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PROBABILISTIC ANALYSIS SOFTWARE FOR STRUCTURAL SEISMIC RESPONSE

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SUMMARY

The evaluation of structural reliability under earthquake loading is a particularly complex and computationally demanding process with only limited, often ill-suited software available to engineers and researchers. To address this deficiency, a new software application called *PSResponse* was developed using a Monte Carlo approach to determine the probabilistic distribution of selected dynamic response quantities of single-degree-of-freedom and simple multiple-degree-of-freedom stochastic structures subjected to a large ensemble of generated earthquake motions. Knowing the probabilistic response of a structure an accurate assessment of a specific reliability measure can be made from the probability of exceeding a chosen threshold. A brief description of the components and user-interface of *PSResponse* is presented along with an example application that examines the importance of the hysteretic assumption in inelastic dynamic reliability analysis.

INTRODUCTION

The evaluation of seismic reliability of building structures requires, at the most fundamental level, the evaluation of the probabilistic dynamic response of a given structure to the stochastic dynamic action of an earthquake. Because of the difficulty of determining the dynamic response of a structure in a statistical sense past estimates of the seismic reliability of existing structures, and typical structural systems, have been largely qualitative or anecdotal in nature. With the movement of many national building codes towards more performance-based design measures, a need was identified for a more quantitative method of evaluating structural reliability under seismic loading. To address this need, a new software application called *PSResponse* was developed following a comprehensive review of the available random vibration methods and numerical procedures that have been developed for analyzing the probabilistic response of non-linear systems. This review (Sjoberg 2003) assessed the limitations of each method with regard to the level of complexity in the system model, the nature of the variability in stochastic system properties, the degree of non-linearity in system restoring forces and the nature of the random excitation process. To provide an accurate, robust and practical means of evaluating the reliability of civil engineering structures

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subjected to seismic loading, the chosen procedure had to allow for highly non-linear, hysteretic response behaviour, non-Gaussian stochastic structural models with multiple degrees of freedom and realistic earthquake motions, rather than mathematically tractable idealizations. These requirements tended to eliminate all the frequency-domain based analytical random vibration methods; therefore, the decision was made to forego the efficiency of an analytical method in favour of developing a software application using a robust numerical time-history approach incorporating the Monte Carlo method, which makes no assumptions regarding the aforementioned requirements. This approach, while more computationally demanding than the analytical procedures, provides the most accurate possible evaluation of the probabilistic response of linear and non-linear systems under stochastic dynamic loading, including the ability to evaluate first-passage probabilities and the temporal evolution of a response distribution.

NUMERICAL MODELS AND SOLUTION METHODS

Having chosen to use a Monte Carlo time-history approach in *PSResponse* to determine the probabilistic seismic response of a structure, several types of component models needed to be incorporated into the overall architecture of the software, which was developed in C++. Models for generating and modulating artificial ground motion time-histories, structural models along with a means of simulating a non-linear, hysteretic restoring force, as well as an overall numerical time-stepping method to solve the differential equation of motion were linked together to form the computational foundation of the software. In addition, the overall framework of *PSResponse* required algorithms for Fourier analysis and power spectrum estimation (Press et al. 1999), a frequency filtering algorithm to ensure that input ground accelerations were truly representative of a real earthquake, a long period random number generator to ensure a reliable source of random numbers essential for Monte Carlo analysis, and algorithms for solving the eigensystem representing the natural frequencies and mode shapes of a multiple-degree-of-freedom structure. In total, the computational engine of *PSResponse* consists of approximately 79 algorithms linked together in an object-oriented framework. A brief summary of the major elements within that framework is given in the following sections.

