13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 1675

TITLE

THE LINEAR FORMULATION FOR THE EQUATIONS OF THE DYNAMIC RESPONSES OF RIGID AND FLEXIBLE STRUCTURES SUPPORTED BY FRICTION PENDULUM SLIDING (FPS) BEARINGS

Authors NAMES[1]

MUTLU OZER

SUMMARY

Friction Pendulum Sliding (FPS) bearings are very efficient and cost effective seismic protection devices, which simply alter the force-response characteristics of the structures by a large displacement response at isolation level. Maximum dynamic displacement response of an isolated structure is a crucial parameter for designing a reliable seismic protection system. The governing equation of the dynamic response of an isolated structure supported by (FPS) bearings consists of a non-linear trigonometric term that makes the analysis of the isolated structure extremely difficult. Therefore this paper proposes a linear second-order ordinary differential equation (ODE) that represents the governing equation of the motions of isolated structures by (FPS) bearings.

The procedure basically starts to design a mathematical model of a two-DOF system that is used to analyze the motion of an isolated structure excited by harmonic ground acceleration. In this study, it is proven that the non-linear trigonometric term of governing equation is the function of radius of curvature of friction pendulum bearings (FPB), damped-natural frequency of isolated structure and relative velocity of supported system at isolation level. This functional relationship is basically used to eliminate non-linear trigonometric term from the governing equation and to derive linear second-order ordinary differential equations of the motions of rigid and flexible structures supported by (FPS) bearings. Parameter studies are also performed to verify these equations with the results of a recent experimental study[2]. The primary parameters of parameter study are the excitation frequency of $w = 3\pi$ rad/s, peak ground accelerations of 1.0g and 0.5g, and radius of curvatures of friction pendulum bearings, which vary from R=30 in to R= 250 in. The figures of parameter study illustrate the relative displacement responses, absolute acceleration responses and absolute velocity responses of isolated structures. By using the values of maximum acceleration responses, the inertia reductions of isolated rigid structures are calculated.

As an example, dynamic responses of a 300 kips-rigid structure supported by (FPS) bearings subjected to harmonic ground acceleration are also calculated and the results of seismic design properties are tabulated. Those results are consistent with the results of experimental studies.

[1] 1600 Holloway Ave. San Francisco State University, School of Engineering, San Francisco, CA 94132

INTRODUCTION

The Spherically shaped, articulated sliding bearings are placed at foundation level to support complete structures of buildings, bridges and heavy industrial equipment such as heavy electrical transformers and large liquid storage tanks. The isolated structure supported by (FPS) bearings undergoes friction pendulum motion when excited by ground motion. The magnitude of the lateral force that causes the lateral displacement of the supported structure depends primarily upon the curvature of the spherical sliding surface, the coefficient of friction mobilized during sliding and the vertical load on the bearings. The lateral force is proportional to the vertical load, which minimizes torsional motions in structures. The following mathematical model is designed to derive the governing equations of the dynamic responses of structures isolated by (FPS) bearings. With this model dynamic response of isolated structures are analyzed when such system undergoes harmonic ground excitation. Parameter studies are performed for the sizes of radius of curvature of friction pendulum bearings (FPB) of R=30, R=60, R=90, R=120, R=150, R=180, R=210, R=250 inches. In this parameter study, the excitation frequency is used as of $w = 3\pi$ since the dominant frequency of most major earthquakes is close to $w = 3\pi$, which corresponds to the period of 0.7s. Peak ground accelerations are used 0.5*g and 1.0*g.

The governing equation of the dynamic response of a structure supported by (FPS) bearings

The mathematical model shown in Fig. (a) represents the motion of the structures as a single-DOF system excited by harmonic ground acceleration. With this model, M_b denotes of the total mass of the structure if structure is totally rigid; if not it denotes the mass portion of the structure that vibrates at the frequency of friction pendulum bearings. In this case the structure behaves as a flexible structure, and the rest portion of the mass of the structure, M_s, which vibrates at its own frequency. The equations of the motion are derived as follows.

