



## **EVALUATION OF MODAL PUSHOVER ANALYSIS USING VERTICALLY IRREGULAR FRAMES**

**Chatpan CHINTANAPAKDEE<sup>1</sup>, and Anil K. CHOPRA<sup>2</sup>**

### **SUMMARY**

Recently, modal pushover analysis (MPA) has been developed to improve conventional pushover procedures by including higher-mode contributions to seismic demands. This study compares the seismic demands for vertically irregular frames determined by MPA procedure and the rigorous nonlinear response history analysis (RHA), due to an ensemble of 20 ground motions. Forty-eight irregular frames, all 12-story high with strong-columns and weak-beams, were designed with three types of irregularity—stiffness, strength, and combined stiffness and strength—introduced in eight different locations along the height using two modification factors. Next, the median and dispersion values of the ratio of story drift demands determined by modal pushover analysis (MPA) and nonlinear RHA were computed to measure the bias and dispersion of MPA estimates leading to the following results: (1) the bias in the MPA procedure does not increase, i.e., its accuracy does not deteriorate, in spite of irregularity in stiffness, strength, or stiffness and strength provided the irregularity is in the middle or upper story; (2) the MPA procedure is less accurate relative to the reference “regular” frame in estimating the seismic demands of frames with strong or stiff-and-strong first story; soft, weak, or soft-and-weak lower half; stiff, strong, or stiff-and-strong lower half; (3) in spite of the larger bias in estimating drift demands for some of the stories in particular cases, the MPA procedure identifies stories with largest drift demands and estimates them to a sufficient degree of accuracy, detecting critical stories in such frames; and (4) the bias in the MPA procedure for frames with soft, weak or soft-and-weak first story is about the same as for the “regular” frame.

### **INTRODUCTION**

Current structural engineering practice evaluates the seismic resistance of buildings using the nonlinear static procedure (NSP) or pushover analysis described in FEMA-273 (BSSC [1]). Recently, modal pushover analysis (MPA) has been developed to improve conventional pushover procedures by including higher-mode contributions to seismic demands (Chopra [2]). This MPA procedure offers several attractive features. It retains the conceptual simplicity and computational attractiveness of current pushover procedures with invariant force distributions—now common in structural engineering practice. The computational effort involved in MPA including the first few—two or three—modes is comparable to that required in FEMA procedures using two or three lateral-force distributions. With the roof

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displacement determined from the elastic design spectrum and empirical equations for the ratio of peak deformations of inelastic and elastic systems, pushover analysis for each mode requires computational effort similar to one FEMA force distribution.

The accuracy of MPA must be evaluated for a wide range of structural systems and ground motions to identify the conditions under which it is applicable for seismic evaluation of structures. To this end, it has been applied to SAC buildings (Goel [3]), and generic frames (Chintanapakdee [4]) designed according to the static force distribution specified in the International Building Code (IBC) (International Code Council [5]). By studying the bias and dispersion of this approximate procedure, MPA has been shown to be accurate enough in estimating seismic demands for seismic evaluation of such “regular” buildings. Because vertical irregularities significantly influence the seismic demands on buildings, the next logical step is to determine whether MPA can estimate seismic demands on irregular buildings to a degree of accuracy sufficient for practical application. Furthermore, as all pushover analyses aim to detect any deficiency in the structure that results in localizing large seismic demands, MPA’s potential in this regard remains to be evaluated.

The objective of this investigation to evaluate the accuracy of MPA in estimating seismic demands and detecting weakness in vertically irregular frames by documenting the bias and dispersion of the ratio of the seismic demands on irregular frames determined by MPA procedure to their “exact” values computed by nonlinear RHA. It is demonstrated in this paper that the MPA procedure has a similar degree of accuracy for estimating seismic demands for some types of irregular frames as it does for “regular” frames. In addition, the MPA procedure detects which stories will be subjected to large seismic demands; irregular frames for which MPA does not work well are identified.

## STRUCTURAL SYSTEMS

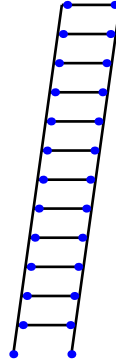
### Reference “Regular” Frame

The MPA procedure has ready been evaluated for “regular” frames of six different heights of 3, 6, 9, 12, 15, and 18 stories, each designed for five different strength levels (Chintanapakdee [4]). The second phase of the overall investigation concerned irregular frames, which is the subject of this paper. To focus on the issue of height-wise irregularity, the frame height was fixed at 12 stories, a mid-rise frame for which pushover analyses are appropriate.

While the MPA procedure has already been evaluated using code-designed buildings (Goel [3]), this investigation is concerned with generic frames to cover a wider range of building properties. These frames are described fully in Chintanapakdee [6]; a summary is included next. To evaluate the accuracy of MPA for irregular frames, a 12-story “regular” frame was defined as the reference case for comparison. The reference frame was designed so that the height-wise distribution of stiffness achieves equal drifts in all stories under the IBC lateral forces. Assuming that the second moment of cross-sectional area for each beam and its supporting columns in the story below are the same, numerical values for the flexural rigidities of structural elements were selected such that the fundamental vibration period  $T_1$  is 2.40 sec. This is the value determined from  $T_u = 0.045H^{0.8}$ , where  $H$  is the total height of the frame in feet, an equation that defines the mean-plus-one-standard-deviation of measured periods of steel moment resisting frames (Goel [7]).

