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DESIGN SPECTRA FOR BURIED PIPELINES

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SUMMARY

For a buried pipeline system, the maximum responses along the pipe segments and the maximum differential deformations across the joints are of design concern. A simple model of buried pipeline system with uniform properties is used to evaluate the modal parameters and the static design terms. When the modal parameters of the pipeline system are combined with the homogeneous soil properties into the equivalent modal frequencies and damping ratios, the modal dynamic effect can be found from the design spectra proposed for buildings at the same site. Almost all the static design terms depend on the rotational and lateral stiffness ratios in the lateral vibration, or on the axial stiffness ratio in the axial vibration. As a result, the first mode dominates the design-related responses in the most cases of lateral and axial vibrations.

INTRODUCTION

The main concern in the seismic damage evaluation of a buried pipeline system is probably the differential motions across the joints. Nelson [1] introduced a discrete model containing two long pipe segments connected by a joint to study the so-called interference response spectrum. Each segment is linked to the ground via a spring and a dashpot. The same model was used by Hadid [2] in the stochastic response analysis of pipelines.

On the other hand, if the seismic responses along the pipe segments are of major interest, the pipelines are usually modeled as uniform beams rested on elastic foundations. The length of the pipeline could be finite (Hindy [3], Zerva [4]), semi-infinite (Novak [5]), or infinite (Ogawa [6]).

The buried pipeline model considered here is similar to that proposed by Wang [7], except that the infinitely rigid segment is replaced by a uniform beam and the lateral vibration is included additionally. In this buried pipeline model, the beam segments of equal length are aligned and connected by identical joints, and the pipe-soil interaction is simulated by an elastic foundation with uniformly distributed springs and dashpots. Several design-related responses, such as the maximum deformation and force along the segments and the maximum differential deformation across the joints, are studied when this buried pipeline system is excited by the axial and lateral ground motions.

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PIPELINE MODELS

In accounting for both the responses along the pipe segments and the differential responses across the joints in a buried pipeline system, the analysis model for axial vibration is shown in Fig. 1. Uniform pipe segments of equal length connected by identical joints are excited by the ground motion in the longitudinal direction. Hence, the equations of axial motion are

$$\bar{m}\frac{\partial^2 u_j(x_j,t)}{\partial t^2} - EA\left[\frac{\partial^2 u_j(x_j,t)}{\partial x_j^2} + c\frac{\partial^3 u_j(x_j,t)}{\partial x_j^2 \partial t}\right] + \left[\bar{k}_S u_j(x_j,t) + \bar{c}_S \frac{\partial u_j(x_j,t)}{\partial t}\right] = -\bar{m}\frac{\partial^2 u_g(x_j,t)}{\partial t^2} \tag{1}$$

where j is the number of segments in the range of $1 \le j \le N$, N is the total number of the segments used for analysis, $\bar{m}$, E, A and c are the mass per unit length, Young's modulus, cross-sectional area and damping coefficient of the segments, respectively, u_g is the ground displacement in the longitudinal direction, u_j is the axial deformation of the jth segment, x_j is the axial local coordinate in the range of $0 \le x_j \le L$, and L is the length of a single pipe segment.

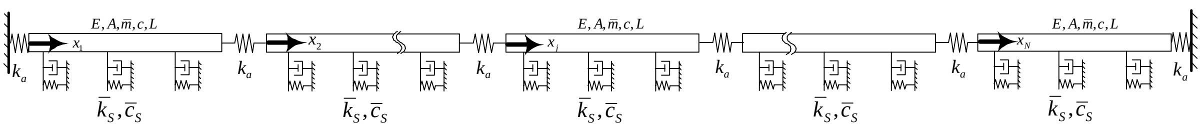

Figure 1 Pipeline model for axial vibration analysis

For simplicity in the vibration analysis, the effect of pipe-soil interaction is simulated by an elastic foundation with continuous springs of stiffness $\bar{k}_s$ and continuous dashpots of damping coefficient $\bar{c}_s$, as shown in Figure 1 and Eq. (1).