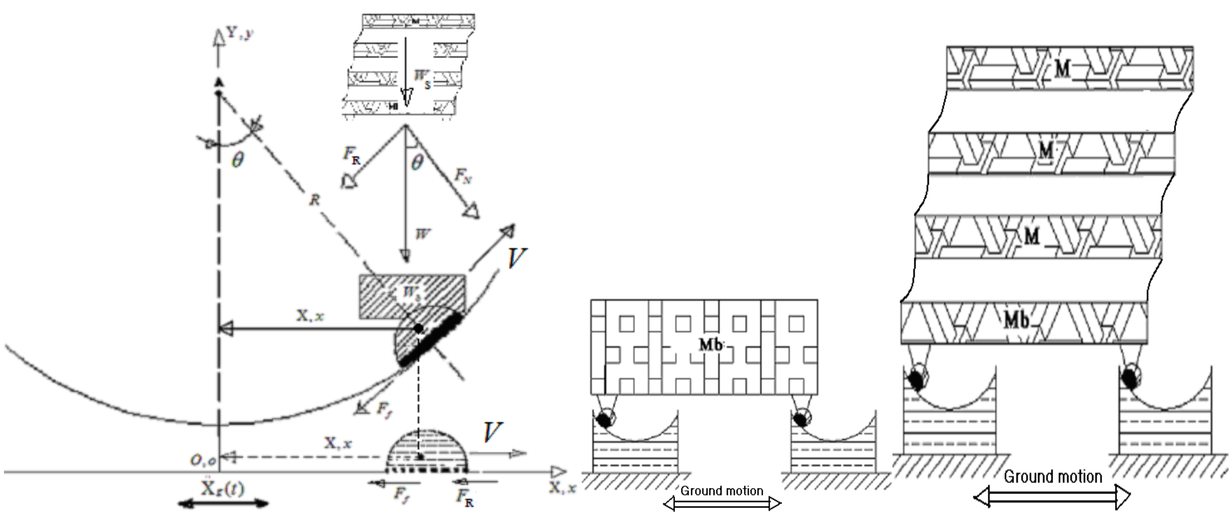

Fig 1. (a) The mathematical model (two-DOF system) for analyzing the dynamic response of the structure supported by (FPS) bearings, (b) schematic of an isolated rigid structure by (FPS) bearings (c) schematic of an isolated flexible structure by (FPS) bearings

$$\sin\theta = x/R \qquad (1)$$
$$\sin\theta = F_R/W$$

$\cos\theta = F_N / W$

$F_f = \mu F_N$

$$F_R = W(x/R) \qquad (2)$$

$$F_f = \mu W \cos\theta \qquad (3)$$

$\ddot{X}_g(t) = \ddot{X}_{g0}\sin(wt)$

$\ddot{X} = \ddot{x} + \ddot{X}_g(t)$

$W = W_b + W_s$

$V = F_R + F_f$

Dynamic equilibrium of the structure supported by (FPS) bearings is as follows:

$M_b\ddot{X} + V = 0$ or,

$$M_b(\ddot{x} + \ddot{X}_g) + F_R + F_f = 0 \qquad (4)$$

by substituting equation (2) and equation (3) into equation (4), the governing equation of structure, which is supported by FPB and subjected to harmonic ground excitation would be derived as follows:

$$M_b\ddot{x} + \mu W\cos\theta + (W/R)x = -M_b\ddot{X}_g \qquad (5)$$

It is extremely difficult to solve the above second-order non-linear differential equation (5). Therefore, nonlinear trigonometric term, $\cos\theta$, is eliminated by redefining it with the terms of $R, \dot{x}$ and w_D. In order to obtain a linear second-order (ODE) equation from this non-linear differential equation (5), the procedure starts by taking derivative of equation (1) as follows:

$d(\sin\theta) = d(x/R)$

$$\cos\theta = \dot{x} / (\dot{\theta}R) \qquad (6)$$

$\dot{\theta}$, the angular velocity, which is the natural vibration frequency, w_D, of damped structure supported by friction pendulum bearings (FPB) subjected to harmonic ground acceleration.

The equation (5) is reorganized by substituting $W = W_b + W_s$, $M = M_b + M_s$, $\cos\theta = \dot{x} / (\dot{\theta}R)$, $\dot{\theta} = w_D$ and $w_n = \sqrt{g/R}$ as follows:

$$\ddot{x} + \mu\frac{(M_b + M_s)}{M_b}\left(\frac{w_n^2}{w_D}\right)\dot{x} + \frac{(M_b + M_s)}{M_b}w_n^2 x = -\ddot{X}_g$$

Since $w_D = w_n\sqrt{1-\xi^2}$, and $w_D \cong w_n$ when $\xi << 1$, the motion of the supported structure with (FPS) bearings is derived as a linear second-order ODE as follows.

$$\ddot{x} + \mu\left(1 + \frac{M_s}{M_b}\right)w_n\dot{x} + \left(1 + \frac{M_s}{M_b}\right)w_n^2 x = -\ddot{X}_g \qquad (7)$$

