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ACCOUNTING FOR SHEAR IN SEISMIC ANALYSIS OF CONCRETE STRUCTURES

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SUMMARY

Techniques for modelling the seismic response of concrete structures are limited by the accuracy of the material models. Current models for reinforced concrete subjected to shear typically do not account for the effects of cracking and yielding. Diagonal cracking has a very pronounced effect on the shear stiffness of concrete structures, however, recommendations for cracked section shear stiffness are not readily available. The plastic strain of reinforcement is another important parameter that must be considered in the nonlinear seismic shear analysis of reinforced concrete. There is a strong relationship between plastic strain of reinforcement and plastic shear strain, e.g. yielding of the reinforcement results in yielding in shear of the element. Pinching of hysteresis loops is directly linked to the plastic strain in reinforcement, as is the deviation of principal compression stress and principal compression strain angles in concrete.

The authors have recently developed a general model to predict the complete load-deformation response of reinforced concrete elements subjected to reverse-cyclic shear. A unique feature of the model is that deformations at the cracks are separated from deformations of concrete between cracks, and crack deformations are assumed to be a consequence of strain compatibility between concrete and reinforcement.

This paper presents simplified methods for modelling the non-linear seismic shear response of reinforced concrete based on the underlying principles of the general model. The methods include an effective cracked section shear stiffness determined from the shear strength and the shear strain at yield. The shear strain at yield is primarily a function of the yield strain of the horizontal reinforcement and strain of the vertical reinforcement. The cracked section shear stiffness can be used for linear analysis. For non-linear static analysis, a complete envelope is provided where the shear response is assumed to be elastic-plastic. The ultimate shear strain is determined from the shear strain at yield and the shear strain ductility. The latter is a function of the ratio of shear stress to concrete compression strength. Simple hysteretic rules are also provided to define the complete reverse-cyclic shear response for non-linear dynamic analysis.

INTRODUCTION

Techniques for modelling the seismic response of structures have become very sophisticated thanks to

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advancements in solution methods and computing power. The accuracy of these techniques is however limited by the accuracy of the models for the response of individual elements in the structure. Models to capture the flexural response of individual elements are advanced and rational. For example, non-linear fibre models, which capture the complete moment-curvature response including the effects of cracking and yielding of the reinforcement, are now widely used. By contrast, models currently used for the seismic shear response of individual elements are crude.

The simplest type of seismic analysis is linear analysis (static or dynamic). The issue with this analysis is what effective stiffness to use to account for the non-linear behaviour. Design codes such as UBC-97 [1] state that "stiffness properties of reinforced concrete elements shall consider the effects of cracked sections." For flexure, reduced cracked section stiffnesses expressed as a fraction of the gross, or uncracked, section stiffness are widely available in the literature. For example, FEMA 356 Prestandard and Commentary for the Seismic Rehabilitation of Buildings [2] recommends $0.8E_cI_g$ and $0.5E_cI_g$ for uncracked and cracked walls respectively.

For shear, simple recommendations for the cracked section stiffness are not readily available, instead, the gross section shear stiffness is often used. For example, FEMA 356 suggests using the gross section shear stiffness for both uncracked and cracked walls. Diagonal cracking has a pronounced effect on the shear stiffness that should be taken into account, e.g. for the heavily reinforced element shown in Fig. 1, the cracked section shear stiffness is about one-tenth that of the gross section. A simple equation for estimating the cracked section shear stiffness based on the quantity of reinforcement and the applied axial stress is proposed here.

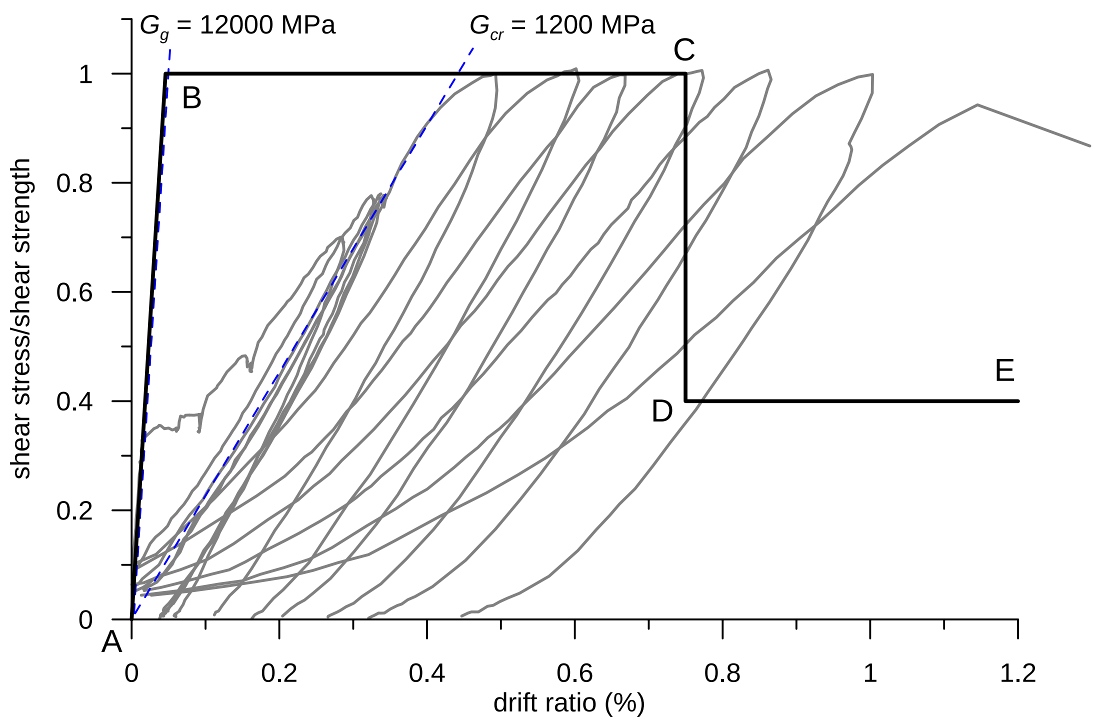

Figure 1 FEMA 356 generalized load-deformation function for shear walls compared to results from a membrane element test (Stevens et al. [3])