On the other hand, the pipeline model proposed for lateral vibration analysis is shown in Figure 2. Again, the elastic foundation and the uniform properties in pipe segments and joints are assumed. The equations of lateral motion are

$$\bar{m}\frac{\partial^2 u_j(x_j,t)}{\partial t^2} + EI\left[\frac{\partial^4 u_j(x_j,t)}{\partial x_j^4} + c\frac{\partial^5 u_j(x_j,t)}{\partial x_j^4 \partial t}\right] + \left[\bar{k}_S u_j(x_j,t) + \bar{c}_S \frac{\partial u_j(x_j,t)}{\partial t}\right] = -\bar{m}\frac{\partial^2 u_g(x_j,t)}{\partial t^2} \tag{2}$$

where EI is the flexural rigidity of pipe segments, u_g is the ground displacement in the lateral direction, u_j is the lateral deformation of the jth segment.

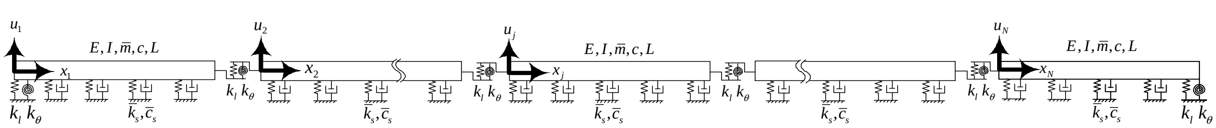

Figure 2 Pipeline model for lateral vibration analysis

MODAL ANALYSIS

Both Eq. (1) and Eq. (2) could be decomposed into modal equations if the damping ratio of each mode is assumed to dissipate the energy. Only the modal analysis performed to Eq. (2) is discussed in detail here. For the case of free vibration, the equation of lateral motion of the *j*th pipe segment is

$$\bar{m}\frac{\partial^2 u_j(x_j,t)}{\partial t^2}+EI\left[\frac{\partial^4 u_j(x_j,t)}{\partial x_j^4}+c\frac{\partial^5 u_j(x_j,t)}{\partial x_j^4\partial t}\right]+\left[\bar{k}_S u_j(x_j,t)+\bar{c}_S\frac{\partial u_j(x_j,t)}{\partial t}\right]=0 \tag{3}$$

By virtue of the separation of variables, i.e., $u_j(x_j,t)=\sum_{k=1}^{\infty}\phi_{jk}(x_j)q_k(t)$, Eq. (3) becomes

$$\frac{EI\phi_{jk}^{\mathrm{IV}}(x_j)}{\bar{m}\phi_{jk}(x_j)}=-\frac{\ddot{q}_k(t)+\dfrac{\bar{c}_S}{\bar{m}}\dot{q}_k(t)+\dfrac{\bar{k}_S}{\bar{m}}q_k(t)}{c\dot{q}_k(t)+q_k(t)} \tag{4}$$

If the constant obtained from the two expressions in Eq. (4) is ω_k^2, then we have two ordinary differential equations:

$$EI\phi_{jk}^{\mathrm{IV}}(x_j)-\omega_k^2\bar{m}\phi_{jk}(x_j)=0 \tag{5}$$

$$\ddot{q}_k(t)+\left(\frac{\bar{c}_S}{\bar{m}}+c\omega_k^2\right)\dot{q}_k(t)+\left(\frac{\bar{k}_S}{\bar{m}}+\omega_k^2\right)q_k(t)=0 \tag{6}$$

