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DYNAMIC RESTORING FORCE MODEL OF RC-COLUMN UNDER SEISMIC LOAD

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SUMMARY

Dynamic loading tests of reinforced concrete columns (RC columns) under varying axial load were carried out and the complicated restoring force characteristics including the varying axial load effect and loading rate effect have been obtained. On the basis of the test results the dynamic restoring force model of RC column under seismic load is derived. The model expresses the varying axial load effect and loading rate effect and can be applied easily to seismic response analysis. It is also ascertained that the presented model is useful by comparing it quantitatively with the dynamic loading test results.

1. INTRODUCTION

When reinforced concrete frame (RC frame) is subjected to strong ground motion, the columns of the frame are subjected not only to the lateral force but also to the axial force which varies extremely with time. Accordingly the bending restoring force characteristics of column are effected by the loading rate and the varying axial force simultaneously. To analyze the seismic response of RC frame strictly the restoring force model which can calculate the effects of the loading rate and the dynamically varying axial force on the bending restoring force is required.

From this reason dynamic loading tests have been carried out[1)-3)]. On the basis of the test results the dynamic restoring force model of RC column which is subjected to the repeated lateral force and dynamically varying axial force simultaneously and easily applied to seismic response analysis is obtained.

2. DYNAMIC AND STATIC LOADING TEST

2.1 RC column specimen

The specimens of dynamic and static loading test are the cantilever RC columns shown in Fig.1 and explained in Table-1 -Table-2. Table-1 and Table-2 show the test conditions and the material properties of

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reinforcing steel respectively.

All specimens are fully reinforced by hoop to prevent shear failure of column. The concrete strength, the reinforcement ratio, the axial force, the loading rate and the loading hysteresis of the specimens are different among them and the effects of them on the restoring force characteristics of RC column have been examined.

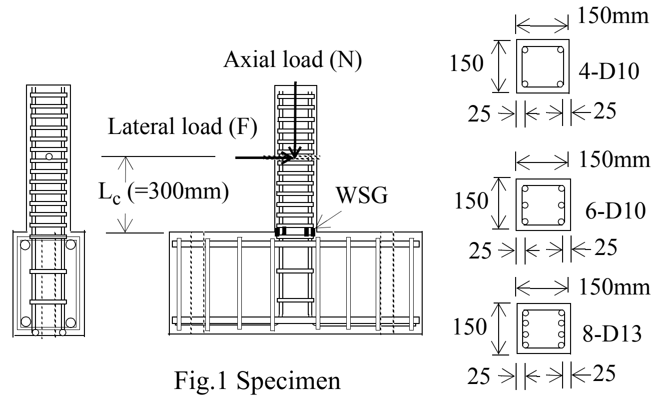

Fig.1 Specimen

Table-1 Specimens and test conditions

Specimen	Load	N_o/N_y	p_t(%)	p_w(%)	σ_c	ε_{cu}(%)
(Dynamic test)						
SCDI-L-40A	D I	0.40	0.95	2.29	44.3	0.45
SCDI-L-40B	D I	0.40	"	"	"	"
SCDC-L-30	D C	0.33	0.63	1.60	35.8	0.46
SCDI-L-30A	D I	0.32	0.95	1.37	38.7	0.56
SCDI-L-30B	D I	0.32	"	"	"	"
SCDR-L-30	D R	0.33	0.63	1.60	35.8	0.46
SCDI-L-20A	D I	0.22	0.95	2.29	38.4	0.43
SCDI-L-20B	D I	0.21	0.63	1.26	29.7	0.49
SCDR-L-20	D R	0.22	"	"	33.2	0.51
SCDI-S-15	D I	0.15	2.26	3.20	112.4	0.37
SCDR-S-15	D R	0.15	"	"	"	"
(Static test)						
SCSC-L-30	S C	0.33	0.63	1.60	35.8	0.46
SCSR-L-30	S R	0.33	"	"	"	"
SCSI-L-20	S I	0.21	"	1.26	32.3	0.53
SCSR-S-15	S R	0.15	2.26	3.20	112.4	0.37

p_t: tension reinforcement ratio
p_w: hoop reinforcement ratio
σ_c: compression concrete strength (N/mm^2)
ε_{cu}: ultimate strain

Name of specimen:
SC D I- L-40A
1 2 3- 4- 5

1: RC-column specimen (L_c=300mm)
2: Static loading (S), dynamic loading (D)
3: Forced deformation
(C-wave, I-wave, R-wave)
4: Concrete strength
(S: super-high strength, L: ordinary strength)
5: Initial axial force ratio (N_o/N_y)

Table-2 Material properties of reinforcing steel

Steel bar	σ_y	σ_u	ε_y(%)
D10	351	520	0.19
D13	352	576	0.19

σ_c: yield stress (N/mm^2)
(=P_y/A_s, P_y: yield axial force, A_s: sectional area)
σ_u: tensile strength (N/mm^2)
(=P_u/A_s, P_u: ultimate axial force)
ε_y: yield strain

2.2 Loading conditions

The specimens are subjected to axial load (N) and lateral load (F) as explained in Fig.1. The test setup is shown in Fig.2. The specimen was set horizontally and fixed to rigid and strong steel frame. The lateral load was given by an actuator which was connected to the specimen by loading beam with universal joints. The axial force of column was loaded by pulling PC bars connected to the top of specimen. The principle of the lever was utilized to pull the PC bar as shown in Fig.2.

