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DATA CONDENSATION FOR ANALYTICAL NONSTATIONARY PROBABILISTIC DYNAMIC RESPONSE ANALYSIS: AN APPLICATION USING THE 1999 CHI-CHI EARTHQUAKE GROUND MOTION RECORDINGS

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SUMMARY

A method recently developed by the authors for the condensation and compact analytical description of measured nonstationary random processes into a particular form for probabilistic nonstationary dynamic response analysis is presented through a large-scale application. The application data-set is the measured ground-motion excitations from the 1999 Chi-Chi earthquake (Taiwan). The data condensation technique involves Karhunen Loeve spectral decomposition and orthogonal polynomial analytical approximation. By dividing the country into geographical regions relative to the epicenter location, clearly defined data features emerge from the condensed version of the data subsets. "Probabilistic response spectra" are derived from these data, and the effect of structural design parameters are readily explored with respect to the expected response level. It is hoped that the underlying technique will provide a new set of tools for the development of design spectra based on an acceptable level of risk, that is, a true performance-based design standard.

INTRODUCTION

The ground motion recordings from almost 500 stations from the M7.6 Chi-Chi earthquake (21 September, 1999) in Taiwan comprises one of the best available dense data sets of a strong earthquake. Previously, authors Smyth and Masri [1-3] have developed tools for the condensation of large nonstationary excitation data ensembles in a particular format which allows highly efficient analytical probabilistic dynamic response analysis. The Chi-Chi data-set affords the opportunity to fully exercise the condensation tools to illustrate their potential in a data-rich environment.

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THE 1999 CHI-CHI EARTHQUAKE DATA-SET

The ground motion recording data used in this study was taken from the October 2001 special issue of the Bulletin of the Seismological Society of America (BSSA, 2001) and comprises the final release of the Taiwan Central Weather Bureau (CWB) data-set. A map of the station locations is shown in Figure 1.

