13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 1678

COMPARATIVE STUDY OF DIFFERENT METHODS OF STRUCTURAL DAMAGE ASSESSMENT

Rustem V. SHAIKHUTDINOV [1], James L. BECK[2], Keith A. PORTER[3]

SUMMARY

In a modern performance-based earthquake engineering (PBEE) design framework, component fragility functions are used to relate parameters of structural response to damage. Since common structural response parameters are coupled to structural member properties, the usage of fragility functions in damage analysis after performing the structural analysis is inconsistent. The errors arising from such inconsistency do not seem to have been addressed previously. In the present study, we investigate this issue by comparing the results of damage estimation performed by three different methods. The first method, considered to be the most accurate, represents a coupled structural and damage analysis and utilizes randomized structural properties in the simulations for both the dynamic structural analysis and the damage analysis. The second, uncoupled, method samples the randomized structural properties twice during dynamic simulations, once for structural analysis and once for damage estimation. The third method is analogous to the second method but it uses a deterministic structural model for the dynamic simulation and uncertain structural properties for the damage analysis. Comparison of the results for a reinforced-concrete frame shows that relative to the first method, the second method provides reasonable estimates of the expected values and variances of the damage, while the third method underestimates the damage uncertainty. The variance estimated by a common approach based on a deterministic load and deterministic structure complemented by a probabilistic damage analysis is much less than the variance estimated by the first method.

INTRODUCTION

Today, the earthquake engineering community faces new challenges brought about by the needs of the real estate development and management industries. The safety of buildings and structures is no longer the only concern of their owners. Assuming that safety requirements are met by satisfying the building code, many owners are now asking: “how much is it likely to cost to repair after an earthquake?” and “how long is it likely to be shut down in case of an earthquake?” These questions are directed at the evaluation of the economic performance of the structure rather than just its engineering performance. Therefore, a formal

[1] Doctoral Candidate, California Institute of Technology, Pasadena, California, USA. Email: rustem@caltech.edu
[2] Professor, California Institute of Technology, Pasadena, California, USA.
[3] GW Housner Senior Research Fellow, California Institute of Technology, Pasadena, California, USA.

analytical framework is desirable to evaluate both the safety and economic performance parameters that are of major concern for real estate owners and managers. An essential element of this framework is the probabilistic assessment of the damage states of the structural and non-structural components.

Some of the proposed analytical frameworks are assembly based vulnerability (ABV) in Porter [1], Beck [2,3] and the Performance Based Earthquake Engineering (PBEE) framing equation of the Pacific Earthquake Engineering Research (PEER) Center (Krawinkler [4], Miranda [5]). These techniques perform structural analysis to obtain structural response and then use fragility functions to relate this response to inflicted damage. However, if the objects of damage evaluation are structural components, such as beams, columns, etc., using fragility functions after structural analysis leads to inconsistencies since, in reality, damage to structural components affects the structural response. (By fragility function, we mean the probability of a component reaching or exceeding a particular level of damage as a function of the structural response to which the component is subjected.) To eliminate these inconsistencies, we propose a coupled analysis that performs the damage analysis concurrently with the structural analysis. We compare the two approaches by performing damage estimation for a reinforced-concrete moment frame.

PROBABILITY OF PERFORMANCE CRITERIA BEING MET

In dealing with the seismic performance of a structure, one should consider a broad range of uncertain factors. The number of factors involved in the analysis varies depending on the chosen performance criteria. In general, we have to consider such variables as ground motion, building properties, the cost of material and labor, etc. Because of the uncertainty of these independent variables, the performance criteria can not be determined precisely. Consequently, it is important to address this uncertainty by estimating the probability of a performance parameter being in a certain range. Following PEER practice, we refer to performance parameters as decision variables, emphasizing that the parameters are for use in a decision-making process for seismic design or mitigation. The probability of a decision variable being greater than some critical value is given by the following expression:

$$P(DV > DV_l) = \int\limits_{DV(\underline{q},\underline{x},\underline{m}) \le DV_l} f_{\underline{Q},\underline{X},\underline{M}}(\underline{q},\underline{x},\underline{m})\, d\underline{q}\, d\underline{x}\, d\underline{m} \tag{1}$$

where the decision variable (*DV)* represents a chosen performance parameter such as repair cost or downtime; the vector $\underline{Q}$ encompasses the group of variables that define the ground motion time history; $\underline{X}$ contains structural and nonstructural properties; $\underline{M}$ is composed of variables that we collectively call market conditions (labor and material prices, availability, contractors etc.); and $f_{\underline{Q},\underline{X},\underline{M}}(\underline{q},\underline{x},\underline{m})$ is a joint probability density function (PDF) of all the uncertain variables.

The integral (1) can be written in a more convenient form that has been proposed to be a basis for PBEE (Irfanoglu [6], Krawinkler [4], Miranda [5]):

$$P(DV > DV_l) = \sum_{i=1}^{N} \int_0^\infty \int_0^\infty \{P(DV > DV_l \mid \underline{DM} = \underline{dm}_i) P_{\underline{DM}|\underline{EDP}}(\underline{dm}_i \mid \underline{edp}) \\ f_{\underline{EDP}|IM}(\underline{edp} \mid im) f_{IM}(im)\} d(\underline{edp}) d(im) \tag{2}$$

where damage measure ($\underline{DM}$) is a vector containing discrete damage states of all damageable components; $\underline{EDP}$ is a vector of engineering demand parameters containing structural response characteristics such as inter-story drift ratio (IDR), peak diaphragm acceleration (PDA) etc.; $\underline{IM}$ is an intensity measure of the ground motion such as spectral acceleration (S_a), peak ground acceleration (PGA), etc.; $P(DV > DV_l \mid \underline{DM} = \underline{dm}_i)$ is the probability of the decision variable being greater than DV_l conditioned on knowledge of the component damage states, $\underline{dm}_i$, $i = 1...N$, where N is the number of possible damage states of vector $\underline{DM}$;

$P_{\underline{DM}|\underline{EDP}}(\underline{dm}_i \mid \underline{edp})$ is the probability that the structure suffered the damage defined by $\underline{dm}_i$ given that it has been subjected to the $\underline{EDP}$ equal to $\underline{edp}$; $f_{\underline{EDP}|\underline{IM}}(\underline{edp} \mid \underline{im})$ is the conditional PDF of the structural response ($\underline{EDP}$), given that intensity of the ground motion is $\underline{im}$; $f_{\underline{IM}}(\underline{im})$ is the joint PDF of the vector of seismic event intensity measures ($\underline{IM}$). Equation (2) gives the probability of a decision variable being greater than some threshold value given that an earthquake has happened.

Note that (2) is another way of formulating (1) under the assumption that each analysis stage: hazard, structural, damage and loss can be done separately. The integration over state space variables $\underline{Q}$, $\underline{X}$, $\underline{M}$ is implicit in (2) as opposed to the explicit integral form (1). For example, probability $P(\underline{EDP} > \underline{edp} \mid IM = im)$ is found by integration of the joint PDF over the region of values of $\underline{Q}$ corresponding to the intensity im and values of $\underline{X}$ providing $\underline{EDP} > \underline{edp}$.

