



## **HYBRID CONTROL OF SEISMICALLY EXCITED STRUCTURES BY VIBRATION CONFINEMENT**

**Mustapha OULED CHTIBA<sup>1</sup>, Slim CHOURA<sup>2</sup>, Sami EL-BORGI<sup>1</sup>, Ali H. NAYFEH<sup>3</sup>**

### **SUMMARY**

A hybrid (active and passive) control strategy, aiming at confining and suppressing simultaneously the vibratory motion in flexible structures, is proposed. These structures are assumed to be composed of parts that are sensitive to vibrations that result from an initial energy distribution and/or external disturbances. The design objective is to devise a feedback scheme that leads to transferring the vibrational energy in the sensitive parts to the remaining ones (non-sensitive parts) of the structure. In order to keep away from the build-up of transferred energy into the non-sensitive parts, the feedback scheme considers, along with the confinement, the suppression of vibrations in the whole spatial domain of the structure. A case study for a three-story building is considered to examine the effect of the number of the added bracing elements and their spatial placement on the vibration confinement of the structure. It is demonstrated that the use of a single bracing element attached directly to the floor gives a relatively better reduction of vibrational amplitudes of the floors. A parametric study is carried out to observe the influence of the various parameters of the control strategy on the simultaneous confinement and suppression of structural vibrations. The robustness of the proposed control strategy is analyzed and then tested for the three-floor structure.

### **INTRODUCTION**

Most flexible structures are subject to vibration, which can excite the unwanted structural resonances or to be propagated to regions where they cannot be tolerated. Therefore, it may be of interest to reduce the vibrations in the more sensitive parts of the structure and to transfer the vibrational energy to the less sensitive parts. These systems include communications antennas in flexible space structures, and payload masses in flexible robot manipulators, etc. The common practice is to locate force and/or torque actuators with appropriate time-varying magnitudes at the span of the structure. These actuators must provide sufficient amount of damping for energy dissipation. The rate of vibration reduction in all parts of the structure is approximately the same; i.e., it would take the same time to bring to rest both the sensitive parts and those that are irrelevant to certain performance specifications. In some cases, it is desirable to

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<sup>1</sup>Applied Mechanics and Systems Research Laboratory, Tunisia Polytechnic School

<sup>2</sup>Preparatory Institute for Engineering Studies, Sfax, Tunisia

<sup>3</sup>Department of Engineering Sciences and Mechanics, Virginia Polytechnic Institute and State University, Blacksburg, VA, USA

suppress the vibrations at a faster rate at those sensitive regions or stations. A variable suppression rate throughout the structure can be achieved by altering the mode shapes of the structure in addition to the eigenvalues.

In recent years, considerable effort has been devoted to devise effective control design methodologies for flexible structures. Several researchers, such as Snyder et al. [1], El Naschie [2], Bendiksen [3] and Xie [4], have studied the effect of localization on the performance of flexible structures. Choura [5] and Choura and Yigit [6-7] have developed a strategy of the vibration confinement. Their strategy consists of assigning the eigenstructure (both eigenvalues and eigenvectors) of the controlled flexible structure to confine and suppress simultaneously the vibratory motion. The row assignment of the new modal matrix has the leader role on the way the vibration will be redistributed and, thus, confined in the structure. They have shown that the proposed control strategy guarantees the vibration confinement in the structure and its stability.

In this paper, we develop a hybrid control strategy for simultaneous confinement and suppression of vibrations in flexible structures. The proposed strategy is applied to a building that is modeled as a one-dimensional shear-type structure with a linear elastic behavior. In particular, a control design is developed for a three-story scaled building, tested by Kobori et al. [8], with added bracing elements and applied force actuators. The paper presents simulations of the three-story building under the seismic excitation known as El Centro Earthquake (North-South component) with a peak ground acceleration (PGA) of 0.112g.

## PROBLEM FORMULATION AND OBJECTIVE

Consider the  $(n+m)$ -dimensional discretized model of flexible structures governed by :

$$M\ddot{x} + C\dot{x} + Kx = B_u u(t) + E_w w(t) \quad (1)$$

where  $M$  is an  $(n + m) \times (n + m)$  symmetric positive definite mass matrix,  $x$  is an  $(n + m)$ -dimensional generalized vector,  $C$  is an  $(n + m) \times (n + m)$  internal damping matrix and  $K$  is an  $(n + m) \times (n + m)$  symmetric positive stiffness matrix.  $u(t)$  is an  $(m + p)$  vector of control force inputs supplied by the actuators.  $B_u$  is the  $(n + m) \times (m + p)$  matrix, which characterizes the manner the actuators are distributed along the span of the structure,  $w(t)$  is a  $q \times 1$  vector of the external excitations whose locations are given by the  $(n + m) \times q$  matrix  $E_w$ . Let  $(n + m)$  sensors generate the output given by  $Dx$ , where  $D$  is nonsingular square matrix of appropriate dimension. Without loss of generality, let  $D$  be the identity matrix.

