *Equivalent EP curve up to* d_{max} *= 1% height*	μ = 2.61	77.0	103.0	**-9.4**
Constant ductility spectra *Equivalent EP curve up to* $d_{max} \approx$ *solution*	μ = 2.90	68.0	90.96	-20.0
Italian Seismic Code [13] *Equivalent EP curve up to horizontal slope*	M*=1333 K*=87500 T* = 0.7755	105.3	139.5	+22.7
Eurocode 8 *Equivalent EP curve up to* $d_{max} \approx$ *solution*	M*=1333 K*=99740 T* = 0.7264	98.6	130.6	+14.9
Dynamic analysis			113.7	

CONCLUSIVE REMARKS

The analyses presented herein show that the use of constant ductility spectra in non linear static analysis procedures leads to a substantial stability of results, also with respect to the pushover curve conversion issue, providing that parameters defining the bilinear converted capacity curve vary in the selected range.

In any case, it must be noted that although the influence of the capacity curve conversion on the displacement corresponding to the performance point appears to be almost negligible, the ductility demands are affected by a huge variability. This leads to the fact that, as the conversion varies, different ductility demands, and so different damage conditions, correspond to the same performance point. Though this last consideration might seem trivial considering the definition of ductility itself, it turns out to be very important from a design point of view when capacity design has to be adopted. Such influence appears to be even more important from a retrofit point of view.

As far as the reliability of the different procedures is regarded, the numerical investigations have shown that the use of constant ductility spectra generally leads to more accurate results, with errors often 50% smaller than the ones obtained with other procedures.

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