One of the essential parts of the PBEE framing equation is the conditional probability of being in a particular damage state given the response: $P(\underline{DM} = \underline{dm}_i \mid \underline{EDP} = \underline{edp})$. The vector $\underline{DM}$ is a collection of damage states of every damageable component, where damage state is chosen as a discrete variable defining the severity of the damage for each damageable building component. In practice, the conditional probability for the vector $\underline{DM}$ can usually be expressed in terms of scalar conditional probabilities of its members: $P(DM_j = n \mid EDP_i = z)$, where $n = 1...k$, is the damage state number, k being the number of damage states of the j-th component; EDP_i is a member of $\underline{EDP}$ relevant to damage of the j-th component; z is the known value of EDP_i. This scalar conditional probability can be found from:

$$P(DM_j = n \mid EDP_i = z) = P(DM_j \geq n \mid EDP_i = z) - P(DM_j \geq n+1 \mid EDP_i = z) \tag{3}$$

where $P(DM_j \geq n \mid EDP_i = z)$ is the fragility function of the j-th component with respect to the n-th damage state, expressed in terms of EDP_i. By definition, the fragility function of an arbitrary component is the probability of the component being in n-th or higher damage state, given that the relevant EDP is equal to z:

$$F^n(z) = P(DM \geq n \mid EDP = z) \tag{4}$$

In the next section, we consider fragility functions in detail.

FRAGILITY FUNCTIONS OVERVIEW

We shall apply definition (4) to the structural members of a facility, meaning all the components that affect structural response, such as beam, columns, slabs, etc. Thus, consider a structural member under seismic loading, which has k damage states. In general, the occurrence of damage to the element depends on various conditions: the earthquake characteristics, design of the structure, properties of the elements of the structure including the properties of the element under consideration. As before, we denote the variables that define the earthquake properties (time history) by the vector $\underline{Q}$, and variables that define the structural properties by the vector $\underline{X}$. We write $\underline{X} = [\underline{X}_1, \underline{X}_2, \ldots, \underline{X}_i, \ldots]$, where vector $\underline{X}_j$ contains the properties of the j-th element, $j = 1,2\ldots m$, and m is the number of elements in the structure. Suppose there exists a function $g_n(\underline{X}, \underline{Q})$ with the following property:

$$DS \geq n \Leftrightarrow g_n(\underline{X}, \underline{Q}) < 0 \tag{5}$$

The function $g_n(\underline{X}, \underline{Q})$ is called a limit state function for the n-th damage state. Given that (5) holds we can rewrite (4) as follows:

$$F^n(z) = P(g_n(\underline{X}, \underline{Q}) < 0 \mid EDP = z) \tag{6}$$

Consider the conditioning part $EDP = z$. Normally, EDP is chosen in a way to provide maximum information about the damage state, that is, about the event $g_n(\underline{X}, \underline{Q}) < 0$. Therefore, the conditional probability of the event $g_n(\underline{X}, \underline{Q}) < 0$ given that $EDP = z$ is different from the probability of the event $g_n(\underline{X}, \underline{Q}) < 0$ without knowledge of the value of EDP, implying that the limit state function and EDP are probabilistically related. It is reasonable to assume that the limit state function and EDP are also functionally related, meaning that the limit state function $g_n(\underline{X}, \underline{Q})$ can be written as an explicit function of EDP: $g_n(EDP, \underline{X}, \underline{Q})$. Further, in practice, a simplified form of the limit state function is often assumed:

$$g_n(\underline{X}, \underline{Q}) = C(\underline{X}_i) - EDP(\underline{X}, \underline{Q}) \tag{7}$$

where $C(\underline{X}_i)$ is a capacity of the i-th member with respect to n-th damage state, formulated in terms of EDP. The capacity is assumed to be the only property of the member that defines its ability to resist the loading without experiencing the n-th damage state. In general, the capacity depends on all remaining properties of the element: X_i. We can use this model to estimate the fragility function. Substituting (7) into (6):

$$\begin{aligned} F^n(z) &= P(C(\underline{X}_i) - EDP(\underline{X}, \underline{Q}) < 0 \mid EDP = z) \\ &= P(C(\underline{X}_i) < z \mid EDP = z) \end{aligned} \tag{8}$$

then assuming that EDP = z contains no further information relevant to $C(\underline{X}_i)$ < z, that is, these events are independent, we can find the fragility function as:

$$F^n(z) = P(C(\underline{X}_i) < z\,) \tag{9}$$

This shows that if all of the aforementioned conditions are satisfied, then the fragility function of the component is the cumulative distribution function (CDF) of its capacity.

For structural members, the assumption of independence of the events $C(\underline{X}_i)$ < z and $EDP = z$ is not satisfied. We show this by exploring the function $EDP(\underline{X}, \underline{Q})$. In general, it depends on both the excitation $\underline{Q}$ and the structural properties $\underline{X}$. Examples of such EDPs are: inter-story drift ratio (IDR), ductility, floor acceleration, various damage indices, etc.; this list includes practically all structural response parameters that are being used by the earthquake engineering community. Thus, the properties of each structural member ($\underline{X}_i$) affect the values of these "structure-dependent" EDPs. Otherwise, if a component does not have any effect on the structural response (EDP does not depend on $\underline{X}_i$), it is not a structural member. Therefore, we can write:

$$EDP(\underline{X}, \underline{Q}) = EDP(\underline{X}_1, \underline{X}_2, \ldots, \underline{X}_i, \ldots, \underline{Q}) \tag{10}$$

Substitute (10) into (8):

$$F^n(z) = P(C(\underline{X}_i) < z \mid EDP(\underline{X}_1, \underline{X}_2, \ldots, \underline{X}_i, \ldots, \underline{Q}) = z) \tag{11}$$

When seismic performance evaluation is based on structural analysis, the value of EDP is obtained as a result of a structural simulation. To perform this simulation, it is necessary to have: first, a structural model of the facility, and second, specific numerical values of all the parameters of the model. Therefore, some subset of the structural member properties ($\underline{X}_j$, $j = 1 \ldots m$) has to be specified before the EDP value is obtained. We denote the subset of the member properties that defines the structural behavior by $\underline{X}_i^S$ ($\underline{X}_i^S \subset \underline{X}_j$). In general, the properties of a member that are used for the calculation of the EDP do not fully coincide with the properties controlling the occurrence of damage, although, in case of structural members, there is usually an overlap. We define the subset of $\underline{X}_i$ that controls the capacity of the element by $\underline{X}_i^C$. We assume that subsets $\underline{X}_i^S$ and $\underline{X}_i^C$ overlap: $\underline{X}_i^C \cap \underline{X}_i^S \neq \varnothing$. The size of overlapping depends on the definition of damage states, on the choice of EDP and on the structural model. Incorporating all the available information into the conditioning part of (11), we can obtain the probability of an element being in the n-th or higher damage state as follows:

$$F_{SM}^n(z) = P(C(\underline{X}_i^C) < z \mid EDP(\underline{X}_1^S, \ldots, \underline{X}_i^S, \ldots, \underline{Q}) = z, \underline{X}_1^S = \underline{x}_1^S, \ldots \underline{X}_i^S = \underline{x}_i^S, \ldots) \tag{12}$$