The equation of motion given in (1) describes the global dynamics of an  $n$ -DOF structure to which  $m$  passive elements are added for the purpose of vibration confinement. The passive elements are considered to be vibration absorbers and thus allow the reduction of the vibration amplitudes in the  $n$  sensitive elements of the flexible structure.

In this study, the emphasis is on synthesizing a family of force vectors  $u(t)$  for confining and suppressing simultaneously the vibrational energy in nonsensitive parts of a flexible structure. Therefore, it is necessary to devise a mechanism for shielding such parts from incoming vibrations waves with a reduced number of actuators. In real applications, reducing the number of actuators for simultaneous confinement and suppression is commonly preferable.

## STRATEGY OF VIBRATION CONFINEMENT AND SUPPRESSION

The flexible structure described by (1) is proposed to be rearranged and decomposed such that it consists of two subsystems; one includes all sensitive parts as well as some neighboring nonsensitive parts, totaling to  $(n+p)$ , and another contains the remaining  $(m-p)$  nonsensitive parts ( $m \geq p$ ). Therefore, without the influence of the external excitation, equation (1) is simplified to :

$$\begin{bmatrix} M_1 & M_2 \\ M_2^T & M_3 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} C_1 & C_2 \\ C_2^T & C_3 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} K_1 & K_2 \\ K_2^T & K_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{bmatrix} I_{n+p} \\ 0_{m-p} \end{bmatrix} u(t) \quad (2)$$

where the matrices  $M_1$ ,  $C_1$  and  $K_1$  are  $(n+p) \times (n+p)$ ;  $M_2$ ,  $C_2$  and  $K_2$  are  $(n+p) \times (m-p)$ ;  $M_3$ ,  $C_3$  and  $K_3$  are  $(m-p) \times (m-p)$ ; and  $I_{n+p}$  and  $0_{(m-p)}$  are the  $(n+p) \times (n+p)$  identity and the  $(m-p) \times (m-p)$  zero matrices, respectively. At this stage, it should be noted that the control law  $u(t)$  is synthesized in the absence of all external excitations. One actuator is assigned to each element in the first subsystem, and more control emphasis will be put on the sensitive elements located in the first subsystem in the sense that their convergence to the desired steady state is attained at fast rates.

The sensitive elements of the structure are included in the first subsystem which consists of  $(n+p)$  parts and the  $(n+p)$ -dimensional feedback vector input  $u(t)$  is responsible of redistributing the vibration energy among the first subsystem. Such a redistribution has a major role in confining the vibration in certain parts of the structure. Of course, controllability is a condition for closed loop stability, and it is mainly determined by the criterion the  $(n+p)$  actuators are placed in the structure.

The major role of the proposed confinement strategy is to decouple the first subsystem from the rest of the structure and to convert the original vibratory modes into a set of modes that allow the vibrational energy to be trapped in prescribed regions of the structural domain. This can be accomplished by a proper selection of the vector force whose structure is given by:

$$u(t) = M_2 \ddot{x}_2 + C_2 \dot{x}_2 + K_2 x_2 - \bar{M}_1 \ddot{x}_1 - \bar{C}_1 \dot{x}_1 - \bar{K}_1 x_1 \quad (3)$$

The substitution of equation (3) into equation (2) yields :

$$(\bar{M}_1 + M_1) \ddot{x}_1 + (\bar{C}_1 + C_1) \dot{x}_1 + (\bar{K}_1 + K_1) x_1 = 0 \quad (4)$$

$$M_2^T \ddot{x}_1 + M_3 \ddot{x}_2 + C_2^T \dot{x}_1 + C_3 \dot{x}_2 + K_2^T x_1 + K_3 x_2 = 0 \quad (5)$$

We note that the feedback  $u(t)$ , presented in equation (3), decouples the  $x_1$ -vector from the  $x_2$ -vector. It can be seen that there exists one-way coupling between  $x_1$  and  $x_2$ , and therefore the vibratory motion of the first subsystem can be regulated separately. Next, the development of the proposed feedback input  $u(t)$  is outlined adopting an earlier design by Choura and Yigit (2001).