Non-linear static analysis includes yielding of elements to provide a more complete representation of the load-deformation response. FEMA 356 defines a generalized force-displacement curve (see Fig. 1) which includes the uncracked stiffness up to the yield point and an ultimate shear strain of 0.0075 (drift ratio of

0.75%). While this value is reasonable for the example shown in Fig.1, it is not appropriate for all cases. A simple method that accounts for the quantity of reinforcement and the applied axial stress is presented in this paper.

Non-linear dynamic analysis of structures includes the complete hysteretic response for each element. Until recently, no general rational model was available for the complete load-deformation response of elements subjected to seismic, or reverse-cyclic, shear. As a result, FEMA 356 recommends that "unloading and reloading stiffnesses and strengths, and any pinching of the load-versus-rotation hysteresis loops, shall reflect the behavior experimentally observed for wall elements similar to the one under investigation." However, this is not practical when considering the wide range of conditions found in existing and new structures. This paper presents a simple hysteretic model derived from the more general rational model developed by Gérin and Adebar [4].

GENERAL MODEL FOR SEISMIC SHEAR

The methods presented in this paper are derived from a general rational model for predicting the complete load-deformation response of reinforced concrete membrane elements subjected to reverse-cyclic shear (Gérin and Adebar [4]). The general model is based on strain compatibility and force equilibrium, combined with simple material models, to determine the actions of the reinforcement and the concrete.

Collins [5] developed the concept of using membrane elements – elements with uniformly distributed reinforcement in two directions and subjected to uniform biaxial stress and strain – to study the fundamental behaviour of reinforced concrete subjected to shear. Membrane elements have direct application in structures such as shear walls, and can be considered fundamental "building blocks" for understanding shear in any concrete structure.

To better understand reverse-cyclic shear, the results from membrane element tests conducted by Stevens et al. [3], Meyboom [6], and Villani and Vecchio [7] were studied in detail (Gérin [9]). Data from an earlier series of monotonic shear tests by Vecchio and Collins [8] were also examined. The study of experimental data focussed on the relationships between various stress and strain components. Understanding and modelling of these relationships lead to an understanding of the fundamental mechanisms of the reverse-cyclic shear response. Some of these relationships and mechanisms are outlined below; full details are provided elsewhere (Gérin [9]).

> There is a nearly linear relationship between shear strain and strain of the weaker reinforcement (first reinforcement to yield). When the weaker reinforcement yields, the shear deformations increase proportionally.
>
> The pinching of hysteresis loops is a function of plastic strain in the reinforcement. At the end of an unloading segment, cracks remain open in proportion to the plastic strain accumulated in the reinforcement from previous yield cycles. As loading is applied in the reverse direction, the previous-direction cracks close and the new-direction cracks open. This occurs with very little stiffness due to the "gap" created by the plastic strain in the reinforcement. The pinching becomes more pronounced as additional plastic strain accumulates in the reinforcement.
>
> Principal stress and principal strain angles deviate during the load reversal stage of each cycle. Before yielding, the principal strain angle follows the principal stress angle closely throughout the cycle (the response is essentially linear elastic). After yielding, when there is plastic strain in the reinforcement, the cracks remain open at the end of unloading. As loading is applied in the new direction, the orientation of the principal stress changes to the new direction, however, the orientation of the principal strain does

not fully change direction until the previous-direction cracks close and the new-direction cracks open. As a result, the principal strain angle lags the principal stress angle. This lag is a function of plastic strain in the reinforcement.

To capture the observed behaviour, a general model was formulated where deformations at the cracks are separated from deformations of concrete between cracks. While the concrete and reinforcement strains are directly related to the applied loads, the crack deformations are a consequence of maintaining strain compatibility between the concrete and reinforcement. This approach is a significant departure from existing models where cracked concrete is treated as a single homogeneous material.

The importance of separating the crack deformations from the deformations of concrete between cracks is illustrated in Fig. 2. The strain components were separated using the procedures of the general model: the crack strain normal to the cracks is equal to the total strain normal to the cracks, the crack strain parallel to the cracks is zero, and the principal angle of concrete stress is equal to the principal angle of strain in the concrete between cracks. Deformations at the cracks are defined entirely in terms of average strains, avoiding the need for an empirical crack-slip function to define local strains.