where we denote a fragility function with the known structural model properties included in the conditioning part by $F_{SM}^{n}(z)$ and $\underline{x}_j$ is the specified values of member properties $\underline{X}_j$ for j = 1…m. Assuming that the intersection of the sets $\underline{X}_i^S$ and $\underline{X}_i^C$ is not a null set, we denote by $\underline{X}_i^{CO}$ the subset of properties that affects only the capacity, by $\underline{X}_i^{CS}$ the parameters that enter both the structural model and the capacity function ($\underline{X}_i^{CS} = \underline{X}_i^C \cap \underline{X}_i^S$) and by $\underline{X}_i^{SO}$ the properties that are required only for the structural model. Then vector $\underline{X}_i$ can be rewritten in terms of these sub-vectors: $\underline{X}_i = [\underline{X}_i^{CO}, \underline{X}_i^{CS}, \underline{X}_i^{SO}]$, and (12) takes the form:

$$F_{SM}^{n}(z) = P(C(\underline{X}_i^{CO}, \underline{X}_i^{CS}) < z \mid EDP(\ldots, \underline{X}_i^{S}, \ldots, \underline{Q}) = z, \ldots, \underline{X}_i^{CS} = \underline{x}_i^{CS}, \underline{X}_i^{SO} = \underline{x}_i^{SO}, \ldots) \qquad (13)$$

Now, besides EDP = z, the conditioning part contains another piece of information ($\underline{X}_i^{CS} = \underline{x}_i^{CS}$) that is relevant to the event $C(\underline{X}_i^C) < z$. Using it, we can find a probability of the n-th or higher damage state as:

$$\begin{aligned} F_{SM}^{n}(z) &= P(C(\underline{X}_i^{CO}, \underline{x}_i^{CS}) < z \mid EDP(\ldots, \underline{X}_i^{S}, \ldots, \underline{Q}) = z, \ldots, \underline{X}_i^{CS} = \underline{x}_i^{CS}, \underline{X}_i^{SO} = \underline{x}_i^{SO}, \ldots) \\ &= P(C(\underline{X}_i^{CO}, \underline{x}_i^{CS}) < z) \end{aligned} \qquad (14)$$

The fragility function with conditioning on structural properties becomes a CDF of the capacity of the element where some part of the elements properties is known and the other part is uncertain (random). Therefore, the uncertainty in the damage estimation is reduced by the knowledge of some parameters.

In the limit, it is possible to have a case where we have complete knowledge about the damage. To see this, consider an element for which the following condition holds:

$$\underline{X}_i^C \subset \underline{X}_i^S \qquad (15)$$

The condition (15) states that all the properties that define the element's capacity are also needed for development of the structural model, meaning that knowledge of structural properties implies a full knowledge of capacity properties. Then, the probability of being in the n-th or higher damage state can be found in a way similar to (14) as follows:

$$\begin{aligned} F_{SM}^{n}(z) &= P(C(\underline{X}_i^{C}) < z \mid EDP(\underline{X}_1^{S}, \ldots, \underline{X}_i^{S}, \ldots, \underline{Q}) = z, \underline{X}_1^{S} = \underline{x}_1^{S}, \ldots \underline{X}_i^{S} = \underline{x}_i^{S}, \ldots) \\ &= P(C(\underline{x}_i^{C}) < z) \\ &= \begin{cases} 1, & C(\underline{x}_i^{C}) < z \\ 0, & C(\underline{x}_i^{C}) > z \end{cases} \end{aligned} \qquad (16)$$

Depending on the known value of $C(\underline{x}_i^{C})$, such a fragility function takes on the value of 0 or 1, providing full knowledge about occurrence of the damage.

In summary, using both EDP and structural properties, we can either reduce or eliminate parameter uncertainty in damage prediction. If we do so, it should lead to a better overall seismic performance evaluation. We shall compare the traditional fragility functions approach and the proposed approach using structural model based fragility functions by considering a damage of a reinforced-concrete moment frame. The model is chosen to represent the case where (15) holds. This case requires the use of (16) for damage estimation. It also should provide the most benefit from the reduction of uncertainty in the damage estimation when using the proposed approach.

METHODS OF DAMAGE ESTIMATION

The probability of damage is estimated by integration over the failure region in the corresponding state space. Given the complexity of the earthquake engineering application, the integration is usually conducted by simulation. Figure 1 shows the structure of the state space and relation between different groups of variables in the space. Notations in this figure are consistent with those used previously: $\underline{Q}$ – properties of the earthquake, $\underline{X}$ – properties of the structural members in the model, $\underline{EDP}$ – engineering demand parameters, $\underline{DM}$ –damage measure.

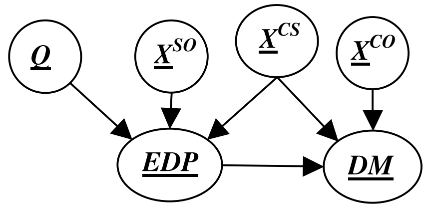

Figure 1. Relations between the variables in the state space.

In the case where the structural response defining properties overlap with damage defining properties ($\underline{X}_i^{CS} \neq \varnothing$), one should use care while disaggregating the analysis into two separate modules, one for structural analysis and one for damage analysis. A disaggregated analysis assumes that only the vector of *EDP*s is transferred from the structural analysis to the damage analysis. Utilizing relation (9) instead of (14) or (16) assumes that there is no knowledge about $\underline{X}^{CS}$ when doing the damage analysis, contradicting the fact that these properties have already been defined during the structural analysis. Therefore, in the case of a disaggregated analysis using Monte Carlo simulation, two distinct samples of $\underline{X}^{CS}$ are used: one for structural analysis and another one for damage analysis, as shown in Figure 2a. It is foreseeable that such approach could reduce the accuracy of damage estimation. If equations (14) or (16) are used for damage analysis, the integration does not have inconsistencies. But this approach requires that damage analysis is performed, in part, together with structural analysis as shown in Figure 2b. We shall investigate the difference between coupled damage analysis (Figure 2b) and uncoupled damage analysis (Figure 2a) by studying three separate simulation methods for damage assessment. For the present study, we consider the case where the structural properties include all the damage-related properties, so (15) holds.

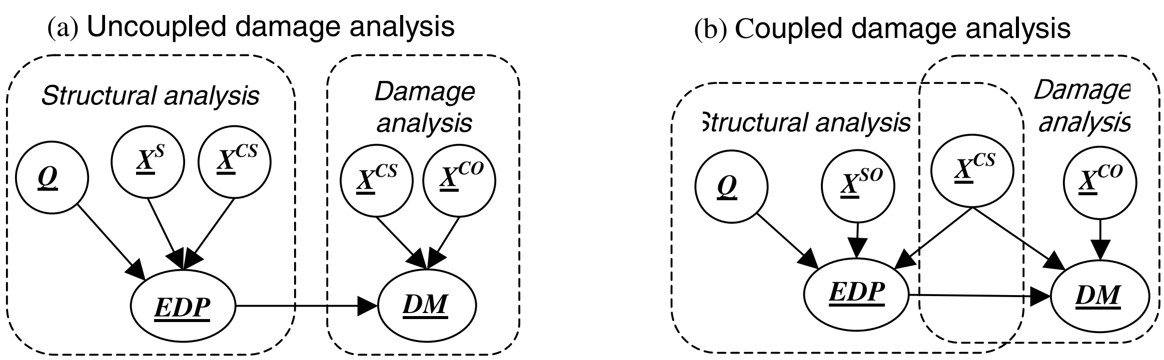

Figure 2. Relations between variables for (a) traditional damage analysis and (b) coupled damage analysis.