It is obvious in Eqs. (5) and (6) that the modal shape, $\phi_{jk}(x_j)$, is independent of the soil properties, but the modal dynamic effects, $q_k(t)$, should be referred to the compound frequencies and damping coefficients. The *k*th-mode shape of the *j*th segment, $\phi_{jk}(x_j)$, solved in Eq. (5) is

$$\phi_{jk}(x_j)=C_{jk1}\sin\beta_k x_j+C_{jk2}\cos\beta_k x_j+C_{jk3}\sinh\beta_k x_j+C_{jk4}\cosh\beta_k x_j \tag{7}$$

where $\beta_k^4=\omega_k^2\bar{m}/EI$. Furthermore, the modal properties in Eq. (7), such as β_k and C_{jk}, can be determined by the boundary conditions at both ends:

$$EI\frac{\partial^2\phi_{1k}(0)}{\partial x_1^2}=k_\theta\frac{\partial\phi_{1k}(0)}{\partial x_1} \tag{8}$$

$$EI\frac{\partial^3\phi_{1k}(0)}{\partial x_1^3}=-k_l\phi_{1k}(0) \tag{9}$$

$$EI\frac{\partial^2\phi_{Nk}(L)}{\partial x_N^2}=-k_\theta\frac{\partial\phi_{Nk}(L)}{\partial x_N} \tag{10}$$

$$EI\frac{\partial^3\phi_{Nk}(L)}{\partial x_N^3}=k_l\phi_{Nk}(L) \quad (11)$$

and the compatibility conditions at joints:

$$EI\frac{\partial^2\phi_{jk}(L)}{\partial x_j^2}=k_\theta\left[\frac{\partial\phi_{(j+1)k}(0)}{\partial x_{j+1}}-\frac{\partial\phi_{jk}(L)}{\partial x_j}\right] \quad (12)$$

$$EI\frac{\partial^2\phi_{(j+1)k}(0)}{\partial x_{j+1}^2}=k_\theta\left[\frac{\partial\phi_{(j+1)k}(0)}{\partial x_{j+1}}-\frac{\partial\phi_{jk}(L)}{\partial x_j}\right] \quad (13)$$

$$EI\frac{\partial^3\phi_{jk}(L)}{\partial x_j^3}=k_l\left[\phi_{jk}(L)-\phi_{(j+1)k}(0)\right] \quad (14)$$

$$EI\frac{\partial^3\phi_{(j+1)k}(0)}{\partial x_{j+1}^3}=k_l\left[\phi_{jk}(L)-\phi_{(j+1)k}(0)\right] \quad (15)$$

where k_θ and k_l are the rotational and lateral stiffness of a joint, respectively. It is assumed that the pipeline system is connected to two fixed ends by the two identical joints. This assumption is reasonable for the response analysis in the middle pipe segments for a large pipeline system. However, the selection of the number of segments in the pipeline system is limited by the assumptions of equal length and straight alignment of segments, and uniform soil properties.

In view of Eqs. (8) through (15), the modal parameters depend only on two stiffness ratios defined as

$$\rho_\theta=\frac{k_\theta}{EI/L} \quad (16)$$

$$\rho_l=\frac{k_l}{EI/L^3} \quad (17)$$

The values of the rotational and lateral stiffness ratios vary in a wide range and are quite different between DIP and PVC pipes. For comparison with the exact modal frequencies in the pipeline systems of simple boundary conditions, such as rollers, hinges, fixed supports, and even free ends, the modal frequencies are non-dimensionized as follows:

$$\tilde{\omega}_k=\frac{\omega_k}{\sqrt{2k_l/\overline{m}}}\sqrt{1+2\rho_l}=\frac{\omega_k}{\sqrt{EI/\overline{m}L^3}}\sqrt{1+\frac{1}{2\rho_l}} \quad (18)$$

For example, the variation of the first three $\tilde{\omega}_k$ in a pipeline system of three segments is shown in Figure 3, where the dash lines represent the exact results for the case of simple boundary conditions. It is noted that the modal frequencies are so well-separated that the SRSS rule for the modal combination is expected to provide excellent response estimates. Obviously, the well-separated modal frequencies come from the assumption of uniform properties of the segments and joints.

