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DEVELOPMENT OF A HYBRID EXPERIMENTAL METHOD FOR DYNAMIC TESTING OF LARGE SCALE STRUCTURAL MODELS

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SUMMARY

In this research using shake table and dynamic actuators at the same time a new hybrid testing method is presented. The original idea of the method was proposed by Kausel in 1998 and developed by the author thereafter. Here the input motion is divided between the shake table and actuators and is applied at the same time on the experimental model. It is shown that using this method it is possible to simulate the rotational input motion at the foundation level, to apply effects of all three components of earthquake simultaneously even on 1-D shake tables, and to minimize the total power required for the experiment compared with when each of the testing apparatuses are used separately. Different techniques for dividing the ground motion between the two facilities are presented which include dividing the amplitude by a constant factor, dividing the frequency content, and dividing the amplitude on a timely basis based on a minimization scheme, which is called the dynamic optimization method in this research. It is shown that the last method, i. e., the dynamic optimization, is the preferred technique that results in minimum testing power.

INTRODUCTION

Various methods for earthquake engineering tests have been developed in the recent decades, including pseudo-static experiments using hydraulic actuators, pseudo-dynamic testing with force actuators, and dynamic experiments with shake table.

In the pseudo-static testing the specimen is subjected to forces and deformations produced by hydraulic actuators and changing so slowly that can be regarded as constant in a given period of time. Therefore the effects of dynamic inertial forces on the specimen cannot be visualized in this technique. The main idea behind this method is to evaluate the hysteresis behavior of different structural members rather than systems. In pseudo-dynamic testing similar to pseudo-static method rate of loading on the model is slow, but the inertial forces are simulated by the aid of a computer controlling the progress of the test. This method is usually applied on structural systems that can be as large as a full-scale building. In this method in a certain time step lateral displacements of the stories are theoretically calculated and applied by the force actuators at the roof levels. The forces in the actuators required to produce the above displacements are directly recorded by the load cells during the experiment resulting in the values of the shear forces in

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the columns. The latter values are needed in the nonlinear dynamic equations of the system using which the lateral displacements for the next time step are calculated by the controlling computer of the testing setup. Because of the lack of need to exert forces at high speeds in this method, the experimental apparatus is simpler and cheaper and it is practical to test larger and even full-scale specimens. However, modeling inertial forces raised by rapid motions of building under earthquake is only theoretically feasible and their effects are not visible in the test. In the dynamic method of testing using shake tables, usually it is not possible to pick up large experimental samples or to test the models at their full scales. On the other hand, using this method makes it possible to simulate the real shaking of ground.

Other than the above classic methods of testing, two different methods can be explored, at least theoretically. In one recent method, named the effective force method, while keeping the base of the structure unmoved dynamic forces equal to the acceleration of ground motion multiplied by mass of the story are applied by actuators at each story level. This way, the effect of ground motion is simulated exactly by external dynamic forces. Because of the need to apply force without limiting the displacements of the structural model and without interference of the actuators with the structure at its natural frequencies of vibration, practical implementation of the effective force method has yet to be fulfilled. Recently, some remedies have been suggested to make this method feasible by Dimig et al. [1]. The next method that is the focus of this paper is called the hybrid testing method. Here, by simultaneous use of force actuators and a shake table each simulating a part of the ground motion, the advantages of both testing apparatuses are benefited in combination avoiding their weak points. This idea was first given by Kausel [2]. After that, the idea was improved by the author of this paper and several ways for dividing the ground motion between the two testing facilities were examined resulting in an enhanced method named the dynamic optimization technique. This paper is a summary of the research done on this new method giving the basic corresponding equations and some numerical examples.

DYNAMIC EQUATIONS FOR THE EXPERIMENTAL MODEL

The governing equations for different testing methods in earthquake engineering are described here for a single degree of freedom model.

The dynamic equation of equilibrium for an SDOF oscillator with possible nonlinear behavior can be shown as follows:

$$m\ddot{u} + f\left(y, \dot{y}\right) = 0 \tag{1}$$

In Eq. (1), m shows the mass and u is the absolute lateral displacement of the oscillator, and the dots represent derivation respect to time. Also $u = y + u_g$ in which y is the relative displacement respect to the base and u_g is the ground (base) displacement. As is seen in Eq. (1), the base shear is shown by $f\left(y, \dot{y}\right)$ as a function of relative displacement and velocity of the oscillator. This function can be linear or nonlinear based on the level of shaking. For a linear system, the well-known relation $f\left(y, \dot{y}\right) = ky + c\dot{y}$ exists in which k and c represent the lateral stiffness and the damping of the system.