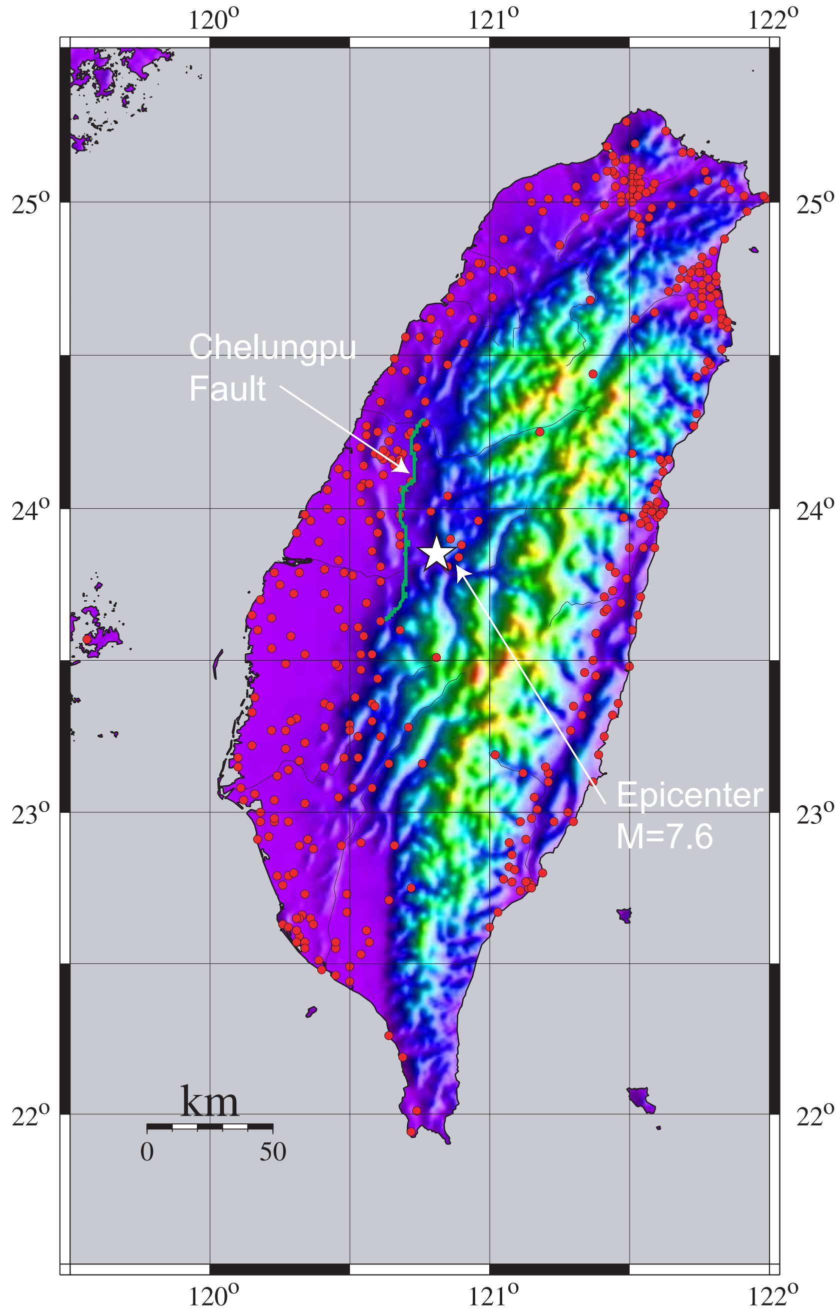

Figure 1 Locations of the Taiwan CWB seismic recording stations

In order to create an ensemble of records to represent a nonstationary process, a 'trigger' threshold level of 1% g was chosen in order to synchronize the records (i.e., not in absolute time, but rather by site ground motion commencement). In total, 397 stations were used to create the data ensemble. These represent the most up to date digital recorders in the CWB network.

EARTHQUAKE GROUND MOTION CONDENSATION

The condensation of the ensemble begins with the construction of the covariance matrix of the data ensemble. The covariance matrix $\mathbf{C}$ of ground acceleration $\ddot{s}(t)$ is defined by

$$\mathbf{C}_{\ddot{s}\ddot{s}}(t_1,t_2) = E[(\ddot{s}(t_1) - \mu_{\ddot{s}}(t_1))(\ddot{s}(t_2) - \mu_{\ddot{s}}(t_2))] \tag{1}$$

where $E[\ ]$ is the expectation operator and $\mu_{\ddot{s}}(t)$ is the mean value of the acceleration at time *t*. The covariance matrix derived from the 3 component directions at all 397 stations is shown in Figure 2. This covariance matrix can be condensed using the Karhunen Loeve spectral decomposition as follows:

$$\mathbf{C} = \sum_{i=1}^{k} \lambda_i \mathbf{p}_i \mathbf{p}_i^T + \mathbf{E}_k = \mathbf{C}_k + \mathbf{E}_k \tag{2}$$

where $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N \geq 0$ and $\mathbf{C}_k$ represents the truncated covariance matrix by using the largest eigenvalue contributions. $\mathbf{E}_k$ is the truncation error matrix.

The second stage of the condensation procedure is also a conversion from a databased representation to an analytical approximation of the data ensemble. This is accomplished by fitting the eigenvectors with a set of orthogonal polynomial basis functions. Chebyshev polynomials are used for their equal error/equal ripple properties for fitting finite duration functions. This fitting step is written as

$$\mathbf{p}_i(t_l) \approx \hat{\mathbf{p}}_i(t_l) = \sum_{j=0}^{m_i-1} H_{ij} T_j(t'_l) \tag{3}$$

where $t'_l = \dfrac{2t_l}{t_{\max}} - 1$, and m_i is the number of Chebyshev polynomials used in the series to fit $\hat{\mathbf{p}}_i$. The *T*'s are the Chebyshev polynomials and H_{ij} is the coefficient of the Chebyshev polynomial of order *j* associated with eigenvector $\mathbf{p}_i$.

Finally, reassembling the analytical approximations of the eigenvectors together in a truncated series as in (2) yields a compact analytical form of the original covariance matrix:

$$\hat{\mathbf{C}}_k(t_1,t_2) = \sum_{i=1}^{k} \lambda_i \sum_{j=0}^{m_i-1} \sum_{l=0}^{m_i-1} H_{ij} T_j(t'_1) T_l(t'_2) \tag{4}$$

where $0 \leq t_i \leq t_{\max}$; $i = 1, 2$. The level of data reduction is dramatic, for example, for $m_i = 400$ and taking $k = 100$, the data storage required is about 1/60$^{\text{th}}$ of the original 1191 x 2000 data points. Not only is the data reduction significant, but the format is in a special analytical format which has a closed-form response solution for general linear multi-degree-of-freedom (MDOF) systems [1-2]. Although a review of the details of the closed form solution is beyond the scope of this brief paper, it allows one to rapidly generate the complete second order probabilistic description of the transient response of MDOF linear dynamic systems. The response comparison of the estimated response versus the exact response from direct simulation of all of the excitations and subsequent response averaging for a 1Hz single-degree-of-freedom oscillator with 5% of critical damping is shown in Figure 3. The agreement is very good, and the visible discrepancy is only due to a premature truncation of the series in (4) to show that truncation will of course affect accuracy.

