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SIMPLIFIED ESTIMATION OF ECONOMIC SEISMIC RISK FOR BUILDINGS

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SUMMARY

A seismic risk assessment is often performed on behalf of a buyer of a commercial building in a seismically active region. One outcome of the assessment is that a probable maximum loss (*PML*) is computed. *PML* is of limited use to real-estate investors as it has no place in a standard financial analysis and reflects too long a planning period. We introduce an alternative to *PML* called probable frequent loss (*PFL*), which is defined as the mean loss resulting from shaking with 10% exceedance probability in 5 years. *PFL* is approximately related to expected annualized loss (*EAL*) through an economic hazard coefficient that can be tabulated or mapped. *PFL* and *EAL* offer three advantages over *PML*: (1) their planning period is meaningful to investors; (2) they can be used in financial analysis (making seismic risk a potential market force); and (3) rather than relying on expert opinion, one can estimate *PFL* and *EAL* using rigorous performance-based earthquake engineering (PBEE) principles, and yet produce a good approximation using a single linear structural analysis. We illustrate using 15 buildings, including a 7-story nonductile reinforced-concrete moment-frame building in Van Nuys, California, and 14 buildings from the CUREE-Caltech Woodframe Project.

INTRODUCTION

Seismic risk enters into several important real-estate decision-making processes: purchase of investment property, performance-based design of new structures, seismic rehabilitation of existing buildings, and decisions regarding the purchase of earthquake insurance, for example. In such situations, it matters who the decision-makers are, how they make decisions, what aspects of seismic risk most concern them, how long their planning horizon is, and other parameters. We focus on one of the most-common seismic risk decision situations: the purchase of existing commercial property by real-estate investors in seismic regions. (The most common situation is probably purchasing a home in seismically active regions.)

Economic seismic risk to these properties is assessed every time the property changes hands, on the order of every five to ten years. By contrast, a building is designed and built only once. Thus the most common

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opportunity for market forces to bring about seismic-risk mitigation for commercial properties is at times of sale. Anecdotal evidence suggests that these are mostly missed opportunities: risk is typically not mitigated, even in more-vulnerable buildings.

This can be partly explained by considering the context in which seismic assessments are performed. During virtually every sale of an existing commercial building, the buyer analyzes the building's investment value in terms of net present value, internal rate of return, or capitalization rate, considering projected revenues and expenses, rent roll, market leasing, physical condition, and other property information, and using a planning period on the order of 5 yr. Uncertainties in vacancy rate, future market rents, and other important parameters are dealt with using best-estimate inputs and then again with deterministic sensitivity studies to probe conditions that would lead to poor performance (higher future vacancy rates, for example). Costs that occur rarely and unpredictably, such as future earthquake repair costs, are commonly ignored in the financial analysis. This is important: seismic risk is not a market quantity.

PML: POOR METRIC OF LOSS

The market is not wholly without forces to influence seismic-risk mitigation. After a bid is accepted and before the property changes hands, during a process called the due-diligence study, the bidder performs or contracts for an engineering assessment of the property's condition, which typically includes an estimate of the earthquake probable maximum loss (*PML*). The earthquake *PML* has no commonly accepted quantitative definition, as pointed out by Zadeh [1]. ASTM [2] grappled with and abandoned an effort to standardize seismic *PML*, producing instead some new terminology. The *PML* nonetheless lingers on. Most working definitions of *PML* involve the level of loss associated with a large, rare event, as is Rubin [3]. One common definition is that *PML* is the 90th percentile of loss given the occurrence of what building codes until recently called the design basis earthquake, or *DBE*—an event producing a shaking intensity with 10% exceedance probability in 50 years. This is an upper-bound loss given shaking with a mean recurrence time of 475 years.

The *PML* is used primarily to satisfy commercial mortgage underwriters that the risk of borrower default due to earthquakes is low. It is common, for example, for a lender to refuse to underwrite a mortgage if the *PML* exceeds 20% to 30% of the replacement cost of the building, unless the buyer purchases earthquake insurance—a costly requirement that often causes the investor to decide against bidding. Once the *PML* hurdle is passed, the bidder usually proceeds to ignore seismic risk. There are at least three good reasons for this:

1. *Irrelevant planning period*. Investors plan on the order of 5 years, and view loss corresponding to shaking intensity with a 500-year recurrence time as irrelevant.
2. *Incompatibility with financial analysis. PML* is a scenario loss, not an ongoing cost. Investors cannot use it in a cashflow analysis.
3. *Custom.* Investors are not required by custom or regulation to include seismic risk in the financial analysis.

Lacking any measure of economic risk beyond *PML*, the bidder has no basis for assessing how the seismic risk of a building should influence the purchase price or for judging whether seismic risk mitigation might be worth exploring. Faced with a high *PML*, the bidder might either pass on the investment opportunity (rather than purchase earthquake insurance) or increase the discount rate used in the financial analysis to reduce the present value of the future net income stream, but no analysis informs the adjustment. This typically closes the matter.

EXPECTED ANNUALIZED LOSS

There is another common term in earthquake loss estimation, namely expected annualized loss (*EAL,* [2]), which measures the average yearly amount of loss when one accounts for the frequency and severity of various levels of loss. If one knew *EAL*, one could include it as an operating expense in a financial analysis. We present three ways to estimate *EAL*, from an exhaustive approach (labeled method 1) to two successively simpler ones (methods 2 and 3).