Earthquake Generation

Generation of artificial acceleration records in *PSResponse* uses simple, well-known ground motion models that reproduce the probabilistic characteristics of a specified frequency spectrum. These methods are; spectral representation with random phases or random phases and frequencies, the filtered Poisson process and the inverse FFT method. The input frequency spectrum is specified as either a filtered white-noise that is shaped by user-defined Kanai-Tajimi and Clough-Penzien filter parameters or as the power spectrum of an input earthquake time-history. In both cases, temporal amplitude modulation of the generated ground acceleration records is based on a deterministic envelope function that is used for each record generated. For filtered white-noise records, the envelope function is based on user-defined parameters specifying a second–order increasing function followed by a period of constant maximum acceleration and then an exponentially decreasing function. For a real earthquake input, the deterministic envelope function is automatically determined from the input time-history.

Time-Stepping Method

Since analytical solution of the equation of motion is not possible for arbitrarily varying excitations and non-linear systems, a numerical time–stepping method is required to integrate the differential equation, or system of equations, governing the response of the system. For response analysis of multiple-degree-of-freedom systems, unconditionally stable procedures are generally necessary to avoid the excessive computational demands of conditionally stable procedures, which require an extremely short time-step to remain within the stability limit of higher modes of response. With this in mind, the Newmark method was selected for response analysis since it is unconditionally stable when using the average acceleration interpolation of response. However, due to its superior accuracy (Chopra 1995), Newmark's linear acceleration method is the default time-stepping procedure with provision made to automatically switch to the average acceleration method when required for stability reasons.

Integration of system response in any time-stepping procedure can give rise to significant error when transitions in the force-deformation relationship associated with response velocity sign changes are not followed closely. If the point at which response velocity went to zero during a time-step is not identified in the numerical procedure then the step-by-step path of the force-deformation relationship will "overshoot" the true path and the calculated displacement at the end of the time-step will be either too large or too small. To minimize the overshoot problem, the Newmark time-stepping algorithm developed in *PSResponse* uses a variable time-step when velocity sign changes are detected. The time-step is repeatedly bisected until the absolute value of the response velocity at the end of the reduced time-step is less than a preset fraction of the peak response velocity.

Structural Model

The type of structures that *PSResponse* is intended to model are those that may be represented as typical lumped mass idealizations consisting of shear walls or frames as the lateral load resisting elements. These idealized structures are further simplified by static condensation to lateral-degree-of-freedom only models prior to response analysis to facilitate rapid calculation of the response time-history. Also, calculation of the displacement time-history is based solely on integration of the governing differential equation of motion, consequently, the second-order (P-Δ) effect on lateral displacement produced by the vertical load acting on the structure in its displaced configuration is not considered in the displacement calculation.

The distribution of damping in the structural model is determined by whether the system is linear or non-linear. Classical damping is assumed for linear systems to allow for modal analysis while for non-linear systems, damping may be specified as Rayleigh or as a multiple of a single baseline damping value for each storey.

Randomization of the structural model, including damping, is accomplished by selecting one of six probability distributions for each of the properties that describe the idealized structure. For example, floor mass, storey height, bay length, column stiffness etc. may each be assigned individual distributions, which are then used to generate a random realization of the structure for each random earthquake in the Monte Carlo analysis.

Hysteresis Model

Modeling of the hysteretic restoring force in non-linear structures is done using the well-known Bouc-Wen hysteresis model (Bouc 1967, Wen 1976, Baber et al. 1981 & 1985, Foliente 1995), which was modified to reduce the number of pinching parameters that need to be identified. The Bouc-Wen model is used because it is able to produce a wide variety of hysteresis shapes, including the pinching and degradation behaviour exhibited by many hysteretic systems, without the use of piece-wise linear equations governed by numerous empirical rules relating stiffness to displacement. It also allowed for the development of an algorithm within *PSResponse* that identifies the parameters governing hysteretic behaviour from experimental cyclic test or shake-table data provided by the user. Thirteen separate parameters must be identified in the original Bouc-Wen model, which is a computationally demanding process since system identification problems rapidly increase in difficulty as the number of parameters increases. To simplify the process, the Bouc-Wen model was modified to reduce the number of parameters controlling pinching behaviour from six to three using the assumption that overall structural hysteretic pinching begins at or very near zero restoring force in each loading cycle. This assumption, in effect, reduces the pinching function from a five parameter model to a three parameter model in its modified form. A discussion of this modification and examples comparing the original and modified pinching function is given in the doctoral dissertation of the first author (Sjoberg 2003).