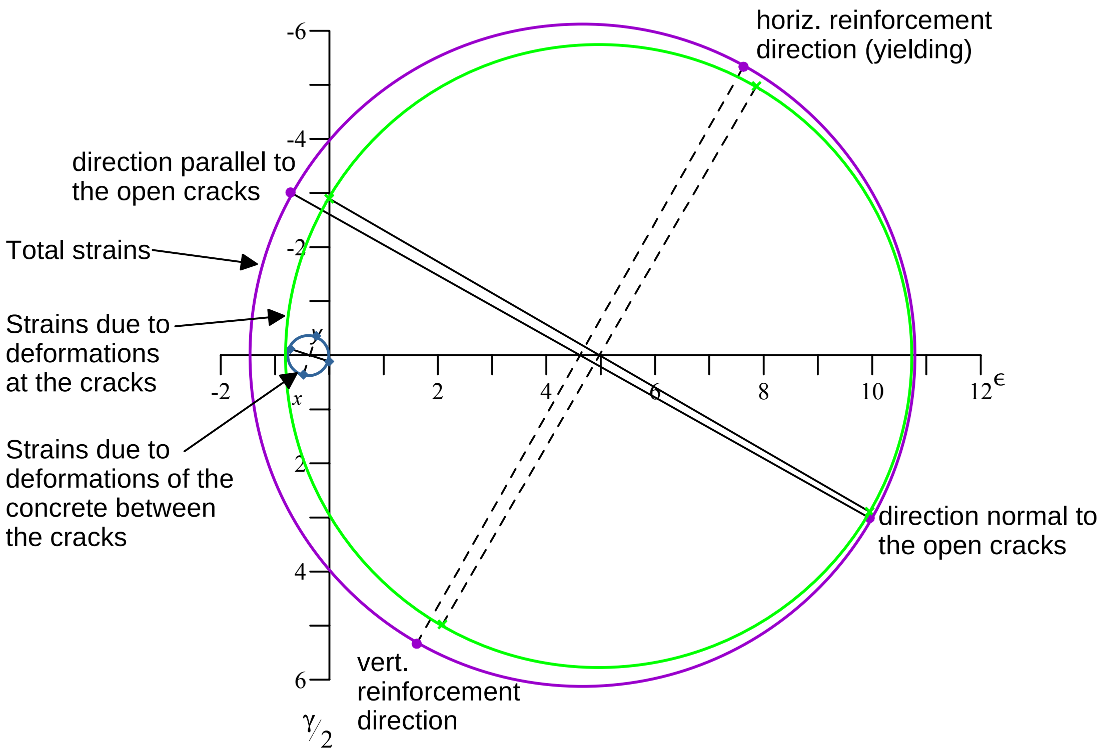

Figure 2 Crack and concrete components of total strains for a point after yielding of the weak reinforcement

Separating deformations at the cracks from deformations in the concrete between the cracks enables the shear strain to be explicitly linked to strains in the reinforcement. The pinching of the hysteresis loops is automatically captured by having the reinforcement plastic strains determine the crack closing/opening during the reversal. The principal stress and strain angles are allowed to deviate as required to maintain equilibrium and compatibility. The complex concrete stress-strain behaviour seen when considering cracked concrete as a single material is automatically captured by a simple concrete model coupled with deformations at the cracks. This eliminates the need for the complex empirical formulations seen in previous models to describe the concrete response (Gérin and Adebar [4]).

The model components are summarized in Table 1. Complete details are in Gérin [9]. Figure 3 shows the predicted load-deformation response for a membrane element subjected to pure shear (Stevens et al. [3]). The model captures well the principal characteristics of the experimental response.

Table 1 Key Components of the General Model

Component	Assumption
Strain compatibility	concrete strains + crack strains = reinforcement strains = total strains
Equilibrium	concrete normal force + reinforcement force = applied normal force concrete shear force = applied shear force
Crack angle	fixed direction: may be estimated from principal stress direction at cracking
Principal concrete stress angle	Before yielding: fixed direction estimated from a modified version of Baumann's equation (fn of element properties and loading characteristics) After yielding: angle rotates as stress increases in x direction (concrete stress in direction of weak reinforcement is constant)
Principal strain angle	Before yielding: independent of principal concrete stress angle After yielding: stress and strain angles rotate simultaneously (constant difference)
Concrete compression model	parabola for envelope, linear unloading/re–loading; softening due to transverse tensile strains
Concrete tension model	after cracking, concrete tension stress reduces as a function of crack width
Crack closing function	empirical relationship between crack opening and normal compression stress
Reinforcement model	bi–linear stress–strain relationship; bare bar yield strength used; plastic strains are cumulative from one shear direction to the other
Failure modes	concrete compression failure: concrete strength exceeded concrete shear failure: shear strain reaches limit reinforcement failure: both direction reinforcement yield

SIMPLIFIED MODEL

The general model summarized briefly above predicts the complete hysteretic response of reinforced concrete in shear. The simplified model, developed from the general model, provides an estimate of the envelope of the shear response including the cracked-section shear stiffness and the maximum shear strain. The model can treat fully-cracked or initially uncracked sections. Simple rules are presented to allow an approximate prediction of the hysteretic response from the basic envelope.