Method 1: integrate seismic vulnerability function and hazard

Method 1 involves evaluation of the seismic vulnerability function and seismic hazard function, and integrating to calculate *EAL*. Assuming independence of intensity and of losses between events, *EAL* can be calculated as

$$EAL = V \int_{s=0}^{\infty} y(s)\left|G'(s)\right| ds \tag{1}$$

where V denotes value exposed to loss (here, replacement cost of the building); s refers to an intensity measure such as damped elastic spectral acceleration; $y(s)$ is referred to here as the mean seismic vulnerability function, giving the average loss as a fraction of V given the occurrence of s; and $G'(s)$ is first derivative with respect to s of the mean annual frequency of exceeding intensity s. $G(s)$ is referred to here as the hazard function.

In most practical situations, $y(s)$ and $G(s)$ would be evaluated at a set of n+1 discrete intensity values s_0, s_1, … s_n. We denote these values by y_0, y_1, … y_n, and G_0, G_1, … G_n, respectively. We define

$$\Delta s_i \equiv s_i - s_{i-1} \quad i = 1, 2, \ldots n \tag{2}$$

$$\Delta y_i \equiv y_i - y_{i-1} \quad i = 1, 2, \ldots n \tag{3}$$

We approximate $G(s)$ varying exponentially and $y(s)$ varying linearly between values of s, i.e.,

$$G(s) = G_{i-1} \exp\left(m_i\left(s - s_{i-1}\right)\right) \quad \text{for } s_{i-1} < s < s_i \tag{4}$$

$$y(s) = y_{i-1} + \frac{\Delta y_i}{\Delta s_i}\left(s - s_{i-1}\right) \quad \text{for } s_{i-1} < s < s_i \tag{5}$$

$$m_i = \frac{\ln\left(G_i / G_{i-1}\right)}{\Delta s_i} \quad i = 1, 2, \ldots n \tag{6}$$

where m_i is a (negative) constant for $s_{i-1} < s < s_i$. Then

$$G'(s) = m_i G(s) \quad \text{for } s_{i-1} < s < s_i \tag{7}$$

We define

$$\tau \equiv s - s_{i-1} \quad s_{i-1} < s < s_i \tag{8}$$

Then *EAL* is given by

$$\begin{aligned} EAL &= V\sum_{i=1}^{n}\left(\int_{0}^{\Delta s_i}\left(y_{i-1} + \frac{\Delta y_i}{\Delta s_i}\tau\right)\left(-m_i G_{i-1}\exp\left(m_i\tau\right)\right)d\tau\right) + R \\ &= V\sum_{i=1}^{n}\left(y_{i-1}G_{i-1}\left(1-\exp\left(m_i\Delta s_i\right)\right) - \frac{\Delta y_i}{\Delta s_i}G_{i-1}\left(\exp\left(m_i\Delta s_i\right)\left(\Delta s_i - \frac{1}{m_i}\right) + \frac{1}{m_i}\right)\right) + R \end{aligned} \tag{9}$$

where R is a remainder term for values of $s > s_n$, and has an upper bound of $VG(s_n)$ if $y(s) \le 1$.

$G(s)$ is increasingly available, e.g., from software produced by the US Geological Survey [4]. Software such as HAZUS [5], USQUAKE [6], and ST-RISK [7] contain pre-evaluated $y(s)$ and $G(s)$ and can calculate $y(s)$ and *EAL*. However, these canned seismic vulnerability functions rely to a significant extent on expert opinion, do not perform structural analysis on a building-specific basis, and thus are insensitive to many of the details that cause performance differences between distinct buildings of the same building type.

One can use performance-based earthquake engineering (PBEE) methodologies such as assembly-based vulnerability (ABV, [8]) to calculate $y(s)$ on a building-specific basis without relying on expert opinion. Details of ABV have been described elsewhere. Briefly, it is a simulation procedure in five stages:

Full ABV

1. *Facility definition*. To define the facility one must know its location (latitude and longitude) and design, including site soils, substructure, structural and nonstructural assemblies. One creates an inventory of the damageable assemblies and identifies the *EDP*—story drift ratio, member force, etc.—that would cause damage to each assembly.

2. *Ground-motion selection.* Select a ground-motion time history and scale to an intensity value of interest.

3. *Structural analysis.* Create a stochastic structural model and perform a nonlinear time-history structural analysis to determine structural response, quantified via engineering demand parameters (*EDP*s, meaning member forces and deformations). By "stochastic structural model," we mean a model where masses, damping, and force-deformation behavior are uncertain, having prescribed probability distributions. In the past, we have created stochastic structural models starting with a deterministic (best-estimate) structural model and multiplying all masses by a Gaussian variable e_M, multiplying viscous damping by a Gaussian variable e_β, and multiplying all strengths by a single Gaussian variable e_{FD}. Parameters of these variables are discussed in the illustration portion of this work.

4. *Damage analysis.* Simulate damage to each damageable assembly via assembly fragility functions. It is assumed that after an assembly is subjected to a certain *EDP*, it will be in an uncertain damage state D, indexed by $d = 0, 1, 2, \ldots N_D$, where $d = 0$ indicates the undamaged state. We assume that the damage states can be sorted in increasing order, either because an assembly in damage state $d = i + 1$ must have passed through damage state i already, or because the effort to restore an assembly from damage state $d = i + 1$ necessarily restores it from damage state $d = i$. The threshold level of *EDP* causing an assembly to reach or exceed damage state d is uncertain (we refer to it as the assembly's capacity to resist damage state d), and is denoted by X_d. The cumulative distribution function of capacity is denoted by $F_{Xd}(x)$. Then, given the response x to which an assembly is subjected, the probability distribution of the damage state is
$$\begin{aligned} p\left[D = d \mid EDP = x\right] &= 1 - F_{X_1}\left(x\right) && d = 0 \\ &= F_{X_d}\left(x\right) - F_{X_{d+1}}\left(x\right) && 1 \le d < N_D \\ &= F_{X_{ND}}\left(x\right) && d = N_D \end{aligned} \tag{10}$$
where $d = 0$ refers to the undamaged state. In the past [8 – 12], we have taken all capacities as lognormally distributed.