Method 1. Vector $\underline{X}^{CS}$ is randomly sampled according to its probability distribution at the start of each simulation. $\underline{EDP}$ is calculated as a result of a nonlinear dynamic time history structural analysis. Then $\underline{DM}$ is calculated according to (16) by using the obtained values of $\underline{EDP}$ and $\underline{X}^{CS}$. For each damage calculation, only one sample of the structural properties is used. To perform the integration over the state space of random variables, $\underline{X}^{CS}$, they are generated a statistically significant number of times. In essence, the method performs a coupled damage analysis with a randomized structural model and a structural model based fragility function (14) that in this case becomes deterministic (16). It results in an implementation of the scheme presented by Figure 2b.

Method 2. Vector $\underline{X}^{CS}$ is randomly sampled according to its probability distribution. $\underline{EDP}$ is calculated as a result of a nonlinear dynamic time history structural analysis. Then $\underline{DM}$ is estimated from $\underline{EDP}$ by using (9). This is equivalent to ignoring the previous sample of $\underline{X}^{CS}$ and estimating these damage properties from a new random sample of $\underline{X}^{CS}$. The samples are independent and identically distributed. Effectively, the method implements the structure shown by Figure 2a, where a part of the structural model is randomized twice: one time for the purpose of structural analysis and the other time for the purpose of damage analysis. The method is inconsistent because of the double-sampling of the structural properties $\underline{X}^{CS}$. However, it provides a desirable disaggregation of the problem.

Method 3. The uncertainty in the structural properties is ignored by taken them to be equal to their expected values $\underline{X}^{CS} = E[\underline{X}^{CS}]$, for the purpose of the structural analysis. Everything else is the same as in Method 2. The method is the easiest of all three methods to implement in practice, since in the case of fixed excitation ($\underline{Q}$), it uses the computationally intensive dynamic structural simulation only once. For this reason, it is often used and so it is included in the present study along with Methods 1 and 2.

In addition to the structural properties, there exist two ways to apply the earthquake load $\underline{Q}$. One way is to randomly generate the ground motion time history (or to use a set of appropriate recorded time histories and randomly select records from the set) and the other way is to use a single time history for all simulations. In the latter case, the particular properties of the chosen ground motion can be a factor in the final damage estimation. Therefore, three methods together with the two ways to apply the earthquake load constitute six different cases of analysis that will be studied further. Figure 3 illustrates all cases that are considered in the present study; *i.i.d.* in the figure stands for independent identically distributed random variables.

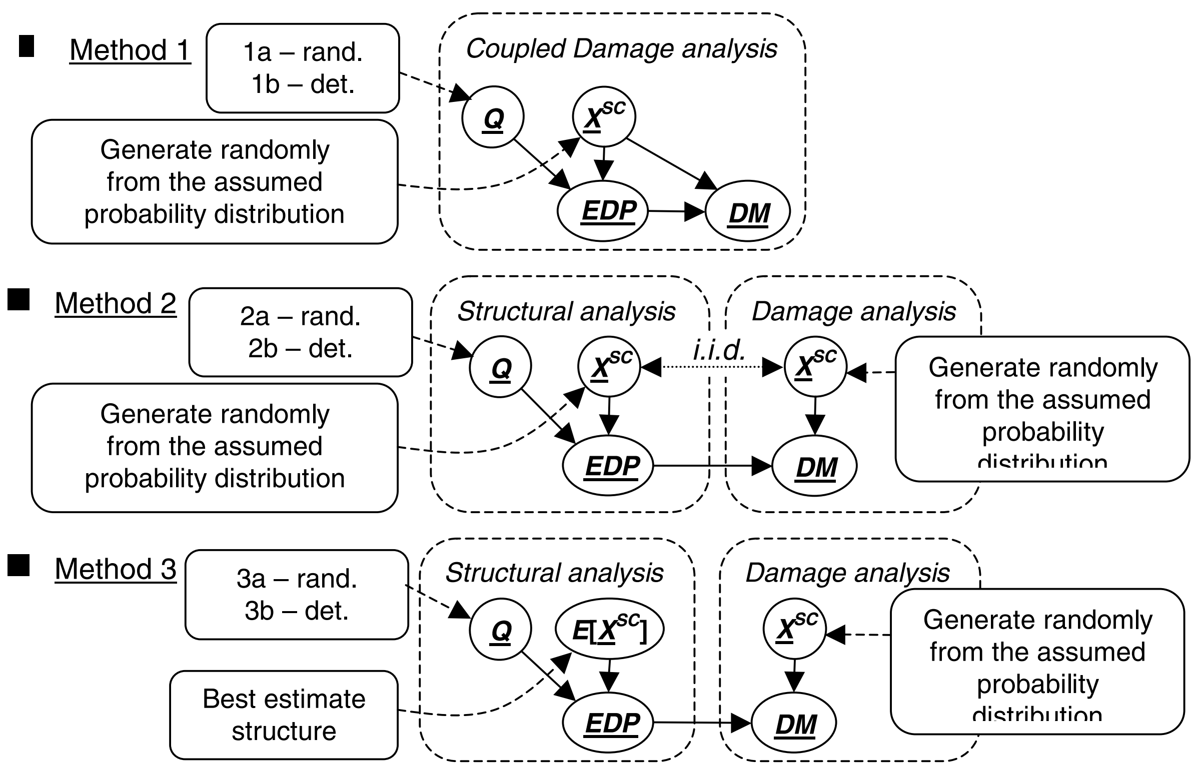

Figure 3. Methods used for the sample case study damage estimation

STRUCTURAL MODEL DESCRIPTION

The reinforced concrete moment frame shown in Figure 4 is chosen as a case study. The frame represents the south frame of the 7-story Van Nuys hotel building. For a detailed building description, see Beck [3] or Li [7]. The 2D model of the frame is developed for the present study. The flexural behavior of the beams and columns is represented by one-component Giberson beam with plastic hinges at the ends (Sharpe [8]). Shear deformation for the beams and columns is assumed to be elastic and is incorporated into the flexural elements. The Q-HYST bi-linear hysteresis (Saiidi [9]) is used to model stiffness degradation of reinforced concrete members in flexure as shown in Figure 5. Properties of the reinforced-concrete members are taken from the original structural drawings (Rissman [10]). The software program for reinforced-concrete members cross-section analysis UCFyber (ZEvent [11]) is used to calculate parameters of the Q-HYST hysteretic rule for the force-deformation curves for each flexural member. The inelastic dynamic analysis program Ruaumoko (Carr [12]) was used for performing the structural analyses.