Consider the following linear transformation:

$$x_1 = Q \eta_1 \quad (6)$$

where  $Q$  is a transformation matrix whose major role is the confinement of the vibrational energy in regions of the subsystem's spatial region. The choice of its row elements decides the way the vibration

energy will be rearranged in the spatial domain associated with the first subsystem. In order to guarantee lower energy levels in the sensitive parts, their associated rows in the matrix  $Q$  should have all of its elements lower than those associated with the nonsensitive elements. For this reason, the matrix  $Q$  is suggested be in the following form:

$$Q = \begin{bmatrix} q_{11} & q_{12} & \cdot & \cdot & q_{1(n+p-1)} & q_{1(n+p)} \\ q_{21} & q_{22} & \cdot & \cdot & q_{2(n+p-1)} & s_{2(n+p)} q_{2(n+p)} \\ q_{31} & q_{32} & \cdot & \cdot & s_{3(n+p-1)} q_{3(n+p-1)} & s_{3(n+p)} q_{3(n+p)} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ q_{(n+p-1)1} & q_{(n+p-1)2} & \cdot & \cdot & s_{(n+p-1)(n+p-1)} q_{(n+p-1)(n+p-1)} & s_{(n+p-1)(n+p)} q_{(n+p-1)(n+p)} \\ q_{(n+p)1} & s_{(n+p)2} q_{(n+p)2} & \cdot & \cdot & s_{(n+p)(n+p-1)} q_{(n+p)(n+p-1)} & s_{(n+p)(n+p)} q_{(n+p)(n+p)} \end{bmatrix} \quad (7)$$

The columns of the matrix  $Q$  must preserve a pattern of sign changes, which is commonly known to exist for the mode shapes associated with the class of systems described by equation (4). The sign changes in  $Q$  are characterized by the scalar  $s_{ij}$  which is defined as:

$$s_{ij} = \begin{cases} (-1)^{n+p+1} & \text{if } i+j \text{ is even} \\ (-1)^{n+p} & \text{if } i+j \text{ is odd} \end{cases}$$

Substituting equation (6) into equation (4) yields:

$$\ddot{\eta}_1 + \Gamma \dot{\eta}_1 + \Lambda \eta_1 = 0 \quad (8)$$

where

$$\Gamma = Q^{-1} (M_1 + \bar{M}_1)^{-1} (C_1 + \bar{C}_1) Q \quad (9)$$

$$\Lambda = Q^{-1} (M_1 + \bar{M}_1)^{-1} (K_1 + \bar{K}_1) Q \quad (10)$$

The matrices  $Q$ ,  $\Gamma$  and  $\Lambda$  are considered to be the design parameters of the control law  $u(t)$ . The matrices  $\Gamma$  and  $\Lambda$  must be properly chosen to ensure the stability of the first subsystem. They are chosen as diagonal matrices given by:

$$\Gamma = \text{diag}\{2\xi_1 \omega_1, 2\xi_2 \omega_2, \dots, 2\xi_{n+p} \omega_{n+p}\} \quad (11)$$

$$\Lambda = \text{diag}\{\omega_1^2, \omega_2^2, \dots, \omega_{n+p}^2\} \quad (12)$$

and thus:

$$\bar{C}_1 = (M_1 + \bar{M}_1) Q \Gamma Q^{-1} - C_1 \quad (13)$$

$$\bar{K}_1 = (M_1 + \bar{M}_1) Q \Lambda Q^{-1} - K_1 \quad (14)$$

According to Choura and Yigit [6], the set of matrices which decouple the dynamics of the first subsystem are expressed by:

$$\bar{M}_1 = (I_{n+p} - \alpha)^{-1} (K_1 Q \Lambda^1 Q^{-1} - M_1) \quad (15)$$

$$\bar{C}_1 = (M_1 + \bar{M}_1) Q \Gamma Q^{-1} - C_1 = (I_{n+p} - \alpha)^{-1} \{K_1 Q \Lambda^1 Q^{-1} - \alpha M_1\} Q \Gamma Q^{-1} - C_1 \quad (16)$$

$$\bar{K}_1 = \alpha \bar{M}_1 Q \Lambda Q^{-1} = \alpha (I_{n+p} - \alpha)^{-1} \{K_1 - M_1 Q \Lambda Q^{-1}\} \quad (17)$$

If the matrices  $\alpha$ ,  $Q$ ,  $\Gamma$  and  $\Lambda$  are specified then the gain matrices  $\bar{M}_1$ ,  $\bar{C}_1$  and  $\bar{K}_1$  can be determined from equations (15-17). The gain matrices can be interpreted physically as modifications of mass, stiffness and damping of the structure. Choura [5] has demonstrated that the primary task of  $\alpha$  is to scale the magnitude of the feedback gain matrices.