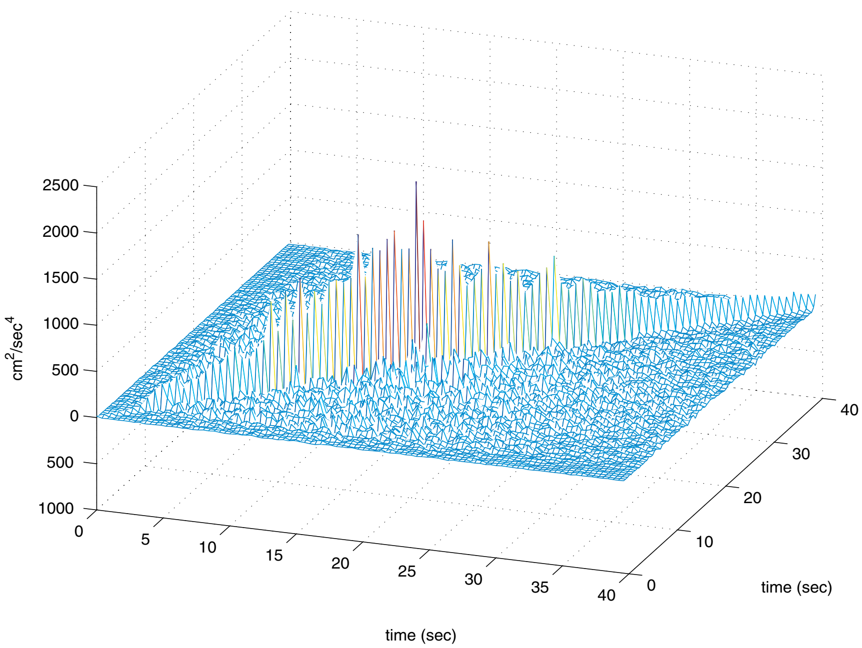

Figure 2 The covariance matrix of the ground motion acceleration records from the entire CWB data-set.

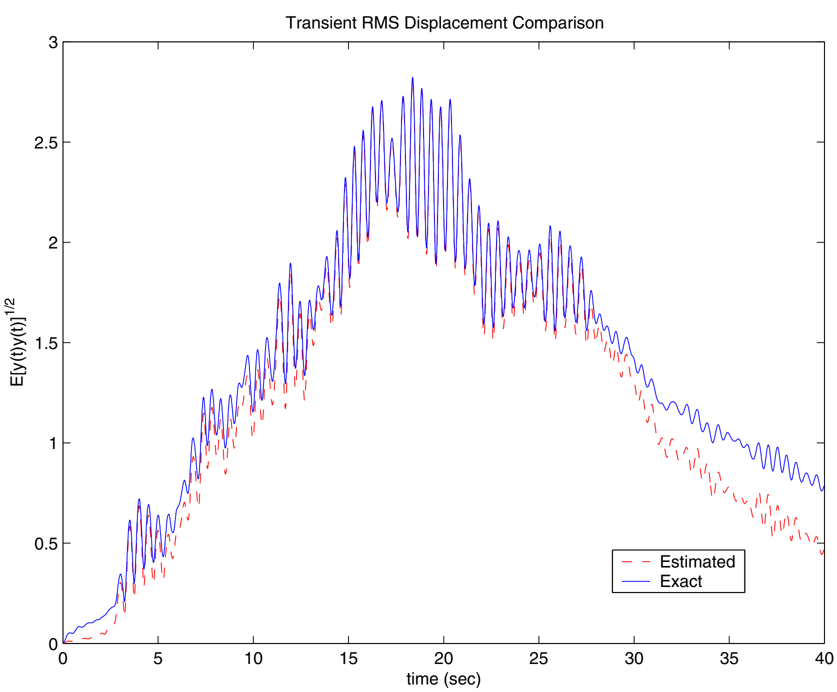

Figure 3 Response comparison of estimated versus ‘exact’ rms transient displacement of a SDOF oscillator. This is the square root of the diagonal of the response covariance matrix.