Where,

W, M The total weight, mass of the supported structure

W_b, M_b The weight, mass of the structure that vibrates at the frequency of FPB

W_s, M_s The weight, mass of the structure that vibrates at its own frequency

X, Y Imaginary fix coordinate system

x, y Translating coordinate system, which is attached to (FPS) bearings

V The lateral force or shear force of bearing at the isolation level

F_R Restoring force

F_f Friction force

$\dot{\mathrm{X}}, \dot{x}$ Absolute, relative velocity of bearing

$\ddot{\mathrm{X}}, \ddot{x}$ Absolute, relative acceleration of bearing

$\ddot{\mathrm{X}}_g(t)$ Harmonic ground acceleration($\ddot{\mathrm{X}}_g(t) = \ddot{\mathrm{X}}_{g0} \sin(wt)$)

$\ddot{\mathrm{X}}_{g0}$ Peak ground acceleration

R The radius of curvature of bearing

μ The coefficient of friction mobilized during sliding (assumed constant)

w_n Natural frequency of (FPS) bearing

w_D Damped-natural frequency of isolated structure

w Excitation frequency

ξ Damping

ϖ_n Dominate-natural frequency of flexible structure supported by (FPS) bearings

The equation of the motion for rigid structure in the form of a linear second-order ODE

$M_s = 0$, $M_b =$ The mass, which is also total mass, since $M_s = 0$ of the supported structure that vibrates at the frequency of friction pendulum bearings (FPB). The governing equation of the motion of a supported rigid structure by (FPB) would be derived from equation (7) as follows:

$$\ddot{x} + 2\xi w_n \dot{x} + w_n^2 x = -\ddot{X}_g \qquad (8)$$

Where, $\xi = \mu/2$

The equation of the motion for flexible structure in the form of a linear second-order ODE

$M_s > 0$, and ϖ_n, which is dominate-natural frequency of system is $\varpi_n^2 = (1 + \frac{M_s}{M_b}) w_n^2$. The governing equation of the motion of a supported flexible structure by (FPB) would be derived from equation (7) as follows:

$$\ddot{x} + 2\xi \varpi_n \dot{x} + \varpi_n^2 x = -\ddot{X}_g \qquad (9)$$

where, $\xi = \frac{\mu}{2}\sqrt{(1 + \frac{M_s}{M_b})}$, Since $(1 + \frac{M_s}{M_b}) > 0$ then $\varpi > w_n$, $\xi_{rigid} < \zeta_{flexible}$

Indeed, the dynamic response of flexible structure is a combination of two vibrations. Those vibrations are the vibration of some mass of structure that vibrates at the frequency of FPB, and the vibration of the rest of the mass of the structure that vibrates its own dominant frequency.

The obvious question would be asked about what fraction of the total mass of structure vibrates at the frequency of friction pendulum bearings (FPB), or what fraction of the total mass of structure vibrates at its own dominant frequency? The stiffness level of the structure would be the answer of this question. For example, if stiffness of structure is too large, then whole structure would be assumed rigid, and $M_s = 0$. But an explicit, formulated answer to this question is not in the scope of this study.

It is also shown from the above equation that damping, for flexible structure, $\xi > \mu/2$, and so the displacement response of flexible structure would be expected less than displacement response of the rigid structure supported by (FPS) bearings excited by harmonic ground motion.

The complete solution of the above linear second-order ordinary differential equation (8) and equation (9) would be as follows:

$x = e^{-\xi w_n t}(A\cos w_D t + B\sin w_D t) + C\sin wt + D\cos wt$ (For rigid structure)

$x = e^{-\xi w_n t}(A\cos \varpi_D t + B\sin \varpi_D t) + C\sin wt + D\cos wt$ (For flexible structure)

This general solution contains two distinct vibration components: Transient vibration and steady state vibration, where the constants A, B, C and D are defined as follows:

$$A = (-\ddot{X}_{g0}/w_n^2)\frac{2\xi(w/w_n)}{\left[1-(w/w_n)^2\right]^2 + \left[2\xi(w/w_n)\right]^2}(\frac{M_b}{M_s + M_b})$$

$$B = (\ddot{X}_{g0}/w_n^2)\frac{w\left[1-(w/w_n)^2\right] - 2\xi^2 w_n (w/w_n)}{\left[1-(w/w_n)^2\right]^2 + \left[2\xi(w/w_n)\right]^2}(\frac{M_b}{M_s + M_b})(\frac{1}{w_D})$$

$$C = (-\ddot{X}_{g0} / w_n^2) \frac{1-(w/w_n)^2}{\left[1-(w/w_n)^2\right]^2 + \left[2\xi(w/w_n)\right]^2} (\frac{M_b}{M_s + M_b})$$

$$D = (\ddot{X}_{g0} / w_n^2) \frac{2\xi(w/w_n)}{\left[1-(w/w_n)^2\right]^2 + \left[2\xi(w/w_n)\right]^2} (\frac{M_b}{M_s + M_b})$$