In pseudo-static testing, the force in the hydraulic actuator is $f\left(y, \dot{y}\right)$ in which by lengthening the time of the experiment and lowering the rate of change of actuator force, dependence of f on $\dot{y}$ is minimized. On the other hand, in pseudo-dynamic testing at each time step Eq. (1) is used to calculate the relative displacement y to be applied by the actuator on the model and then the force in the actuator $f\left(y, \dot{y}\right)$ to make this displacement is recorded and used in Eq. (1) again to calculate the relative displacement for the next time step.

If Eq. (1) is used for analyzing a system vibrating on a shake table, the term $m\ddot{u}$ or $f(y,\dot{y})$ shows the force exerted by the shake table on the oscillator and vice versa. Equation (1) can be rewritten this time in effective force format by resorting to relative displacements, as shown in Eq. (2):

$$m\ddot{y} + f(y,\dot{y}) = -m\ddot{u}_g \tag{2}$$

In Eq. (2), the right side exhibits the effective force needed to be applied on the model if the base of the model is kept unmoved. This is the basic idea behind the effective force method of testing. The hybrid testing method concerned in this paper is a combination of testing with shake table and the effective force method. Here, the ground motion is divided in two parts as follows:

$$u_g = u_{g1} + u_{g2} \tag{3}$$

Substituting Eq. (3) in Eq. (1) results in:

$$m(y + u_{g1} + u_{g2}) + f(y,\dot{y}) = 0 \tag{4}$$

Equation (4) can further be written as Eq. (5):

$$m\ddot{u}_1 + f(y,\dot{y}) = -m\ddot{u}_{g2} \tag{5}$$

in which:

$$u_1 = y + u_{g2} \tag{6}$$

Equation (5) governs the response of the oscillator in the hybrid testing method. In Eq. (5), the term $m\ddot{u}_{g1}$ represents the effective force to be applied by the force actuators on the system and the term $m\ddot{u}_1 + m\ddot{u}_{g1} = m\ddot{u}$ again shows the force on the oscillator by the shake table as discussed above for testing with shake table only.

THE TESTING POWER

In the hybrid method the main question is what is the best criterion for dividing the ground motion between the two apparatuses in order to have the minimum testing power. The electric power needed for implementing a dynamic experiment is defined as the product of the external force applied on the system by the testing facility and the velocity at which this force is applied. The equations of testing power in different experimental methods described above are presented here as a basis for numerical calculations illustrated afterwards.

Power for Testing with Shake Table

Referring to Eq. (1), the force exerted by, or on the shake table is $m\ddot{u}$ which is applied at the velocity of the original ground motion, $\dot{u}_g$. Multiplying these two terms together results in the testing power, W, as follows:

$$W = m\ddot{u}\dot{u}_g \tag{7}$$

Power for Testing with Actuators (the Effective Force Method)

As is shown by Eq. (2), the force in the actuators is $-m\ddot{u}_g$ applying at a velocity of $\dot{y}$. Then the power needed is:

$$W = -m\ddot{u}_g\dot{y} \tag{8}$$

Power for Hybrid Testing Method

The governing relation for this method is Eq. (5). As is shown in this equation, the force produced by the actuators in the system is $P_1 = -m\ddot{u}_{g1}$ applying with a velocity of $\dot{u}_1$. On the other hand, the external force of the shake table is $P_2 = m(\ddot{u}_1 + \ddot{u}_{g1}) = m\ddot{u}$ that is applied at a velocity equal to $\dot{u}_{g2}$. The testing power in this case is computed by the addition of powers needed for each facility in the test, as follows:

$$W = -m\ddot{u}_{g1}\dot{u}_1 + m\ddot{u}\dot{u}_{g2} \tag{9}$$

Substituting the relation $u_1 = y + u_{g2}$ in Eq. (9) results in:

$$W = -m\ddot{u}_{g1}\dot{y} + m(\ddot{u} - \ddot{u}_{g1})\dot{u}_{g2} = -m\ddot{u}_{g1}(\dot{y} + \dot{u}_{g2}) + m(\ddot{y} + \ddot{u}_g)\dot{u}_{g2} \tag{10}$$

