

VULNERABILITY OF AGED CONCRETE GRAVITY DAMS

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SUMMARY

The ageing of a concrete gravity dam presents a new challenge for the development of a methodology that can adequately predict the stability of the dam under seismic excitations and identify those structures where remedial measures are to be taken. A numerical algorithm is presented for the seismic analysis of a concrete dam in the vicinity of an infinite reservoir with the application of damage mechanics considering fluid-structure interaction. The time dependent degradation of concrete owing to environmental factors and mechanical loading in terms of isotropic damage index is considered. The results obtained from the analyses can indicate the vulnerability of the dam to seismic excitation. This procedure can be very effective for practicing engineers to assess the structural safety and to decide the importance of retrofitting or decommissioning the dam.

INTRODUCTION

In seismic prone area, massive water retaining structures like concrete gravity dams pose a hazard to the community at the time of earthquake. Concrete gravity dams are large and complicated structures, which are expected to withstand earthquakes without unacceptable damages. Most of the concrete dams are designed in seismic prone areas to resist the unpredictable earthquakes. Due to ageing, the dams are subjected to severe environmental conditions that affect the strength of the concrete. Therefore, it is required to analyze the dam with some initial damage in it. The behavior of huge concrete gravity dams is also affected on the presence of reservoir. Hence, safety assessment of dams is important for new as well as old dams considering the dam reservoir interaction in a coupled manner.

An ideal mathematical model of a concrete gravity dam should include the dam and the reservoir. The dam-reservoir interaction if incorporated in the analysis, would give more realistic structural responses. The dam-reservoir system has been extensively modeled using finite element technique (Zienkiewicz *et al.* [1] and Saini [2]) to obtain the two-dimensional response due to horizontal ground motion, considering the compressibility of water. The dam-reservoir interaction was accounted by the substructure method (Zienkiewicz *et al.* [1]), coupled equation (Saini [2]), boundary element method (Hanna and Humar [3]) and finite difference scheme (Hung and Wang [4]). The far boundary of the

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reservoir has been modeled using infinite elements (Saini [2]), radiation damping effect (Sharan [5]; Maity & Bhattacharyya [6]).

Though various analysis procedures have been developed to determine the effect of damage on the responses of the dam due to seismic excitation, primary mechanisms or factors that can produce premature deterioration of concrete dams needs to be identified. The damage index used by El-Aidi & Hall [7,8], Cervera *et.al.* [9] and Ghaemian & Ghobarah [10] in a dam-reservoir analysis is mainly evaluated based on relation between the strain energy of the damaged material and elastic energy of the undamaged material. The durability of concrete in dam is considerably affected by the accumulation of damage induced by the time variant external loading in conjunction with environmental loading processes such as moisture and heat transport, freeze-thaw action, dissolution processes such as calcium leaching, chemical expansive reactions (alkali-silica reaction) due to presence of silt deposited on the upstream face. Although considerable progress has been achieved in carrying out dynamic analysis by modeling concrete gravity dam considering the nonlinear behavior of concrete including cracks, environmental effects are still accounted for by more or less heuristic evaluations of the degradation process and its influence on the residual structural safety. Since the dam face is in constant touch with water, concrete degradation due to hygro-mechanical loading is inevitable and should be considered in the analysis procedure.

The present work is focused on a damaged model of an ageing concrete gravity dam. To emphasize the importance of dynamic analyses of an aged and damaged dam, comparisons are presented for responses of dams with and without damage, with and without fluid-structure interaction; subjected to horizontal component of Koyna earthquake, 1967. Two types of damage models: orthotropic and isotropic have been selected to compare the responses of the dam when subjected to seismic excitation. In the orthotropic model, the damage percentage along mutually perpendicular directions in the dam is not considered to be equal; whereas in an isotropic model it is considered to be equal. In this paper evolution of damage index within the elastic limit is derived considering environmental factors due to exposure to water, mechanical loading and chemical reaction. The dam-reservoir model is analyzed with orthotropic and isotropic damage in dam and considering the reservoir as compressible. The infinite reservoir-domain is truncated by boundary condition as proposed by Maity and Bhattacharyya [6]. The displacements at the top of the dam and the pressure coefficient at the bottom of the dam-reservoir interface are evaluated for a comparison.