The solution of equation (8) can be expressed as:

$$\eta_h(t) = G(t)\eta_h(0) + H(t)\dot{\eta}_h(0) \quad (18)$$

where  $G(t)$  and  $H(t)$  are diagonal matrices whose elements are given by

$$g_i(t) = \exp(-\xi_i \omega_i t) \left\{ \cos \omega_i \sqrt{1-\xi_i^2} t + \frac{\xi_i}{\sqrt{1-\xi_i^2}} \sin \omega_i \sqrt{1-\xi_i^2} t \right\} \quad (19a)$$

$$h_i(t) = \exp(-\xi_i \omega_i t) \left\{ \frac{\xi_i}{\omega_i \sqrt{1-\xi_i^2}} \sin \omega_i \sqrt{1-\xi_i^2} t \right\} \quad (19b)$$

Therefore, the vector  $x_1$  can be expressed by:

$$x_1(t) = Q \{G(t)Q^{-1}x_1(0) + H(t)Q^{-1}\dot{x}_1(0)\} \quad (20)$$

The presence of the matrix  $Q$  in front of the right-hand side term equation (20) justifies the ability of the proposed strategy to redistribute the vibrational energy in the structure. This is possible only if  $Q$  is constructed according to the format given in equation(7). With the presence of the external excitation  $w(t)$ , the response of the first subsystem can be shown to be:

$$x_1(t) = Q \{G(t)Q^{-1}x_1(0) + H(t)Q^{-1}\dot{x}_1(0)\} + Q \int_0^t H(t-\tau) (K_1 Q \Lambda^{-1} - \alpha M_1 Q)^{-1} (I_{n+p} - \alpha) E_1 w(\tau) d\tau \quad (21)$$

Again, the presence of the matrix  $Q$  in front of the right-hand side terms, including the integral term resulting from the external excitation, of equation (21) proves the ability of the proposed strategy to redistribute the vibrational energy in the structure. Because the first subsystem is decoupled, the stability of the dynamics associated with the second subsystem is maintained provided that the system formed by  $M_3$ ,  $C_3$  and  $K_3$  is asymptotically stable.

## ROBUSTNESS OF THE PROPOSED STRATEGY

It is commonly known that parameter uncertainties result from neglecting higher order dynamics in modeling flexible structures. In control design, the issue of robustness of the controlled structure against parameters uncertainties and disturbances should be addressed. Assume that the parameters of the mass, stiffness and damping of the structure are uncertain. Therefore, equation (4) should be modified to account for the assumed uncertainties. It becomes

$$(M_1^* + \Delta M_1 + \bar{M}_1)\ddot{x}_1 + (C_1^* + \Delta C_1 + \bar{C}_1)\dot{x}_1 + (K_1^* + \Delta K_1 + \bar{K}_1)x_1 = 0 \quad (22)$$

respectively, where  $\Delta M_1$ ,  $\Delta C_1$  and  $\Delta K_1$  represent the uncertainties associated with the nominal matrices  $M_1^*$ ,  $C_1^*$  and  $K_1^*$ . Equation (22) can be simplified as:

$$(I + (M_1^* + \bar{M}_1)^{-1} \Delta M_1)\ddot{x}_1 + (Q\Gamma Q^{-1} + (M_1^* + \bar{M}_1)^{-1} \Delta C_1)\dot{x}_1 + (QAQ^{-1} + (M_1^* + \bar{M}_1)^{-1} \Delta K_1)x_1 = 0 \quad (23)$$

where

$$\Gamma = Q^{-1}(M_1^* + \bar{M}_1)^{-1}(C_1^* + \bar{C}_1)Q \quad A = Q^{-1}(M_1^* + \bar{M}_1)^{-1}(K_1^* + \bar{K}_1)Q \quad (24)$$

From equation (23), note that if the feedback matrix  $\bar{M}_1$  is chosen such that the magnitude of  $M_1^* + \bar{M}_1$  is adequately large, the time response of the uncertain system will resemble that of the nominal system in the sense that both eigenvalues and eigenvectors are comparable. This can be achieved by making the diagonal matrix  $\alpha$  approximately equal to the identity matrix, but none of its elements can be set to unity. In control design, it is known that large feedback gains are commonly employed for reducing the sensitivity of the structure to uncertainties and disturbances. Here, a similar design is adopted in concept to achieve simultaneous confinement and suppression of vibrations provided that the selected gains must conform to the identities given by equations (15-17). It should be noted that the proposed design becomes feasible only if the matrix  $M_1^*$  is large in magnitude.