5. *Loss analysis.* Given damage, assess loss via probabilistic construction cost-estimation. In particular,
$$C_T = \left(1 + C_{OP}\right)\left(\sum_j \sum_d N_{j,d} C_{j,d}\right) \tag{11}$$

where C_{OP} refers to contractor's overhead-and-profit factor; $N_{j,d}$ refers to the number of assemblies of type *j* in damage state *d* (determined in the damage analysis); and $C_{j,d}$ refers to the uncertain cost to restore one assembly of type *j* from damage state *d*. In the past, we have treated C_{OP} as uncertain, with uniform distribution between 0.15 and 0.20, and $C_{j,d}$ as lognormal with mean and standard deviation varying by assembly type and damage state.

Steps 2-5 are repeated many times at each of many intensity levels, each time sampling each uncertain variable once. One uses ABV to calculate average loss at each intensity level and produce $y(s)$, then integrates with $G'(s)$ to calculate *EAL*.

ABV has proven to be a useful research tool, and has been used to evaluate seismic risk and to perform benefit-cost analysis of seismic-risk mitigation for steel-frame, woodframe and concrete buildings and to explore major contributors to the uncertainty in economic seismic risk [8 – 12]. However, it is difficult to use in professional practice for estimating $y(s)$ because of the special skills, software, and data required. It is not particularly computationally costly. Once set up, the structural analyses for a typical building can be performed overnight, and the subsequent damage and loss analyses can be performed in an hour or so. It is the setup that is time-consuming, principally the creation of the structural model.

Method 2: one-step approximation of vulnerability and hazard

The extreme simplification of method 1 is to use $n = 1$ in Equations (2) through (9). That is, $y(s)$ and $\ln(G(s))$ are approximated as linear functions of s. To do so requires only two values of $y(s)$ and of $G(s)$, and s_0 is chosen as the point at which $y(s)$ just becomes nonzero, i.e., the initiation of damage. We denote this intensity as s_{NZ}, and approximate $y(s_{NZ}) = 0$. It is common, for example, to assume damage initiates at intensities on the order of 0.05g.

As noted above, $G(s)$ is readily available, so the simplification is primarily valuable in that only s_{NZ} and a single scenario value of vulnerability, $y(s_1)$, are additionally required. For this special case, we choose s_1 as s_U, where $y(s_U)$ denotes an upper-bound loss. We denote $y(s_U)$ by y_U. Thus, we approximate

$$\begin{aligned} y(s) &= 0 && s \le s_{NZ} \\ &= a(s - s_{NZ}) && s_{NZ} \le s \le s_U \\ &= y_U && s_U \le s \end{aligned} \tag{12}$$

$$G(s) = G_{NZ} \exp\left(m\left(s - s_{NZ}\right)\right) \tag{13}$$

where a and m are constant over s for a particular building. We calculate a and m based on $y(s_{NZ})$, $G(s_{NZ})$, $y(s_{EBE})$ and $G(s_{EBE})$, where s_{EBE} is the intensity measure in some intermediate event ($s_{NZ} < s_{EBE} < s_U$) referred to here as the economic-basis earthquake (*EBE*), in an effort to evoke the design-basis earthquake, *DBE*, of older codes. We refer to $y(s_{EBE})$ as the probable frequent loss, *PFL*, to imitate the *PML*, but at a lower level of shaking. We make the notation more compact by denoting $G(s_{NZ})$ as simply G_{NZ}, and $G(s_{EBE})$ as G_{EBE}.

$$a = \frac{PFL}{V\left(s_{EBE} - s_{NZ}\right)} \tag{14}$$

$$m = \frac{-\ln\left(G_{NZ}/G_{EBE}\right)}{s_{EBE} - s_{NZ}} \tag{15}$$

$$
\begin{aligned}
s_U &= s_{NZ} + \frac{y_U}{a} \\
&= s_{NZ} + \frac{y_U V\left(s_{EBE} - s_{NZ}\right)}{PFL}
\end{aligned}
\tag{16}
$$

We can now evaluate EAL. Defining $\tau \equiv s - s_{NZ}$ and $\sigma \equiv s - s_U$, and recalling that $m < 0$,

$$
\begin{aligned}
EAL &= V\left(\int_{s_{NZ}}^{\infty} a\left(s - s_{NZ}\right)\left(-mG\left(s\right)\right)ds - \int_{s_U}^{\infty}\left(a\left(s - s_{NZ}\right) - y_U\right)\left(-mG\left(s\right)\right)ds\right) \\
&= V\left(\int_{0}^{\infty}\left(a\tau\right)\left(-mG_{NZ}\exp\left(m\tau\right)\right)d\tau - \int_{0}^{\infty}\left(a\sigma\right)\left(-mG_U\exp\left(m\sigma\right)\right)d\sigma\right) \\
&= Va\left(-\left(G_{NZ} - G_U\right)\left(\exp\left(m\tau\right)\left(\tau - \frac{1}{m}\right)\right)\Bigg|_{\tau=0}^{\infty}\right) \\
&= -Va\frac{\left(G_{NZ} - G_U\right)}{m} \\
&= \frac{PFL}{\left(s_{EBE} - s_{NZ}\right)}\frac{\left(G_{NZ} - G_U\right)\left(s_{EBE} - s_{NZ}\right)}{\ln\left(G_{NZ}/G_{EBE}\right)} \\
&= \frac{\left(G_{NZ} - G_U\right)}{\ln\left(G_{NZ}/G_{EBE}\right)}PFL
\end{aligned}
\tag{17}
$$

