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# INFLUENCE OF LATERAL PRESTRESS ON RESIDUAL SHEAR CRACK OF REINFORCED CONCRETE COLUMN

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## SUMMARY

This paper describes the investigation of the influence of lateral prestress on the crack opening behavior within reinforced concrete columns by experiments. From the experiments, it is found that not only shear crack strength but also allowable shear force which makes residual shear crack opening under a target value increased with lateral prestressing.

## INTRODUCTION

According to the building design based on structural performance, damage on reinforced concrete buildings such as crack must be controlled to implement the required performance. From views of durability, waterproof and appearance, the shear crack within the reinforced concrete column should be prevent or its residual opening should be controlled. The shear crack opening also should be controlled to keep the residual strength of the column, since its transverse reinforcements stresses and residual deformation become larger due to increasing of the crack width.

When the design was performed to control the residual shear crack opening within reinforced concrete columns, it is seemed that allowable shear force for temporary loading formula in Architectural Institute of Japan Standard of Reinforced Concrete Structures [1] might be available. However, AIJ Standard does not describe clearly on the residual crack width control. Based on experimental results in beam, the residual shear crack width was predicted by using the allowable shear force formula above as index of damage by Fukuyama et al. [2]. Crack widths could not be controlled in some member, and verification based on other index was required. An axial load could not be neglect, although the effect of the axial load was not considered in the formula, when the shear crack strength was examined. Thus, it might be needed:

a) Method of restricting residual shear crack opening.
b) To control residual crack opening, evaluation index at design.

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This paper describes the effect of lateral prestressing into column [3] [4] on the shear crack behavior for moderate earthquake motion, and develops the methods of preventing a shear crack initiation, or controlling the residual shear crack width. An evaluation method of allowable shear force which makes residual shear crack width within the column under the control target value, as control index, was also considered.

**Table 1 List of Test Specimens**

| No. | $b$, $D$ (mm) | $d_w$ (mm) | $\sigma_0/f_c$ | $s$ (mm) | $p_w$ (%) | $p_{wp}$* (%) | $f_c$ (N/mm$^2$) | $f_{ct}$ (N/mm$^2$) | $f_{wp}$ (N/mm$^2$) | $\sigma_L$ (N/mm$^2$) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 340 | 6.4*, and D16** | 0.30 | 60 | 2.54 | 0.29* | 40.1 | 2.09 | 859 | 2.5 |
| 2 | | | | | | | 45.0 | 2.21 | 528 | 1.6 |
| 3 | | | | | | | 48.0 | 2.29 | 0 | 0.0 |
| 4 | | 6.4*, and D13** | | | 0.29 | | 35.4 | 1.96 | 876 | 2.6 |
| 5 | | | | | | | 35.3 | 1.96 | 0 | 0.0 |

*pretensioned hoop, **sub hoop, $b$ is breadth of column, $D$ is depth of column, $d_w$ is nominal diameter of transverse hoop, $\sigma_0$ is axial stress of column, $f_c$ is compressive strength of concrete, $s$ is spacing of transverse hoops in longitudinal direction, $p_w$ is ratio of transverse hoop(=$(A_{wn}+A_{wp})/(b \cdot s)$), $p_{wp}$ is ratio of transverse hoop used in prestressing(=$(A_{wp})/(b \cdot s)$), $f_{ct}$ is tensile strength of concrete, $A_{wn}$ is cross area of one pair of transverse reinforcement without prestressing, $A_{wp}$ is cross area of one pair of transverse reinforcement with prestressing and $\sigma_L$ is lateral prestress(=$p_{wp} \cdot f_{wp}$)