For flexible structure, the natural frequency of w_n in the above equations will be replaced by

$$\varpi_n^2 = (1 + \frac{M_s}{M_b}) w_n^2$$

Parameter study

Parameter studies performed for the rigid and flexible structures when the structures are excited by harmonic ground acceleration. Excitation frequency is used as $w = 3\pi$. Maximum peak accelerations are 0.5g and 1.0g. For flexible structure mass ratio is assumed $(M_s / M_b) = 3$. Mat lab figures of Fig. 2, Fig. 3 and Fig. 4 illustrate dynamic and free responses- relative displacements, absolute velocities and absolute accelerations-of rigid and flexible structures supported by (FPS) bearings for radius of curvature of R=60 in., R=120 in.,

As an example, seismic design properties of a 300 kips rigid structure, which supported by (FPS) bearings excited by harmonic ground acceleration are calculated, and the results of seismic design properties are tabulated in table1.

w=3*pi (excitation frequency), $\mu = 0.10$ (the coefficient of friction mobilized during sliding-assumed constant-), X_{g0}=1.0g (peak ground acceleration)							
Radius of Curvature of FPB R (in.)	Natural period of FPB $T_n = 2\pi\sqrt{\frac{R}{g}}$ (s)	Nat. Freq. of FPB $w_n =$ $(*\pi)$ rad./s	Maximum displacement of structure @ isolation level D(in.)	Inertia reduction %	Effective Stiffness $K_{eff} = \frac{V}{D}$ (lb/in.)	Effective Period $T_{eff} = 2\pi\sqrt{\frac{W}{K_{eff}\, g}}$ (s)	Effective damping Ratio $\zeta_{eff} = \frac{2}{\pi}(\frac{\mu}{\mu + \frac{D}{R}})$ ….%
30	1.8	1.14	17.5	44	9627	1.8	9
60	2.5	0.81	21	64	5102	2.5	14
90	3.0	0.66	22.5	74	3451	3.0	18
120	3.5	0.57	24	78	2717	3.4	21
150	3.9	0.51	27.5	81	2033	3.9	22
180	4.3	0.47	32	82	1650	4.3	23
210	4.6	0.43	34	84	1439	4.6	24
250	5.1	0.40	36	85	1229	5.0	26

Table 1. Design properties of the dynamic responses of a 300 kips rigid structure supported by (FPS) bearings subjected to harmonic ground acceleration.