MATHEMATICAL MODELING

Modeling of Infinite Reservoir

Assuming the water to be frictionless, the hydrodynamic pressure generated in the reservoir in excess of the hydrostatic pressure during motions of small amplitude is governed by the wave equation.

$$\nabla^2 p(x, y, t) - \frac{1}{c^2} \ddot{p}(x, y, t) = 0$$

where ∇^2 is the Laplacian operator in two dimensions, *C* is the velocity of sound in water and dots represent differentiation with respect to time. Eqn (1), together with the appropriate boundary conditions (Fig. 1), defines completely the hydrodynamic aspects of the problem.

(1)

(a) At the free surface (Γ_t) :

Considering the effects of surface waves of the fluid, the boundary condition of the free surface is taken as

$$\frac{1}{g}\ddot{p} + \frac{\partial p}{\partial y} = 0 \tag{2}$$



Fig.1 Geometry of dam-reservoir system

(b) At the dam-reservoir interface (Γ_s):

Considering the dam to vibrate with an acceleration of $ae^{i\omega t}$ in which, ω is the circular frequency of vibration; and $i = \sqrt{-1}$, the condition along the dam-reservoir interface may be specified as:

$$\frac{\partial p}{\partial n}(0, y, t) = -\rho_f a e^{i\omega t} \tag{3}$$

Where ρ_f is the mass density of water, *n* is the outwardly directed normal to the elemental surface along the interface, and *a* is the acceleration of the dam-reservoir interface in the direction of *n*.

(c) At the water-reservoir bed interface (Γ_r): Assuming reservoir floor to be rigid, the condition adopted is

$$\frac{\partial p}{\partial n}(x,0,t) = 0 \tag{4}$$

(d) At the truncation surface (Γ_t) :

In the finite element analysis, difficulties arise when the domain becomes infinite. The researchers have proposed different types of truncation boundary conditions for finite element analysis. The far boundary condition proposed by Maity and Bhattacharyya [6] seems to be very effective and straightforward for implementation in finite element algorithm. Thus, the boundary condition at the truncation surface will be:

$$\frac{\partial p}{\partial n} = \frac{\partial p}{\partial x} = -\frac{p}{H}\zeta$$
(5)

where, p is the hydrodynamic pressure of the water reservoir and ζ is given by:

$$\zeta = -\frac{\sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{2m-1} e^{\left(-f_m \frac{x}{H}\right)} \cos\left(\lambda_m \frac{y}{H}\right)}{\sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{(2m-1)f_m} e^{\left(-f_m \frac{x}{H}\right)} \cos\left(\lambda_m \frac{y}{H}\right)}$$
(6)

To get the effect of unbounded water domain in the truncation surface, ζ is determined numerically assuming *m* to be a large number.

FINITE ELEMENT IMPLEMENTATION

The water domain is discretized as an assemblage of finite elements, assuming pressure to be the nodal unknown. By the use of the Galerkin process, the discretized form of eqn. (1) is obtained as

(7)

(9)

 $[E]\{\ddot{p}\} + [A]\{\dot{p}\} + [G]\{p\} = -\rho_f[S]\{a\}$

where $\{p\}$ represents the vector of nodal pressures for the water domain. Expressions for the matrices [E], [A] [G] and [S] are as given below. The derivation in similar approach may be found in Zienkiewicz and Newton [11].