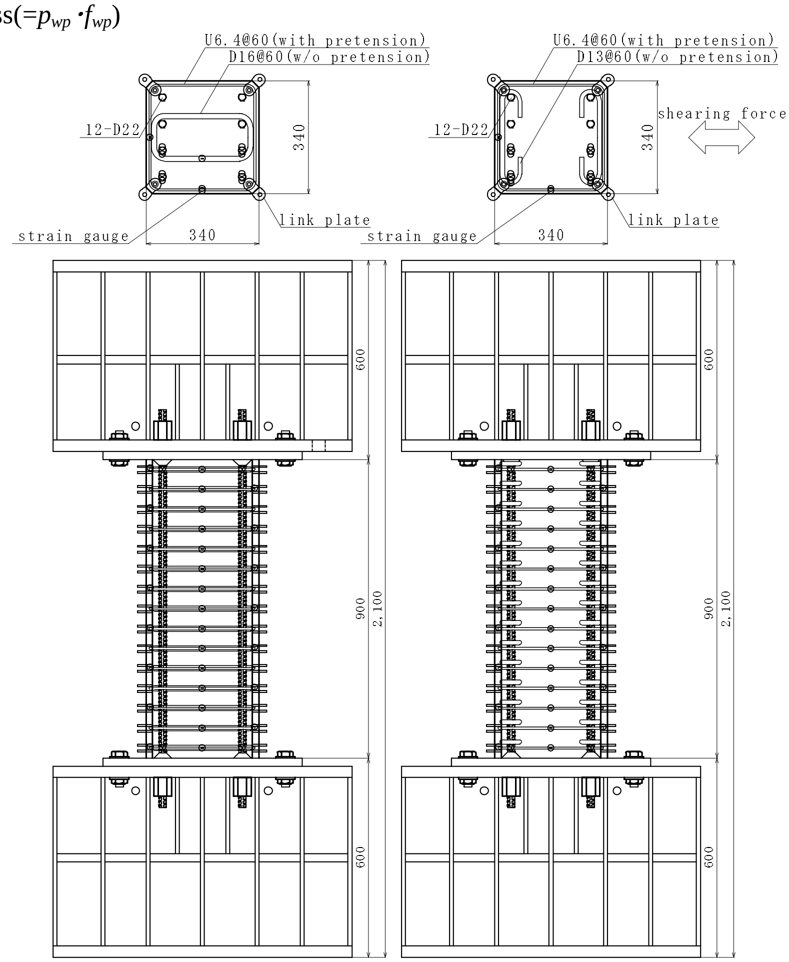



**Fig. 1 Details of Test Specimens: (a) No.1-3, and (b) No.4 and 5 (unit: mm)**

**Table 2 Mechanical Properties of Steel used in the Experiments**

| Type | Material | $f_{sy}$, $f_{wy}$ (N/mm$^2$) | $f_{st}$ (N/mm$^2$) | $E_s$ (kN/mm$^2$) |
|---|---|---|---|---|
| D22 | – | 1016* | 1162 | 206 |
| U6.4 | – | 1441 | 1465 | 197 |
| D13 | SD345 | 378 | 535 | 198 |
| D16 | SD295A | 344 | 513 | 202 |

*0.2% offset, $f_{sy}$ and $f_{wy}$ are yield strength of steel, $f_{st}$ is tensile strength of steel, and $E_s$ is steel Young's modulus

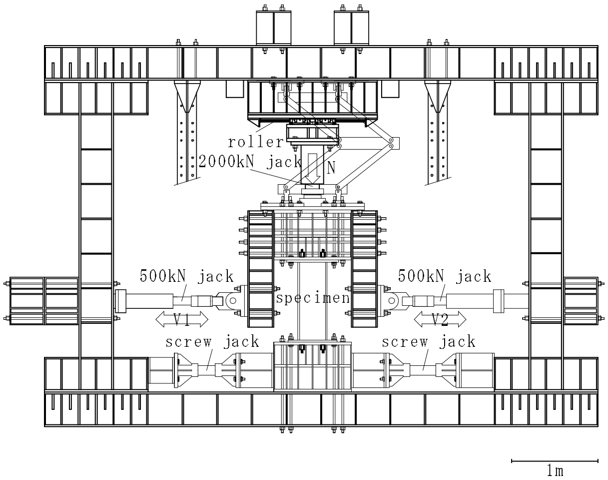



**Fig. 2 Sketch of Loading Apparatus**

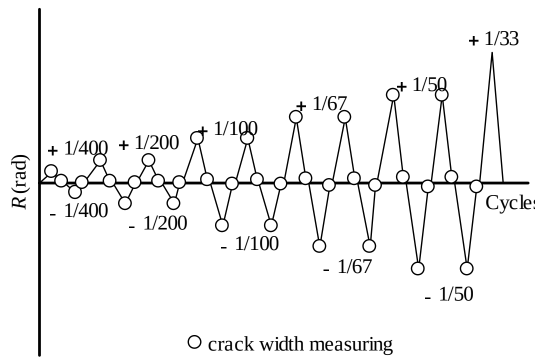



**Fig. 3 Horizontal Forces Loading Path**

## TEST PROGRAM

### Test Specimens

Table 1 lists test specimens, and Fig. 1 shows details of the specimens. The test specimens were total five specimens which have square shaped section 340mm×340mm, height 900mm. Five specimens were designed as columns which occur the shear failure in ultimate condition, before the longitudinal reinforcement had been yielded, without bond splitting failure, based on AIJ Guidelines for Reinforced Concrete Buildings [5]. Principal variables were effective tensile stress into a piece of transverse reinforcement $f_{wp}$ (60, 37 and 0% of its yield). Two types of sub reinforcements were used (see Figs. 1(a) and (b)). In this experiment, the transverse hoop used for prestressing was only outer one. The cover to transverse reinforcements was 12mm. The maximum particle size of coarse aggregate was 25mm. Cement was high early strength Portland cement. Mechanical properties of steel used in the experiments were shown in Table 2.