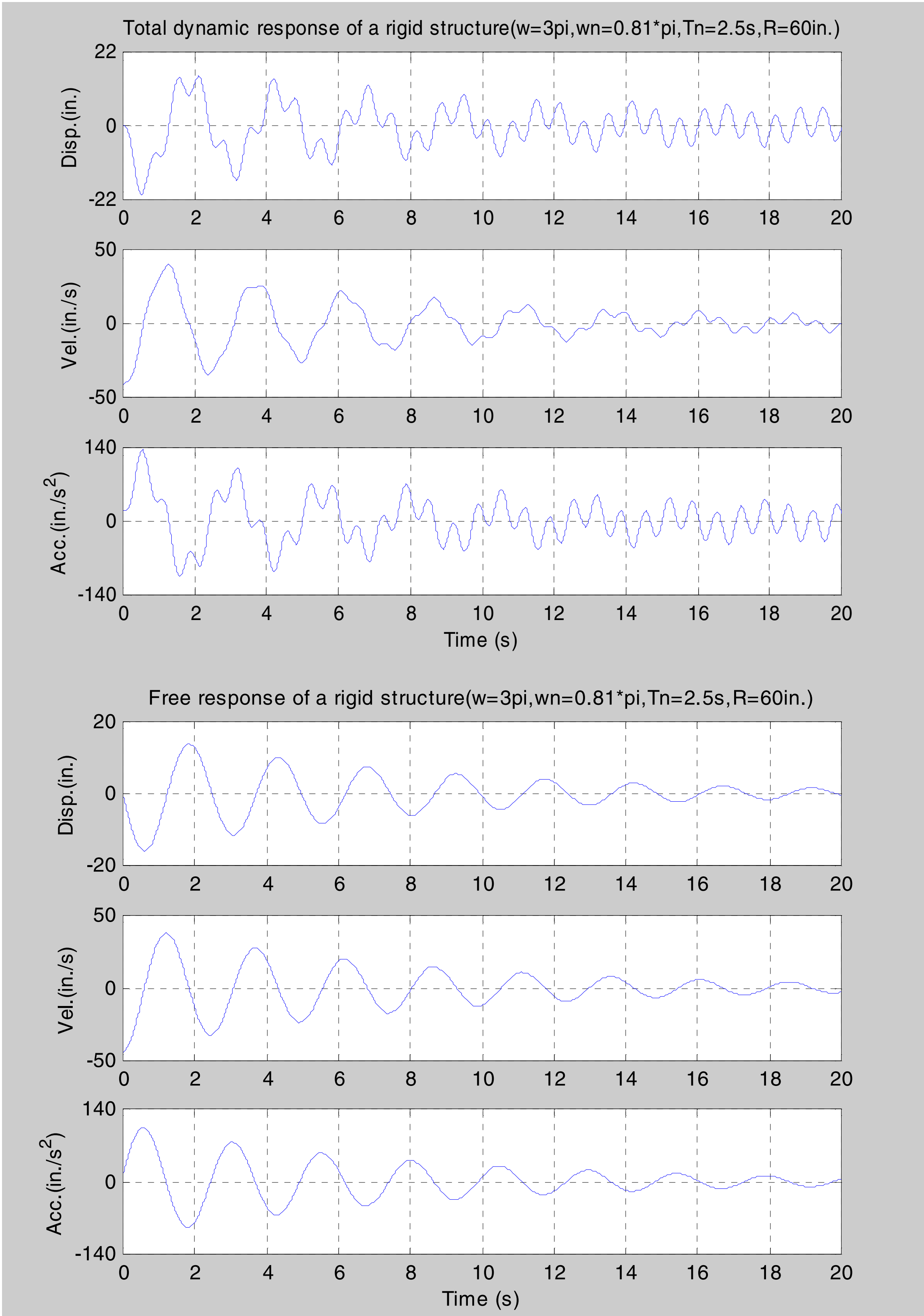

Fig. 2 Total dynamic and free responses of a rigid structure supported by (FPS) bearings (Radius of curvature of FPB, R=60 in.)