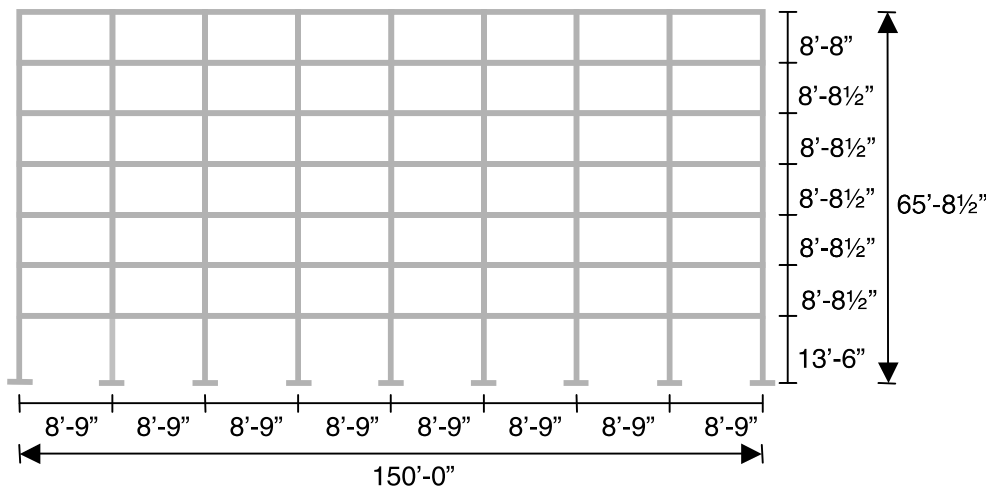

Figure 4. Reinforced concrete moment-resisting frame chosen for the case study.

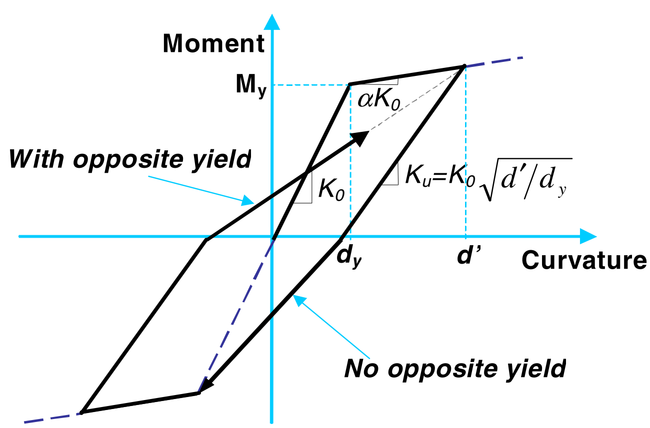

Figure 5. Flexural members hysteretic rule: Q-HYST.

DAMAGE MODEL

For the purpose of this exploratory study, it is assumed that each structural member has only two damage states: "undamaged" (DS = 0) and "yield" (DS =1). A member is considered to be in damage state "yield" if the yielding point in the moment-curvature relations has been reached ((dy, My) in Figure 5). The "yield" damage state of the *i*-th member can be described by the following damage model:

$$DS = \text{"yield"} \Leftrightarrow d_y(\underline{X}_i) < d^i_{\max}(\underline{X}, \underline{Q}) \tag{17}$$

where $d_y(\underline{X}_i)$ is the yield curvature of the reinforced-concrete member, $d^i_{\max}(\underline{X}, \underline{Q})$ is the maximum curvature attained by the element during the simulation. Yield curvature depends only on the properties of the member $\underline{X}_i$, implying that the limit state function defined in (17) is exact, therefore (17) is valid. Note that (17) is the problem-specific version of (5), where the limit state function has been formulated according to the particular features of the component under consideration.

Comparing the limit state model (17) with the one used in (7) – (9), it is noted that $d_y(\underline{X}_i)$ is by definition the capacity of the structural member with respect to "yield" damage state, formulated in terms of maximum curvature $d^i_{\max}$. Therefore, maximum curvature $d^i_{\max}$ is the *EDP* chosen for the damage analysis. According to (9), the fragility function for this case is a CDF of capacity:

$$F^1(z) = (d_y(\underline{X}_i) < z) \tag{18}$$

This is the fragility function to be used for the damage estimation according to Methods 2 and 3.

In order to obtain the maximum curvature ($d^i_{\max}$), it is necessary to perform a dynamic simulation. To do so, we need to specify the parameters of the structural model. From the flexural hysteresis rule (Figure 5), it can be observed that some of the required parameters are: yield moment M_y and initial stiffness K_0. From these two parameters, the yield curvature can always be derived: $d_y = M_y \diagup K_0$. Therefore, as a result of the dynamic simulation, the value of the maximum curvature for each element and the value of the yield curvature are obtained. Substituting this information into the conditioning part of (12) for the *i*-th element:

$$\begin{aligned} F^n(z) &= P(d^i_y(\underline{X}_i) < z \mid d^i_{\max}(\underline{X}, \underline{Q}) = z,\ d^i_y(\underline{X}_i) = \tilde{d}^i_y, \ldots) \\ &= P(\tilde{d}^i_y < z \mid d^i_{\max}(\underline{X}, \underline{Q}) = z,\ d^i_y(\underline{X}_i) = \tilde{d}^i_y, \ldots) \\ &= \begin{cases} 1, & \tilde{d}^i_y < z \\ 0, & \tilde{d}^i_y > z \end{cases} \end{aligned} \tag{19}$$

where $\tilde{d}^i_y$ is the value of the yield curvature of the *i*-th element that has been used during the dynamic simulation. Thus, whenever the maximum curvature attained during the dynamic simulation exceeds the yield curvature of the element, the element is considered to be in damage state "yield". This is the binary (deterministic) fragility function that is used for damage estimation by Method 1. Note that $\tilde{d}^i_y$ is one of the properties of the element that is known from the input structural data and it is also the "capacity" of the element with respect to the "yield" damage state, formulated in terms of the maximum curvature. Therefore, the model satisfies (15), as intended for the present study.

Global damage is estimated as the number of components in the "yield" damage state (N_t), where the number of components is equal to the number of ends of the flexural members (beams or columns), since either end of a member may yield. For the chosen frame, the total number of flexural members is 119, hence there are 238 damageable components in the frame. For each dynamic structural simulation, N_t is calculated by the previously described three different methods.