If $s_U >> s_{NZ}$ (as is likely), then $G_U << G_{NZ}$, which leads to:

$$
EAL \approx \frac{G_{NZ}}{\ln\left(G_{NZ}/G_{EBE}\right)}PFL \tag{18}
$$

Defining

$$
H \equiv \frac{G_{NZ}}{\ln\left(G_{NZ}/G_{EBE}\right)} \tag{19}
$$

leads to the final form:

$$
EAL \approx H \cdot PFL \tag{20}
$$

where H is referred to as the economic hazard coefficient. Thus, EAL can be approximated using a single scenario loss estimate, the probable frequent loss, and a parameter H that contains only hazard variables, and can therefore be mapped or tabulated. Later, we evaluate the quality of this approximate formula for EAL based on some case studies.

The challenge remains of how to select s_{EBE} and how to calculate PFL. We address selection of s_{EBE} later. PFL can be calculated with ABV (as in method 1) but using only a single intensity level, which saves substantial computational effort (one value of s versus many) but little labor, since as noted above it is the setup of the nonlinear stochastic structural model that is most costly in terms of labor.

Method 3: PFL and linear ABV

If s_{EBE} is low enough, a linear structural model and modal analysis might adequately capture structural response at s_{EBE}, and the subsequent loss analysis, as will be shown, can then be greatly simplified. We refer to this next simplification as linear assembly-based vulnerability (LABV), which proceeds as follows:

Linear ABV

1. *Facility definition*. Same as under ABV.

2. *Hazard analysis*. Invert the hazard function at the exceedance frequency of the EBE (discussed later) to determine $s_{EBE} = G^{-1}(G_{EBE})$. The hazard function is readily available in the United States, e.g., using software such as Frankel [4] and adjusting to account for site classification such as by using F_a or F_v as appropriate from the International Building Code [13].

3. *Structural analysis*. In this simplification, the *EDP* for each damageable assembly is calculated considering only the first-mode spectral response. We denote by ϕ_1 the mode shape of a building at its small-amplitude fundamental period of vibration, T_1. Let the modal excitation and modal mass for the first mode be denoted by L_1 and M_1, respectively. Each damageable assembly k is assumed to be sensitive to an *EDP*, characteristic of that assembly type, whose value we denote by x_k, and which can be calculated as a function of ϕ_1, L_1 and M_1. For example, considering one frame direction, the *EDP* for a segment of wallboard partition on the m^{th} story would be the interstory drift along that wall line, estimated as

$$x \approx \frac{s_{EBE}}{\omega_1^2}\left(\frac{\phi_{1(m+1)} - \phi_{1m}}{h_m}\right)\frac{L_1}{M_1} \tag{21}$$

where $\omega_1 = 2\pi/T_1$, ϕ_{1m} refers to the component of the fundamental mode shape at floor m, and h_m refers to the height of story m.

4. *Damage and loss analysis*. This step combines steps 4 and 5 of ABV. The expected value of the cost to restore a damaged assembly of a specified type from damage state d is denoted by c_d; it can be calculated by standard construction-cost estimation principles. Then, given the response x to which an assembly is subjected, the mean cost to repair the damageable assembly is

$$\overline{y}(x) = \sum_{d=1}^{N_D} c_d\, p\left[D = d \mid EDP = x\right] \tag{22}$$

where probability p[$D = d \mid EDP = x$] is given by Equation (10). We refer to Equation (22) as a mean assembly vulnerability function. Kustu [14] referred to it as a component damage function, expressing the same information as a fraction of the replacement cost of the assembly. (We prefer the non-normalized costs because doing so avoids unnecessary consideration of an additional uncertainty, i.e., the replacement cost of the assembly.) Introducing the index k to refer to individual assemblies in the building, y_k to indicate the mean repair cost of that assembly, and x_k to indicate the value of the *EDP* to which that assembly is subjected under the EBE,

$$PFL = \left(1 + \overline{C}_{OP}\right)\sum_{k=1}^{N} \overline{y}_k\left(x_k\right) \tag{23}$$

where $\overline{C}_{OP}$ refers to contractor's mean overhead-and-profit factor. The *EAL*, as before, is calculated per Equation (20). As we will show, libraries of mean assembly vulnerability functions can be accumulated and used in subsequent studies.

Hence, method 3 is very simple: one need only create an inventory of the damageable assemblies in the building, perform a linear modal structural analysis at s_{EBE}, and perform some straightforward calculations to estimate *PFL* and hence *EAL*. This level of effort is probably practical within the budget and schedule of due-diligence studies for commercial real-estate investment purchases. The questions remain, what is an appropriate definition of the EBE, and how accurate is method 3 compared with methods 1 and 2?