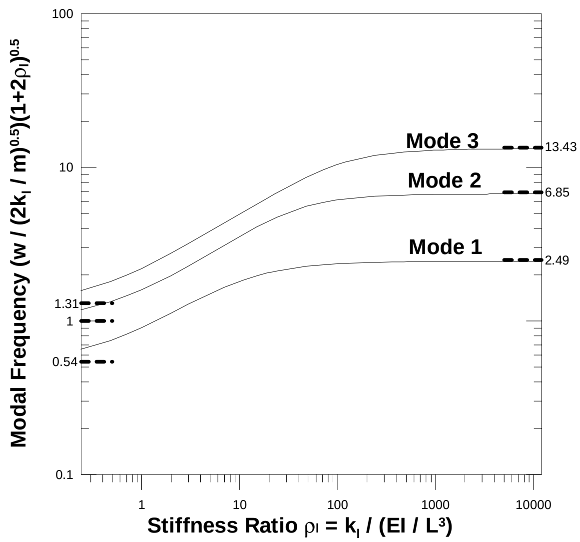

Figure 3 Non-dimensionized modal frequencies in lateral vibration (*N*=3, ρ_θ=100)

Similarly, the axial stiffness ratio is defined as

$$\rho_a = \frac{k_a}{EA/L} \tag{19}$$

where k_a is the axial stiffness of a joint. The variation of non-dimensionized modal frequencies in the axial vibration of a pipe system of $N=3$ is shown in Fig. 4.

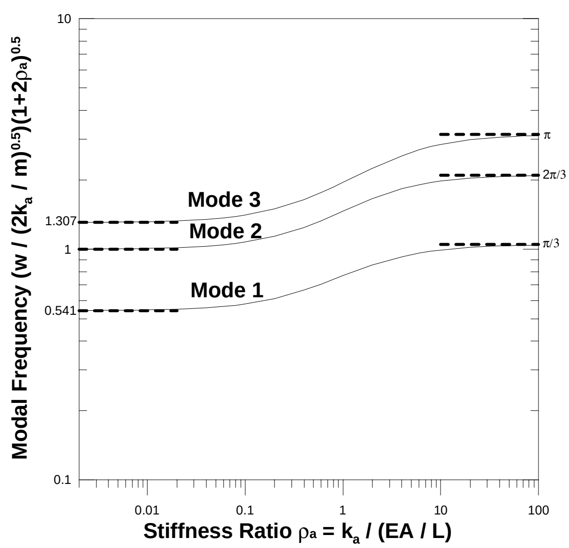

Figure 4 Non-dimensionized modal frequencies in axial vibration (*N*=3)

Once ω_k is found by a given pair of ρ_θ and ρ_l, the equivalent modal frequencies including the pipe-soil interaction are obtained by

$$\bar{\omega}_k = \sqrt{\frac{\bar{k}_S}{\bar{m}} + \omega_k^2} \tag{20}$$

The compound modal damping ratios can also be obtained by

$$\bar{\zeta}_k = \frac{\bar{c}_S / \bar{m} + 2\zeta_k \omega_k}{2\sqrt{\bar{k}_S / \bar{m} + \omega_k^2}} \tag{21}$$

where ζ_k is the modal damping ratio for the pipe segments only. The modal equation of motion is then specified by $\bar{\omega}_k$ and $\bar{\zeta}_k$, from which the modal dynamic effect could be found in the design spectrum at the site of the buried pipeline system.

DESIGN-RELATED RESPONSES

For the buried pipeline system under the lateral or lateral vibration, there are several maximum responses of design concern, such as the deformations and stresses along the pipe segments and the differential motions across the joints. When the ground motion is involved and the orthogonality properties of modes are applied, the modal equation of motion becomes

$$\ddot{q}_k(t) + 2\bar{\zeta}_k \bar{\omega}_k \dot{q}_k(t) + \bar{\omega}_k^2 q_k(t) = -\frac{\sum_{j=1}^{N} \int_0^L \phi_{jk}(x_j) \ddot{u}_g(x_j, t) dx_j}{\sum_{j=1}^{N} \int_0^L \phi_{jk}^2(x_j) dx_j} \tag{22}$$