PARAMETERS OF THE DAMAGE MODEL AND GROUND MOTIONS

Here we define the probability distributions of the uncertain parameters that are used in the analysis. In the present case, we assume that the only uncertain parameter is yield curvature. We adopt a lognormal probability distribution where the parameters are found as follows. The yield moment is assumed to be equal to:

$$M_y^i = \overline{M}_y^i x \tag{20}$$

where $\overline{M}_y^i$ is the best estimate of the yield moment of the i-th element, as calculated by UCFyber (ZEvent [10]) and x is a lognormally distributed random variable with expectation $E[x] = 1$ and the coefficient of variation $\delta[x] = 0.08$, making the yield moment a lognormal random variable with expectation $E[M_y^i] = \overline{M}_y^i$ and coefficient of variation $\delta[M_y^i] = 0.08$. The study by Ellingwood [13] suggests that coefficient of variation 0.08 is a reasonable estimate of the uncertainty in the flexural strength of reinforced-concrete members. The stiffness K_0^i is assumed to be deterministic and equal to the value calculated by UCFyber (ZEvent [10]). Therefore, the yield curvature is a lognormal random variable with the following parameters:

$$F_{d_y^i}(z) \sim LN\,(\mu_x + \ln(\overline{M}_y^i / K_0^i), \sigma_x) \tag{21}$$

where $\mu_x = -\,0.0032$ and $\sigma_x = 0.08$ are mean and standard variation of $ln(x)$ that has normal distribution. These values of μ_x and σ_x provide the required expectation and coefficient of variation of x. Formula (21) is used for generating a randomized structural model and for the fragility function (18).

The stiffness properties K_0^i of the structural model are assumed to be deterministic. Therefore, the natural frequencies of the original (undamaged) structural model are the same for all randomly generated samples of the structural model. The first natural frequency of the present model is $T_1 = 1.5$ sec., which agrees with the value exhibited by the Van Nuys 7-story hotel in the longitudinal direction during the 1994 Northridge earthquake, as reported by Islam [14]. Software program Bispec (Hachem [15]) is used to determine S_a. Ground motions for the analysis are taken from the set of the ground motions developed for the SAC Steel Project (WCFS [16]).

RESULTS

For the first analysis, we collect the statistics of N_t that are calculated by the three different methods for the same ground motion. For this acceleration time history, we performed 40 dynamic simulations with the structural model where yield curvature is randomly generated for every member. These simulations are used to calculate N_t by Method 1 and Method 2. One dynamic simulation is also performed with the structural model that has yield curvature equal to its expected value. Then the total damage N_t is estimated 40 times by Method 3.

Table 1 gives the results of damage estimation for 40 simulations using the ground motion time history LA15 scaled to the level of intensity $S_a = 0.5$g at $T_1 = 1.5$ sec and 5% damping. For this particular ground motion, Method 2 overestimates the damage on average by 6.0% relative to Method 1, and Method 3 overestimates the damage on average by 7.7%. Also, Method 2 gives a variance for N_t that is 42% of that of Method 1, while Method 3 produces a damage estimate with a variance that is 6.7% (about 15 times less) of the variance of the Method 1 estimates.

Table 1. Results of damage estimation for LA15 ground motion at 0.5g.

Simulation number	N_t Method 1	N_t Method 2	N_t Method 3	Simulation number	N_t Method 1	N_t Method 2	N_t Method 3
1	81	97	106	21	102	110	106
2	94	97	104	22	88	102	104
3	95	97	103	23	103	106	103
4	89	103	104	24	101	103	103
5	97	104	103	25	101	98	101
6	106	104	97	26	90	102	103
7	94	102	103	27	81	89	102
8	91	96	108	28	100	96	105
9	98	105	105	29	109	112	105
10	88	103	102	30	105	105	102
11	103	101	104	31	95	105	106
12	85	92	108	32	95	97	103
13	105	104	103	33	89	103	104
14	82	105	101	34	97	104	103
15	115	112	106	35	106	104	97
16	80	91	104	36	94	102	103
17	96	95	106	37	91	96	108
18	86	95	104	38	98	105	105
19	103	103	101	39	88	103	102
20	85	104	106	40	103	101	104
$E[N_t]$	Method 1		96.3	Method 2	102.1	Method 3	103.7
$\mathrm{Var}[N_t]$			76.7		32.2		4.9

The effects that have been observed for one particular ground motion might be occasioned by some particular features of this ground motion. To ensure that similar phenomena take place in general, independently of the individual characteristics of the input excitation, we conduct an analogous analysis for 30 other acceleration time histories. All time histories are scaled to provide S_a = 0.5g at T_1 = 1.5 sec and 5% damping. The scaling factor does not exceed 2 for all time histories. Statistical properties (mean and variance) of N_t are based on 40 dynamic simulations for each ground motion. The results are shown in Table 2.

Table 2. Statistical properties of damage estimation for 30 different ground motion time histories.

	$E[N_t]$ Method 1	$E[N_t]$ Method 2	$E[N_t]$ Method 3	$\mathrm{Var}[N_t]$ Method 1	$\mathrm{Var}[N_t]$ Method 2	$\mathrm{Var}[N_t]$ Method 3
Average values for 30 ground motions	**96.3**	**100.5**	**99.3**	**84.2**	**42.2**	**6.3**

For the chosen set of earthquake ground motions, the damage estimated by Method 2 is on average 4.4% greater than for Method 1. Method 3 overestimates the mean $E[N_t]$ by 3.1%, compared to Method 1. As far as the dispersion of the damage estimates is concerned, Method 2 gives a variance for N_t on average 50% less than Method 1. The variance estimated by Method 3 is on average 7.5% of that calculated by Method 1. These results are roughly consistent with those in Table 1 for the LA15 record.

Next, we perform damage estimation at different intensity levels by scaling one ground motion time history. We choose the ground motion LA15 since it provides the values of damage that are the closest to the average values. We scale it to provide values of S_a at T_1 = 1.5 sec from 0.1g to 1.0g at a 0.1g step. The results are shown in Figures 6 - 7. Figure 6a depicts the average values of N_t as a function of S_a. As before, N_t is calculated by the three different methods. The difference between the estimates of $E[N_t]$ from Methods 2 and 3 and those from Method 1 (relative to Method 1) is plotted in Figure 6b. For Method 2, the difference decreases from 40.5% to 3.2% over the range from 0.1g to 1.0g. For the range between 0.4g and 1.0g, the difference does not exceed 6.8%, averaging at 4.6%. Figure 7a displays the variance of N_t as a function of S_a. Method 2 underestimates the variance by approximately 50% for the lower end and provides roughly the same results as Method 1 for the high values of spectral acceleration (S_a > 0.5g). Method 3 underestimates the variance quite significantly: the variance of the damage estimate is on average about 10% of that obtained by Method 1. Figure 7b shows the coefficient of variation of the damage estimate. The average value of the coefficient of variation of the damage estimate for Method 1 is 14.6%. The coefficient of variation of the flexural members' capacity is 8%. Thus, uncertainty in building structural properties is approximately doubled in the damage estimate. Figure 7b also reveals that at lower S_a values, the uncertainty in the damage estimate from Method 1 is much higher than the average value: in the range S_a < 0.5g, the coefficient of variation is on average 30%, while for higher S_a values it is 6.8%, which is lower than the coefficient of variation of the source of uncertainty, the uncertain yield capacity. The other two methods give much lower estimates of the coefficient of variation, essentially repeating the behavior shown by the variances: Method 2 gives 8.6% and Method 3 gives 4.3% on average over the whole range of S_a.