DEFINITION OF ECONOMIC-BASIS EARTHQUAKE

To test the life-safety of a structural design, structural engineers have historically considered upper-bound shaking (10% exceedance probability) during the design life of the building (50 years), referring to this level of shaking as the design-basis earthquake (DBE). If one wants to examine an upper-bound event during the owner's planning period, then it is consistent to define the EBE using same exceedance probability (10%) during the owner's planning period (5 yr). This EBE corresponds to a return period of approximately 50 years (more accurately, 47.5 yr, assuming Poisson arrivals of earthquakes). Again, the mean loss given the EBE is referred to here as the *probable frequent loss* (*PFL*), in imitation of *PML*, the probable maximum loss.

Why not use the shaking intensity with 50% exceedance probability in 50 years, a scenario shaking level treated for example by FEMA 356 [15], and which would be only slightly stronger than the EBE? The reason is effective risk communication: EBE is defined for its meaning to the investor, for whom 50 years is too long a planning period and 50% exceedance probability does not bespeak an upper-bound intensity. Our definition of EBE more simply and directly addresses the concerns of the investor. We now test how well this definition works in methods 2 and 3 for sample facilities.

CASE STUDIES

To test whether method 3 and the EBE defined as above produce an acceptably accurate estimate of *EAL*, we begin by analyzing an actual highrise hotel building located in Van Nuys, California. The hotel is a seven-story, eight-by-three-bay, nonductile reinforced-concrete moment-frame building constructed in 1966. It suffered significant damage in the 1971 San Fernando and 1994 Northridge Earthquakes. We performed an ABV analysis of the building as it existed prior to the 1994 Northridge Earthquake. We used as an intensity measure the 5%-damped elastic spectral acceleration at the building's small-amplitude fundamental period, S_a(1.5 sec). We used Frankel [4] to determine seismic hazard, adjusting for soil conditions using provisions of the International Building Code [13].

For details of the structural model and development of the assembly fragility functions and unit-cost distributions, see Beck [9]. For the ABV analysis, we examined 20 levels of ground motion: S_a = 0.1g, 0.2g, … 2.0g. At each S_a level, 20 ground-motion time histories were selected at random (within scaling limitations and other preferences) from 100 provided by Somerville [16] and randomly paired with a sample of the stochastic structural model to perform a nonlinear time-history structural analysis. In each of the 400 structural analyses, all structural, damage, and cost parameters were allowed to vary according to prescribed probability distributions.

Masses were taken as perfectly correlated, normally distributed, with coefficient of variation equal to 0.10, as suggested by Ellingwood [17]. Damping was taken as normally distributed with mean value of 5% and coefficient of variation equal to 0.40, as derived in Beck [9]. Structural members were taken as having deterministic stiffnesses (including post-yield, unloading, etc.) but with yield and ultimate force and deformations that are perfectly correlated, normally distributed, with coefficient of variation of 0.08, as suggested in [17].

Component capacities were taken as lognormally distributed, with median (denoted by x_m) and logarithmic standard deviation (denoted by β) summarized in Table 1. Repair-cost distributions for individual damaged components (referred to here as unit-repair costs) were taken as lognormally distributed with median (x_m) and logarithmic standard deviations (β), also summarized in Table 1, with mean values estimated by a professional cost estimator. Contractor overhead and profit were taken as uniformly distributed between 15% and 20% of total direct costs (the sum of the costs to repair individual assemblies). Unit costs are in 2001 US dollars.

Two limitations of the model should be acknowledged. First, it did not capture collapse. Second, it employed uncoupled structural and damage analyses, that is, damage was taken as conditionally independent of structural characteristics, conditioned on structural response. Shaikhutdinov [18] has recently found that such an uncoupled analysis can significantly underestimate uncertainty in repair costs, among other effects.

Table 1. Summary of assembly fragility parameters and cost distributions [9].

Assembly description	Unit	Limit state; repair	EDP(1)	*Capacity* x_m	*Capacity* β	*Cost*, $ x_m	*Cost*, $ β
Stucco finish, 7/8", 3-5/8" metal stud, 16"OC	64 sf	1.Cracking; patch	PTD	0.012	0.5	125	0.2
Drywall fin., 5/8-in., 1 side, metal stud, screws	64 sf	1. Visible dmg; patch	PTD	0.0039	0.17	88	0.2
Drywall fin., 5/8-in., 1 side, metal stud, screws	64 sf	2. Signif. dmg; replace	PTD	0.0085	0.23	253	0.2
Drywall ptn, 5/8-in., 1 side, metal stud, screws	64 sf	1. Visible dmg; patch	PTD	0.0039	0.17	88	0.2
Drywall ptn, 5/8-in., 1 side, metal stud, screws	64 sf	2. Signif. dmg; replace	PTD	0.0085	0.23	525	0.2
Nonductile CIP RC beam or column	ea	1. Light; epoxy	PADI	0.080	1.36	8000	0.42
Nonductile CIP RC beam or column	ea	2. Moderate; jacket	PADI	0.31	0.89	20500	0.4
Nonductile CIP RC beam or column	ea	3, 4. Severe or collapse; replace	PADI	0.71	0.8	34300	0.37
Window, Al frame, sliding, hvy sheet glass…	ea	1. Cracking; replace	PTD	0.023	0.28	180	0.2
Paint on exterior stucco or concrete	sf	Paint	(2)	N/A		1.45	0.2
Paint on interior concrete, drywall, or plaster	sf	Paint	(2)	N/A		1.52	0.2

(1) PTD = peak transient drift ratio; PADI = Modified Park-Ang damage index (displacement portion): $(\phi_m - \phi_y)/(\phi_u - \phi_y)$, where ϕ_m = maximum curvature, ϕ_y = yield curvature, ϕ_u = curvature at maximum moment for the element in question, considering the element's own material and geometric properties
(2) Paint entire room, hallway, etc. to achieve reasonable uniform appearance if any component in the line of sight requires painting.