REGIONAL FEATURE EXTRACTION

One of the attractive properties of the spectral decomposition component of the data condensation procedure is that it extracts the dominant spectral contributions. For example, by grouping the data into regional subsets, one can observe averaged regional characteristics of the ground motions ensembles. Consider for example the data from a subset of stations just to the west of the Chelungpu fault, and contrast that with a subset selected from the Taipei basin which is further from the epicenter.

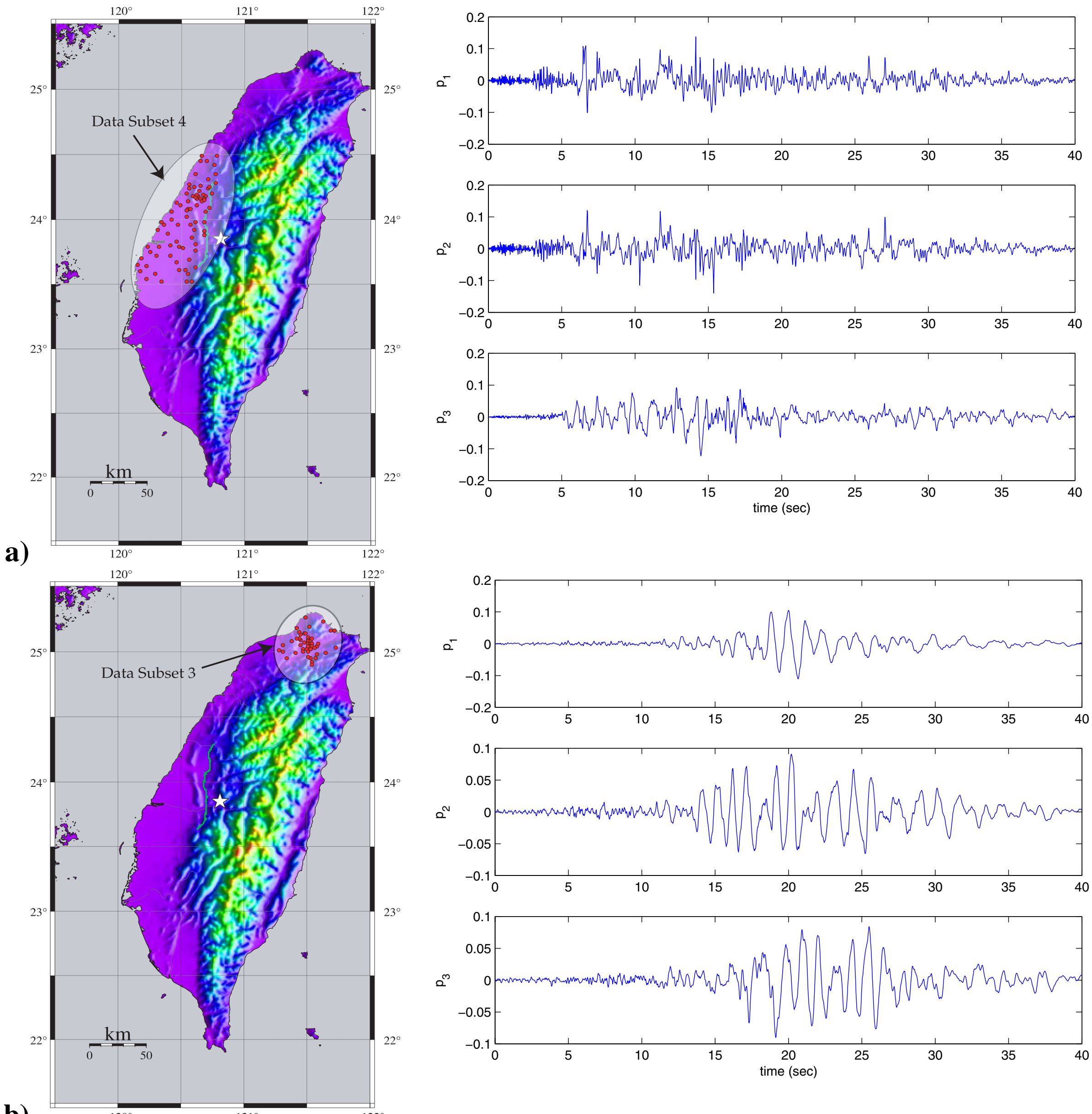

Figure 4 The dominant eigenvectors of the earthquake process covariance matrices from two different data subset from the Chi-Chi earthquake.
(a) Subset from just west of the Chelungpu fault, (b) Subset from the Taipei basin.

From inspection of Fig. 4 and a comparison of the dominant three eigenvectors from each of the regional subsets, it is immediately evident that the Taipei basin subset contains much more low-frequency ground motion input than the relatively near-fault Taichung area just west of the Chelungpu fault.