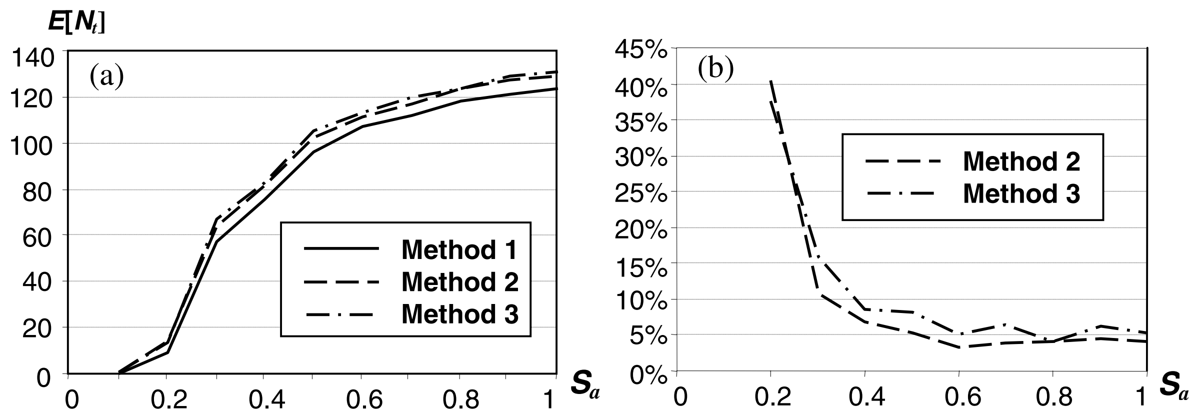

Figure 6. Expectation of damage (a) and relative difference (b), ground motion LA15.

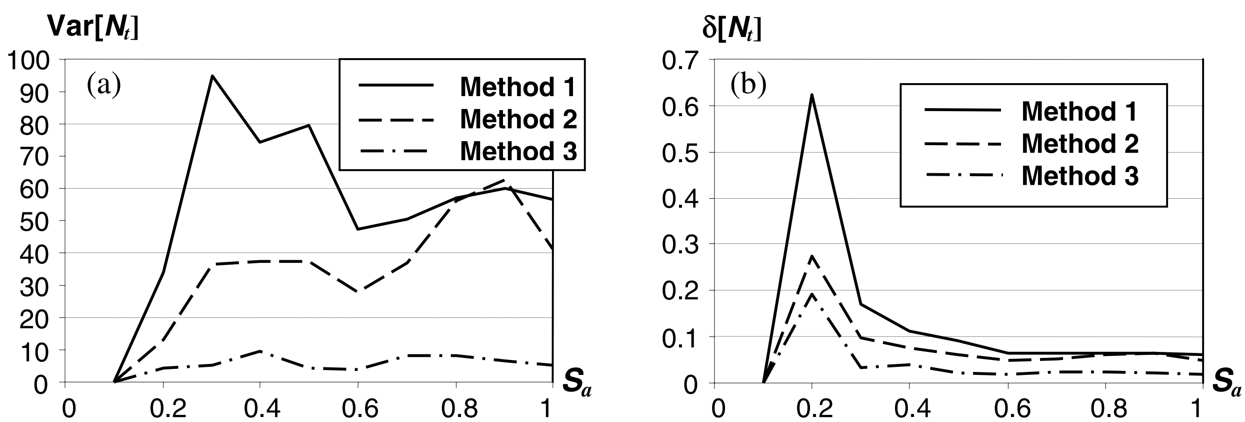

Figure 7. Variance of the damage estimate (a) and coefficient of variation (b), ground motion LA15.

In order to examine the possible influence of the particular characteristics of the ground motion, we repeat the analysis at the same levels of the spectral acceleration using a set of 40 different ground motions taken from the SAC Steel Project database. At each value of S_a, we perform 40 dynamic simulations (one for each ground motion record) with randomly generated structural models when Method 1 and Method 2 are used for damage estimation and 40 dynamic simulations with the best estimate structure when Method 3 is used. The results are presented in Figures 8 - 9. Note that for this case, Method 3 does not have any computational advantage over Method 1 and 2, since we need to perform a dynamic simulation for each sample ground motion record. Method 2 overestimates the expected value of damage with the relative difference in the range 3% to 21%, where the maximum error occurs at 0.2g. The average error for higher end (between 0.4g and 1.0g) is 4.1%. Method 3 provides damage estimates that differ less than 9.8% from those of Method 1.The average error at the high end (0.4g – 1.0g) is very low: 0.8%. Figure 9a shows the variances of the damage estimates produced by the three methods for the set of ground motion time histories. There is no significant difference between the variance estimates provided by the different methods, which differ appreciably from the case of a fixed excitation (LA15) where the discrepancy between the variance estimates is apparent. Figure 9b shows the coefficient of variation of the damage estimates for the three methods. The results are practically identical for all three methods. The value of the coefficient of variation is considerably higher than for the case of fixed excitation (Figure 7b), reflecting the variability introduced by taking a set of ground motion time histories.

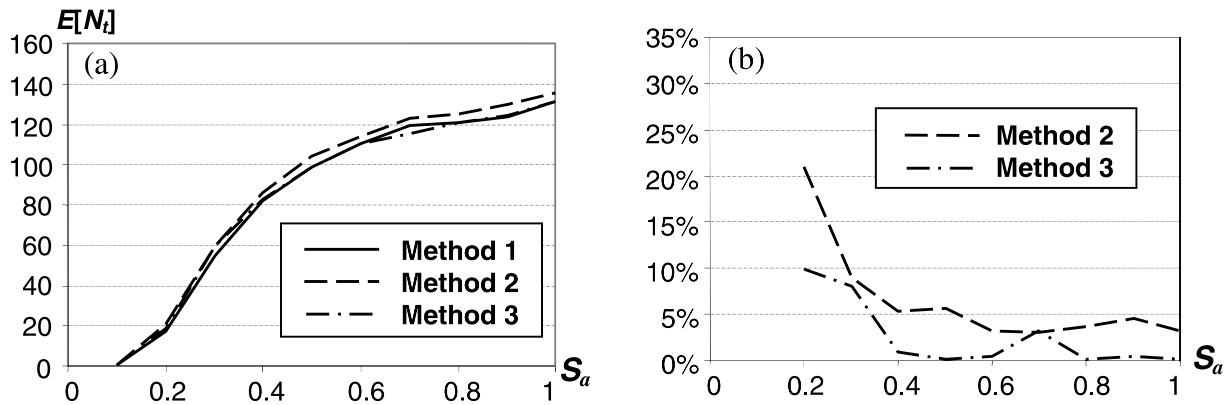

Figure 8. Expectation of damage (a) and relative difference (b), set of ground motions.