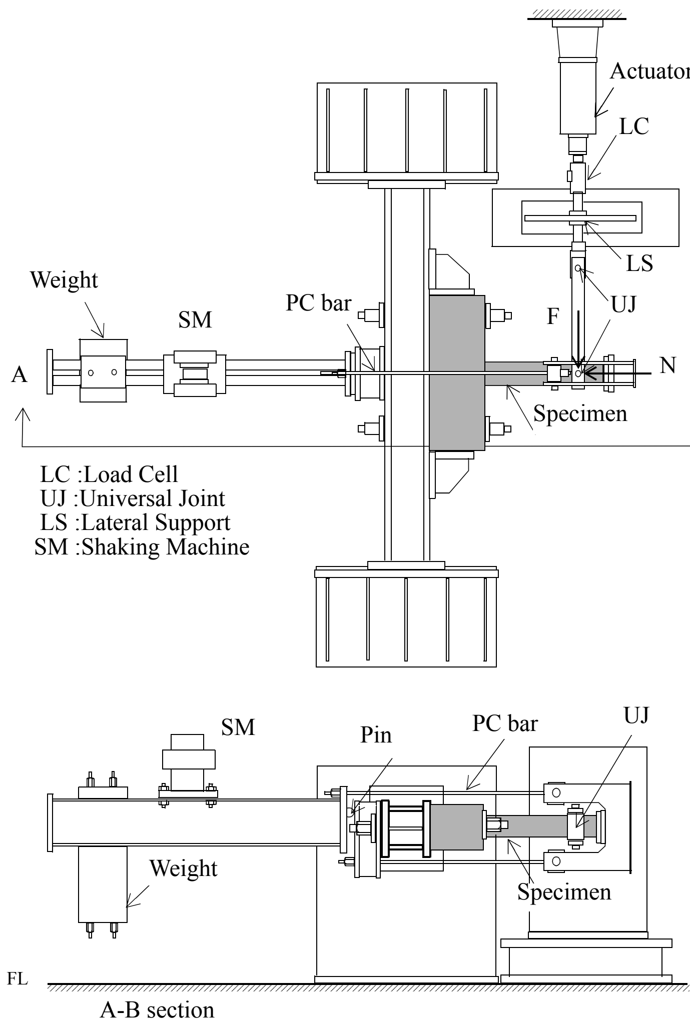

Fig.2 Test setup

In dynamic loading test both the lateral load (F) and the varying axial load (N) were applied dynamically. The dynamically varying axial load was given by the inertia force of the weight attached to the loading beam which vibrated up and down caused by the dynamic axial deformation of column. Fig.3 and Fig.4 show the time histories of the applied lateral deformation at the loading point (δ) and the time histories of the strain rate (dε/dt) of reinforcing steel in the column-end section respectively. There are the time histories of the dynamically varying axial force of column in Fig.5 in which N_o/N_y shown by the dashed line is the initial axial force ratio of column.

As shown in these figure the axial force vibrated nearly with the natural period of the loading beam and there was no relation between the lateral deformation and the varying axial force. In dynamic loading test the maximum velocity of lateral deformation at the loading point was 100mm/second and the maximum strain rate of main reinforcing steel of the specimen was about 0.1/second -0.3/second.

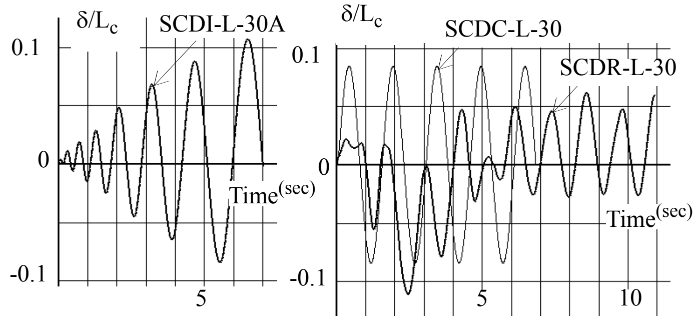

Fig.3 Time histories of loading point displacement

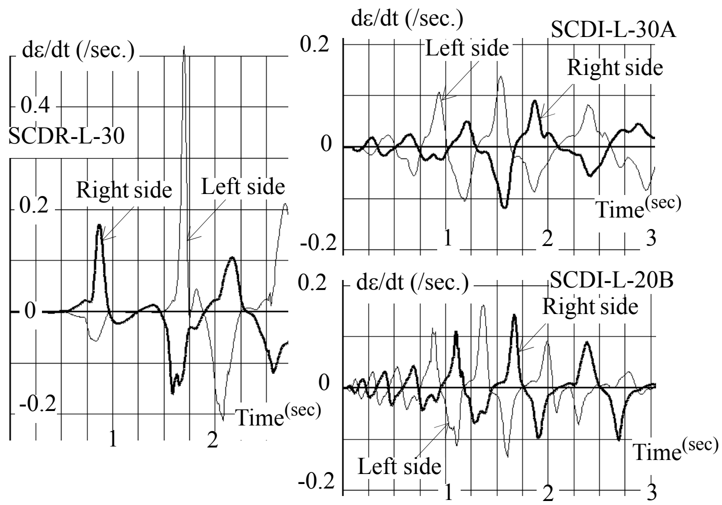

Fig.4 Strain-rate of reinforcing steel in the column-end section

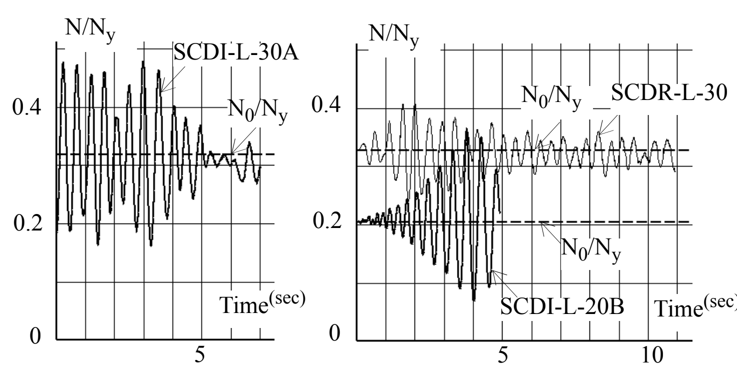

Fig.5 Varying axial force of column specimen

The loading rate effect of restoring force characteristics can be obtained by comparing the dynamic loading test result with static loading test result. From this reason static loading test was also executed to examine the loading rate effect on the restoring force characteristics of RC column.