Incorporation of the modified Bouc-Wen hysteresis model into the Newmark time-stepping procedure required that a separate numerical solution algorithm be linked with the Newton-Raphson iteration scheme of the Newmark method since the Bouc-Wen equation describing a degrading or pinched hysteresis loop is a first-order, non-linear ordinary differential equation with no exact solution. Therefore, to solve for the hysteretic restoring force within each Newton-Raphson iteration, an Adams predictor-corrector method incorporating a Runge-Kutta algorithm (Burden & Faires 1985) and variable step-size was linked to the Newmark procedure.

SOFTWARE USER-INTERFACE

Following development of the solution algorithms and numerical components that form the computational engine of *PSResponse*, a Windows user–interface was developed to provide easy access to the software and to ensure the integrity of the input data prior to analysis. The user-interface is based on a *wizard manager* architecture that guides the user through a sequence of input and output dialog boxes that depends on the type of analysis selected. The wizard manager algorithm, which acts as the link between the dialog boxes and the computational framework, determines which of the 26 dialog boxes are required, passes information between dialog boxes, passes input data to the computational algorithms and stores both input data and output arrays. Figures 1 & 2 illustrate a few of the input and output dialog boxes.

EXAMPLE APPLICATION

As an example application of the *beta* release of *PSResponse*, the effect of hysteresis properties on the probabilistic peak displacement response of a single-degree-of-freedom structure is examined. To quantitatively assess the influence of an assumed hysteresis model, the reliability index associated with structural drift limits of: 0.5%, 1%, 2% and 4% was evaluated for a sequence of generated earthquakes using four different hysteretic behaviors and four natural periods with 2% damping.

The single-degree-of-freedom structures were simulated with the same initial stiffness, which was derived from experimental data taken from a cyclic lateral displacement-controlled test of a Parallam® column. The four hysteresis models were derived from: (a) the best-fit Bouc-Wen hysteresis model corresponding to the experimental data, (b) the best-fit hysteresis model with strength, stiffness and pinching degradation parameters quadrupled, (c) the best-fit hysteresis model with no degradation and 100% of experimental yield strength, and (d) the best-fit hysteresis model with no degradation and 50% yield strength. Figure 3 (left) shows the best-fit Bouc-Wen hysteresis model corresponding to the experimental data and Figure 3 (right) shows the first five seconds of SDOF displacement response to the El Centro ground motion for T = 8.0 seconds and 2% damping for each of the four hysteresis models.