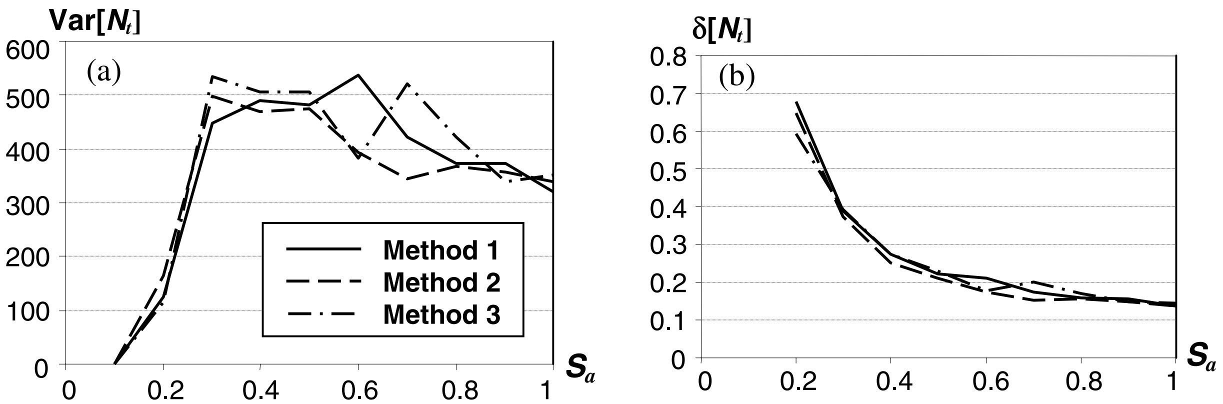

Figure 9. Variance of damage estimation (a) and coefficient of variation (b), set of ground motions.

CONCLUSIONS

The purpose of this study is to compare two different approaches to structural damage estimation: the uncoupled approach based on fragility function that predicts damage from the knowledge of *EDP* alone, and a proposed approach of coupled damage estimation that predicts damage using both knowledge of *EDP* and knowledge of structural properties. In particular, we explore the inconsistency that is present whenever there exist properties that are used for both structural damage (capacity) and structural response. The uncoupled approach in this case results in using two samples of such properties instead of one. We study the effects of this inconsistency by developing a damage model (damage state, EDP, limit state function) that maximizes the possible discrepancy between the two approaches by maximizing the set of overlapping properties. Then we use three different methods of damage estimation. The coupled damage analysis approach is implemented through Method 1. Method 2 and Method 3 use uncoupled structural and damage analyses; Method 2 uses two randomly generated sets of structural properties, one for structural analysis and one for damage analysis, while Method 3 uses the best-estimate structural properties for the structural analysis and randomly generated properties for the damage analysis.

The results have shown that all three methods provide fairly close estimates of the expected damage, implying that double sampling of structural properties may have insignificant impact in some cases. However, the effects of other possibly important factors, such as the level of uncertainty in the structural properties, the form of the probability distribution of the structural properties or redundancy in the structural model, have not been studied. These factors should be investigated in future research.

The variance in the damage estimates exhibited a more diverse pattern of behavior. While the estimates from Methods 1 and 2 in general agree, the variance estimated by Method 3 is significantly lower for a fixed excitation, as expected, since it does not include uncertainty in the structural properties. Therefore, the uncertainty estimates from the usual uncoupled approach are adequate only when a randomized structure or random excitation is used. When the deterministic, best-estimate structure is employed together with a deterministic load, the dispersion in the damage estimate is significantly underestimated. Therefore, Method 3 should only be used for the purpose of calculating mean damage estimates and not the variance of the damage estimates or the probability of exceeding (or not exceeding) some damage threshold value. This is important because Method 3 can be viewed as a particular implementation of a general family of methods that can be defined in the following way: deterministic load – deterministic structure – probabilistic damage model. For example, the deterministic load can be a monotonic lateral force used in push-over analysis. Therefore, it might be possible to extend the present conclusions to a common damage estimation technique where a deterministic push-over analysis is complemented with a damage analysis using fragility curves, implying that the results obtained by such techniques should be treated with caution.

ACKNOWLEDGEMENT

This work was supported in part by the Earthquake Engineering Research Centers Program of the National Science Foundation under Award Number EEC-9701568 through the Pacific Earthquake Engineering Research Center (PEER). This support is gratefully acknowledged. Any opinions, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect those of the National Science Foundation.

REFERENCES

1. Porter KA, Kiremidjian AS, LeGrue JS. "Assembly-based vulnerability of buildings and its use in performance evaluation." Earthquake Spectra 2001; 17(2): 291-312.

2. Beck JL, Kiremidjian AS, Wilkie S, Mason A, Salmon T, Goltz J, Olson R, Workman J, Irfanoglu A, Porter KA. “Social, economic and system aspects of earthquake recovery and reconstruction, final report for CUREe-Kajima Phase III Project.” Consortium of Universities for Research in Earthquake Engineering. Richmond, CA, December, 1999.
3. Beck JL, Porter KA., Shaikhutdinov RV, Au SK, Moro T, Tsukada Y, Masuda M. “Impact of seismic risk on lifetime property values, final report.” Consortium of Universities for Research in Earthquake Engineering. Richmond, CA, 2002.
4. Krawinkler H. “A general approach to seismic performance objectives.” International Conference on Advances and New Challenges in Earthquake Engineering Research, First Annual Meeting of ANCEER, Hong-Kong, China, August 19-20, 2002.
5. Miranda E, Aslani H. “Probabilistic response assessment for building-specific loss estimation.” Report No. PEER-2003/03, Berkeley, CA: Pacific Earthquake Engineering Research Center, University of California, 2003
6. Irfanoglu A, “Structural design under seismic risk using multiple performance objectives.” Report No. EERL 2002-02, Pasadena, CA: Earthquake Engineering Research Laboratory, California Institute of Technology, 2000.
7. Li YR, Jirsa JO. “Nonlinear Analyses of an instrumented structure damaged in the 1994 Northridge earthquake.” Earthquake Spectra 1998; 14(2): 245-264.
8. Sharpe RD. “The nonlinear response of inelastic structures.” Ph.D. Thesis, Christchurch, New Zealand: Department of Civil Engineering, University of Canterbury, 1974.
9. Saiidi M, Sozen MA. “Simple and complex models for nonlinear seismic response of reinforced concrete structures.” Report UILU-ENG-79-2031, Urbana, IL: Department of Civil Engineering, University of Illinois, 1979.
10. Rissman and Rissman Associates. “Holiday inn van nuys structural drawings.” Pacific Palisades, CA, 1965.
11. ZEvent. UCFyber Version 2.4.1. Berkeley, CA, 2000.
12. Carr AJ. Ruaumoko. Christchurch, New Zealand: University of Canterbury, 2001.
13. Ellingwood B, Galambos TV, MacGregor JG, Cornell CA. “Development of a probability-based load criterion for American national standard A58, special publication 577.” Washington, DC: National Bureau of Standards, 1980.
14. Islam MS. “Holiday Inn.” 1994 Northridge Earthquake Buildings Case Study Project Proposition 122: Product 3.2. Sacramento CA: Seismic Safety Commission, 1996, 189-233.
15. Hachem MM. BiSpec 1.2. Berkeley, California: University of California at Berkeley, 2003.
16. Woodward-Clyde Federal Services. “Suites of earthquake ground motions for analysis of steel moment frame structures.” Pasadena, CA, 1997. http://nisee.berkeley.edu/data/strong_motion/sacsteel/ground_motions.html