Now a parameter α is defined to identify the two parts of the ground motion as follows:

$$\alpha = \frac{\ddot{u}_{g1}}{\ddot{u}_g} \Rightarrow \frac{\ddot{u}_{g2}}{\ddot{u}_g} = 1 - \alpha \tag{11}$$

Substitution of Eq. (11) in Eq. (10) results in:

$$W = (1-\alpha)^2\dot{u}_g\ddot{u}_g + (1-\alpha)\dot{u}_g\ddot{y} - \alpha\dot{y}\ddot{u}_g \tag{12}$$

Obviously, it is desired to make the testing power needed a minimum value. This is the key point behind calculating the value of the dividing factor α. Calculation of α can be done in different ways that is the subject of discussion in the next section.

METHODS FOR DIVIDING THE GROUND MOTION

In this section three different methods are presented for dividing the ground motion between the force actuators and the shake table in the hybrid testing method. These methods are compared numerically in the next section. The methods are dividing in time domain, dividing in frequency domain, and the dynamic optimization, which are described one by one.

Dividing in Time Domain

In this method, the dividing factor α is varied between zero and unity calculating each time the testing power required. Here simply the amplitude of the acceleration time history of the ground motion is decreased by the factor α and $(1-\alpha)$ separately that results in two different time histories, which are input to the actuators and the shake table, respectively. The instance of α for which the power reaches a minimum value, gives the desired dividing factor for the ground motion between the force actuators and the shake table according to Eq. (11).

Dividing in Frequency Domain

As will be seen in the examples, generally the shake table needs less power for being vibrated with low frequency records while on the contrary the force actuators consume the minimum power when the frequency content is highest. Therefore it will be to the benefit of the experiment if the record is broken up into two parts, namely the low frequency part and the high frequency part, and the low part is input to the shake table and the remaining to the actuators. This is done conveniently through a direct and an inverse fast Fourier transform in turn. However, the cutoff frequency is to be selected that is the main challenge in this method. The simplest way is calculating the maximum required power each time choosing a cutoff frequency and repeating the computations for different values of that. The desired value of the cutoff frequency for the experimental model under a certain earthquake record is the amount for which the largest required power in the experiment becomes minimum.

The Dynamic Optimization

The most important restriction in the method of dividing in time domain is the assumption that the dividing factor α is constant with respect to time. This means that α takes on a constant value all over a ground motion record in power calculations. It can be stipulated that if the value of the dividing factor of the ground motion is "tuned" with the dynamic characteristics of the record and the structure at each time step, i.e. if it changes value by time, the maximum testing power required can become even lower. This is the basic fact constituting the so called dynamic optimization method in this research work.

The implementation of the idea begins with Eq. (12) for computing the testing power in the hybrid experiment. The ideal value for the required power in Eq. (12) is zero that results in:

$$(1-\alpha)^2 \dot{u}_g \ddot{u}_g + (1-\alpha)\dot{u}_g \ddot{y} - \alpha\dot{y}\ddot{u}_g = 0 \tag{13}$$

Expanding and rearranging the above equation with respect to α results in the following:

$$(\dot{u}_g \ddot{u}_g)\alpha^2 - (\dot{y}\ddot{u}_g + 2\dot{u}_g\ddot{u}_g + \ddot{y}\dot{u}_g)\alpha + \ddot{y}\dot{u}_g + \dot{u}_g\ddot{u}_g = 0 \tag{14}$$

Equation (14) can be further simplified to a second order equation for calculating α as follows:

$$\alpha \;\; - \left(\frac{\dot{y}}{\dot{u}_g} + \frac{\ddot{y}}{\ddot{u}_g} + 2 \right)\alpha + \left(\frac{\ddot{y}}{\ddot{u}_g} + 1 \right) = 0 \tag{15}$$

Equation (15) has two real roots if only its Δ factor is not negative; this means:

$$\Delta = \left(\frac{\dot{y}}{\dot{u}_g} + \frac{\ddot{y}}{\ddot{u}_g} \right)^2 + 4\frac{\dot{y}}{\dot{u}_g} \geq 0 \;\Rightarrow\; \frac{\dot{y}}{\dot{u}_g} \geq \frac{-1}{4}\left(\frac{\dot{y}}{\dot{u}_g} + \frac{\ddot{y}}{\ddot{u}_g} \right)^2 \tag{16}$$