### Loading and Measuring Method

The loading apparatus is shown as Fig. 2. Vertical force on the test specimen was supplied by one hydraulic jack with the capacity of 2MN, and axial load ratio (on assumption that this axial load equal to dead load without consideration of steel) was kept constant as 0.30 controlled in the load during test. To facilitate the comparison among the shear crack strength of columns with different prestress, higher axial force ratio was adopted, so that absolute value of difference among specimens become larger. Horizontal forces on test specimen was supplied by two hydraulic jacks with the capacity of 500kN, and controlled in displacement during test. Horizontal forces were applied in cyclic, and the reversal of deformation in the upper and lower in the upper and lower halves of the specimen. Tests repeated, once at deformation angle of member $R=\pm 1/400$, two times, at $R=\pm 1/200$, $\pm 1/100$, $\pm 1/67$ and $\pm 1/50$, once at $R=\pm 1/33$, and finished at $\pm 1/25$. $R$ is the horizontal relative displacement between top and bottom of the column

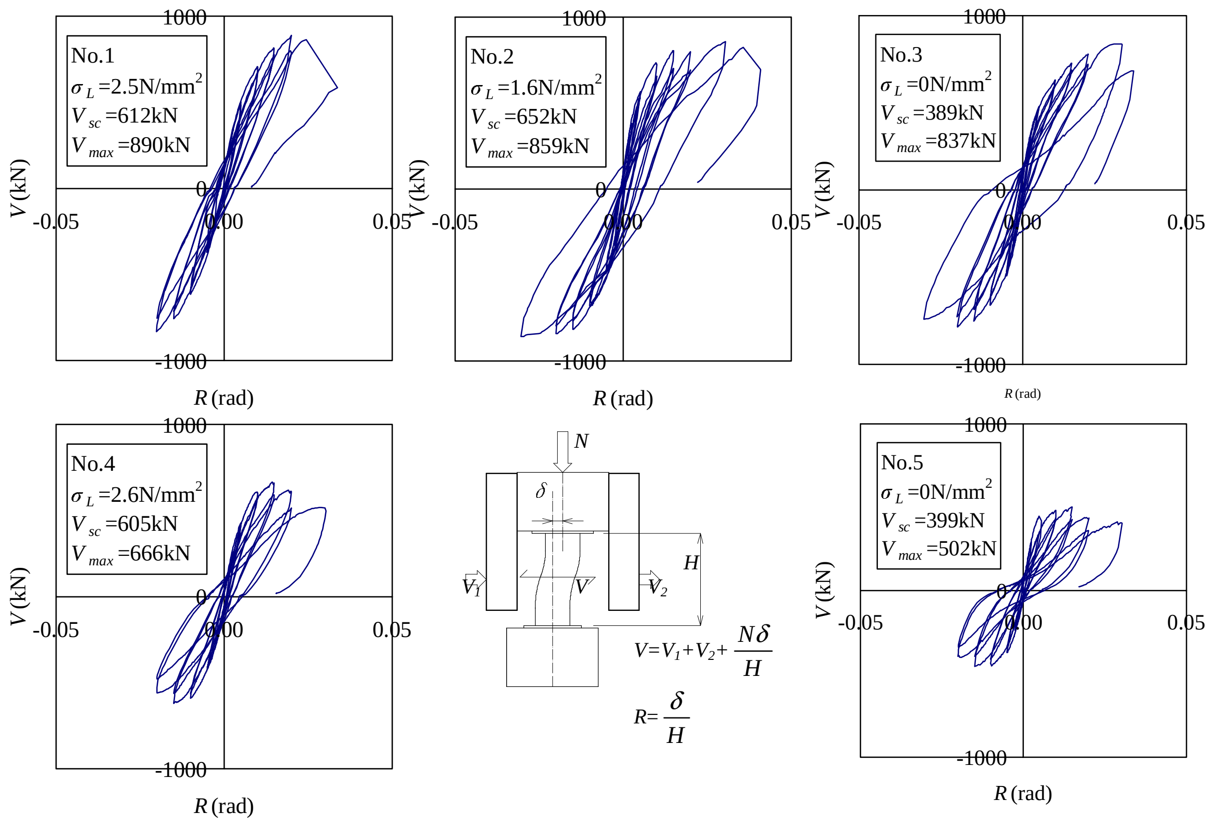