## NUMERICAL RESULTS

A three-story building, tested by Kobori et al. [8] (see figure 1), is used to demonstrate the feasibility of the proposed control strategy by confinement for civil engineering structures. The mass, stiffness and damping of each story are given by:  $m_{si} = 1000$  kg,  $k_{si} = 980$  kN/m and  $c_{si} = 1.407$  kNsec/m ( $i = 1, 2, 3$ ). Figure 2 illustrates the addition of one active bracing element, as considered the nonsensitive part in this structure, between the first story and the ground. Therefore, the modified structure can be modeled as a four-degree-of-freedom oscillator (see figure 3). Here, we assume that all stories are sensitive to vibrations.

We consider three types of modified structures in which the number of active and passive bracing elements varies from one to three with different locations with the respect to the floors and ground (see table 1). An active force actuator is applied to every active bracing element and one to every story level. No force actuators are applied to the passive bracing element. An example of a structure with one active bracing element and one passive bracing element is shown in figure 4. The mass, stiffness and damping of the bracing element are:  $m_{bi} = 100$  kg,  $k_{bi}' = k_{bi}'' = 200$  kN/m and  $c_{bi}' = c_{bi}'' = 0.400$  kNsec/m ( $i = 1, 2, 3$ ). The

scaled building model was subjected to the El Centro earthquake (NS component) scaled to a peak ground acceleration (PGA) of 0.112g.