If Eq. (16) turns out to be true, then there exists two (or a repeated) values for α between them the smaller one can be picked up from the following equation:

$$\alpha = \frac{1}{2}\left(\frac{\dot{y}}{\dot{u}_g} + \frac{\ddot{y}}{\ddot{u}_g} + 2 \pm \sqrt{\left(\frac{\dot{y}}{\dot{u}_g} + \frac{\ddot{y}}{\ddot{u}_g}\right)^2 + 4\frac{\dot{y}}{\dot{u}_g}}\right) \tag{17}$$

The required power in this case is zero. On the other hand, if Eq. (16) is not valid, the power is nonzero regardless of the value of α. Because the equation of power with respect to α is a second order polynomial (refer to Eq. (12)) it has to have an extremum in this case, i.e., a value of α for which the absolute amount of W is minimum, which is the desired value. To find α and the corresponding (minimum) W in this case, the derivation of W with respect to α in Eq. (12) is equalized to zero which after some algebra (details omitted for brevity) results in:

$$\alpha = 1 + \frac{1}{2}\left(\frac{\dot{y}}{\dot{u}} + \frac{\ddot{y}}{\ddot{u}}\right) \tag{18}$$

Substituting this value of α in Eq. (12) results in the minimum value of the testing power W as follows:

$$W = -m\dot{y}\ddot{u}_g - \frac{1}{4}m\dot{u}_g\ddot{u}_g\left(\frac{\dot{y}}{\dot{u}_g} + \frac{\ddot{y}}{\ddot{u}_g}\right)^2 \tag{19}$$

NUMERICAL CALCULATIONS

Using the equations developed in the previous sections, the required experimental power is calculated for single degree of freedom structural models differing in natural frequency. The input motion is selected from between real earthquakes recorded in Khoraasaan province, which is one of four earthquake provinces of Iran and a highly active seismic area. The selected records were chosen to be strong motions, i.e., so as to have relatively large PGA's, medium and long durations, magnitudes larger than 4.5, and spectral intensities larger than 15 cm. Table 1 shows the characteristics of the 10 earthquake records selected.

Table 1. Characteristics of the earthquake ground motions.

Record no.	Soil profile	Freq. cont. (Hz)	PGA (cm/sec^2)	PGV (cm/sec)	PGD (cm)	Duration (sec)	Spectral int. (cm)
1	Hard	0.1-25	941.3	51.66	8.94	35.2	205.87
2	Hard	0.1-25	880.9	73.27	10.7	31.14	321.21
3	Hard	0.2-25	382.4	26.18	4.68	37.58	196.18
4	Soft	1-20	354.2	6.77	0.25	3.02	77.66
5	Hard	0.2-25	319.6	18.1	3.32	37.0	196.1
6	Soft	0.4-25	303.0	13.78	2.6	18.1	72.2
7	Hard	2-25	261.5	7.25	0.26	7.2	42.7
8	Hard	0.4-20	217.1	8.75	1.38	9.42	30.25
9	Soft	0.6-20	158.2	7.18	0.73	7.78	115.44
10	Hard	2-25	157.5	5.24	0.22	5.08	40.92

For doing the numerical calculations with the above 10 records, 10 SDOF structural models are considered having natural frequencies in the range of 1-10 Hz with 1 Hz increments.

In the first set of calculations, dividing in the time domain is examined. Five different values for α (the dividing factor) is considered as: 0, 0.25, 0.5, 0.75, and 1. For example, a zero value for α shows that nothing is input to the actuators and the whole amplitude of the record is sent to the shake table. On the other hand, when α equals unity this means that the shake table is still and the actuators simulate the whole ground motion. The above assumptions amount to 500 cases of calculation of the time history of the testing power. For each time history and structural model, the maximum required power is extracted from the results. Therefore, for each model with a certain natural frequency, 10 values of testing power (corresponding to 10 records) are resulted for each value of α. Figures 1 to 4 show the graphs of the required power against the natural frequency of the model. Then, at each frequency, the value of α for which the power is minimum is extracted out of figures 1 to 4. This process results in the spectrum of α, i.e., the variation of α versus frequency of the model. This is shown in Fig. 5.