**Fig. 4 *V-R* Curves (at –1/50≦R≦＋1/50)**

divided by its height. Shear crack openings were measured by using digital micro scope (which had minimum divisions of a scale, 0.01mm) at the deformation peak and the horizontal force unloaded (additional shear force due to dead load was still residual) of each cycle until ± 1/50, after cracks occurring. Fig. 3 shows the horizontal forces loading path. Shear cracks upon the transverse reinforcement and the middle point between two transverse reinforcements were measured.

## Lateral Prestressing Method

The lateral prestress is applied into the concrete with high strength transverse hoops pretensioned mechanically. Reacting forces of the pretension is taken with the steel cast, and the cast is removed after concrete hardening. The lateral prestress is introduced just before axial force because the dead load of

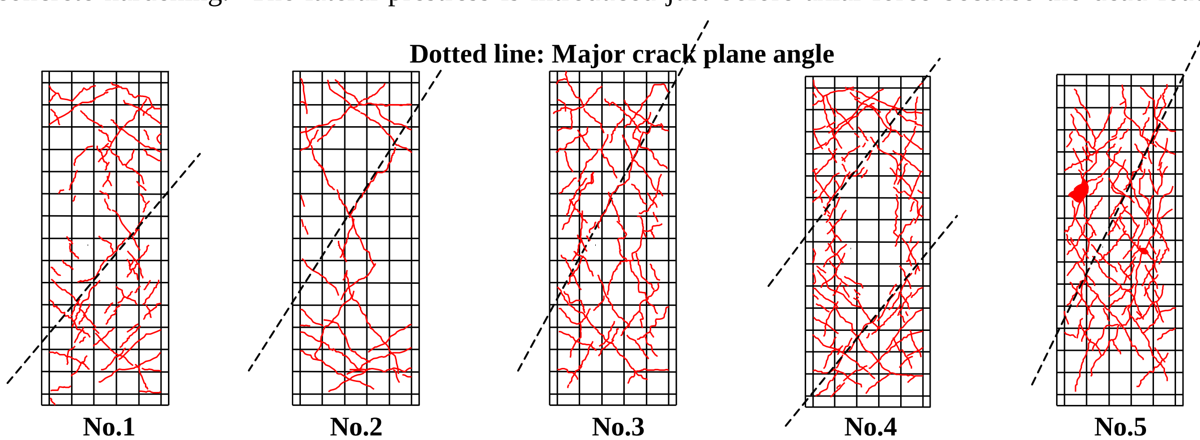



**Fig. 5 Observed Crack Patterns at *R*=1/50**

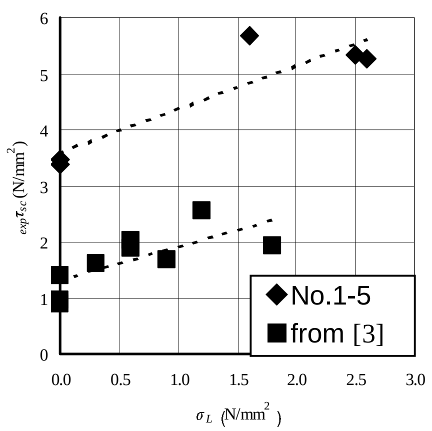



**Fig. 6 Relations Between Shear Crack Stress and Lateral Prestress**

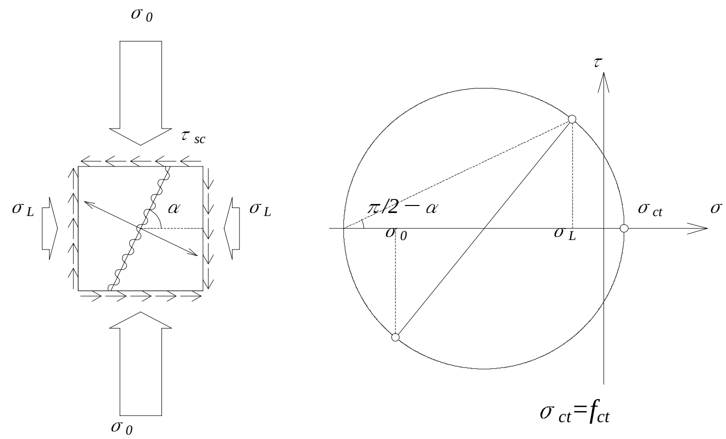


**Fig. 7 Assumption of Mohr's Circle due to Lateral Prestressing**

assumed upper structure is loaded into the column after the construction, such as Precast column. Lateral prestress $\sigma_L$ was defined as value which was the product of the ratio of the transverse reinforcement used in prestressing and the transverse reinforcement stress before column had been loaded axially.