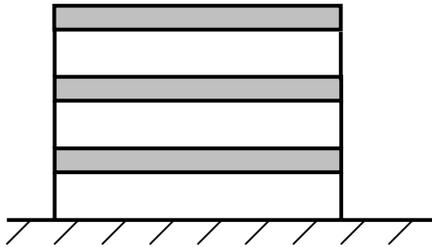


Figure 1. Three-story building

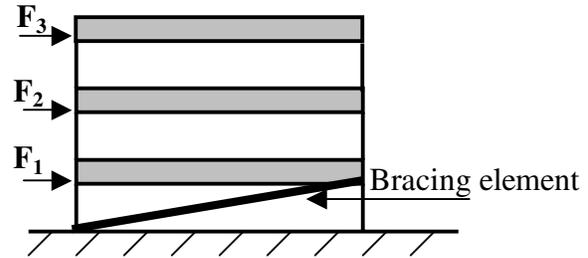


Figure 2: Braced three-story building

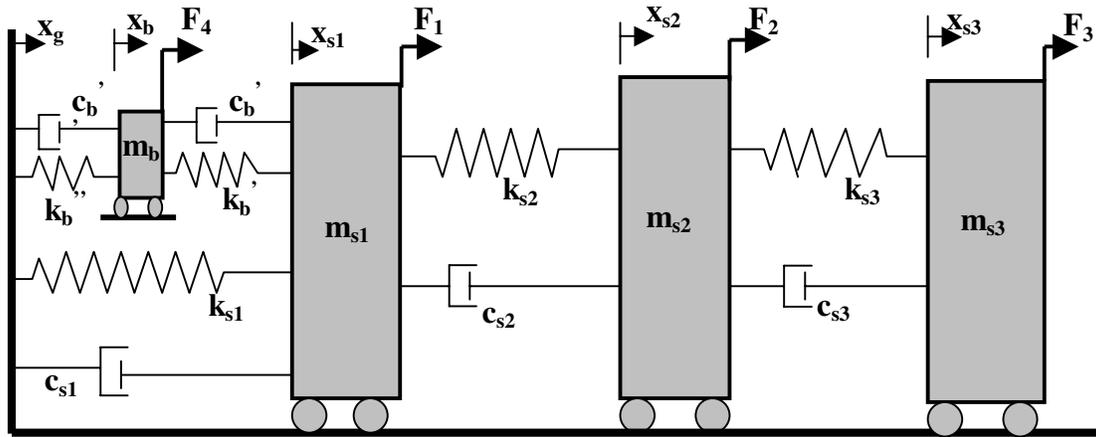


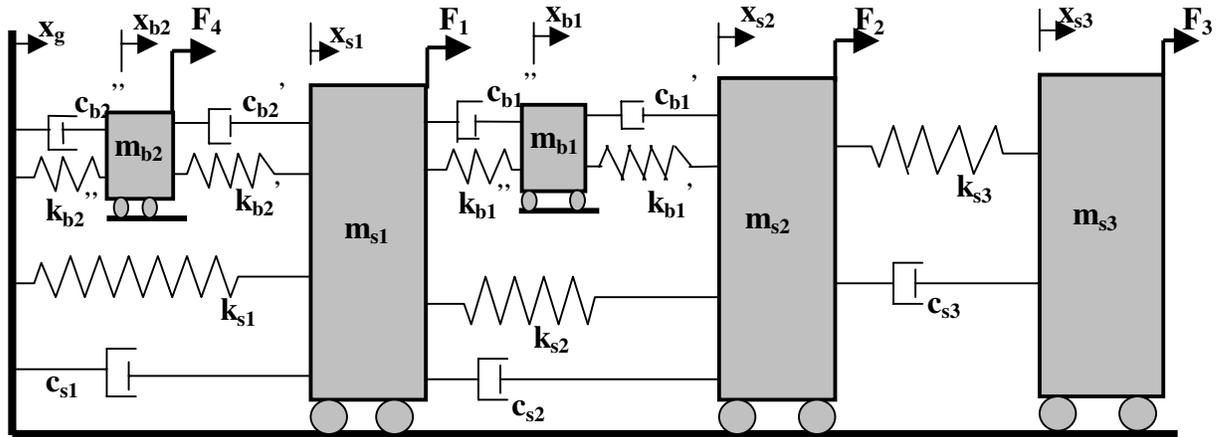
Figure 3: Four DOF oscillator

Table 1: Types of structures

CASES	CASE 1	CASE 2	CASE 3	CASE 4	CASE 5	CASE 6
Structure I	$m_{s1}/\text{ground}^*$	$m_{s2}/m_{s1}^*$	$m_{s3}/m_{s2}^*$	$m_{s2}/\text{ground}^*$	$m_{s3}/\text{ground}^*$	$m_{s3}/m_{s1}^*$
Structure II	$m_{s2}/m_{s1}$ and $m_{s1}/\text{ground}^*$	$m_{s2}/m_{s1}^*$ and $m_{s1}/\text{ground}$	$m_{s2}/m_{s1}^*$ and $m_{s3}/m_{s2}$	$m_{s2}/m_{s1}$ and $m_{s3}/m_{s2}^*$		
Structure III	$m_{s3}/m_{s2}$ ; $m_{s2}/m_{s1}$ and $m_{s1}/\text{ground}^*$	$m_{s3}/m_{s2}$ ; $m_{s2}/\text{ground}^*$ and $m_{s1}/\text{ground}$				

$m_{si}/m_{sj}$  indicates that the bracing element is between the masses  $m_{si}$  and  $m_{sj}$ .

\* indicates the placement of the active bracing element in the structure.



**Figure 4: Three-story structure with one active bracing element and passive bracing element**

In order to validate the feasibility of the proposed strategy of control, we apply a classical control strategy using pole assignment to the braced structure. For comparison purposes, both control strategies are allowed to use the similar closed-loop damping ratios and a similar force norm defined by  $\|\vec{F}\| = \sqrt{F_1^2 + F_2^2 + F_3^2 + F_4^2}$ . The response of the structure was obtained for different cases of : (1) no control and without bracing element (figure 1); (2) classical control using pole assignment (figure 2) and (3) control using vibration confinement (figure 2).

All cases use a maximum force norm of 3.15 kN. The time-drift story response (i.e., inter-story displacement), acceleration and actuator forces for all cases are summarized in table 2 from which it apparent that the proposed strategy yields a better structural performance. It is clear that the bracing elements experience higher vibration amplitudes as opposed to the building stories that are brought to rest at fast rates. In order to show the viability of the proposed control, table 3 summarizes the performance of different types of structures. The lowest vibrational amplitudes associated with the sensitive elements are observed for structure I-case 1.