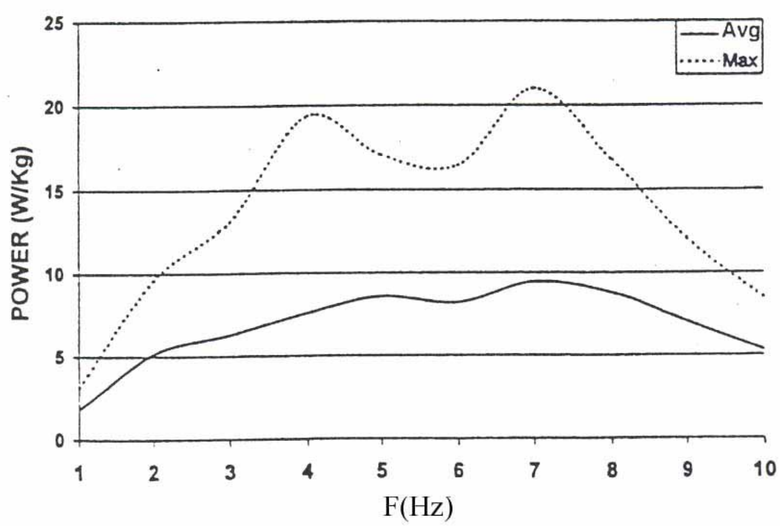

Fig. 1. Maximum instantaneous required power for different models, average curve and the envelope, $\alpha = 0$.

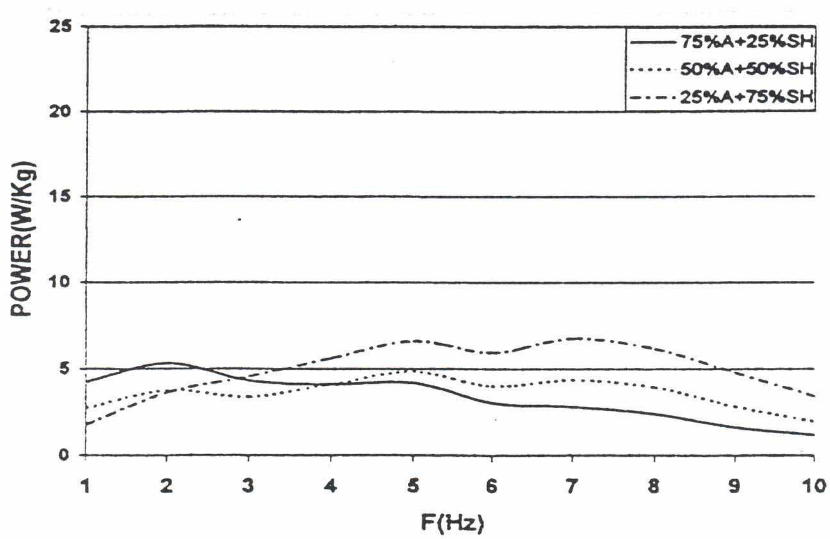

Fig. 2. Maximum instantaneous required power for different models, average curve, $\alpha = 0.25, 0.5, 0.75$.

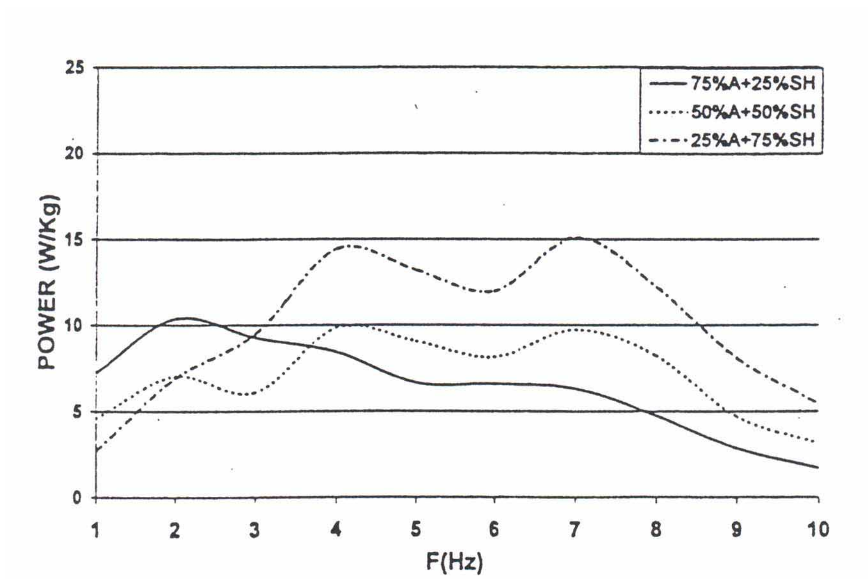

Fig. 3. Maximum instantaneous required power for different models, the envelope, $\alpha = 0.25,\ 0.5,\ 0.75$.