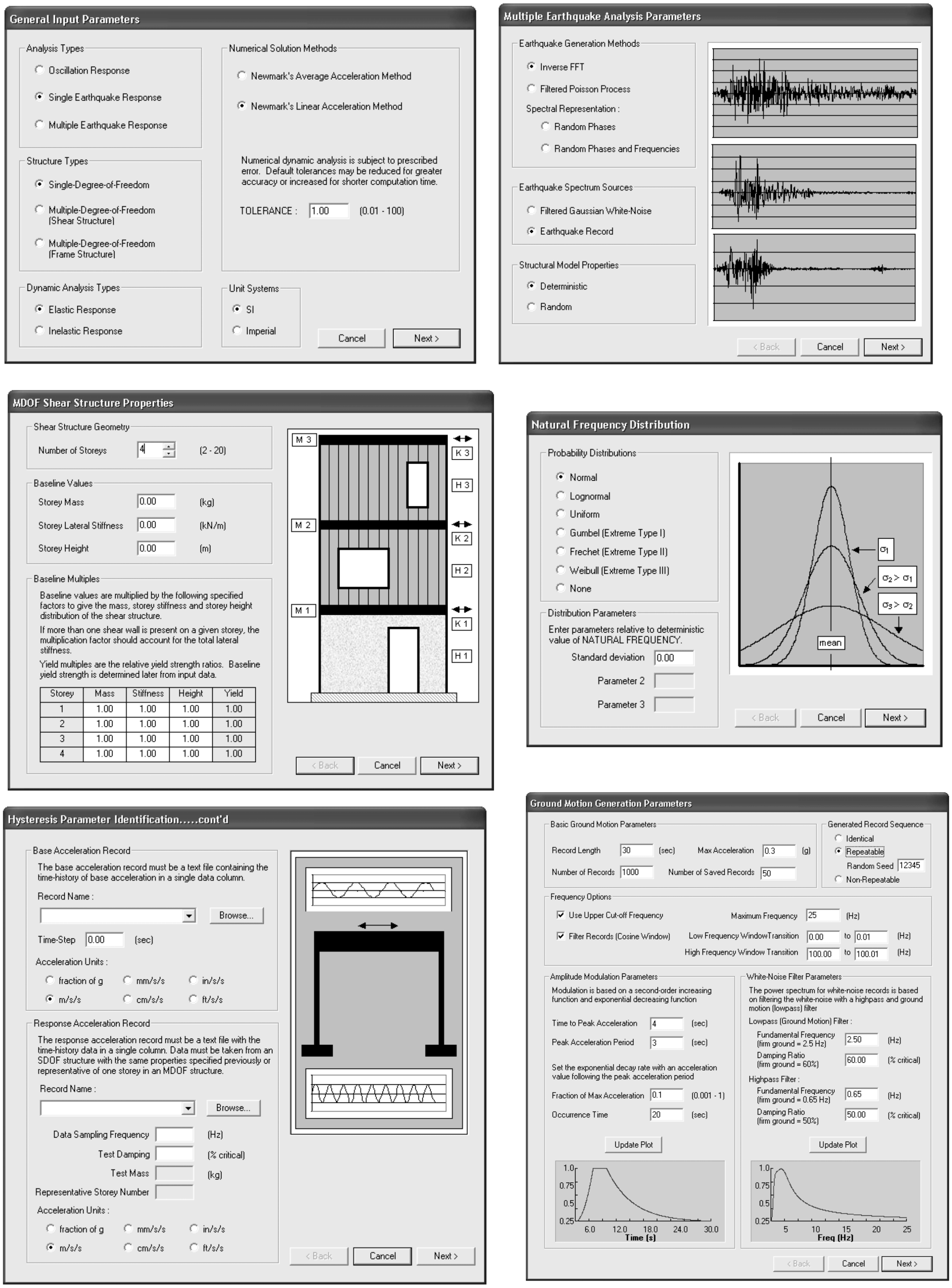

Figure 1: Sample input dialog boxes in *PSResponse*

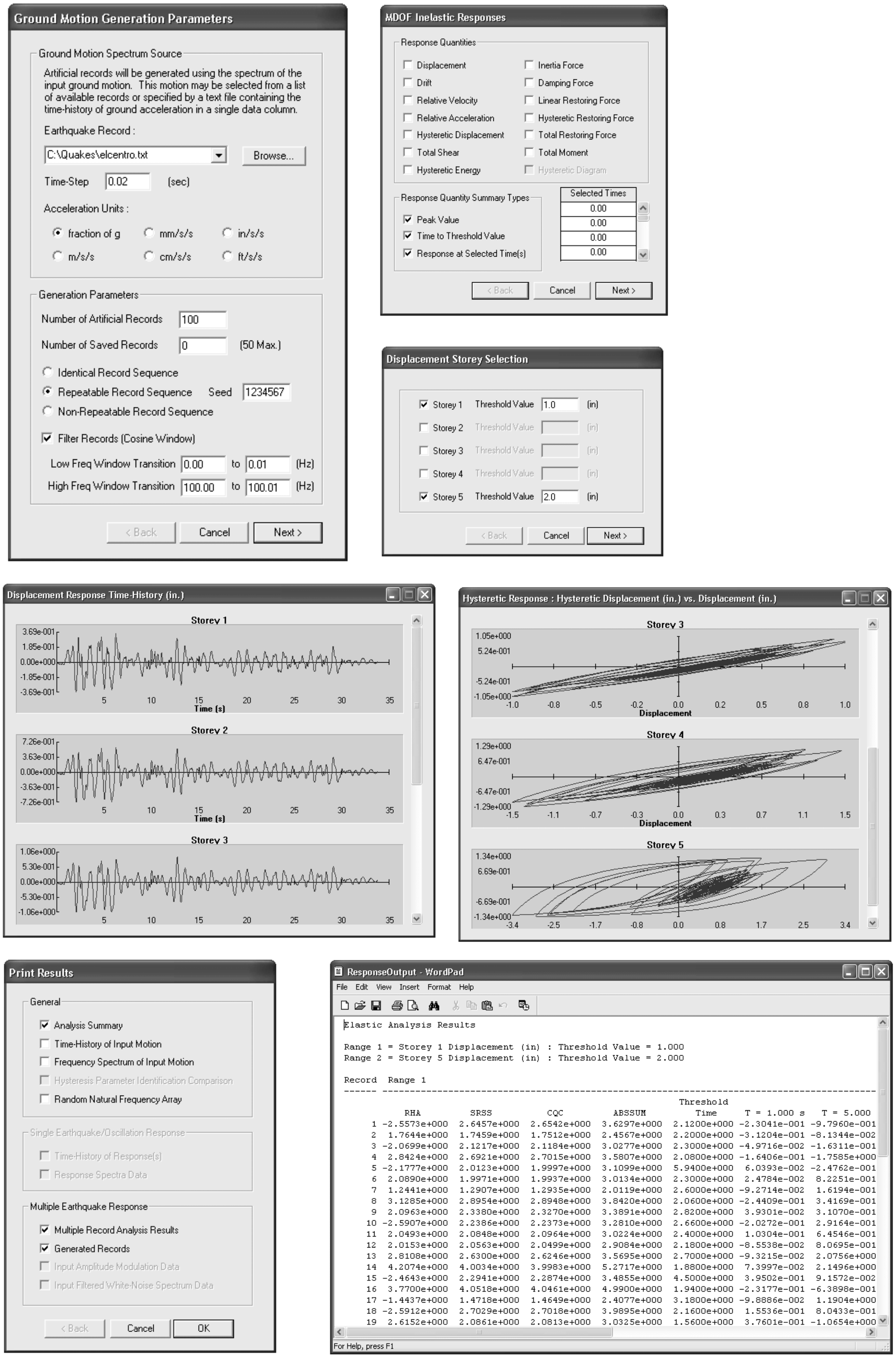

Figure 2: More sample input and output dialog boxes in *PSResponse*

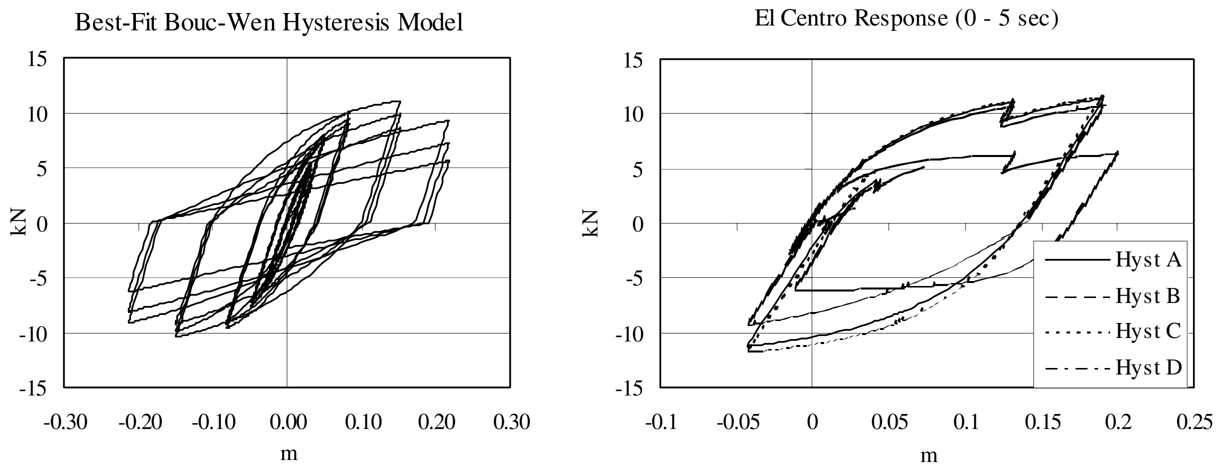

Figure 3: Assumed hysteretic behaviour

Having established hysteresis models to simulate a range of inelastic behaviour, the distribution of peak displacement responses of an SDOF structure with a natural period of: 0.2, 0.5, 1.0, and 3.0 seconds was simulated for each hysteresis model using the same sequence of 1000 earthquake records, which were generated using the El Centro record as the seed. Figure 4 (left) shows a typical histogram plot, overlaid with