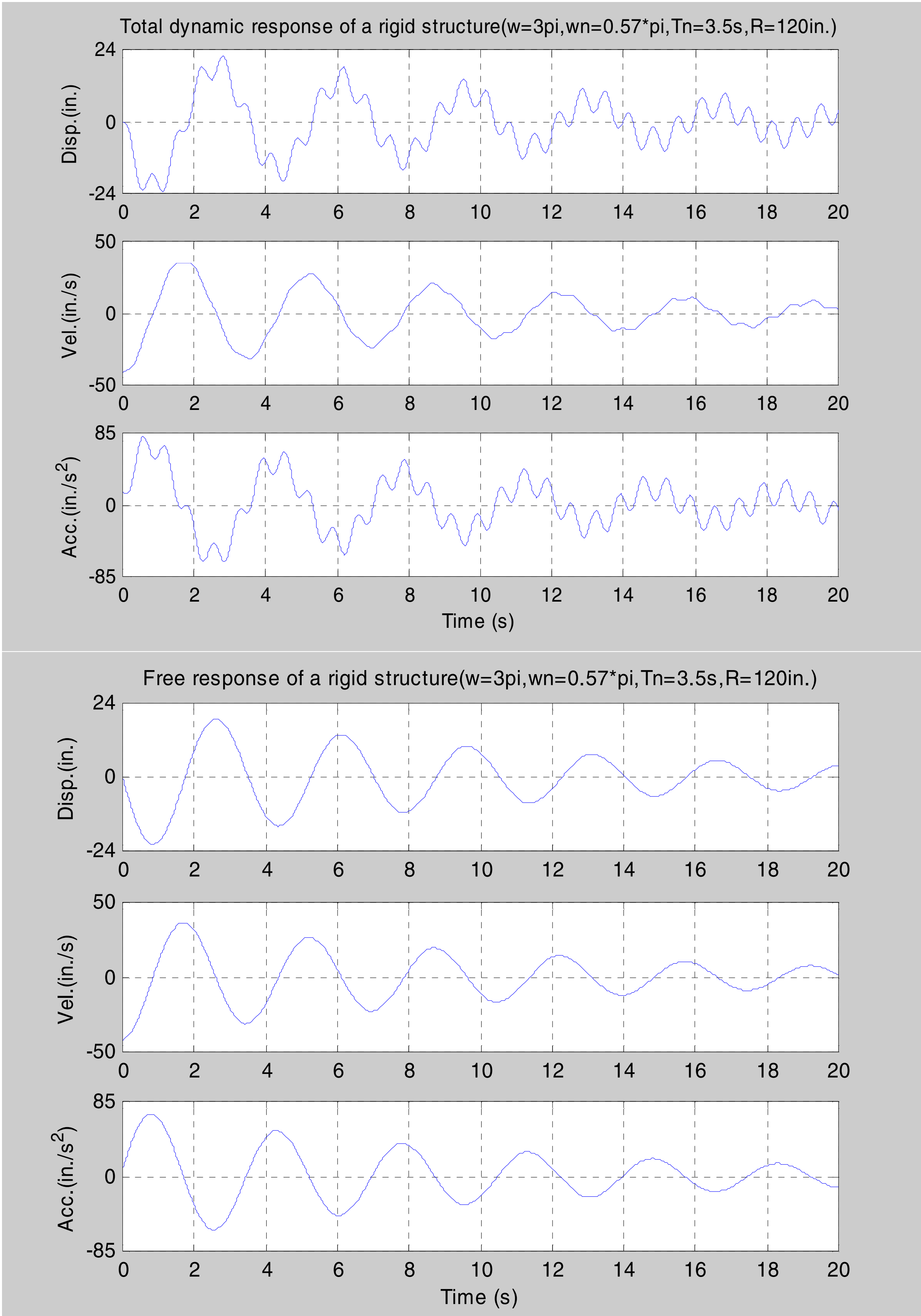

Fig. 3 Total dynamic and free responses of a rigid structure supported by (FPS) bearings (Radius of curvature of FPB, R=120 in.)