In static loading test small incremental deformation ($\Delta\delta$) was given to the specimen and the deformation of column was kept stationary for one minute to exclude the loading rate effect on the restoring force. After the loading rate effect of column was fully removed next incremental deformation was given to the column. Static loading test was carried out by repeating this loading process. In static loading test axial force of column was constant because the weight attached to the loading beam did not vibrate.

2.3 Measurements and data acquisition

The lateral load (F) was measured by a load-meter fixed to the actuator. The initial axial force (N_o) was decided by the weight and the length of loading beam. The dynamically varying axial force was obtained from wire strain gages attached to PC bars. The inertia force of loading beam connected to the actuator and the direction of axial force of PC bars, which changes due to the deformation of column, were considered to obtain the load of specimen. Deformation of column (δ) was measured by linear variable displacement transducers (LVDT). The load and deformation of column were measured in every 0.01 second simultaneously under dynamic load. In static loading test the test data were acquired after the deformation of column by incremental loading was kept stationary for one minute and the loading rate effect was fully excluded.

3. TEST RESULTS

3.1 Load-deformation under static load

The load deformation relations of static test are shown with thick lines in Fig.6. $M(=F\,L_c+N\,\delta)$ is the bending moment at the fixed column-end which includes the P-δ effect of axial force. M_{uo} is the ultimate bending strength under initial axial force (N_o). The ultimate bending strength is calculated under the following conditions.

i) The maximum compression strain of concrete in the section reaches to the ultimate compression strain of concrete (ε_{cu}).

ii) The stress-strain relation of concrete is assumed to be expressed by the Bi-linear model as explained in Fig.7 which shows one of the concrete test results comparing with the Bi-linear model. The stress-strain relation of reinforcing steel is also assumed to be expressed by the Bi-linear model whose plastic strength is equal to the yield stress (σ_y).

According to the bending restoring force (M) divided by the ultimate bending strength (M_{uo}), it is shown that M_{uo} can predict well the maximum bending strength of the specimen and the calculation method of it mentioned above is also useful.

In Fig.6 the restoring force expressed by the modified Clough model (Clough model[4)]) is shown by the thin lines. As well known, the test results are fairly predicted by the Clough model. From this result the dynamic restoring force under varying axial load is also tried to express by the use of the Clough model in the following sections.

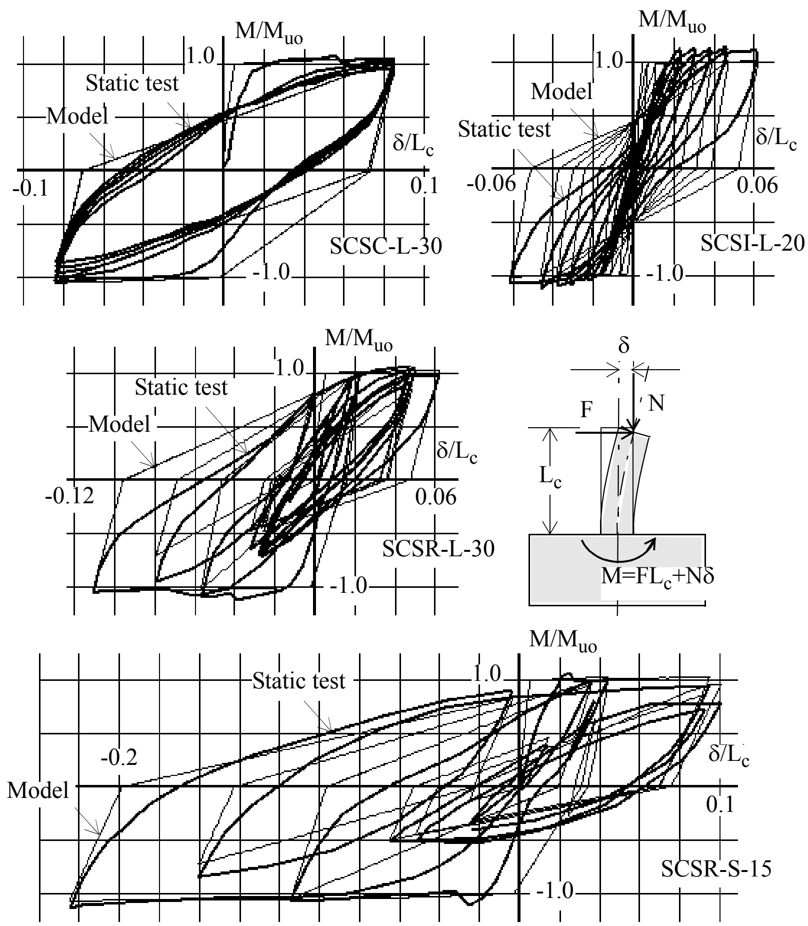

Fig.6 Load-deformation relation of static loading test

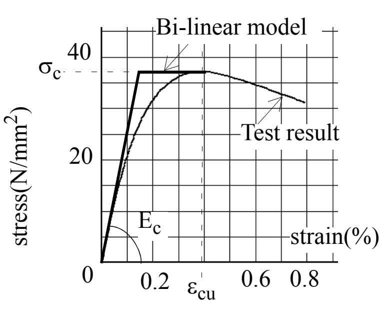

Fig.7 Stress-strain relation of concrete by Bi-linear model comparing with compression test result

3.2 Load-deformation under dynamic load

The load-deformation relations derived from the dynamic loading test are shown in Fig.8. The restoring force characteristics of them are very complicated. The maximum bending strength is quite larger than the calculated ultimate strength (M_{uo}) and the shape of hysteretic load-deformation relation are different from that of static test.