Modeling the spatial variation of the ground excitation is one of the major topics in the vibration analysis of lifeline systems. After the numerical study considering the amplitude decay and phase shift of ground motions, however, the homogeneous ground excitation induces the maximum design-related responses in most cases. Therefore, the homogeneous ground motion is assumed for design purpose. Then, Eq. (22) becomes

$$\ddot{q}_k(t) + 2\bar{\zeta}_k \bar{\omega}_k \dot{q}_k(t) + \bar{\omega}_k^2 q_k(t) = -P_k \ddot{u}_g(t) \tag{23}$$

where the modal participation factor P_k is defined as

$$P_k = \frac{\sum_{j=1}^{N} \int_0^L \phi_{jk}(x_j) dx_j}{\sum_{j=1}^{N} \int_0^L \phi_{jk}^2(x_j) dx_j} \tag{24}$$

Finally, the total deformation after modal superposition is

$$u_j(x_j, t) = \sum_{k=1}^{\infty} \phi_{jk}(x_j) P_k D_k(t) = \sum_{k=1}^{\infty} \psi_{jk}(x_j) D_k(t) \tag{25}$$

where $\psi_{jk}(x_j) = P_k \phi_{jk}(x_j)$ is the effective participation function, $D_k(t)$ is the deformation time history of a single degree-of-freedom system of $\bar{\omega}_k$ and $\bar{\zeta}_k$ subjected to the ground acceleration, $\ddot{u}_g(t)$.

Six design-related responses in the lateral vibration of a pipeline system are considered here. They are the maximum deformation, rotation, shear force, and bending moment along all the pipe segments and the maximum differential deformation and rotation across the joints. The static design responses of the *k*th mode obviously depend on the effective participation function and its first three derivatives, and then depend on ρ_θ and ρ_l as a result. The dynamic effect in each mode due to $\ddot{u}_g(t)$, $(D_k)_{\max}$, could be found in the design spectra used for common building design. It is recommended that the SRSS rule be applied in the modal combination because of the assumption of uniform properties in the pipeline system.

In fact, the contribution of the even modes to the design-related responses in a pipeline system of odd number of segments is negligible because of the symmetry of the pipeline system, the uniform soil properties, and the homogeneous excitation. In most cases for a pipeline system of three segments, only the first and the third modes in the lateral vibration analysis and the first mode in the axial vibration are required to get the satisfactory results of the design-related responses.

For example, for a pipeline system of three segments and $\rho_\theta = 100$ under the lateral vibration, the non-dimensionized static design responses of the first and the third modes for several typical values of ρ_l are listed in Table 1 and Table 2, respectively. In those tables, $\psi_{\max}$, $\psi'_{\max}$, $EI\psi''_{\max}$, and $EI\psi'''_{\max}$ are the static maximum lateral deformation, rotation, shear, and bending moment along the segments, respectively, and $\Delta\psi_{\max}$ and $\Delta\psi'_{\max}$ are the static maximum lateral differential deformation and rotation across the joints, respectively. Probably, the differential motion across the joints is more critical in the seismic damage evaluation of a buried pipeline system than the other design-related responses along the segments. It is also noted in Tables 1 and 2 that the maximum lateral differential deformation across the joints is more sensitive to the variation of ρ_l than the others.