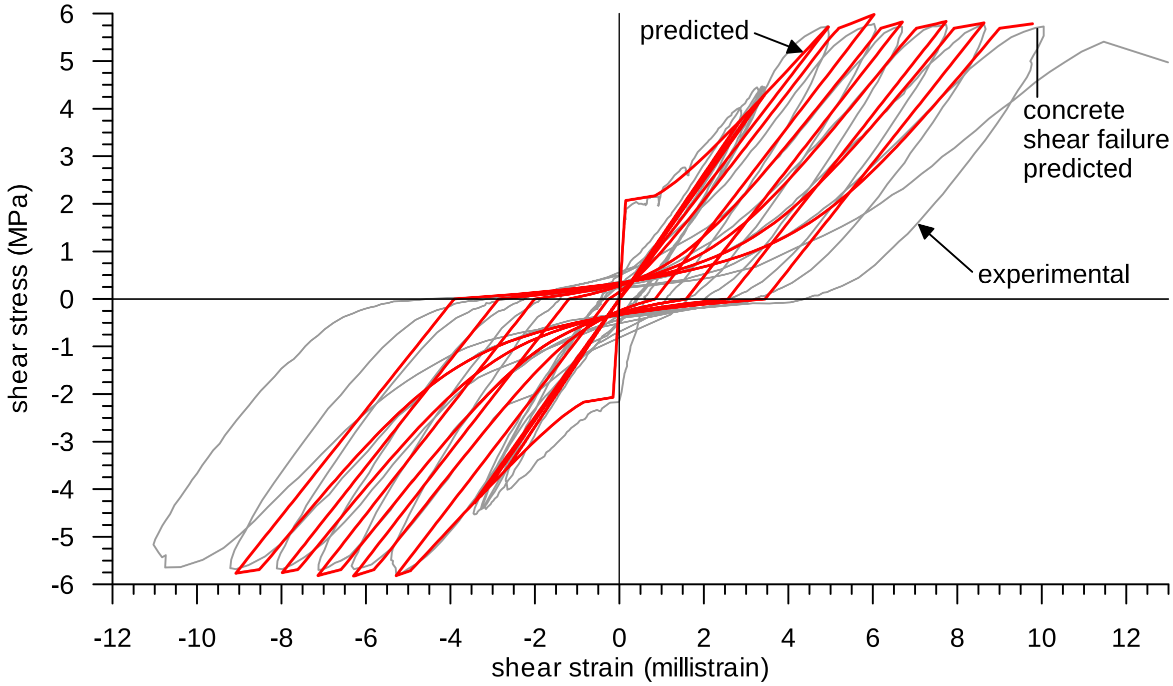

Figure 3 Comparison of General Model prediction (Gérin [9]) and the results from a membrane element test (Stevens et al. [3])

The envelope is defined primarily by the shear stress at yield (assumed to be equal to the shear strength of the element) and the shear strain at yield. These are then used to estimate the cracked section shear stiffness and the ultimate shear strain. Design codes already include equations for the shear strength of walls and these are sufficiently accurate - and well proven - for typical elements. Simplified equations are presented below for the shear strain at yield, the corresponding cracked-section shear stiffness and the ultimate shear strain.

Shear Strain at Yield

As concrete and reinforcement remain compatible on average, the shear strain of reinforced concrete can be determined from the following simple strain transformation:

$$\gamma_{hv} = \varepsilon_h + \varepsilon_v - 2\varepsilon_{45} \tag{1}$$

where ε_h and ε_v are the normal strains in the horizontal and vertical reinforcement directions, respectively; and ε_{45} is the strain at 45° to the reinforcement and in the direction closest to the principal compression strain direction. These normal strain values were chosen since they result in a simple transformation and because the strain values can be easily estimated from the applied stresses as given below.

The shear strain at yielding of the element is defined as when the horizontal reinforcement reaches yield, therefore, the normal strain in the horizontal direction is equal to the yield strain. For simplicity, this strain is assumed to be equal to the bare bar yield strain:

$$\varepsilon_h = \frac{f_y}{E_s} \tag{2}$$

where f_y is the reinforcement yield stress and E_s is the reinforcement modulus of elasticity. For grade 400 MPa reinforcement, $\varepsilon_h = 0.002$.

The sum of the vertical component of the concrete stresses and the stress in the vertical reinforcement must equilibrate the applied vertical axial stress. Assuming for simplicity that the concrete stresses consist of uniaxial compression at 45° to the reinforcement, and assuming a linear stress-strain relationship for the vertical reinforcement, the strain in the vertical reinforcement is given by:

$$\varepsilon_v = \frac{v_y - n}{E_s \rho_v}\ ;\ \geq 0 \tag{3}$$

where v_y is the applied shear stress (at yield), n is the vertical axial compression (compression positive) and ρ_v is the vertical reinforcement ratio.

The derivation of Eq. (3) assumes that the shear force is large enough compared to the vertical compression to cause tension strain in the vertical reinforcement. Thus vertical reinforcement strain given by Eq. (3) is limited to a positive value. Shear critical members such as squat walls would typically not have such high values of axial compression. FEMA 356 limits the axial compression in a ductile member to $0.15A_g f_c'$. Members with greater axial compression are considered to have a brittle response and are dealt with differently.

The strain at 45° to the reinforcement and in the direction closest to the principal compression direction is related to the concrete compression stresses. For simplicity, the concrete stresses are again assumed to be uniaxial compression acting at 45° to the reinforcement. Thus, the principal concrete compression stress is equal to twice the shear stress. For typical elements the principal compression stress in concrete is low compared to the compression strength of concrete and therefore it is reasonable to estimate the compression strain using a linear stress-strain relationship. Concrete softening due to transverse strains is neglected, resulting in a lower bound estimate of the concrete strain. At yield, the strain at 45° is:

$$\varepsilon_{45} = \frac{-2\, v_y}{E_c} \tag{4}$$

where E_c is the tangent stiffness of concrete as defined in building codes.