$$[E] = \frac{1}{C^2} \sum_{\Omega} \int_{\Omega} [N]^T [N] d\Omega + \frac{1}{g} \sum_{\Gamma_f} \int_{\Gamma_f} [N]^T [N] d\Gamma$$

$$[A] = \frac{1}{C} \sum_{\Gamma_r} \int_{\Gamma_r} [N]^T [N] d\Gamma$$

$$[G] = \sum_{\omega} \int_{\omega} \left(\frac{\partial}{\partial x} [N]^T \frac{\partial}{\partial x} [N] + \frac{\partial}{\partial y} [N]^T \frac{\partial}{\partial y} [N] \right) d\Omega + \frac{\zeta}{H} \sum_{\Gamma_r} \int_{\Gamma_r} [N]^T [N] d\Gamma$$

$$[S] = \sum_{\Gamma_r} \int_{\Gamma_r} [N]^T [T] [N_s] d\Gamma$$
(8)

Here, [T] is the matrix that transforms the generalized acceleration of a point on the fluid-structure interface to the acceleration in the direction normal to the interface. In the above equation, Γ and Ω represent the surface and volume of the reservoir respectively. The subscripts *f*, *s*, *r* and *t* represent the free surface, solid-fluid interface, bed-reservoir interface and truncation surface respectively. It is important to note that owing to the implementation of the proposed far-boundary condition, the form of the discretized equation remains unchanged and there is no extra computation required except for the modification of a few elements of the matrix [*G*]. For any prescribed acceleration at the fluid-structure interface, eqn. (7) may be used to solve the hydrodynamic pressure in the fluid domain.

Modeling of Aged Concrete Dam

The equation of motion for the dam-reservoir (Fig.1) can be given as

$$M\ddot{u} + C\dot{u} + Ku = -Ma_{\sigma} + F_{h}$$

where *M* is the mass matrix, *K* is the stiffness matrix of the structure, *u*, \dot{u} and \ddot{u} are displacement, velocity and acceleration vectors respectively, a_g is the vector of ground acceleration, F_h is the vector of nodal point forces associated with hydrodynamic pressures. The damping matrix *C* represents viscous damping in the structure. After the reservoir interaction force, F_h is determined, the dynamic response of the structure can be obtained from eqn. (9).

The structural system considered for the present investigation, has been analyzed using two dimensional plane strain formulations. Since the problem involved here is a long body, whose geometry and loading do not vary in the longitudinal direction, it can be analyzed by this idealization appropriately.

For the plane strain formulations, a constant longitudinal displacement corresponding to a rigid body translation and displacements linear in a direction perpendicular to the cross-section of the dam, corresponding to rigid body rotations do not result in strain. The constitutive relation for elastic isotropic material can be written as

$$\{\sigma\} = [C]\{\varepsilon\} \tag{10}$$

where,

$$[C] = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} (1-\mu) & \mu & 0\\ \mu & (1-\mu) & 0\\ 0 & 0 & (1-2\mu) \end{bmatrix}$$
(11)

Orthotropic Damage Model for Concrete

The concept of damage is based on the material degradation that is induced by distributed micro cracks which leads to a reduction of the net area capable of supporting stresses. The loss of rigidity of the material follows as a consequence of micro cracks in defining a fictitious undamaged material related to the damaged one. The expressions for isotropic and orthotropic damage model are in reference Ghrib and Tinawi [12]. The orthotropic damage index is given by

$$d_i = \frac{\Omega_i - \Omega_i^d}{\Omega_i} = \frac{\Omega_i^n}{\Omega_i}$$
(12)

where Ω_i = tributary area of the surface in direction *i*; and Ω_i^d = lost area resulting from damage. The index *i*(1,2,3) corresponds with the Cartesian axes *x*,*y*,*z*. In this case the ratio of the net area over the geometrical area may be different for each direction. The effective plane strain material matrix can be expressed as

(13)

$$\begin{bmatrix} C_d \end{bmatrix} = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix} (1-\mu)(1-d_1)^2 & \mu(1-d_1)(1-d_2) & 0\\ \mu(1-d_1)(1-d_2) & (1-\mu)(1-d_2)^2 & 0\\ 0 & 0 & (1-2\mu)(1-d_1)^2(1-d_2)^2 / ((1-d_1)^2 + (1-d_2)^2) \end{bmatrix}$$

If $d_1 = d_2 = d$, the damaged isotropic model is expressed as

$$[C_d] = (1-d)^2 [C]$$
(14)

Where *C* is the constitutive matrix of the undamaged model.