**Table 2: Comparative study with and without confinement**

CASE NUMBER		STORY 1	STORY 2	STORY 3	BRACING	
1	No Control	D	1.32	1.00	0.59	-----
		A	308	467	573	-----
		F	-----	-----	-----	-----
2	Classical Active Control (Pole Assignment) $\ \vec{F}\ _{\max} = 3.15 \text{ kN}$	D	0.10	0.18	0.26	0.05
		A	172	193	299	123
		F	1.866	2.241	1.894	0.175
3	C.V.C (1 Bracing) $\ \vec{F}\ _{\max} = 3.15 \text{ kN}$	D	0.21	0.005	0.0005	0.76
		A	327	321	321	612
		F	0.76	1.63	1.63	2.03

D: Maximum interstory-displacement (cm); A: Maximum absolute acceleration (cm/sec<sup>2</sup>);  
F: Maximum control force (kN); C.V.C : control by vibration confinement

A parametric study of the structural response as a function of the control parameters  $\Gamma$ ,  $A$ ,  $\alpha$  and  $Q$  is summarized in table 4. The augmentation of  $\Gamma$  and  $A$  elements reduces the amplitudes of stories and bracing element displacements. The variation of  $Q$  elements influences remarkably the story displacements, accelerations and actuator forces. Through simulations, we have verified that both closed loop stability and confinement of vibrations are preserved to a maximum parameter uncertainty of 17.4% at which point the structure starts to be unstable while the confinement of the vibration is maintained.

## CONCLUSIONS

A feedback strategy for the simultaneous confinement and suppression of vibrations in excited structures was developed. A set of actuators was utilized for modifying the system eigenstructure as a key to redistributing the vibrational energy in the system domain. This yielded a reduction (if not elimination) of the vibration amplitudes of the sensitive elements at the expense of those associated with the nonsensitive parts. The feedback actuator forces were designed to include a mechanism for energy dissipation. A three-floor structure was illustrated to demonstrate the efficiency of the proposed strategy.

**Table 3: Performance of the different types of structures**

		STORY No. 1	STORY No. 2	STORY No. 3	BRACING No. 1	BRACING No. 2	BRACING No. 3
<b>Structure I Case 1</b> $\ \vec{F}\  = 3.15 \text{ kN}$	<b>D</b>	0.20	0.005	0.0005	0.76	-----	-----
	<b>A</b>	327	321	321	612	-----	-----
	<b>F</b>	0.76	1.63	1.63	2.03	-----	-----
<b>Structure I Case 2</b> $\ \vec{F}\  = 3.15 \text{ kN}$	<b>D</b>	0.11	0.018	0.009	0.66	-----	-----
	<b>A</b>	127	117	128	642	-----	-----
	<b>F</b>	0.88	2.21	1.06	2.01	-----	-----
<b>Structure I Case 3</b> $\ \vec{F}\  = 3.15 \text{ kN}$	<b>D</b>	0.082	0.080	0.03	0.79	-----	-----
	<b>A</b>	165	124	142	10410	-----	-----
	<b>F</b>	0.51	2.19	1.41	1.78	-----	-----
<b>Structure I Case 4</b> $\ \vec{F}\  = 3.15 \text{ kN}$	<b>D</b>	0.13	0.19	0.0004	0.78	-----	-----
	<b>A</b>	263	184	184	1117	-----	-----
	<b>F</b>	2.45	0.78	1.61	1.86	-----	-----
<b>Structure I Case 5</b> $\ \vec{F}\  = 3.15 \text{ kN}$	<b>D</b>	0.074	0.18	0.15	1.56	-----	-----
	<b>A</b>	174	168	150	2958	-----	-----
	<b>F</b>	1.65	2.35	1.44	1.99	-----	-----
<b>Structure I Case 6</b> $\ \vec{F}\  = 3.15 \text{ kN}$	<b>D</b>	0.21	0.007	0.004	0.38	-----	-----
	<b>A</b>	264	256	252	449	-----	-----
	<b>F</b>	0.40	1.68	2.44	1.04	-----	-----
<b>Structure II Case 1</b> $\ \vec{F}\  = 3.15 \text{ kN}$	<b>D</b>	0.22	0.006	0.002	0.056	0.66	-----
	<b>A</b>	208	201	198	218	475	-----
	<b>F</b>	0.61	1.88	1.79	-----	1.70	-----
<b>Structure II Case 2</b> $\ \vec{F}\  = 3.16 \text{ kN}$	<b>D</b>	0.25	0.023	0.002	0.50	0.16	-----
	<b>A</b>	239	213	213	617	193	-----
	<b>F</b>	0.47	1.94	2.15	1.33	-----	-----