Substituting Eqs. 2, 3 and 4 into Eq. 1, the shear strain at yield is given by:

$$\gamma_y = \frac{f_y}{E_s} + \frac{v_y - n}{\rho_v E_s} + \frac{4 v_y}{E_c} \tag{5}$$

with the condition that: $0 \leq \dfrac{v_y - n}{\rho_v E_s} \leq \dfrac{f_y}{E_s}$

The first term of Eq. 5 represents the contribution from tension strain of the horizontal reinforcement, the second term the contribution from tension strain of the vertical reinforcement and the third term the contribution from compression strain of concrete.

By substituting typical values into Eq. 5, a range of values for the shear strain at yield for typical shear dominated elements (e.g., squat walls) can be determined. As noted above the contribution from grade 400 MPa horizontal reinforcement is 0.002. The contribution from the vertical reinforcement ranges from 0.0, when the vertical compression stress equals the shear stress, to 0.002 when the vertical reinforcement reaches yield. Assuming a concrete compression strength of 40 MPa, the tangent stiffness E_c is approximately 30,000 MPa. For a typical range of shear stress values, the contribution to the shear strain from the concrete compression strain at 45° ranges from 0.0001 to 0.0007. Adding the three contributions together, the shear strain at yield ranges from 0.0021 to 0.0047. Fig. 4 illustrates the relative contribution of each component to the total shear strain for a range of typical shear stress values.

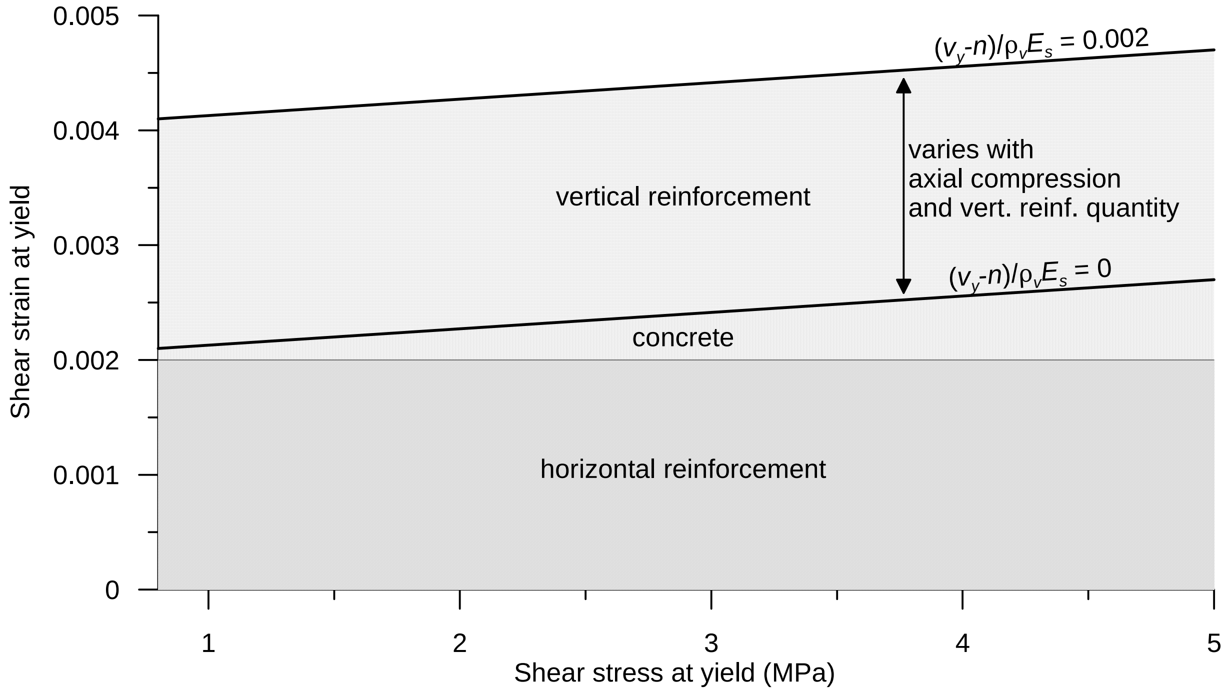

Figure 4 Contributions to shear strain at yield for typical shear walls

Cracked-Section Shear Stiffness

Consistent with FEMA 356, the cracked section shear stiffness is defined as the secant stiffness to the yield point:

$$G_{cr} = \frac{v_y}{\gamma_y} \tag{6}$$

where the shear stress at yield v_y can be taken as the shear strength as given by design codes, and the shear strain at yield γ_y can be estimated from Eq. 5 above. The stiffness given by Eq. (6) represents the fully-cracked condition and thus a lower bound to the stiffness.

To illustrate the results obtained using Eq. (6) for a typical shear dominated element, consider the example of a squat wall with no significant axial force, horizontal and vertical reinforcement ratios of 0.0025 (the minimum for new designs), and 40 MPa concrete. The shear stress at yield is assumed to be the shear strength obtained using ACI 318 Clause 21.7.4 [10] expressed as a shear stress:

$$v_y = 0.25\sqrt{f_c'} + \rho_h f_y \quad \text{[MPa units]} \tag{7}$$

For the values assumed above, the shear stress v_y is 2.58 MPa. From Eq. 5, the shear strain at yield γ_y is 0.002 + 0.002 + 0.0003 = 0.0043. The resulting cracked section shear stiffness is 600 MPa. If the reinforcement ratio is doubled to 0.005 in each direction, the shear strength increases to 3.58 MPa, and the shear strain at yield increases to 0.0045. For this case, the cracked section shear stiffness is 795 MPa. These values of G_{cr} represent 5% and 6.6% respectively of the uncracked section shear stiffness (G_g = 12,000 MPa).