Evaluation of Damage Index

The relation between degraded elastic modulus, E and original modulus, E_0 can be given as

 $E = (1-d)E_0$ (15) To study the effect of time on the degradation process, an analogy given by Atkin [13] is adopted. Ageing process may be described by a normalized process extent with $\gamma = 0$ for a freshly laid concrete and $\gamma = 1$ for its completely aged state. Thus the kinetic law for the ageing process may be stated as

$$\gamma = \frac{1}{\tau_a} (1 - \gamma) \tag{16}$$

where τ_a is the characteristic time of the ageing process, which can be assumed to be the design life of the structure. Integrating eqn.(16),

$$1 - \gamma = \exp(-\frac{t}{\tau_a}) \tag{17}$$

Replacing γ with damage index d in eqn.(18), variation of damage index with time can be given as

$$1 - d = \exp(-\frac{t}{\tau_a}) \tag{18}$$

At a constant ambient humidity, the reaction extent, ξ for the corresponding characteristic time, τ_r as given by Steffens *et al.* [14] is

$$\xi = 1 - \exp\left(-\frac{t}{\tau_r}\right)$$
(19)

Substituting *t* from eqn. (19) in eqn.(18),

$$1 - d = \left(1 - \xi\right)^{\frac{\tau}{\tau_a}} \tag{20}$$

Hence, the variation of degradation or the damage index, d with respect to time can be determined from eqn.(20). The value of τ_a is the characteristic age for which the structure is designed and t is the time corresponding to which the damage index is determined. The gain in compressive strength, f of concrete as obtained from experimental evidences (Washa *et al.* [15]) is predicted by

$$f = 519.07 \ln(t) + 6429.992$$

The value of compressive strength obtained is in psi and t is time in years. The value of static elastic modulus (psi) is obtained from

(21)

$$E_0 = 57000 \sqrt{f}$$
 (22)

SOLUTION SCHEME FOR COUPLING DAM AND RESERVOIR

In fluid-structure interaction problem the fluid and the structure do not vibrate as separate systems under external excitations, but they act together in a coupled way. Therefore, these problems have to be dealt in a coupled way. The discrete structural equation with damping in the presence of fluid may be written as:

$$M\ddot{u} + C\dot{u} + Ku - Qp + f = 0 \tag{23}$$

Where the coupling term arises due to pressure specified on the fluid-structure interface boundary and is

$$\int_{\Gamma_s} N_u^T np d\Gamma = \left(\int_{\Gamma_s} N_u^T n N_p d\Gamma \right) p = Qp$$
(24)

n is the direction vector of the normal to the interface. The discretised fluid equation may be written as:

$$E\ddot{p} + A\dot{p} + Gp + Q^T\ddot{u} + q = 0 \tag{25}$$

The system of eqn.(23) and (25) are coupled second-order ordinary differential equations, which define the coupled fluid-structure system completely. These coupled equations can be written in matrix form as:

$$\begin{bmatrix} M & 0 \\ Q^T & S \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{p} \end{bmatrix} + \begin{bmatrix} C & 0 \\ 0 & G \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{p} \end{bmatrix} + \begin{bmatrix} K & -Q \\ 0 & H \end{bmatrix} \begin{bmatrix} u \\ p \end{bmatrix} = - \begin{bmatrix} f \\ q \end{bmatrix}$$
(26)

The coupled response of fluid-structure system can be obtained under external excitation by solving eqn.(26).