PROBABILISTIC RESPONSE SPECTRA

With the ability to efficiently generate second order statistics of the response using this analytical procedure one is able to readily generate what might be called probabilistic response spectra. The response of a linear single-degree-of-freedom (SDOF) oscillator with 5% critical damping is considered. Figure 5 illustrates the type of result that can be obtained; here the maximum root-mean-square of the transient displacement response is shown for various natural frequencies of the SDOF system. This type of result can then be compared with current code levels as will be done in a forthcoming study [5]. In addition, Fig. 5 shows the effect of polynomial fitting truncation error, as well as eigenvalue/eigenvector pair truncation in the series representation in Eq. (4).

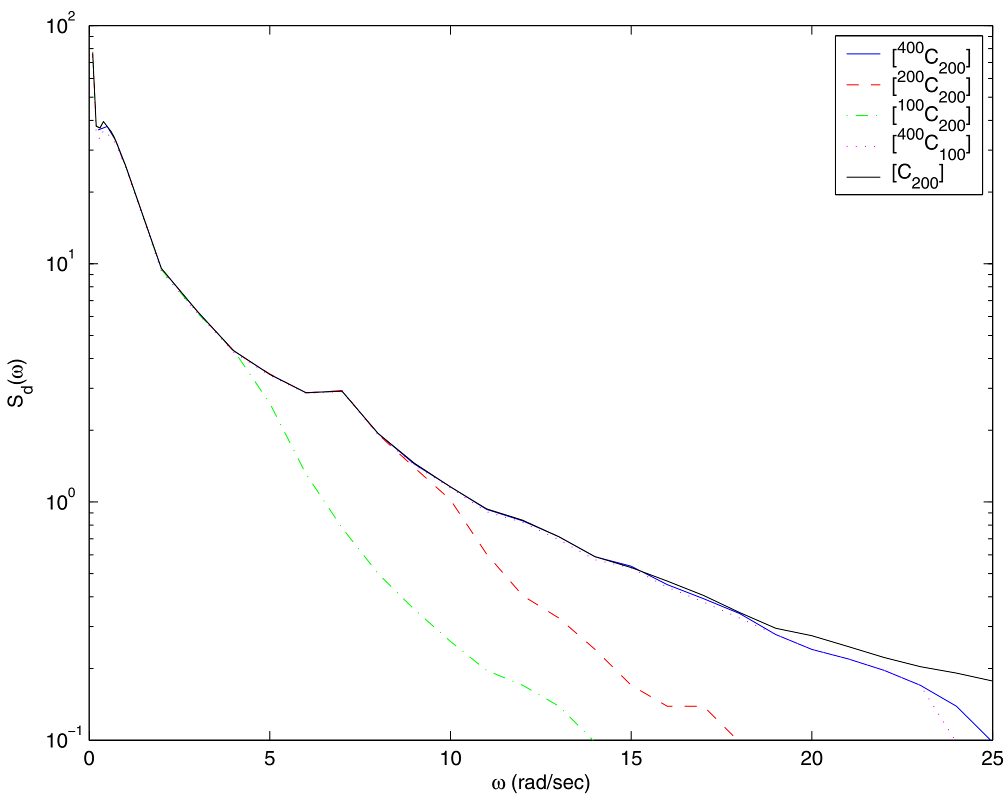

Figure 5 The maximum response rms value of displacement (in cm) versus the natural frequency of the SDOF oscillator. The solid black line is the “exact” simulation spectrum, and the other curves illustrate the effects of truncation error in either the number of eigenvector/eigenvalue pairs included or the number of Chebyshev polynomial terms included in the approximation.

CONCLUSIONS AND FUTURE WORK

An efficient method for the condensation, feature extraction and analytical response analysis of large sets of nonstationary excitation data has been demonstrated using the large 1999 Chi-Chi earthquake data set. Clear differences in the dominant averaged characteristics of the regional ground motion subsets were illustrated. Such a feature extraction tool provides a useful way to interpret large data sets from complex nonstationary phenomena. The ability to efficiently produce probabilistic response spectrum dependent on a variety of system parameters for MDOF systems allows one rapidly assess code levels. In future work these MDOF response spectra will be compared with actual damage levels. The authors Smyth and Masri [3] have also demonstrated through an extension of the technique the applicability to nonlinear system behaviors. Through a combination of these results, future work will include nonlinear response spectra.

ACKNOWLEDGMENTS

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