a fitted Gumbel distribution, for hysteresis model A and T = 0.5 seconds, and Figure 4 (right) shows the fitted Gumbel distributions of all four hysteresis models at T = 0.5 seconds.

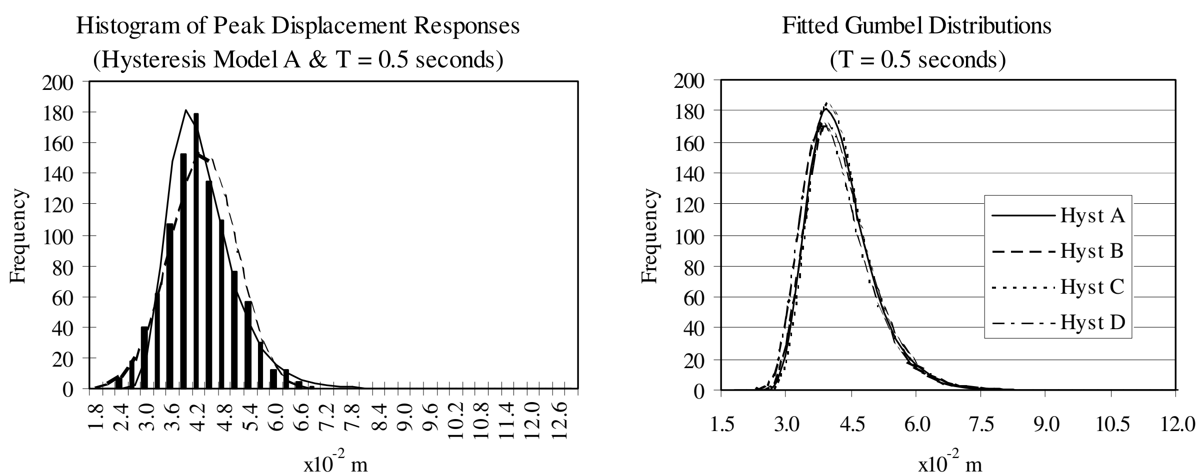

Figure 4: Peak displacement distributions

Using the fitted distributions, the reliability index β associated with a structural drift limit of: 0.5%, 1%, 2% and 4% was calculated for each natural period and hysteresis model combination from the corresponding peak displacement limit exceedence probability. The peak displacement limits corresponding to the chosen drift limits were determined from a structural height that was calculated by assuming the experimental data was taken from a rigid beam portal frame structure with 0.2 m x 0.2 m timber columns. The resulting frame height of 5.34 m gives displacement limits of: 0.027 m, 0.053 m, 0.107 m and 0.214 m, respectively, for the chosen drift limits. The performance function for this reliability analysis may then be simply expressed as:

$$G = X - x \tag{1}$$

where *X* is the displacement limit and *x* is the peak displacement random variable. With that performance function, the reliability index for each drift limit for each hysteresis model at each natural period was calculated using the cumulative distribution function of the fitted Gumbel distribution of peak displacement. Table 1 lists the calculated reliability indices.