The uncracked section shear stiffness is entirely a function of the concrete properties ($G_g = 0.4E_c$), however, the cracked section shear stiffness is governed primarily by the quantity of reinforcement: the shear strength is controlled by the quantity of reinforcement and the shear strain at yield is dominated by the strains of the reinforcement.

Ultimate Shear Strain

Shear dominated elements with typical amounts of reinforcement can deform significantly in shear after yielding of the weaker reinforcement, that is, they have considerable shear strain ductility. The shear strain is limited by concrete failure in one of two modes: concrete compression failure or concrete shear failure. Concrete compression failure is a brittle failure mode that occurs when the diagonal compression stress in concrete exceeds the effective concrete compression strength accounting for the reduction due to transverse tensile strains. Concrete compression failures typically occur in elements that are heavily reinforced and are subjected to large shear stresses. The second failure mode, concrete shear failure, is relatively ductile and results from excessive local damage along the cracks such as concrete splitting and crushing around the reinforcement. Concrete shear failure is associated with large shear displacements along the cracks.

Figure 5 summarizes data from 21 large-scale membrane element tests. The measured shear strain ductility μ_γ is plotted against the shear stress ratio v_y/f_c'. The shear strain ductility is equal to the ultimate shear strain γ_u divided by measured shear strain at yielding γ_y. The ultimate shear strain is defined as the maximum shear strain before any shear strength loss. The figure indicates that there is a very significant interaction between shear ductility and applied shear stress. The concrete compression failures and concrete shear failures are indicated by different symbols in the figure, and tests where both horizontal and vertical reinforcement yielded are indicated by a third symbol.

The general method of shear design developed by Collins et al. [11] limits the shear stress ratio to 0.25 to avoid concrete compression failures for non-seismic cases. From the data in Fig. 5, this limit is appropriate for cases where the shear strain ductility is less than or equal to one 1.0. Based on the data shown in Fig. 5, the following simple conservative limit on the shear strain ductility is proposed:

$$\mu_\gamma = \frac{\gamma_u}{\gamma_y} = 4 - 12\frac{v_y}{f_c'}; \quad v_y \le 0.25 f_c' \tag{8}$$

This proposed limit is shown in Fig. 5.

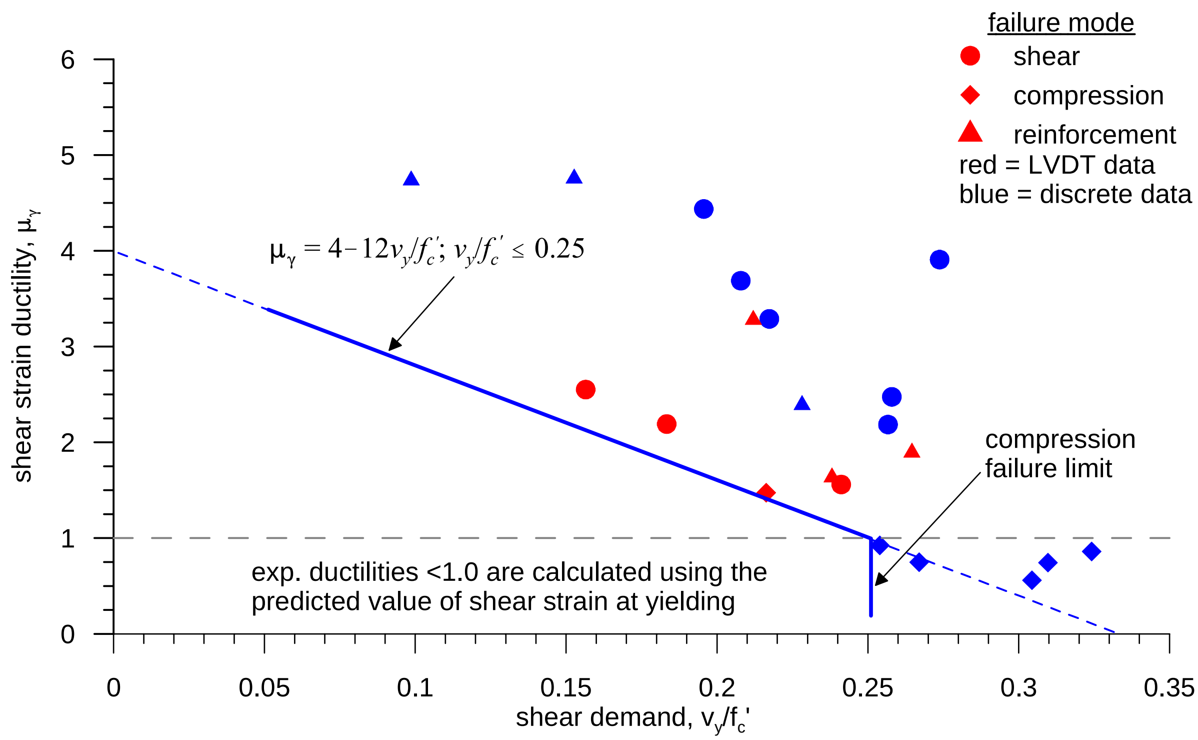

Figure 5 Proposed shear strain ductility

As the ultimate shear strain is determined from the shear ductility and the shear strain at yield, all the parameters which influence the shear strain at yield (i.e., reinforcement ratios, concrete strength, axial load) influence the ultimate shear strain.