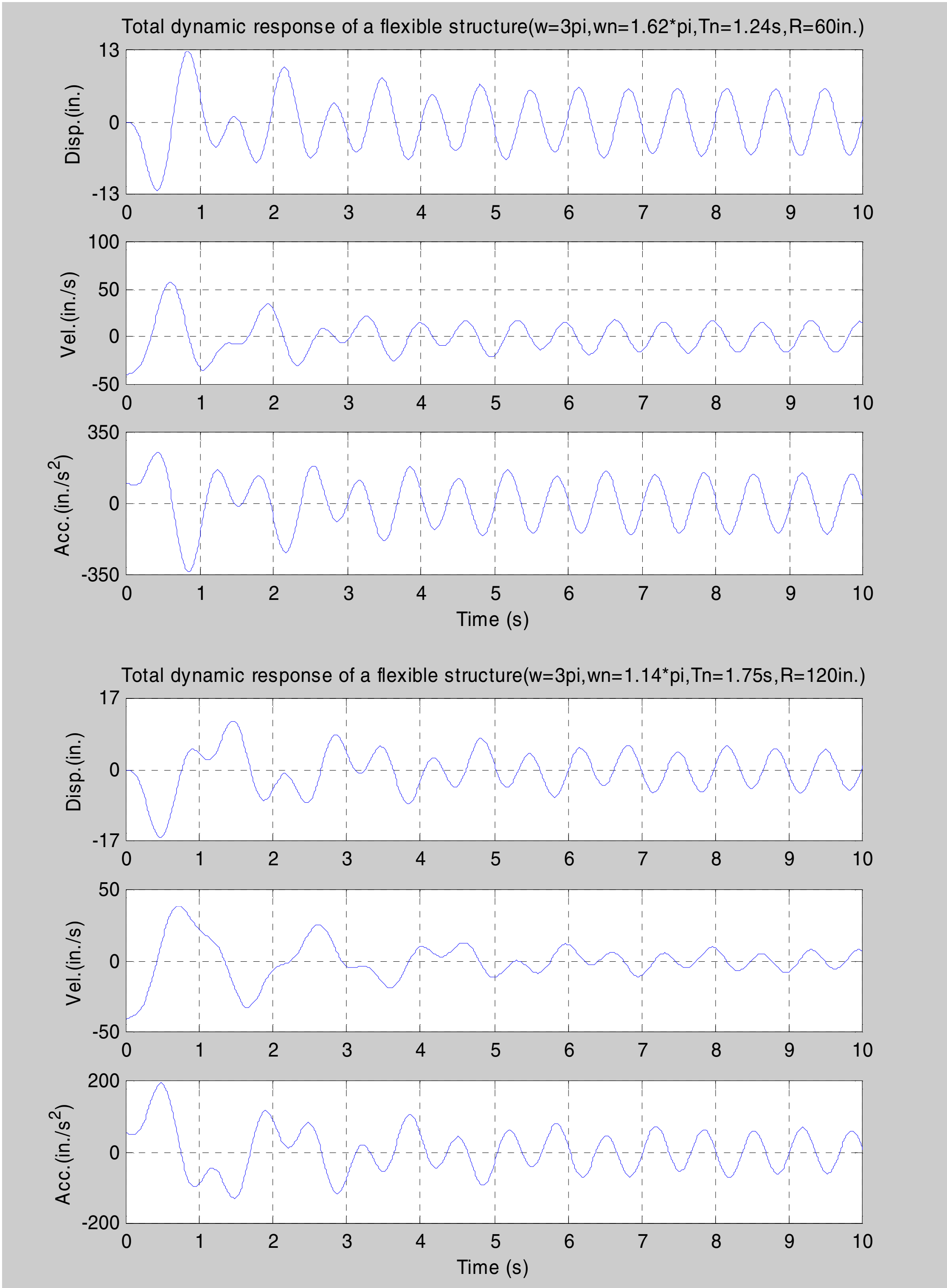

Fig. 4 Total dynamic responses of a flexible structure supported by (FPS) bearings (Radius of curvature of FPB, R=60 in. and R=120in., $(M_s / M_b) = 3$)

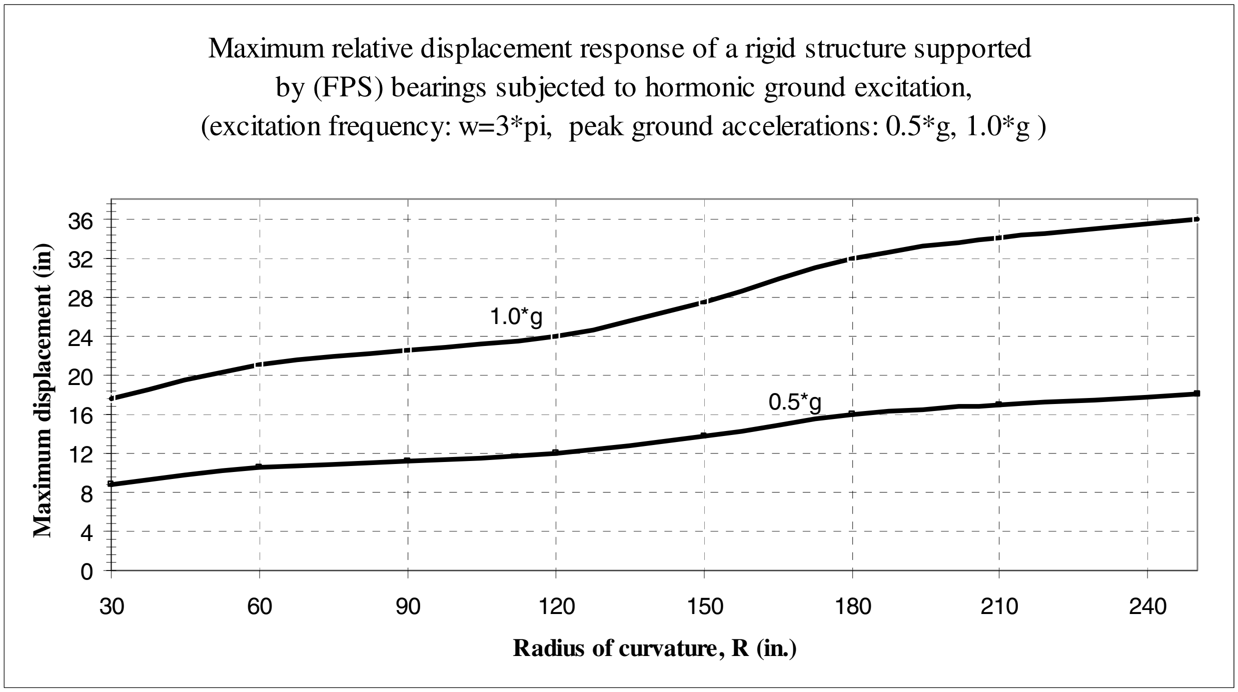

Fig. 5 Maximum relative dynamic displacement response of a rigid structure supported by (FPS) bearings subjected to harmonic ground acceleration.