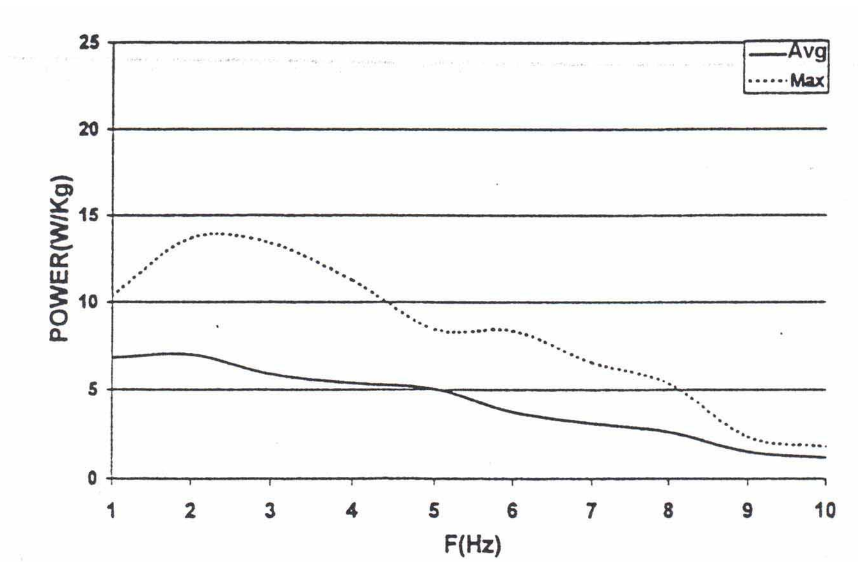

Fig. 4. Maximum instantaneous required power for different models, average curve and the envelope, $\alpha = 1$.

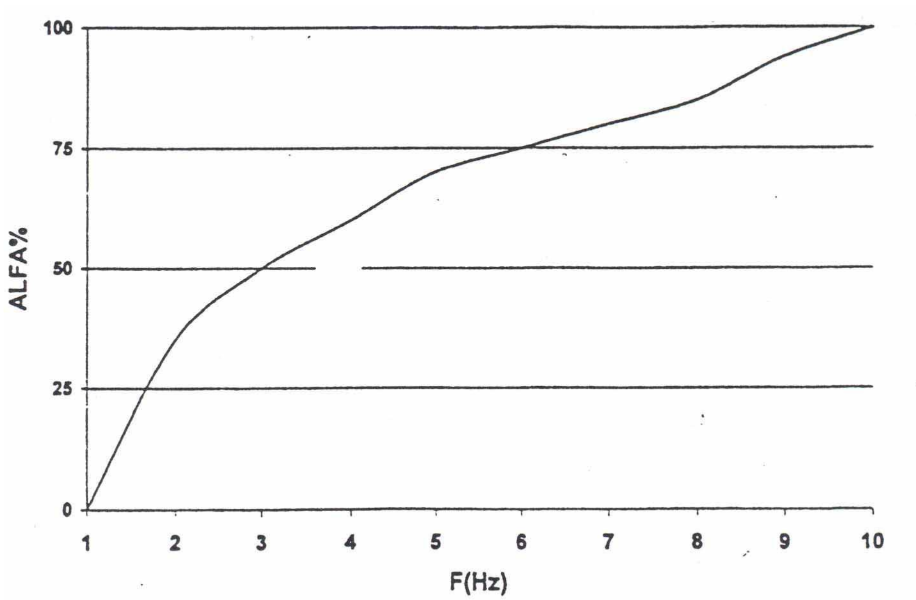

Fig. 5. Variation of α versus the natural frequency of the model (spectrum of α).