Continuing the squat wall example presented above, Eq. 8 gives shear strain ductility μ_γ values of 3.2 and 2.9 for shear strengths v_y of 2.58 MPa and 3.58 MPa. Multiplying by the shear strain at yield determined above (0.0043 and 0.0045) gives ultimate shear strains of 0.014 and 0.013, respectively. For these examples, the FEMA 356 drift limit of 0.75% (a shear strain of 0.0075) is conservative.

The minimum ultimate shear strain would occur at the maximum shear stress level. ACI 318 limits the shear stress in walls to $0.83(f_c')^{½}$ in MPa units. This corresponds to a shear stress ratio of 0.15 for a concrete strength of 30 MPa. According to Eq. (8), the corresponding shear strain ductility is 2.2. For this case, the shear strain at yield given by Eq. (5) varies from 0.0027 to 0.0047 depending on the level of axial compression. The corresponding ultimate shear strain varies from 0.0059 to 0.0103. The FEMA 356 limit of 0.0075 is a reasonable average; but the wide range of values suggests that the more rigorous approach is preferable.

APPLICATION TO SEISMIC ANALYSIS

Linear Analysis

Equations 5 and 6 presented above give designers a simple method for estimating the cracked section shear stiffness for the linear analysis of concrete structures. Similar to the design of flexural members, designers can now consider both uncracked and cracked section properties and obtain a much better estimate of the response.

The use of cracked versus uncracked section properties can be based on whether the objective of the analysis is to obtain forces or displacements, on whether elements are flexure-dominated or shear-dominated and on the estimated stress level in each element. Comparing an initial estimate of the shear stress using uncracked sections can with the shear stress at cracking for each element can indicate which member will be cracked.

One way to estimate the shear stress at cracking is to use the empirical equations given in building codes for the concrete contribution V_c (e.g., ACI 318 Eq. 11-29). Alternatively, the shear stress at cracking can be estimated from first principles:

$$v_{cr} = f_{cr}\sqrt{1+\frac{n}{f_{cr}}} \qquad (9)$$

where f_{cr} is the principal tensile stress at cracking, which can be estimated as $0.33(f_c')^{½}$ [12].

The uncracked section shear stiffness represents an upper bound stiffness while the proposed equations represent a lower bound estimate of the shear stiffness. Given that the lower bound stiffness may be one-twentieth the upper-bound stiffness, in some cases it may be appropriate to select intermediate values based on the shear stress level relative to the cracking stress level.

Non-linear Static Analysis

A simple non-linear curve which captures the change in shear stiffness due to both cracking and yielding is shown in Fig. 6. The gross section stiffness ($0.4E_c$) is applied up to the cracking stress (Eq. 9). Then a straight line is drawn to the yield point, defined by the shear stress at yield (ACI Clause 21.7.4 or similar) and the shear strain at yield (Eq. 5). The response is assumed perfectly plastic after yielding, until the ultimate shear strain is reached (Eq. 8).

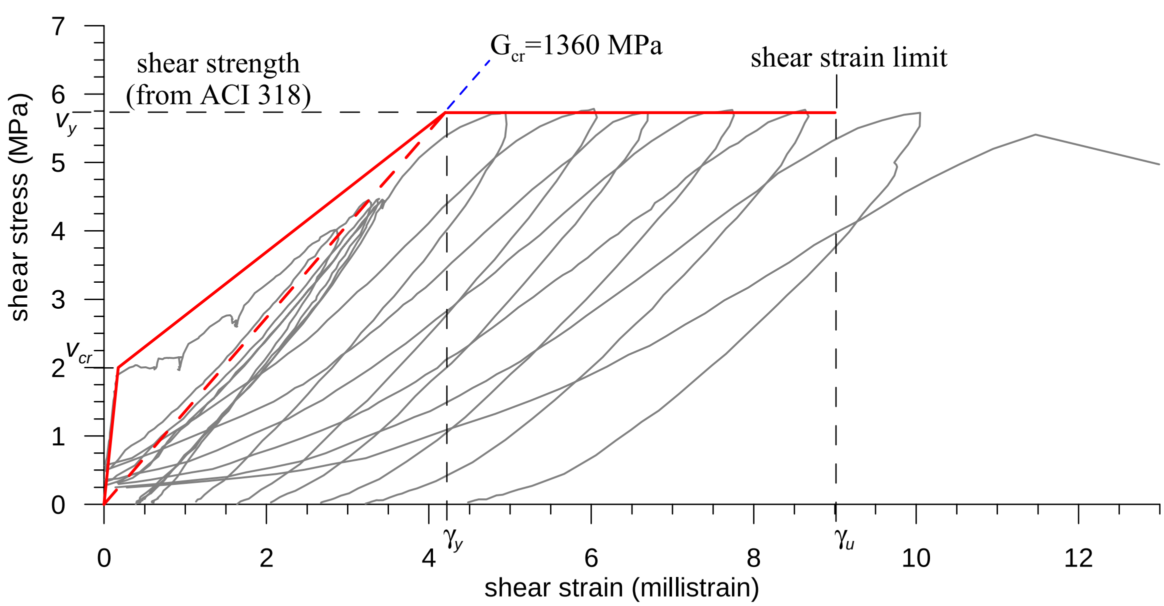

Figure 6 Proposed load-deformation curve for non-linear static analysis (shown for membrane element test SE8, Stevens et al. [3])

Figure 6 compares this approach with test data for a membrane element with relatively high reinforcement ratios of 1% and 3% in the vertical and horizontal directions, respectively (Stevens et. al. [3]). The proposed model improves on the FEMA 356 generalized model by including the reduction in shear stiffness due to cracking and by defining the ultimate shear strain as a function of element properties.