Examining the reliability indices of Table 1 reveals two key trends; the β values for the hysteresis models vary significantly at a given natural period and drift limit, and the range of variability in the β values for the hysteresis models is dependent on the natural period and chosen drift limit. For example, the reliability index for T = 1.0 seconds and 1% drift ranged between -1.3621 and -1.5489, while for 2% drift at the same natural period, the reliability index ranged between 0.9906 and 1.4019. These observations indicate that the characteristics of a hysteresis model have a significant effect on the calculated seismic reliability of a structure, with the effect being more or less pronounced depending on the capacity limit that is used to assess seismic reliability. Therefore, the hysteretic behaviour of a structure needs to be accurately modeled, particularly in shorter natural period structures, to provide an accurate probabilistic description of response and hence a good estimate of seismic structural reliability.

Table 1: Drift limit reliability indices

Period (sec.)	Drift Limit (%)	Hysteresis A	Hysteresis B	Hysteresis C	Hysteresis D
0.2	0.5	4.7531	4.8966	4.6621	5.1166
	1.0	7.7161	*8	7.5916	8
	2.0	8	8	8	8
	4.0	8	8	8	8
0.5	0.5	-2.8584	-2.6016	-2.9694	-2.3180
	1.0	1.4139	1.3439	1.4255	1.4505
	2.0	4.2555	4.0973	4.2970	4.1672
	4.0	7.2567	7.0082	7.3254	7.0775
1.0	0.5	-4.9423	-4.4371	-4.9719	-3.5885
	1.0	-1.5230	-1.4380	-1.5489	-1.3621
	2.0	1.4019	1.3012	1.3836	0.9906
	4.0	3.7652	3.5886	3.7480	3.1064
3.0	0.5	-3.1968	-2.9777	-3.1846	-2.7980
	1.0	-2.4980	-2.3677	-2.4968	-2.3058
	2.0	-1.3396	-1.3339	-1.3529	-1.4422
	4.0	0.1971	0.0873	0.1718	-0.1862

* β = 8 is indicated for exceedence probabilities $< 10^{-15}$

Note that the natural period dependence of the relative importance of an accurate hysteresis model can be seen in the basic equation of motion for a structure, which can be written as follows:

$$\ddot{x} + 2\xi\omega_n\dot{x} + \frac{F(x)}{m} = -\ddot{a}_g \tag{2}$$

From that form of the equation of motion it is clear that the relative importance of the restoring force *F(x)* is reduced with increasing mass for a given structural stiffness. Therefore, the shape of the hysteresis loop for longer period structures is relatively less important when determining the probabilistic response of the structure.

CONCLUSIONS

A new Monte Carlo software application for analyzing the probabilistic response of single-degree-of-freedom and simple multiple-degree-of-freedom structures to random earthquake loading has been presented. The time-history analysis software, developed in C++, links the Newmark method with an Adams predictor-corrector algorithm to solve a modified version of the well-known Bouc-Wen model, which is used to represent the hysteretic behaviour of inelastic structures. Parameter identification of the Bouc-Wen model is accomplished with a least-squares algorithm operating on cyclic test data or shake-table data supplied by the user. The time-stepping algorithms are further linked with well-known earthquake generation models and structural models incorporating random properties based on chosen probability distributions that are used to generate a random realization of the structure for each random earthquake in a Monte Carlo analysis.

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