The resulting seismic vulnerability function is shown in Figure 1(a). The x-axis represents 5%-damped elastic spectral acceleration (denoted by S_a) at the building's small-amplitude fundamental period, 1.5 sec. The y-axis measures repair cost as a fraction of replacement cost. Each circle represents one loss simulation. The jagged line indicates mean damage factor at each S_a level. The smooth curve is a polynomial fit to all of the data. Each simulation includes one nonlinear time-history structural analysis using one ground-motion time history, one simulation of the (uncertain) mass, damping, and force-deformation characteristics of the building, one simulation of the damageability of each of 1,233 structural and nonstructural components, and one simulation of the unit-repair cost for each of 17 combinations of component type and damage state. The analysis included 20 simulations for each of 20 S_a increments from 0.1g to 2.0g. The 400 nonlinear time-history structural analyses took approximately 12 hours of computer time on an ordinary desktop computer; the subsequent loss analysis took less than an hour. The most time-consuming portion of the analysis was creating the structural model.

The jaggedness of the mean-vulnerability curve in Figure 1(a) reflects three effects. First, beam and column repair costs begin to saturate near S_a = 0.5g for some simulations, possibly because of plastic hinges acting as structural fuses. Second, the damage factor begins to saturate near S_a = 0.4g. Repair cost was capped at the replacement cost of the building, and costs were estimated to reach or exceed this value in some simulations beginning at S_a = 0.4g. Third, with a residual coefficient of variation of damage factor as high as 0.50, one would expect to see some jaggedness in the mean vulnerability function from a Monte Carlo simulation with 20 samples per S_a level.

Figure 1(b) provides the site seismic hazard function, denoted by $G(S_a)$ and defined as the mean annual exceedance rate of ground shaking as a function of S_a. We used Frankel [4] to calculate the hazard at T = 1.0 and 2.0 sec, with soil at the B-C boundary, and then linearly interpolated in the log-frequency domain to calculate the hazard at T = 1.5 sec, using International Building Code [13] adjustment factors to account for soil condition.

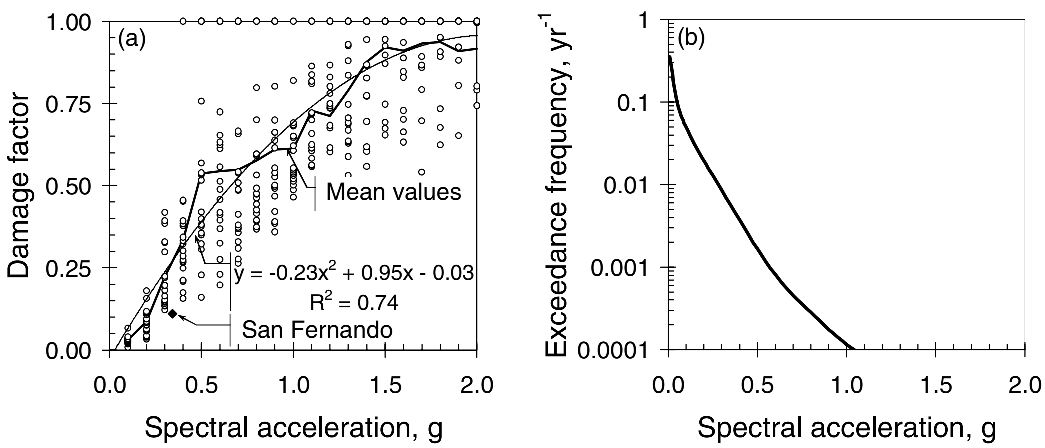

Figure 1(a). Mean seismic vulnerability function, (b) Site-hazard function for Van Nuys building.

For purposes of evaluating *EAL* under methods 2 and 3, we take s_{NZ} = 0.05g and $s_{EBE} \approx$ 0.2g. Table 2 compares the values of *PFL* and *EAL* calculated using the three methods. (Note that *PFL* for Method 2 is taken from the ABV analysis of method 1. The difference between the *PFL* values for methods 1 and 3 is due to the linear approximation of structural response.) Agreement is reasonable: methods 2 and 3 produce *EAL* estimates within about 30% of that of method 1. That method 3 produces a reasonable estimate is particularly promising: at least in this case, one need not create a nonlinear structural model to get a reasonable estimate of *PFL* and *EAL*.

Table 2. Approximation of earthquake loss using probable frequent loss (*PFL*).

	Van Nuys
s_{NZ}	0.05g
s_{EBE}	0.20g
$G(s_{NZ})$, yr^{-1}	0.1026
$G(s_{EBE})$, yr^{-1}	0.0195
H, yr^{-1}	0.0617
PFL methods 1 and 2	$613,000
PFL method 3	$930,000
EAL, method 1	$53,600
EAL, method 2	$37,800
EAL, method 3	$57,400

We performed three tests of whether EBE is defined well. First, we evaluated Equation (9) at each of n = 1, 2, … 20, for Δs = 0.1g. The resulting plot (Figure 2) shows the cumulative contribution to *EAL* considering only $S_a \leq$ 0.1g, then $S_a \leq$ 0.2g, etc. Figure 2(a) shows the results plotted against S_a, while Figure 2(b) shows the same information plotted against mean recurrence time. The plots show that almost half the expected losses for this building result from shaking of S_a = 0.25g or less, i.e., events with a recurrence time of 85 years or less, and that approximately 35% is due to $S_a < s_{EBE}$. Ideally, cumulative loss from $S_a \leq s_{EBE}$ would always be near 50%, which would suggest that s_{EBE} is a good representative scenario shaking level, but of course the fraction will likely vary between buildings, so a cumulative *EAL* fraction of 35% at the s_{EBE} defined this way seems acceptable.