The dynamically varying axial force of each column is shown in Fig.9 expressed with the same abscissa as that used in Fig.8. By comparing Fig.8 with Fig.9, it is shown that there is not clear relation between the restoring force (M/M_{uo}) and the varying axial load (N/N_y).

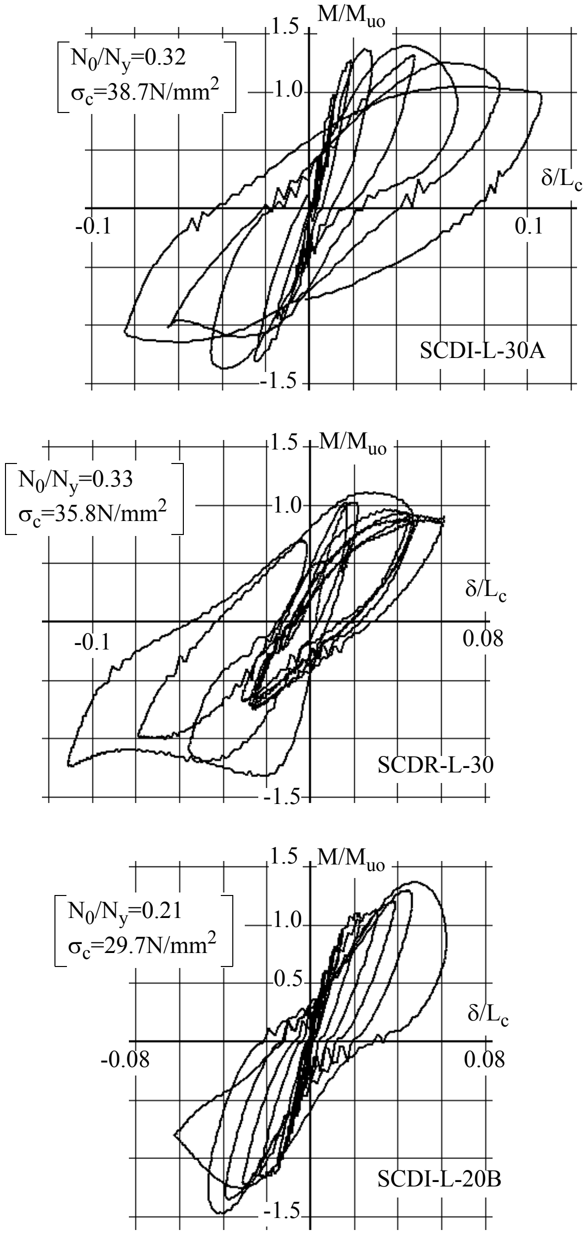

Fig.8 Bending load-deformation relation of dynamic loading test

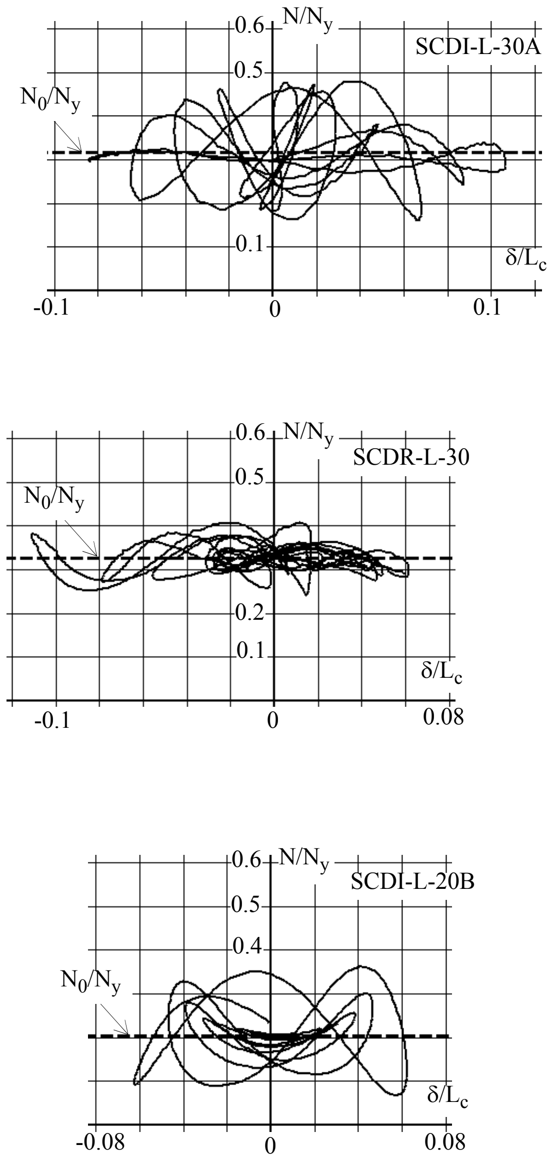

Fig.9 Varying axial force of dynamic loading test

The relations between the column-end moment (M) and the varying axial force (N) are shown in Fig.10. In the figure the dynamic loading test results are expressed by the circles and the ultimate bending strength (M_u) under axial force (N), which is calculated by the same method as the calculation of M_{uo}, are expressed by the real lines. The bending moment of RC column under dynamic load is effected extremely by the varying axial force. The maximum bending moment of specimen is quite larger than the calculated ultimate bending strength at each instance. The clear difference between the test results and the calculated strength is considered to be generated by the loading rate effect on the restoring force (M). From this reason the loading rate effect on the restoring force characteristics of RC column can not be neglected. On the basis of test results the loading rate effect on the restoring force model is obtained in the next section.