NUMERICAL RESULTS

The Koyna dam has been chosen for the extensive analysis using finite element technique. The dimension and the material properties of the dam in the present case are: height of the dam $(H_s)=103$ m; width at the top of the dam is 14.8 m and at the base is 70.0 m, modulus of elasticity $(E)=3.15 \times 10^7$ kN/m²; Poisson's ratio = 0.235 and mass density = 2415.816kg/m³. Structural damping is considered as 5%. The value of *E* is increased with time as obtained by eqn. (22). The increase in damage index with time degrades the concrete strength. The dam is discretized with 8-noded quadratic elements as shown in Fig. 2 and is

analyzed using plain strain formulation. The fluid domain is discretized by two dimensional 4-noded isoparametric quadratic elements. The acoustic wave velocity of water, C is considered as 1438.7 m/sec. Full reservoir condition is considered in the analysis. The responses of the dam are compared with and without initial damage in it and with and without dam-reservoir interaction. The displacement at the top (point A) of the dam and the pressure coefficient 'c' ($c = p/\rho_f aH_f$) at the bottom (point B) of the dam-

reservoir interface is evaluated.

Analysis of the Dam without Reservoir

The crest displacement of the dam due to a static horizontal load of 1000 kN applied at the top of the dam and damage index corresponding to for design life of 10 years, 50 years and 100 years is plotted in Fig.3 for comparison. As expected, at design life of 10 years, there is a sudden increase in displacements after reaching its design lifetime. The effect of isotropic damage on the response of the dam due to horizontal component of Koyna earthquake (Fig. 4) is examined in the absence of reservoir. The analysis is carried



Fig. 2 Finite element mesh of dam-reservoir system

out without any initial damage, after completion of construction, considering one year old concrete strength. In the second case, after 25 years an isotropic damage of 35.4% is estimated for a design life of 50 years. In the third case, after 25 years an isotropic damage of 19.1 % is estimated for a design life of 100 years. The results plotted in Fig. 5 show the effects of damage on displacements of the dam due to seismic excitation. For a lower design life, the percentage of damage and the displacements are comparatively higher. However, it is seen that the peak responses are reached at a later time for a higher design life.

It is observed from Fig.6, that with an increase in damage, the natural frequency of the structure reduces. This behavior is mainly attributed to the reduced stiffness of the structure with increased damage. Increase in damage will thereby increase the time period of the structural system and correspondingly the deflection and stress of the dam will be more. Moreover, the decrement in frequency is high in case of isotropic damage model as compared to orthotropic one, as it affects the stiffness of the dam section in both horizontal and vertical direction. The decrease in the frequency for the model having damage variation along vertical direction is steeper than that of along horizontal direction. Since the cross sectional area along the dam height is more than that along its width, damage along the dam height has prominent and rapid decline on the dam's natural frequency.



Fig. 3 Static horizontal crest displacement due to horizontal load



Fig. 4 Horizontal accelerogram of Kovna earthquake



Fig. 5 Effect of damage on horizontal crest displacement due to seismic



Fig. 6 Variation in frequency of dam with initial damage

The variations of static displacements with respect to varying damages in the two mutually perpendicular directions are studied extensively. The displacement of the structure increases with the increase in damage percentage no matter whether it is an orthotropic or an isotropic damage model. An isotropic damage model exhibits higher displacements as compared to that of orthotropic (Fig. 7), owing to the fact that overall stiffness in an isotropic damage model is reduced along both the mutually perpendicular axes. But in case of orthotropic damage model, stiffness in perpendicular direction is affected more. It is observed from Fig.7 that if the loading and the damage are along the same axis, it reduces the stiffness and influences the displacements in the mutually perpendicular direction. Horizontal displacements are higher for damage along vertical axis. The effect of damage on horizontal and vertical displacements when a horizontal load of 1000 kN is applied at point A of the structure is compared in Fig. 8. In the first case, damage in horizontal direction is considered. The horizontal displacement is found to be higher than the vertical displacement. In the second case, damage in vertical direction is considered and the horizontal displacements are found to be higher than that in the first case. The damage alignment being vertical and horizontal loading incident on the dam section, naturally cause a high horizontal displacement. Damage along the dam height has more pronounced effect on the both the vertical and horizontal displacements. Similar behavior is expected due to vertical application of loading at point A (Fig. 2). An increase in the percentage damage orthotropically results in the increase in displacements in horizontal and vertical direction (Fig. 9). But unlike the previous observation, vertical displacements are higher for damage along the same axis than that along perpendicular axis for vertical loading. Moreover, the horizontal displacements are found to be still higher than the vertical displacements. Hence, it can be concluded that the damage along the dam height will influence the displacements more than damage along the width of the dam. This is because the lost area due to damage along its width is much less than the lost area due to damage along its height.