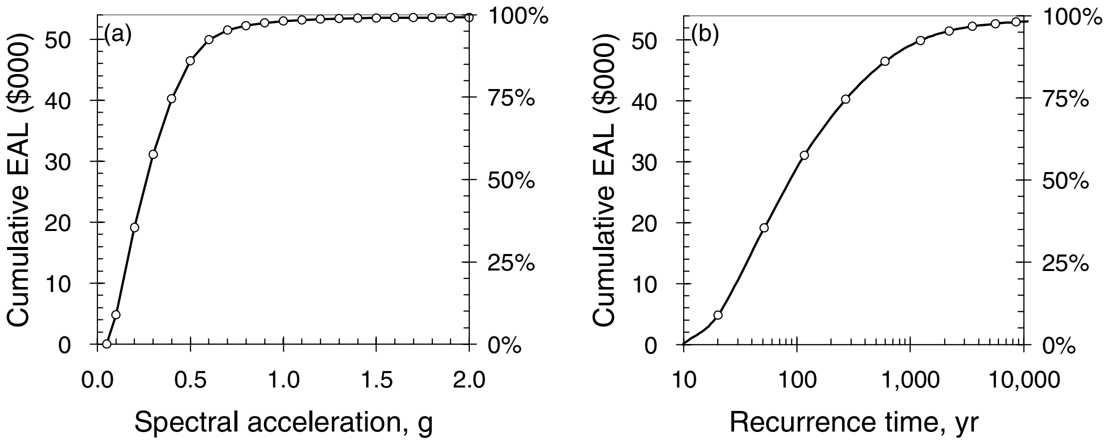

Figure 2. Dominance of frequent events in expected annualized loss for Van Nuys building.

As a second test of EBE, we compared methods 1 and 2 using 14 hypothetical (but completely designed) buildings from the CUREE-Caltech Woodframe Project [12]. The buildings are variants of four basic designs referred to as index buildings [19]. The index buildings include a small house (single story, 1,200 sf, stucco walls, no structural sheathing), a large house (two stories, 2,400 sf, some walls sheathed with plywood or OSB, stucco exterior finish), a three-unit townhouse (two stories, 6,000 sf total, some walls sheathed with plywood or OSB, stucco exterior finish), and an apartment building (three stories, 13,700 sf, 10 dwelling units, and tuck-under parking). Each index building included four or more variants: a poor-quality version, a typical-quality version, a superior-quality version, and one or more alternative designs or retrofits. We considered the Woodframe Project buildings located at an arbitrary site, chosen to be in Los Angeles, California, at 33.9°N, 118.2°W. Again using Frankel [4] to determine site hazard, adjusted for NEHRP soil category D, we find s_{EBE} = 0.4g. Of the 19 buildings examined, 14 have nonzero loss estimates at s_{EBE}. The seismic vulnerability functions for these 14 are shown in Figure 3; the site hazard is shown in Figure 4. The jaggedness of some of the seismic vulnerability functions reflects sensitivity to collapse (which was modeled in these cases).

Figure 5 shows the *EAL* values for these 14 woodframe buildings and for the Van Nuys building calculated by method 1 (referred to in the figure as "exact") and by method 2 (referred to as "approximate"), using EBE as defined above. We denote *EAL* estimated under method 1 by EAL_1, define estimation error as

$$\varepsilon = \frac{EAL_2 - EAL_1}{EAL_1} \tag{24}$$

and take the error for each case-study building as a sample of ε. We find the sample mean and sample standard deviation of this error are $\overline{\varepsilon}$ = 0.12 and s_ε = 0.52, respectively. Thus, for this sample of 15 buildings, the use of s_{EBE} defined as the shaking with 10% exceedance probability in 5 yr produces a fairly modest (12%) error in the estimate of *EAL*, relative to the "exact" method which requires analysis of the complete seismic vulnerability function.

Finally, we tested the error if one defines s_{EBE} as shaking with 50% exceedance probability in 50 yr, and found $\overline{\varepsilon}$ = 0.06 and s_ε = 0.47, respectively—slightly more accurate than the result of using shaking with 10% exceedance in 5 yr.