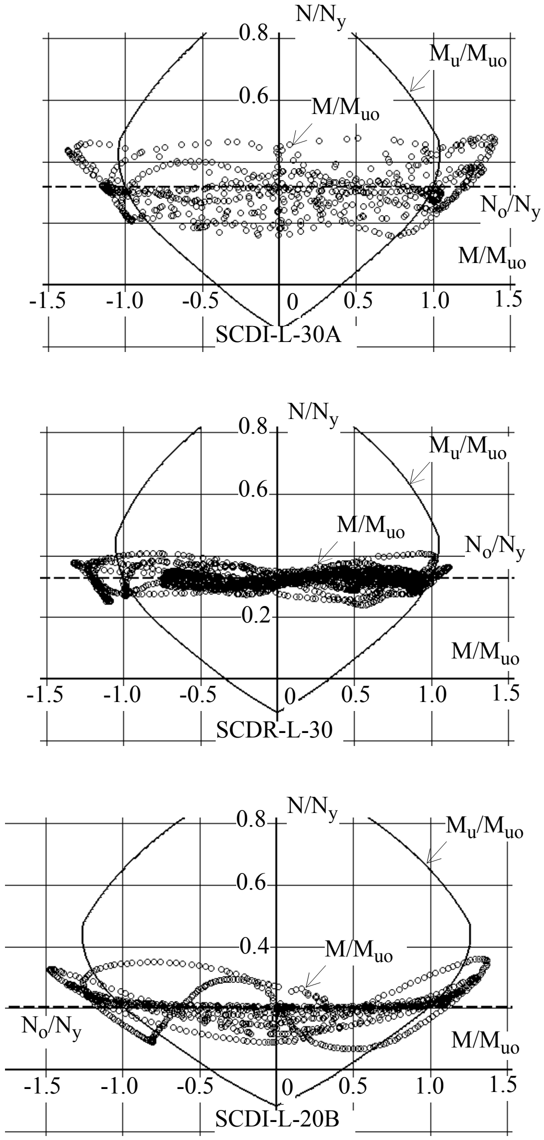

Fig.10 N-M relations of dynamic loading test result compared with calculated result

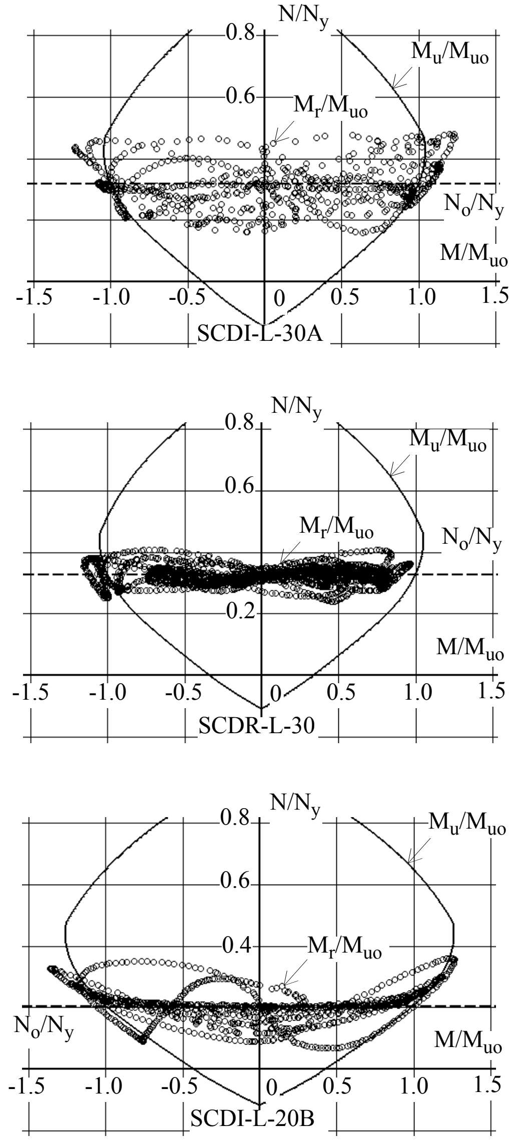

Fig.11 N-M relations of dynamic loading test result when the loading rate effect removed

3.3 Loading rate effect and N-M relation

To express the loading rate effect on the restoring force of RC column, whose loading rate varies irregularly at every instance, the r-value defined by Eq.(1) is introduced in this study.

$$r = ({}_{d}m - {}_{s}m)/{}_{s}m \qquad (1)$$

in which ${}_{d}m$, ${}_{s}m$ are the ratio of dynamic and static restoring force to the calculated ultimate bending strength (M/M_u) under the same hysteretic deformation. The values of ${}_{d}m$, ${}_{s}m$ are defined by the restoring force from the last reversal point of the repeated load as explained in Fig.12. The dynamic and static loading test of RC column under the same hysteretic deformation have been carried out and Eq.(2) to give r-value is obtained from the test result.

$$1/r = C_1 d_A + C_2 |d_B| + C_3 \qquad (2)$$

in which d_A is the accumulated plastic deformation ratio and d_B is the plastic deformation ratio (ϕ/ϕ_u) from the last reversal point as explained in Fig.12. In Eq.(2) the parameters C_1, C_2, C_3 are the constants which are not effected by the hysteresis of deformation, the velocity of deformation and loading conditions. On the basis of the test result, the most suitable parameters C_1, C_2, C_3 to predict the test results by Eq.(2) are obtained and shown in Table-3.