## TEST RESULTS

### *V-R* Curves

Each hysteresis of specimens were shown in Fig. 4. The typical damage process observed on the specimens with increasing the shear force is follow. After flexural crack appeared, flexural shear cracks propagated and shear cracks occurred in the middle of specimens until the maximum strength reached.

### Failure Mode

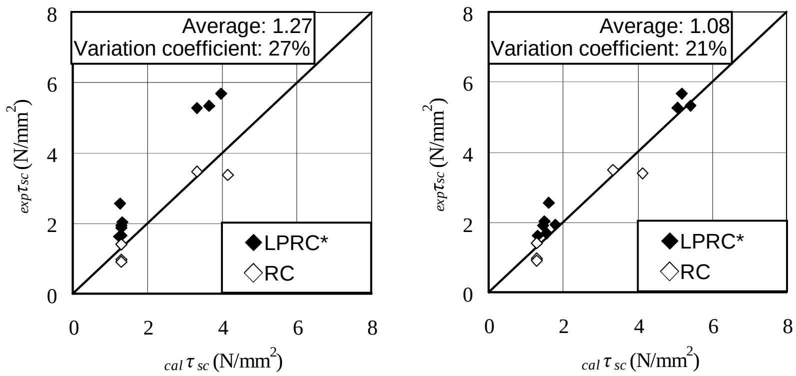
Observed crack patterns at *R*=1/50 were shown in Fig. 5. Here, an angle of crack which reached severe opening was indicated in a broken line. According to an increase of lateral prestress, these angles relative to the axis of the column approximated 45 degrees from the axial direction of the column. Finally, all specimen represented the shear compression failure without flexural yield.

### Effects of Lateral Prestress on Shear Crack Strength

Relations between the shear crack strength $_{exp}V_{sc}$ and the lateral prestress was considered. Relations between the shear crack stress $_{exp}\tau_{sc}$ ($=_{exp}V_{sc}/bD$) and the lateral prestress were plotted in Fig. 6, together with data from literature [3]. The shear crack strength increased with increasing lateral prestress.

### Evaluation of Shear Crack Strength

The verification of precision was estimated by the evaluation equation of the shear crack strength based on the maximum principal stress theory which proposed in literature [4]. The equation was not derived empirically from the statistics of experimental results, based on the hypothesis which could be explained theoretically. Eq. (1) adopted in AIJ Design Guidelines for Reinforced Concrete Buildings [5] and AIJ Standard of Prestressed Concrete Structures [6]. Actually, the lateral prestressed reinforced concrete column was seated under three dimensional stresses condition with combination of the lateral prestress and the axial load, but here, projected into two dimensions like Fig. 7. The comparison to estimate precisions of the calculated values between usual evaluation Eq. (1) of the shear crack strength without consideration of the lateral prestress and the proposed evaluation Eq. (2) with consideration of the lateral prestress, was examined, adding 10 columns from literature [3].




(a) (b) *LPRC (Lateral Prestressed Reinforced Concrete)

**Fig. 8 Comparisons of Shear Crack Stress Between Experimental Results and Calculations: (a) by Eq.(1) and (b) by Eq.(2)**

$$\tau_{sc} = \sqrt{(f_{ct})(\sigma_0 + f_{ct})}\Big/\kappa \quad (1)$$

$$\tau_{sc} = \sqrt{(\sigma_L + f_{ct})(\sigma_0 + f_{ct})}\Big/\kappa \quad (2)$$

where $f_{ct}$ is the concrete tensile strength, $\sigma_L$ is the axial stress of the column and $\kappa$ is constant ($\kappa$ =1.5). $f_{ct}$ was calculated by Eq. (3) which was adopted from literature [7] same as AIJ Design Guidelines [5]. Unit of $f_c$ is in N/mm$^2$.