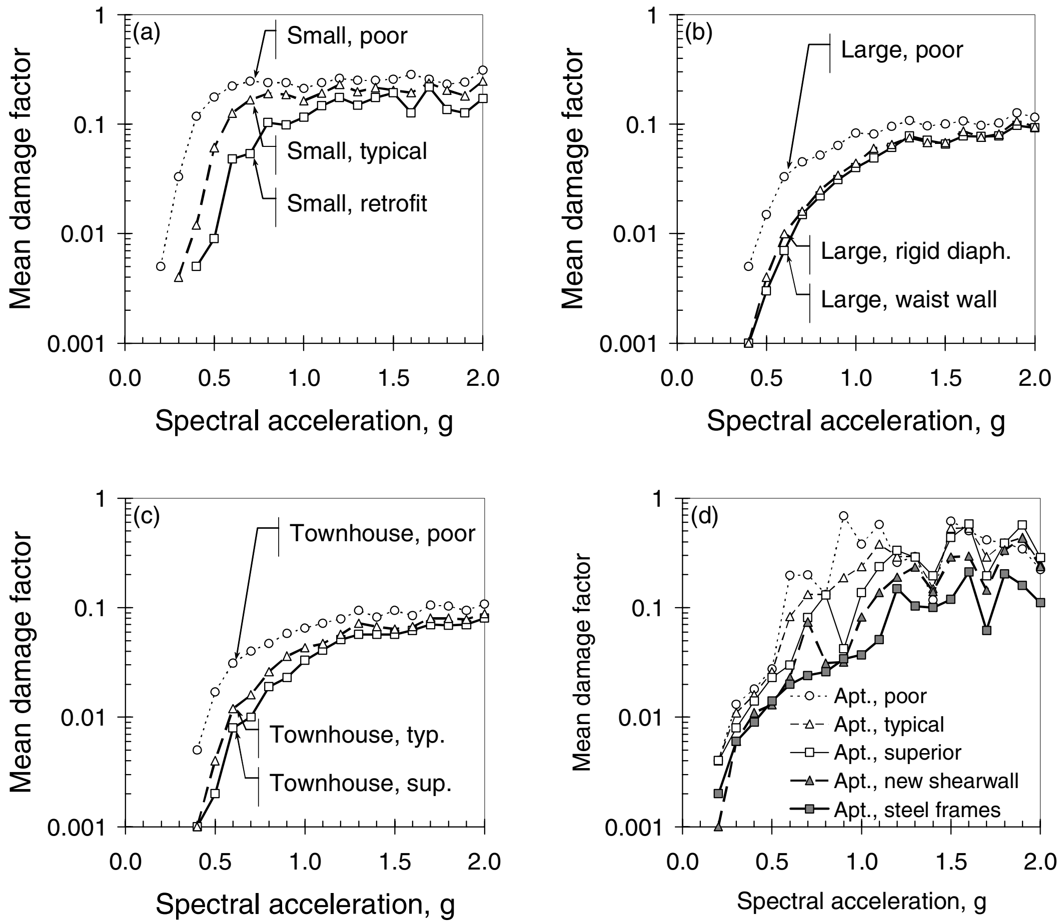

Figure 3. **CUREE-Caltech Woodframe Project index-building mean vulnerability functions.**

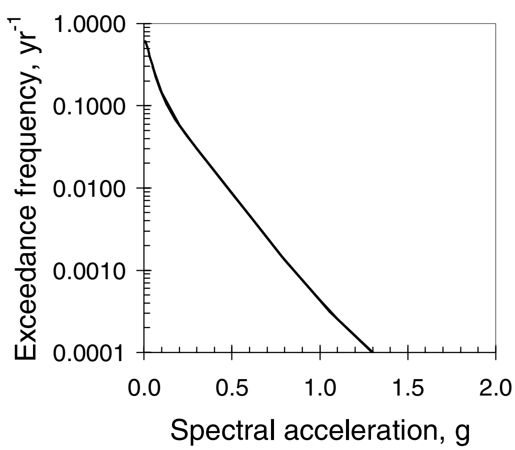

Figure 4. Seismic hazard function for a Los Angeles site.

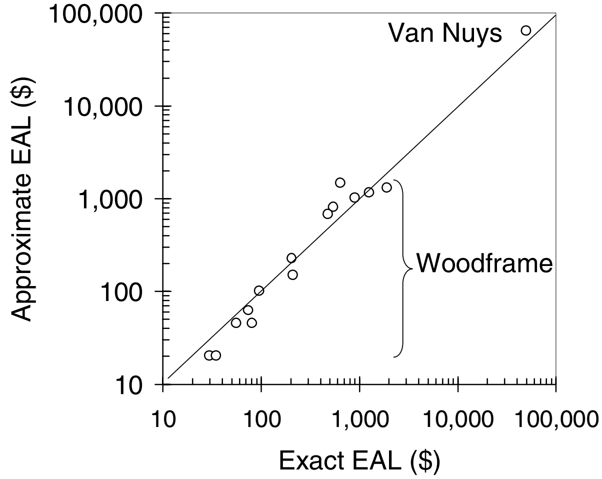

Figure 5. Comparing *EAL* by methods 1 and 2 for 15 sample buildings.

CONCLUSIONS

Through a case study of a nonductile reinforced-concrete moment-frame building, we have shown that probabilistic repair costs can be dominated by small, frequent events, as opposed to rare, *PML*-level losses. Expected annualized loss (*EAL*) is approximately proportional to a scenario loss referred to as the probable frequent loss (*PFL*). *PFL* is defined similarly to *PML*, but with a planning period that is consistent with that of the typical commercial real-estate investor. It is the expected value of loss conditioned on the occurrence of shaking with 10% exceedance probability in 5 years. The constant of proportionality between *PFL* and *EAL*, referred to here as the economic hazard coefficient (*H*), can be mapped or tabulated for ready use by structural engineers or investors. A simplified loss-analysis approach, referred to as linear assembly-based vulnerability (LABV), can produce an estimate of *PFL* and consequently *EAL* that is relatively accurate, compared with an "exact" analysis that involves evaluating the complete seismic vulnerability function and integrating with seismic hazard. The main advantages of the use of *PFL, EAL*, and LABV over current practice (in which *PML* is the dominant metric of seismic risk) is that, while *PML* generally provides little useful information to the bidder for a commercial real-estate investment, both *PFL* and *EAL* can be used directly in the bidder's financial analysis, and can probably be calculated within the budget and schedule of the normal due-diligence study.

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