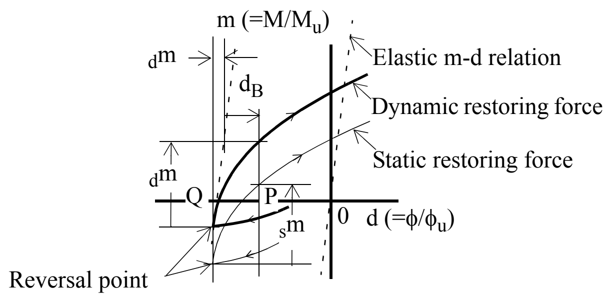

Fig.12 Definitions of non-dimensional force (${}_d m$, ${}_s m$) and non-dimensional deformation (d_B) from the reversal point (Q)

Table-3 Parameters in Eq.(2)

RC specimen	C_1	C_2	C_3
Long specimens (L_c=700mm)	0.0431	0.1043	3.00
Short specimens (L_c=200mm)	0.0104	0.0281	3.00
Middle length specimens (L_c=300mm)	0.0169	0.0433	3.00

By the use of r-value given by Eq.(2), the loading rate effect on the restoring force of RC column under dynamic load can be predicted. By removing the loading rate effect from the bending restoring force of specimen (M) by Eq.(2), the bending restoring force (M_r) is obtained and shown in Fig.11 in relation with the varying axial force ratio (N/N_y) of RC column at each instance. We can see that the bending restoring force in plastic range (M_r) is well predicted by the calculated ultimate strength (M_u) and Eq.(2) is useful to remove the loading rate effect from dynamic restoring force.

The varying axial force effect and the loading rate effect on the restoring force of RC column are also examined in the load-deformation relations shown in Fig.13 and Fig.14. The thick lines in Fig.13 show the ratio of the bending restoring force to the ultimate bending strength under varying axial force at each instance. Accordingly the thick line to show M/M_u is corresponding to the restoring force characteristics of RC column when the varying axial force effect is removed. It is seen that the thick lines to show M/M_u are nearly constant in plastic range and approximated well by regular curves. The thick lines to show the value M/M_u is fairly regular curve than the thin lines to show the value of M/M_{uo}. But the maximum bending strength expressed by thick line is not well predicted by the ultimate bending strength (M_u).

By removing the loading rate effect from the restoring force ratio M/M_u in Fig.13, we get the restoring force ratio M_r/M_u as shown in Fig.14. The obtained M_r/M_u is corresponding to the static restoring force characteristics under constant axial force. It is ascertained that the restoring force ratio M_r/M_u in Fig.14 can be expressed simply by regular curves and the maximum bending strength is well predicted by the calculated ultimate bending strength at each instance (M_u). Accordingly the restoring force ratio M_r/M_u is easily predicted by the static restoring force model and applied to express the dynamic restoring force under varying axial force in the next section.

In this study we call the restoring force ratio M_r/M_u, which is corresponding to the static bending restoring force under constant axial force, as the basic restoring force model (BRF model) to express dynamic bending restoring force model.

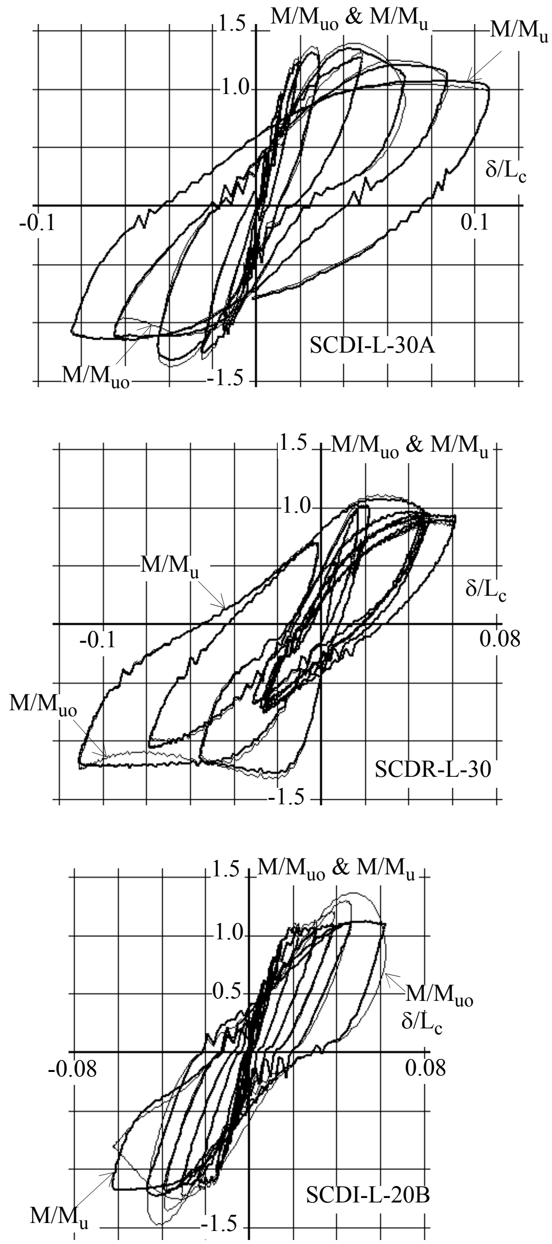

Fig.13
Load-deformation relations of dynamic loading test when the varying axial force effect is removed