Non-linear Dynamic Analysis

Non-linear dynamic analysis requires the complete hysteretic shear response of each element. A simple reverse-cyclic model that extends the envelope presented above was derived from the general model. The main components of the general model have been converted such that the response can be defined directly in terms of shear stress and shear strain.

The simple hysteretic model, shown in Fig. 7, assumes that yielding occurs at v_y for each cycle, unloading occurs at a constant slope equal to G_{cr}, and the plastic shear strain γ_p remaining at the end of each unloading segment is cumulative from one direction of loading to the other. The reloading curve accounts for the closing of diagonal cracks in one direction and the simultaneous opening of diagonal cracks in the other direction in a simple way. In Fig. 7, the envelope to the response accounts for the principal stress orientation (Gérin and Adebar [13]) instead of assuming 45°.

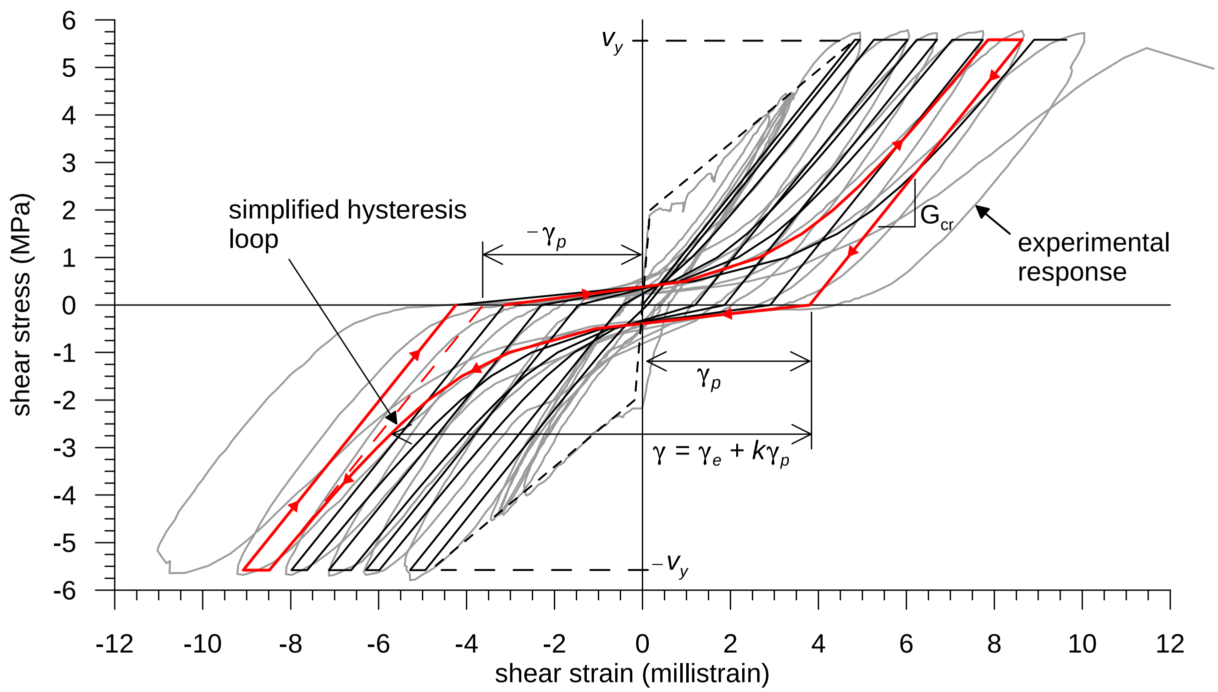

Figure 7 Proposed simple hysteretic model (shown for membrane element test SE8, Stevens et al.[3])

The shear strain at any applied shear stress level is given by:

$$\gamma = \gamma_e + k\gamma_p \tag{10}$$

where γ_e is the elastic shear strain v/G_{cr} and $k\gamma_p$ is the plastic portion of the shear strain, which varies from $+\gamma_p$ to $-\gamma_p$ with the closing/opening of the diagonal cracks. In the general model, the plastic strain depends on the crack angle, the orientation of the principal concrete compression stress and an empirical crack closing function (Gérin [9]). The following simplified expression for the function k in Eq. (10) was developed from the general model by assuming the principal compression stress is at 45° and normal to the closing diagonal cracks, and that the plastic shear strain is proportional to the crack strains:

$$k = 2e^{\left(\frac{2v}{1-0.4v}\right)} - 1 \qquad (11)$$

where v is the applied shear stress.

The response predicted by the simple model is compared to the experimental result from a large-scale membrane element test (Stevens et al. [3]) in Fig. 7. The principal characteristics of the response are well captured. Stiffness decay and pinching of the loops due to the accumulation of plastic strains in the reinforcement are both well represented.

FEMA 356 requires that in the nonlinear dynamic analysis of concrete walls, the hysteretic shear model shall reflect the behaviour of experimentally observed elements similar to the one under investigation. As the model presented here has been verified by comparison with membrane element tests, considered to be a fundamental test of reinforced concrete subjected to pure shear, the model is suitable for a wide range of shear dominated elements.

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