Now, the method of dividing in the frequency domain is examined. The ground motion record no. 2 and the model having a 5 Hz natural frequency are used for the calculations as an example. Through an FFT process, the record is decomposed into its low and high frequency counterparts choosing cutoff frequencies between 1 and 10 Hz each time in sequence. The shake table is vibrated with the low frequency component while the actuator simulates the high frequency component. Figure 6 shows the resulted curve for the required power.

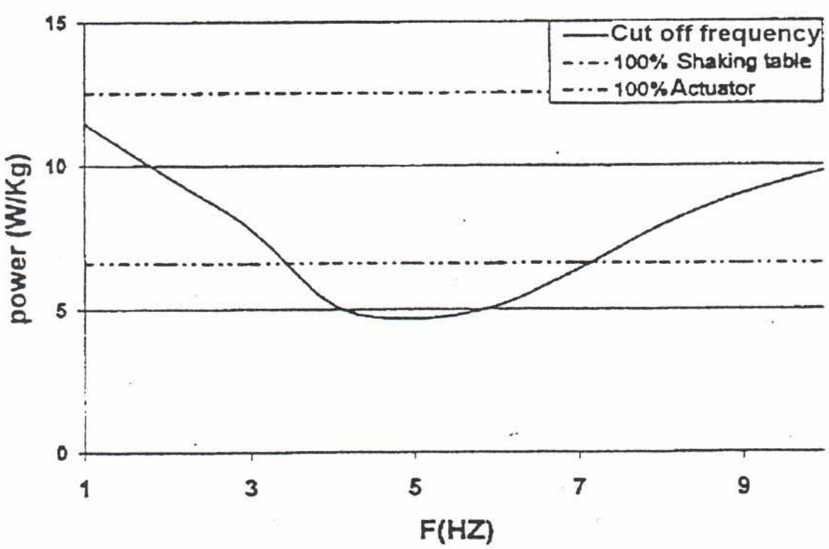

Fig. 6. Variation of the maximum required power against the cutoff frequency, record no. 2, natural frequency of the model is 5 Hz.

In the third set of calculations, the dynamic optimization method is explored as described above. Again, as an example, record no. 2 and the model with a natural frequency of 5 Hz are considered and time histories of the required power are calculated. As can be resulted from Fig. 5, for this model a value of α about

0.75 gives the minimum needed power when using the method of dividing in the time domain. Thus, for the sake of comparison, the time history of power is determined also using the method of dividing in the time domain for the mentioned value of α. The resulting curve is shown in Fig. 7. Then the time history of power using the dynamic optimization method is calculated and depicted in Fig. 8.

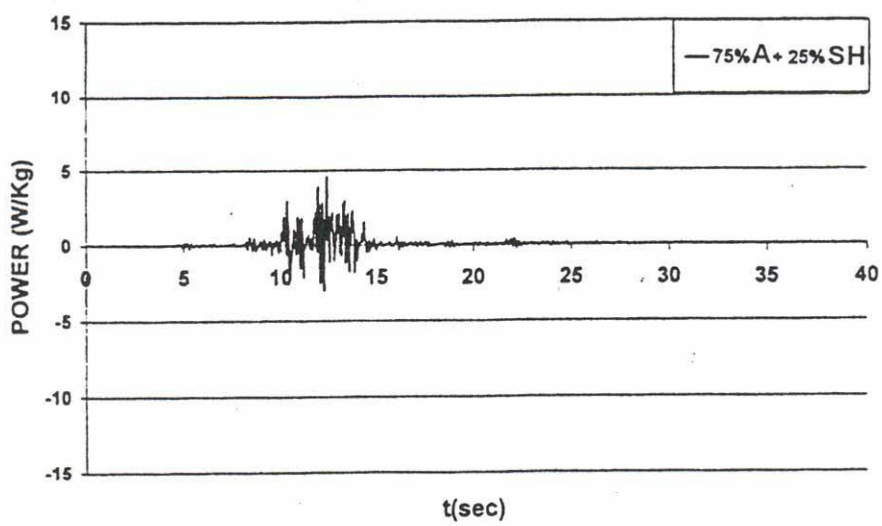

Fig. 7. Time history of the required power, record no. 2, 5 Hz model, dividing in the time domain, $\alpha = 0.75$.