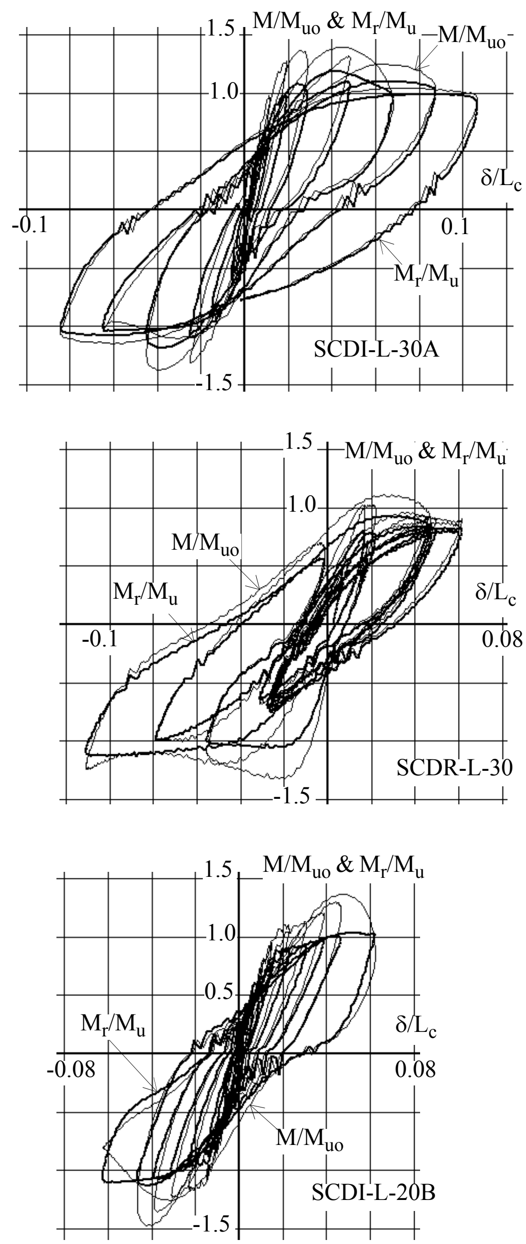

Fig.14
Load-deformation relations of dynamic loading test when the varying axial force effect and the loading rate effect are removed

4. DYNAMIC RESTORING FORCE MODEL

4.1 Static restoring force model

The dynamic restoring force model, which can be easily applied to the numerical seismic response analysis of RC frame under strong ground motion, is obtained here in relation with the loading rate effect and the varying axial force effect mentioned above.
The basic restoring force characteristics (M_r/M_u) mentioned above are corresponding to the static restoring force under constant axial force. We know well that the static restoring force characteristics under constant axial force can be well approximated by the modified Clough model as shown in Fig.15. From this reason the basic restoring force characteristics (M_r/M_u) is assumed to be approximated by the same model of static restoring force. In order to simplify the approximation the restoring force model in the small deformation is assumed to be expressed by the Bi-linear model as shown in Fig.15.
The BRF model shown in Fig.15 is compared with the static test results under constant axial force in Fig.7. Except the restoring force in the small deformation range, the maximum restoring force, the degrading stiffness and shape of hysteresis curve of test results are ascertained to be well predicted by BRF model.

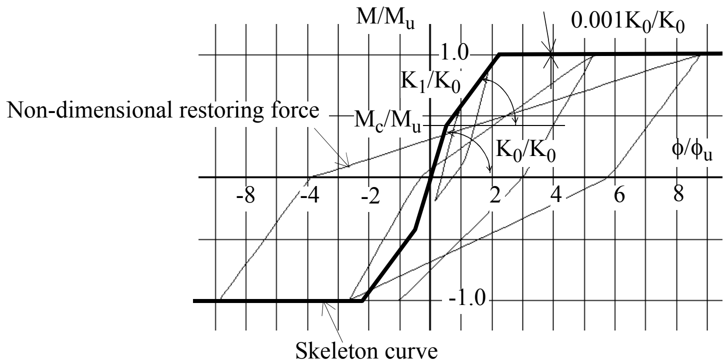

Fig.15 Basic restoring force model (BRF model) and its skeleton curve

4.2 Dynamic restoring force model

By removing the loading rate effect and the varying axial force effect from the dynamic restoring force under varying axial force, the BRF model to express M_r/M_u has been obtained. In reverse by adding to the loading rate effect and the varying axial force effect to the BRF model, we can get the dynamic restoring force under varying axial force. To add the loading rate effect and the varying axial force effect to the BRF model is carried out by the use of Eq.(2) and the ultimate bending strength (M_u). This method to predict the dynamic restoring force under varying axial force is comparatively simple and can be applied to the numerical seismic response analysis.
The dynamic restoring forces predicted by this method are shown in Fig.16 comparing with the dynamic test results under varying axial force. It is shown in Fig.16 that the restoring force calculated by the BRF model predicts well the complicated shape of hysteresis curves and the maximum restoring forces except that in the small deformation range.