Table 1 Static design values of the first mode in lateral vibration (*N*=3, ρ_θ=100)

ρ_l	$\omega_1 / \sqrt{EI/\bar{m}L^3}$	$\psi_{\max}$	$\psi'_{\max} \times L$	$\psi''_{\max} \times L^2$	$\psi'''_{\max} \times L^3$	$\Delta\psi_{\max}$	$\Delta\psi'_{\max} \times L$
1.900E+01	2.019E+00	1.358E+00	9.781E-01	2.986E+00	4.981E+00	2.622E-01	2.986E-02
6.000E+02	2.432E+00	1.322E+00	1.335E+00	3.974E+00	6.213E+00	1.036E-02	3.974E-02
2.640E+03	2.445E+00	1.319E+00	1.345E+00	4.002E+00	6.243E+00	2.365E-03	4.002E-02

Table 2 Static design values of the third mode in lateral vibration (*N*=3, ρ_θ=100)

ρ_l	$\omega_3 / \sqrt{EI/\bar{m}L^3}$	$\psi_{\max}$	$\psi'_{\max} \times L$	$\psi''_{\max} \times L^2$	$\psi'''_{\max} \times L^3$	$\Delta\psi_{\max}$	$\Delta\psi'_{\max} \times L$
1.900E+01	6.192E+00	5.299E-01	8.018E-01	2.212E+00	9.916E+00	5.219E-01	8.987E-03
6.000E+02	1.270E+01	5.751E-01	1.884E+00	9.131E+00	3.640E+01	6.067E-02	9.132E-02
2.640E+03	1.313E+01	5.553E-01	1.901E+00	9.259E+00	3.539E+01	1.340E-02	9.260E-02

As for the axial vibration of a buried pipeline system, the design-related responses would be the maximum axial force in the pipe segments and the maximum axial deformation across the joints. For a pipeline system of three segments under the axial vibration, the non-dimensionized static design responses of the first mode for some typical values of ρ_a are listed in Table 3. In Table 3, $EA\psi'_{\max}$, and $\Delta\psi_{\max}$ are the static maximum axial force along the segments and the static maximum axial differential deformation across the joints, respectively.

Table 3 Static design values of the first mode in axial vibration (*N*=3)

ρ_a	$\omega_1 / \sqrt{EA/\bar{m}L}$	$\psi'_{\max} \times L$	$\Delta\psi_{\max}$
1.000E-02	7.636E-02	8.493E-03	8.493E-01
1.000E-01	2.364E-01	8.120E-02	8.120E-01
1.000E+00	6.260E-01	5.515E-01	5.515E-01
1.000E+01	9.683E-01	1.190E+00	1.190E-01
6.000E+01	1.033E+00	1.308E+00	2.180E-02

On the basis of the above vibration analysis, the procedure for the seismic demands of a buried pipeline system is stated as follows.

1. A pipeline model of three or five pipe segments is preferred because of the assumptions of uniform properties of segments and soil and homogeneous ground excitation used for simplifying the analysis.

2. The axial, lateral, and rotational stiffness of the joints should be carefully evaluated in order to estimate the modal parameters and the static design-related responses.

3. The soil properties are then added to obtain the compound modal frequencies and damping ratios in order to find the spectral deformation in the design spectra proposed for buildings at the same site.

4. Each modal design-related response would be the product of the results in Steps 2 and 3. The SRSS rule is suggested to compose the total design-related responses except that the pipelines are buried in a pretty hard site.

CONCLUSIONS

A simple model of buried pipeline system with finite boundaries is used to develop the procedure for the seismic demands related to the pipe segment and the joint. The main assumptions include the uniform properties in pipes, joints and soil, and the homogeneous ground excitation. On the basis of the assumptions and numerical results, some important conclusions are given in the following:

1. A pipeline model of three or five pipe segments is preferred for the design purpose.

2. The static design-related responses depend on the rotational and lateral stiffness ratios in the lateral vibration, and depend on axial stiffness ratio in the axial vibration.

3. The contributions of the first and the third modes to the static design-related responses are of major concern in the lateral vibration, while the results of the first mode only are good enough in evaluating the static design-related responses in the axial vibration.

4. After combining the modal parameters of the pipeline system and the soil properties to get the equivalent modal frequencies and damping ratios, the modal dynamic amplification effects can be

found in the design spectra used for building design at the same site. Then, the design-related responses are still dominated by the first mode even for a pipeline system of small values of stiffness ratios.

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