**Table 3: (Continued)**

		STORY No. 1	STORY No. 2	STORY No. 3	BRACING No. 1	BRACING No. 2	BRACING No. 3
<b>Structure II Case 3</b> $\ \vec{F}\  = 3.15 \text{ kN}$	<b>D</b>	0.003	0.003	0.045	0.091	0.53	-----
	<b>A</b>	107	107	127	289	369	-----
	<b>F</b>	1.75	2.16	1.13	-----	1.70	-----
<b>Structure II Case 4</b> $\ \vec{F}\  = 3.15 \text{ kN}$	<b>D</b>	0.072	0.051	0.005	0.34	0.096	-----
	<b>A</b>	120	113	113	432	318	-----
	<b>F</b>	2.13	1.36	1.58	1.10	-----	-----
<b>Structure III case 1</b> $\ \vec{F}\  = 3.15 \text{ kN}$	<b>D</b>	0.22	0.005	0.002	0.59	0.057	0.057
	<b>A</b>	206	201	198	413	215	228
	<b>F</b>	0.46	2.04	1.93	1.46	-----	-----
<b>Structure III case 2</b> $\ \vec{F}\  = 3.15 \text{ kN}$	<b>D</b>	0.22	0.005	0.0003	0.059	0.39	0.14
	<b>A</b>	199	194	194	223	282	202
	<b>F</b>	0.66	2.28	1.96	-----	0.81	-----

**D**: Maximum interstory-displacement (cm); **A**: Maximum absolute acceleration (cm/sec<sup>2</sup>);  
**F**: Maximum control force (kN)

**Table 4: Parametric study**

Varied parameters		STORY No. 1	STORY No. 2	STORY No. 3	BRACING No. 1	BRACING No. 2
<b>Structure II Case 1</b>	<b>D</b>	0.22	0.006	0.002	0.056	0.66
	<b>A</b>	208	201	198	218	475
	<b>F</b>	0.61	1.87	1.78	-----	1.70
	$\ \vec{F}\ $	3.15				
$\Lambda = \text{diag}(790, 800, 3000, 3204)$	<b>D</b>	0.21	0.006	0.002	0.05	0.66
	<b>A</b>	197	188	185	211	476
	<b>F</b>	0.60	1.89	1.78	-----	1.70
	$\ \vec{F}\ $	3.13				
$\Lambda = \text{diag}(790, 800, 2500, 3204)$	<b>D</b>	0.22	0.009	0.002	0.054	0.64
	<b>A</b>	207	191	189	212	481
	<b>F</b>	0.53	1.88	1.74	-----	1.67
	$\ \vec{F}\ $	3.10				
$\Gamma = \text{diag}(8.43, 8.48, 16.97, 16.98)$	<b>D</b>	0.18	0.005	0.002	0.04	0.54
	<b>A</b>	202	195	192	210	411
	<b>F</b>	0.56	1.60	1.52	-----	1.41
	$\ \vec{F}\ $	3.15				
$\Gamma = \text{diag}(11.24, 11.31, 22.62, 22.64)$	<b>D</b>	0.16	0.0048	0.002	0.045	0.48
	<b>A</b>	200	192	189	208	399
	<b>F</b>	0.58	1.50	1.41	-----	1.28
	$\ \vec{F}\ $	2.42				

**Table 4: (Continued)**

Varied parameters		STORY No. 1	STORY No. 2	STORY No. 3	BRACING No. 1	BRACING No. 2
$Q = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 12 & -12 & 12 & -12 \end{bmatrix}$	<b>D</b>	0.21	0.005	0.0023	0.05	0.76
	<b>A</b>	204	199	196	215	549
	<b>F</b>	0.80	1.87	1.77	-----	2.02
	$\ \vec{F}\ $	3.3				
$Q = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 16 & -16 & 16 & -16 \end{bmatrix}$	<b>D</b>	0.21	0.0043	0.0023	0.055	0.83
	<b>A</b>	203	198	195	215	609
	<b>F</b>	0.95	1.85	1.75	-----	2.27
	$\ \vec{F}\ $	3.54				
$\alpha = 10 \times \text{diag}(1.1, 1.2, 1.3, 1.4)$	<b>D</b>	0.16	0.0036	0.01	0.047	0.30
	<b>A</b>	188	183	181	198	258
	<b>F</b>	0.40	1.54	1.44	-----	0.65
	$\ \vec{F}\ $	2.20				
$\alpha = 1 \times \text{diag}(1.1, 1.2, 1.3, 1.4)$	<b>D</b>	0.045	0.098	0.067	0.092	0.40
	<b>A</b>	153	164	107	213	771
	<b>F</b>	3.09	0.34	1.63	-----	1.33
	$\ \vec{F}\ $	3.75				

**D:** Maximum interstory-displacement (cm); **A:** Maximum absolute acceleration (cm/sec<sup>2</sup>);  
**F:** Maximum control force (kN)

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