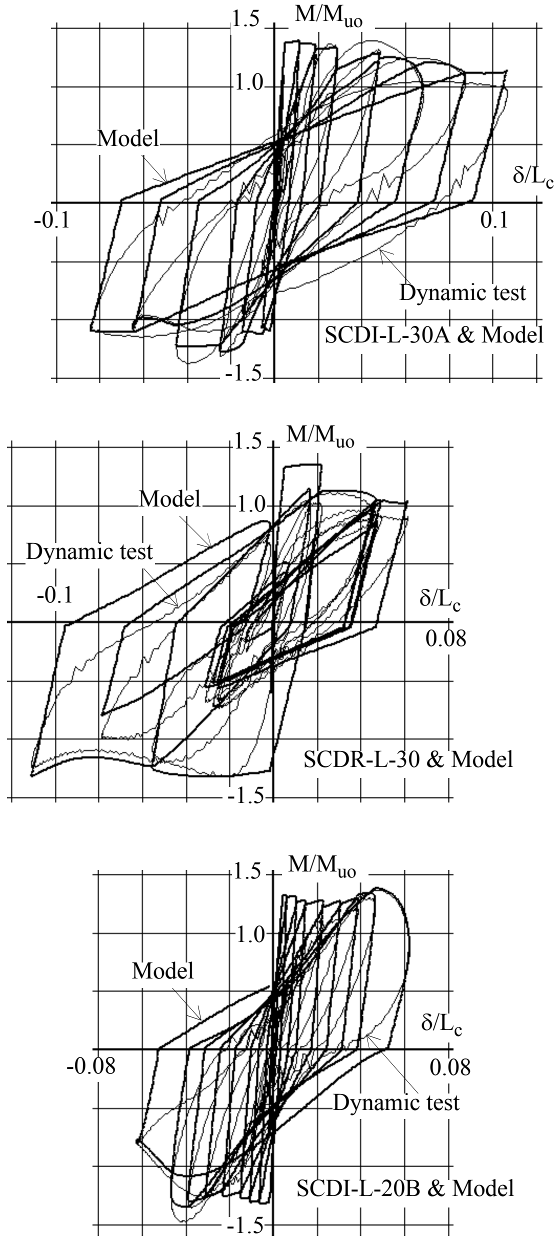

Fig.16 Dynamic restoring force predicted by proposed model
in comparison with dynamic loading test result

4.3 Accuracy of the proposed restoring force model

Comparing with the test results the accuracy of the proposed restoring force model is examined. The absorbing energy of RC column given by Eq.(3) is calculated to show the accuracy of the restoring force model.

$$E_t = \Sigma M \, \Delta\delta/L_c$$

$$E_m = \Sigma M_m \, \Delta\delta/L_c \qquad (3)$$

in which M, M_m: bending restoring force of test and model respectively, δ/L_c: deformation of column, Σ: summation of increment, E_t, E_m: absorbing energy of test and model respectively.

The values of E_t, E_m can not express the difference in the shape of hysteresis curve. To show the accuracy to approximate the hysteresis curve, the summation of the absolute restoring force difference $|M- M_m|$ in every incremental step as shown in Eq.(4) is also calculated.

$$\Delta E=\Sigma|M- M_m|\ |\Delta\delta|/L_c \qquad (4)$$

The predictions of the dynamic restoring force model and the static restoring force model are shown in Table-4 comparing with test results. The maximum restoring forces of test (${}_mM_t$) and model (${}_mM_m$) are also in the table. Concerning with the maximum restoring force, ${}_mM_m/{}_mM_t$=0.91-1.14 are obtained by the proposed dynamic restoring force model and ${}_mM_m/{}_mM_t$=0.70-0.87 are obtained by the static model. From these results we can see that the prediction by the proposed dynamic restoring force model is more accurate than that by the static model.

The energy absorbing of RC column is not well predicted by the proposed model. But the difference between the hysteresis curve of proposed model and test result expressed by $\Delta E/E_t$ is fairly small comparing with that between the static model and test result. These results show the usefulness of the proposed dynamic restoring force model.

Table-4 Accuracy of proposed restoring force model

Specimens	Dynamic model			Static model		
	${}_mM_m/{}_mM_t$	E_m/E_t	$\Delta E/E_t$	${}_mM_m/{}_mM_t$	E_m/E_t	$\Delta E/E_t$
(Dynamic test)						
SCDI-L-40A	1.02	1.18	0.35	0.83	1.15	0.41
SCDI-L-40B	1.08	1.23	0.40	0.85	1.17	0.47
SCDC-L-30	1.07	1.27	0.42	0.80	1.09	0.45
SCDI-L-30A	1.00	1.15	0.42	0.76	1.08	0.48
SCDI-L-30B	0.91	1.10	0.42	0.70	0.98	0.47
SCDR-L-30	1.02	1.26	0.54	0.80	1.08	0.58
SCDI-L-20A	1.13	1.26	0.37	0.78	1.07	0.37
SCDI-L-20B	0.94	1.74	0.84	0.71	1.47	0.85
SCDR-L-20	1.12	1.89	1.04	0.87	1.58	0.88
SCDI-S-15	1.14	1.13	0.39	0.87	1.00	0.42
SCDR-S-15	1.01	1.27	0.49	0.80	1.13	0.55
(Static test)						
SCSC-L-30	0.97	1.26	0.35	0.97	1.26	0.35
SCSR-L-30	0.95	1.29	0.64	0.95	1.29	0.64
SCSI-L-20	0.91	2.13	1.29	0.91	2.13	1.29
SCSR-S-15	0.96	1.07	0.36	0.96	1.07	0.36

5. CONCLUSIONS

The bending restoring force of RC column, which is subjected to dynamic lateral load and varying axial load simultaneously, is effected significantly by the loading rate and the varying axial force and the restoring force characteristics is extremely complicated. The loading rate effect on the restoring force of RC column can be expressed well by Eq.(2). The varying axial force effect on the bending restoring force can be predicted by the ultimate bending strength under the axial force at each instance.

By applying the Eq.(2) and the presented calculation method of the ultimate bending strength to the basic restoring force model (BRF model), which is corresponding to the static restoring force model, the dynamic restoring force model of RC column under varying axial force is obtained. It is ascertained that the derived restoring force model is comparatively accurate in comparison with dynamic loading test results and easy to apply to the seismic numerical analysis under strong ground motion.

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