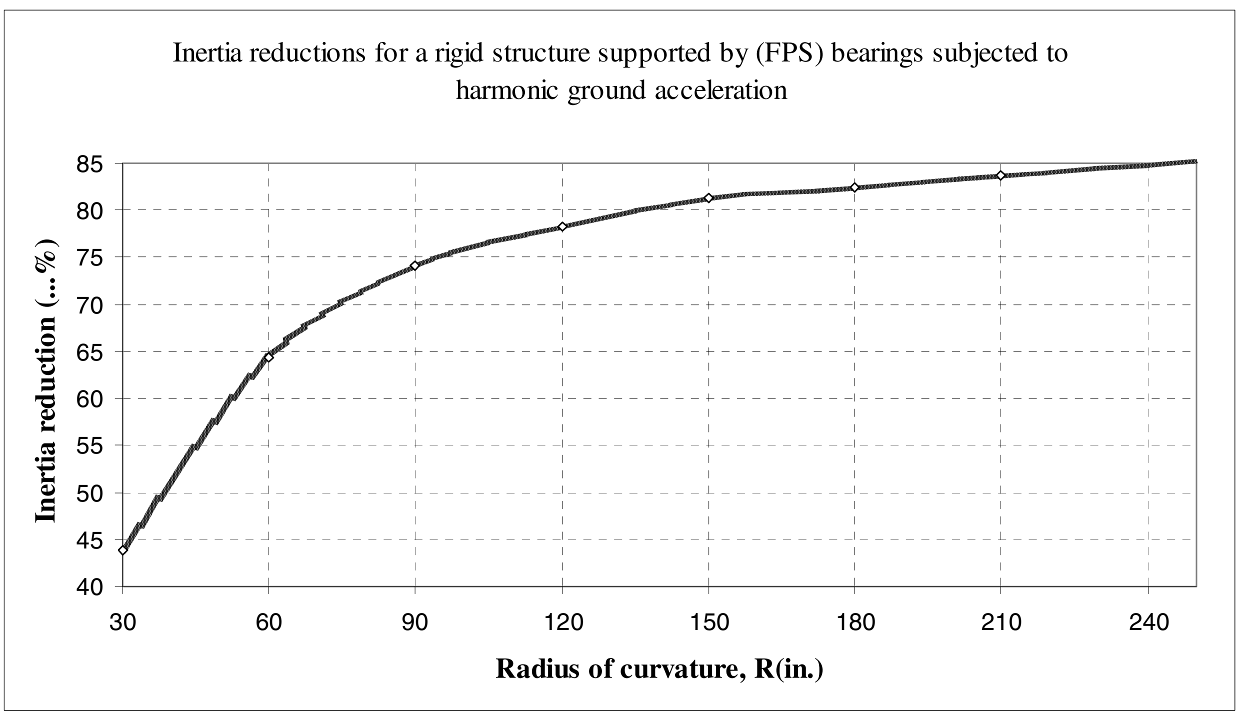

Fig. 6 Inertia reduction of dynamic responses of a rigid structure supported by (FPS) bearings subjected to harmonic ground acceleration

CONCLUSION

The results of dynamic responses of a rigid structure supported by (FPS) bearings, which are illustrated in Fig. 2, Fig. 3, Fig. 5, Fig. 6, and tabulated in Table 1. are obtained by the solution of the proposed linear second-order differential equations (8). Those results are very consistent with the results of the recent experimental study[2]. Experimental study was performed based on real earthquake excitation, which dominant excitation frequency of ground $\approx 3\pi$ rad/s. Therefore, the results of parameter study obtained by the solution of the proposed linear second-order ODE of equation (8) are pretty close to represent the results of dynamic responses of the supported structures by (FPS) bearings when such structures are excited by non-harmonic and non-periodic earthquake motion.

The results of dynamic responses of a flexible structure supported by (FPS) bearings, which are illustrated in Fig. 4 depends on the mass ratio of structure, which is assumed in this parameter study $(M_s / M_b) = 3$. This ratio is fundamentally the function of stiffness matrix of the structure. But the formulated functional relationship between stiffness of the structure and the above mass ratio is out of the scope of this study. The Coefficient of friction used in parameter study is $\mu = 0.10$, which is assumed constant during the vibration of an isolated structure. As a result, the inertia reductions illustrated in Fig. 6 are identical for different peak ground accelerations of 0.5*g and 1.0*g. However, in real application, μ is not constant, and the value of μ is depends upon the magnitude of the relative velocity of the supported structures by (FPS) bearings at isolation level. Therefore, the inertia reduction in experimental study[2] varies with the magnitude of peak ground acceleration.

In short, this study verifies the reliability of those proposed linear second-order ODE of equation (8) and equation (9) for the dynamic analysis of an isolated rigid and flexible structure supported by (FPS) bearings to obtain design properties for preliminary design procedure.

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