Fig. 7 Variation of maximum horizontal displacements with varying damage index under horizontal loading



Fig. 8 Effect of damage with horizontal loading on horizontal and vertical displacement



Fig. 9 Effects of damage with vertical loading on horizontal and vertical displacements

Analysis of the Dam-Reservoir Coupled System

The crest displacements of the Koyna dam under seismic excitation considering the effect of damreservoir interaction is plotted in Fig.10. The effect of dam-reservoir interaction is to increase the crest displacements. The ageing effect of the dam under seismic excitation considering dam-reservoir interaction is also studied. The crest displacements of the dam at 1, 25 and 75 years are presented for isotropic damage under horizontal component of Koyna earthquake in Fig.11. The percentage of damage considering with a design life of 50 years at 1, 25 and 75 years are 1.8, 34.5 and 71.4 respectively. It is observed from the graphical results in Fig 11 that the coupled dam-reservoir system exhibits an increased displacement trend for higher percentage of damage. While comparing the maximum pressure coefficient at point B of the dam due to seismic excitation as seen in Fig. 12, it is observed that there is a decrease in pressure coefficient with an increase in damage percentage. This is because of the reduced stiffness of the dam structure, which causes less acceleration in the adjacent reservoir.

Fig. 13 shows the principal stress at the neck of the dam (point C) without any damage considering reservoir effect. It is observed from the results that the principal stress increases while the interaction effect between dam and reservoir is not neglected. The development of principal stresses through different ages has been plotted in Fig. 14. The stresses are obtained due to horizontal component of Koyna earthquake. The damage percentage considered in the first, tenth and twenty-fifth years are 0.0%, 15.8% and 34.5% respectively. It is noticed that the stresses are more when there is no damage in the dam. This is because of higher stiffness of the dam at initial stage for which energy is not dissipated through damages. The variation in maximum principal stresses are obtained due to seismic excitation and plotted in Fig 15 to observe the variation of stresses for design life of 50 years and 100 years. Thus, if the design life of the dam and the environmental factors are known, the structural response at a certain age can be determined to evaluate its safety.



Fig.10 Effect of dam-reservoir interaction on crest displacements due to seismic excitation (without damage)



Fig.11 Ageing effect on crest displacements of dam with full reservoir due to seismic excitation



Fig.12 Effect of ageing on pressure coefficient at point B











Fig. 15 Effect of design life on principal stress at point C

CONCLUSIONS

A dynamic analysis procedure is presented to predict the ageing effect on the behavior of concrete gravity dams, which may be beneficial to assess the safety of dams during its lifetime. A time dependent isotropic damage index is determined for aged dams. The performance of the dam with increasing age is presented. The results reveal that the degradation process of the structure is dependent on the design life of the structure. For a higher design life, the degradation process will be slower. It is observed that the influence of damage along the dam height is more than the damage along its width.

The behavior of dam during seismic excitation is influenced by the coupled effect of the damreservoir system and the initial damage. The magnitude of the crest displacement of dam increases when the reservoir effect is considered. It is important to note that with the increasing age, the displacement of the dam increases but the hydrodynamic pressure exerted by the reservoir on the dam and the stresses decrease significantly.

The algorithm presented in this paper can be used to forecast the behavior of the existing dam, so that necessary action can be taken either for retrofitting or decommissioning the dam. More accurate behavior may be obtained if coupled effect of the foundation-dam-reservoir interaction with material non-linearity is considered.

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