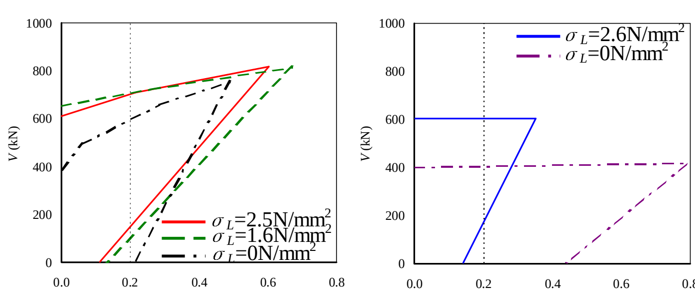
$$f_{ct} = 0.33\sqrt{f_c} \quad (3)$$

Fig.8 shows the estimated precision on the shear crack stress both calculated by original Eq. (1) and proposal Eq. (2). The proposed Eq. (2) takes accounts of the lateral prestress, which had the coefficient of variation 21%, estimate in safely with smaller dispersion than original Eq. (1), which had the coefficient of variation 27%. Thus, prediction accuracy was given by using Eq. (2), more than using present design formula.

## Effect on Damage

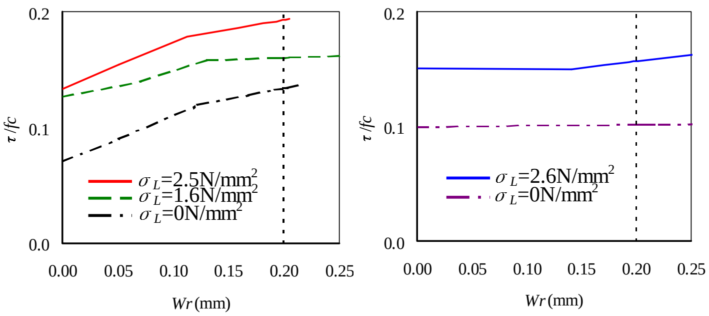
Fig. 9 shows the envelopes of the hysteresis of shear force $V$ - shear crack width $W$. Here the crack width $W$ is the maximum shear crack width, which had been measured on the surface of the specimens. Starting from the shear crack strength, and $W$ at shear force reaching almost zero (additional shearing force due to dead load was still residual) represents the residual shear crack width $W_r$. For the lateral prestressed reinforced concrete column, $W_r$ is prevented in small value, even if the column had experienced larger shear force or crack opening than the usual reinforced concrete column had experienced.

Relations between the residual shear crack widths and the shear stresses which apply the widths are




(a) (b)

**Fig. 9 Relations Between Shearing Force and Shear Crack Width: (a) $p_w$=2.54%, and (b) $p_w$=0.29%**




(a) (b)

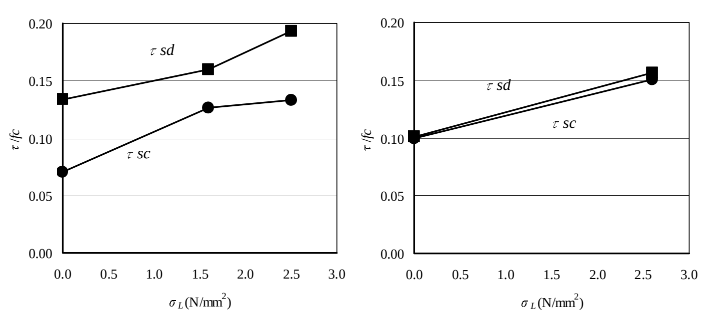
**Fig. 10 Relations Between Shearing Stress and Residual Shear Crack Width: (a) $p_w$=2.54%, and (b) $p_w$=0.29%**

shown in Fig. 10. The shear stress applied residual shear crack width as 0.2mm, which increased with lateral prestress. For the column with larger quantity of transverse reinforcements, the shear stress increased from occurring of shear crack to residual shear crack width reaching 0.2mm, for the column with smaller quantity of transverse reinforcements, shear stress at residual shear crack width reaching 0.2mm was equal to shear cracking stress. For the columns had smaller quantity of transverse reinforcement, when shear crack occurred, then residual crack opening reached 0.2mm.