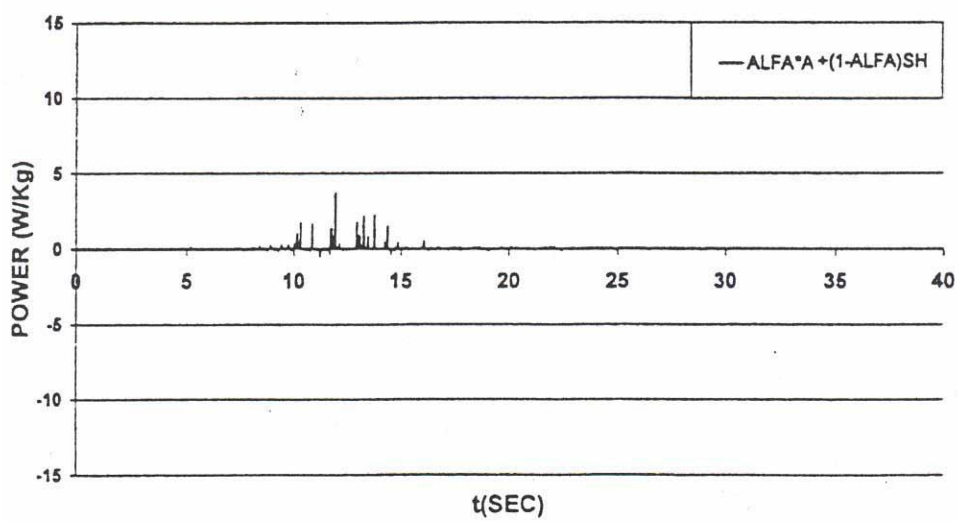

Fig. 8. Time history of the required power, record no. 2, 5 Hz model, dynamic optimization.

INTERPRETATION OF THE RESULTS

In Figs. 1 and 4 only one of the two apparatuses is in use. As is seen in Fig. 1, the shake table generally requires the least power for models with small natural frequencies. For higher frequencies up to about 7 Hz, the required power increases gradually and then decreases to the final point at 10 Hz. For the actuator,

Fig. 4 shows almost an inverse trend. The required power for testing only with the actuator is a minimum for models with high natural frequencies. When the natural frequency decreases, the needed power in this case increases. This is true down to 2 Hz, where the power begins to decrease slightly. Figures 2 & 3 illustrate the results for combinations of the shake table and the actuator. If a curve is to be drawn connecting points with the minimum power in these Figures, it would be seen that up to about 2 Hz $\alpha \leq 0.25$ is the best choice where between 2 and about 4 Hz $0.25 \leq \alpha \leq 0.5$ and from that point up $\alpha \succ 0.5$ are the values resulting in the minimum power. This fact is shown clearly in Fig. 5 where the spectrum of α is depicted. Here it is observed that α increases gradually from zero to unity when the natural frequency of the model varies between 1 and 10 Hz. Therefore, while the shake table and the actuator each best fit for low and high frequency models, respectively, a combination of both will be the best setup in between.

The most important result drawn from Fig. 6, the curve of power versus the cutoff frequency, is that for the case calculated the cutoff frequency giving the least required power is about 5 Hz, but it can well be selected in a wider range between 4 and 6 Hz without causing much difference in the result.

Comparing Figs. 7 & 8 that show the power demand when dividing in time domain with the actuator having a 75% share of the ground motion $(\alpha = 0.75)$, it can be resulted that with the dynamic optimization not only the instantaneous power demand is decreased considerably but also there is a drastic decrease in the total power consumption of the testing facilities. This shows that the dynamic optimization method is very powerful in optimizing a testing program.

CONCLUSIONS

A hybrid method for implementing earthquake engineering experiments was described in this paper. In this method, a combination of shake table and actuators is used to simulate the effect of ground motion on the structural model. Different methodologies for dividing the ground motion record between the two testing facilities were presented. The first method was called dividing in the time domain in which the amplitude of the record was split between the two apparatuses. After calculating for different cases, an spectrum was presented showing the best dividing factor for different natural frequencies of the model resulting in the least instantaneous power demand. The second method or dividing in the frequency domain was a method of decomposing the record into its low and high frequency components. Here the main challenge was finding the cutoff frequency for each record and model for which the required power is a minimum. The last and the most efficient model as per this research was the dynamic optimization method. In this method through a minimization procedure its derivation was presented, the value of the dividing factor resulting in the least power demand at each time step was calculated and input to the testing scheme. It was shown that the dynamic optimization procedure decreases both the maximum instantaneous power demand and the total power consumption of the test.

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