## Definition of Shear Damage Strength

The shear stress is defined as "*shear damage stress*" $\tau_{sd}$ which applies the control target value on the residual shear crack width. Here, the control target value on the crack width was indicated by AIJ Recommendations for Design of Partially Prestressed Concrete [8], as 0.2mm. From a viewpoint of durability, the absolute value of the crack width is adopted as 0.2mm without its reduction in scale-downed specimen used in this experiment. However, when the total depth of a member becomes two times, also residual crack width becomes about two times, even if the member had been experienced the equivalent shear stress, experimental results were reported [9]. The control target value of the crack width should be given great attention to the actual design. Relations between the shear crack stress $\tau_{sc}$ and the lateral prestress $\sigma_L$, and relations between the *shear damage stress* $\tau_{sd}$ and the lateral prestress $\sigma_L$ were shown in Fig. 11. Both shear crack strength and *shear damage strength* increased with lateral prestress. The residual crack width is defined under the following condition, as the inputted horizontal load into column is unloaded and the additional shear force due to dead load is still residual.

In particular, for the specimen with large quantity of transverse reinforcement, under same shear stress, residual shear crack width on the reinforced concrete column with no prestress reached 0.2mm, while lateral prestressed reinforced concrete column ($\sigma_L$ =2.5N/mm$^2$) had no shear crack (see Fig. 10 (a)). *Shear damage stress* increased in lateral prestressed column one and a half times than reinforced concrete




(a) (b)

**Fig. 11 Relations Between *Shear Damage Strength* and Lateral Prestress: (a) $p_w$=2.54%, and (b) $p_w$=0.29%**

column.

## Evaluation of Shear Damage Strength

To control damage on reinforced concrete column, the estimate method of shear damage strength $V_{sd}$ which makes the residual shear crack width into the control target value, is expressed in this paper. It is always after cracking that the residual shear crack occurs. Since the tensile force due to the horizontal load becomes impossible to be subjected by concrete after cracking occur, therefore, be supported by transverse reinforcements instead. The transverse reinforcement subjected tensile force was required the tensile strain. When the transverse reinforcement is in elastic condition, the tensile stress in a single piece of the reinforcement is taken as

$$f_w = W_r \cdot E_s / (j \cdot \sin\alpha) \tag{4}$$

where, $j$ is the distance between corner longitudinal reinforcement bars, and $\alpha$ is an angle at the diagonal crack plane relative to the horizontal plane. The crack width used in Eq. (4) is the crack width at *shear damage strength*. Correctly, the residual crack width differs from the crack width in Eq. (4). Here, the crack width in Eq. (4) is assumed that equal to the residual crack width, since the crack width in Eq. (4) equal to sum total of the crack width and strain distribution in transverse reinforcements is un-uniform.

By assuming *shear damage strength* equals to the total subjected load by all transverse reinforcement across shear crack surface, *shear damage strength* ${}_{cal}V_{sd1}$ given by

$${}_{cal}V_{sd1} = (f_w + f_{wp}) A_{wp} \cdot n + f_w \cdot A_{wn} \cdot n \tag{5}$$

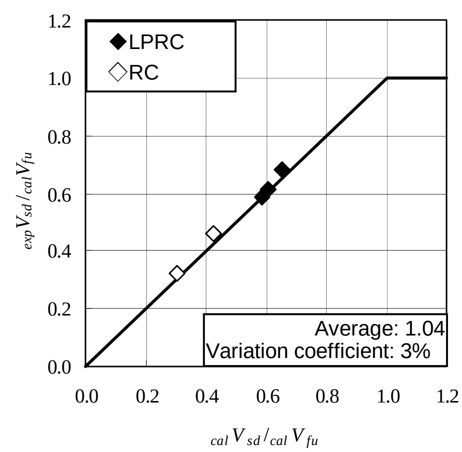
where, $A_{wp}$ is the cross area of one pair of transverse reinforcement with prestressing, $A_{wn}$ is the cross area of one pair of transverse reinforcement without prestressing, and $n$ is the number of pieces of transverse reinforcement crossing the shear crack surface. Eq. (5) takes account of the lateral prestress by including the transverse reinforcement effective tensile stress $f_{wp}$.

Eq. (5) required number of transverse reinforcement crossing the crack surface. Judging from the crack

**Table 3 Calculated Results of Shear Damage Strength**

| No. | $n$ | ${}_{cal}V_{sd1}$ (kN) | ${}_{cal}V_{sc}$ (kN) | ${}_{exp}V_{sc}$ (kN) | ${}_{cal}V_{sd}$ (kN) | ${}_{exp}V_{sd}$ (kN) | ${}_{exp}V_{sd}/{}_{cal}V_{sd}$ |
|---|---|---|---|---|---|---|---|
| 1 | 6 | **850** | 622 | 612 | **850** | 886 | 1.04 |
| 2 | 7 | **819** | 595 | 652 | **819** | 827 | 1.01 |
| 3 | 9 | **738** | 476 | 389 | **738** | 738 | 1.00 |
| 4 | 6 | 384 | **582** | 605 | **582***  | 629 | 1.08 |
| 5 | 10 | 106 | **382** | 399 | **382*** | 407 | 1.07 |

*Bold number shows the bigger value.* (*${}_{cal}V_{sd1} \le {}_{cal}V_{sc}$)



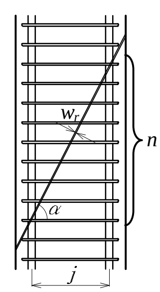




**Fig.12 Comparisons of *Shear Damage Strength* Between Experimental Results and Calculations**

patterns, the number of transverse reinforcement was defined. Since the residual crack width do not occur before crack occur, $V_{sd}$ is calculated as the larger one, ${}_{cal}V_{sd1}$ or ${}_{cal}V_{sc}$ (which calculated by Eq. (2)).

Calculating *shear damage strength* of specimens of this experiment, Fig. 12 and Table 3 were obtained. Both axes are normalized by ${}_{cal}V_{fu}$, the shear force when the bending moment at the column end section reaches the theoretical flexural capacity. $V_{sd}$ was defined as ${}_{cal}V_{sd1}$ for No.1-3, while $V_{sd}$ was defined as ${}_{cal}V_{sc}$ for No.4 and 5. This phenomenon corresponded to results of Fig. 10.

Average of experimental value/calculated value was 1.04 and the coefficient of variation was 3%. Above is evaluated with sufficient accuracy and safely.

## CONCLUSIONS

From the behavior observed during the flexure-shear experiment on the lateral prestressed reinforced concrete column and results presented above, the following conclusions can be drawn:

The shear crack strength on column able to be increased by introduction of lateral prestress. Moreover, based on a maximum tensile stress criterion, the evaluation method of shear crack strength of usual Reinforced Concrete column and Lateral Prestressed Reinforced Concrete column was shown.

By introducing of lateral prestress, improvement of the *shear damage strength* could be recognized, which makes the residual shear crack width to control target value is newly defined. By taking account of the lateral prestress as transverse reinforcements stresses crossing shear crack surface *shear damage strength* was evaluated. The design could allow larger shear force into the columns by lateral prestressing, when the occurrence of the shear crack is prevented or the residual shear crack width is controlled.

By using evaluation method of *shear damage strength* which takes accounts of only load subjected by transverse reinforcements, input shear force into column makes residual crack width to control target value was calculated safely with sufficient accuracy.

## REFERENCES


1. Architectural Institute of Japan. “AIJ Standard for Structural Calculation of Reinforced Concrete Structures –Based on Allowable Stress Concept-.” 1999
2. Fukuyama H, Suwada H, Iso M, Matsuzaki Y, Nakano K, and Kasahara M. “Evaluation of Damage Limit State of RC Elements Due to Shear Crack Width (Part. 1 In case of beams and columns).” Summaries of Technical Papers of Annual Meeting Architectural Institute of Japan, C-2, 2000: 13-14
3. Watanabe H, Makitani E, Ito Y, and Arima H. “Mechanical behaviors of laterally prestressed concrete columns by high strength hoops under compressive or shear conditions.” Journal of Structural and Construction Engineering, No.552, 2000: 133-140
4. Watanabe H, Katori K, Shinohara Y, and Hayashi S. “Effects of Lateral Prestress on Damage of Reinforced Concrete Columns.” Proceedings of the Japan Concrete Institute, Vol.25, No.2, 2003: 193-198
5. Architectural Institute of Japan. “Design Guidelines for Earthquake Resistant Reinforced Concrete Buildings Based on Inelastic Displacement Concept.” 1999: 91-137
6. Architectural Institute of Japan. “Standard for Structural Design and Construction of Prestressed Concrete Structures.” 1998
7. Collins MP. and Mitchell D. “Prestressed Concrete Structures.” Prentice Hall, Englewood Cliffs, NJ, 766, 1991
8. Architectural Institute of Japan. “Recommendations for Design and Construction of Partially Prestressed Concrete (Class III of Prestressed Concrete) Structures.” 2003

9. Honjou M, Nagae T, Yanase T, and Hayashi, S. “Experimental Study on Size Effect in Shear Failure of Reinforced Concrete Pile.” Proceedings of the Japan Concrete Institute, Vol.23, No.3, 2001: 979-984