The frame is designed according to the strong-column-weak-beam philosophy; therefore, plastic hinges form only at beam ends and the base of the first-story columns (Fig. 1). The columns in other stories are assumed to remain elastic. The yield moments of plastic hinges, with bilinear (3% post-yield stiffness) moment-rotation relation, are selected such that yielding occurs simultaneously at all plastic

hinges under the IBC lateral force distribution. The yield base shear is  $V_{by} = (A_y/g)W$ , where  $W$  is the total weight of the frame and  $A_y$  is the median (over 20 ground motions) pseudo-acceleration for an SDF system with vibration period  $T_n = T_1$  and a ductility factor  $\mu=4$ .



**Figure 1. Beam-hinge model of a 12-story frame.**

### **Vertically Irregular Frames**

Forty-eight irregular frames, all 12-stories high, were considered to account for three types of irregularities introduced in eight different locations along the height using two modification factors, described next. Three types of irregularities in the heightwise distributions of frame properties were considered: stiffness irregularity (KM), strength irregularity (SM), and combined-stiffness-and-strength irregularity (KS). Various irregular frames are obtained by modifying the stiffness or/and strength of the reference frame. To obtain a soft or stiff story, the story stiffness was divided or multiplied by a modification factor; and to obtain a weak or strong story, the story strength was divided or multiplied by a modification factor. Two values of the modification factor were considered:  $MF = 2$  or  $5$ . For each of the three types of irregularity, the following eight cases were investigated: (1) soft or/and weak top story; (2) stiff or/and strong top story; (3) soft or/and weak mid-height story; (4) stiff or/and strong mid-height story; (5) soft or/and weak first story; (6) stiff or/and strong first story; (7) soft or/and weak lower half of structure; and (8) stiff or/and strong lower half of structure.

### *Stiffness-Irregular Frames*

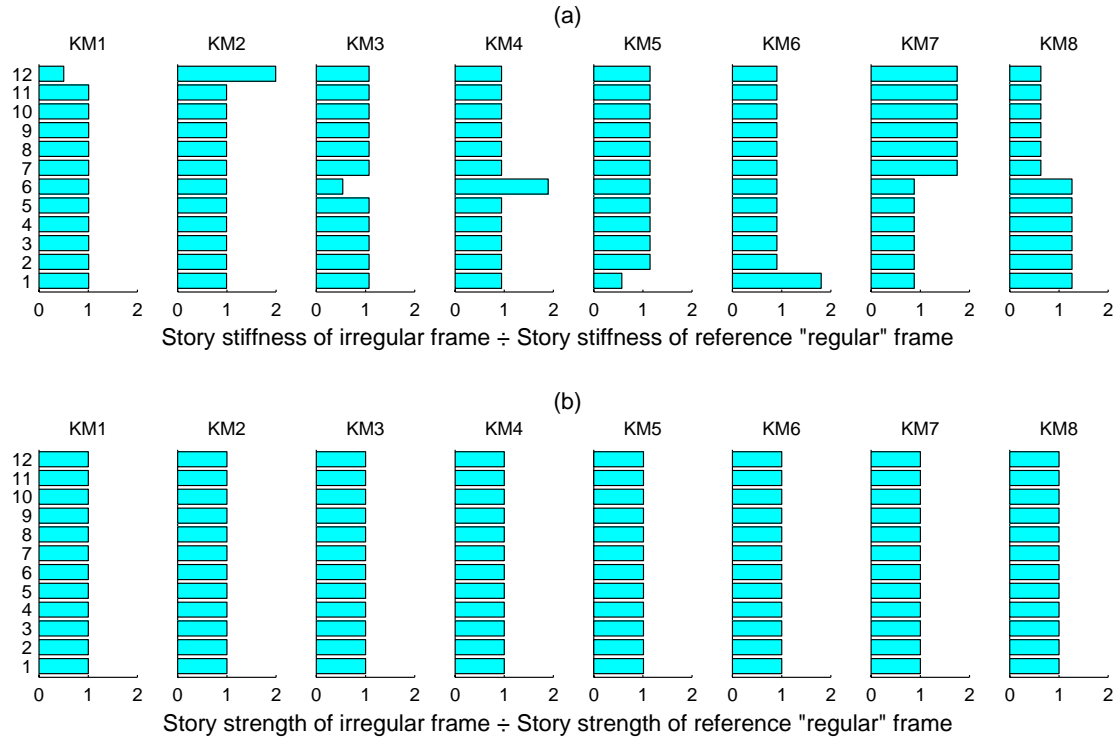
Figure 2 shows the ratio of story stiffness, and of story strength of stiffness-irregular frames (with  $MF=2$ ) to the corresponding properties of the “regular” frame;  $KM_j$  denotes stiffness-irregularity case  $j$  ( $=1, 2, \dots, 8$ ). A total of 16 stiffness-irregular frames are considered corresponding to the 8 cases mentioned above and two values of the modification factor.

The stiffness of a story was modified by changing the stiffness of the columns in that story and the beam they support. To ensure meaningful comparison of seismic demand on “regular” and irregular frames, their fundamental vibration period, yield base shear, and damping properties were kept the same. Modifying the stiffness of one or more stories by the factor  $MF$  obviously affects the vibration period. To maintain the same period as for the “regular” frame, all story stiffnesses were scaled uniformly, causing the ratio of story stiffnesses of irregular and “regular” frames to be different than  $MF$ , as seen in Fig. 2.

The pushover curves using the IBC force distribution show that although stiffness irregularity may influence the initial slope significantly, it affects the yield strength only slightly (Chintanapakdee [6]). All story strengths were scaled uniformly to obtain an irregular frame with the same yield base shear

as the “regular” frame. Note that the post-yield stiffness of irregular frames can be slightly different than the “regular” frame.

The Rayleigh damping matrix for an irregular frame is defined to maintain the modal damping ratio equal to 5% in the first and fourth modes, as for the “regular” frame.



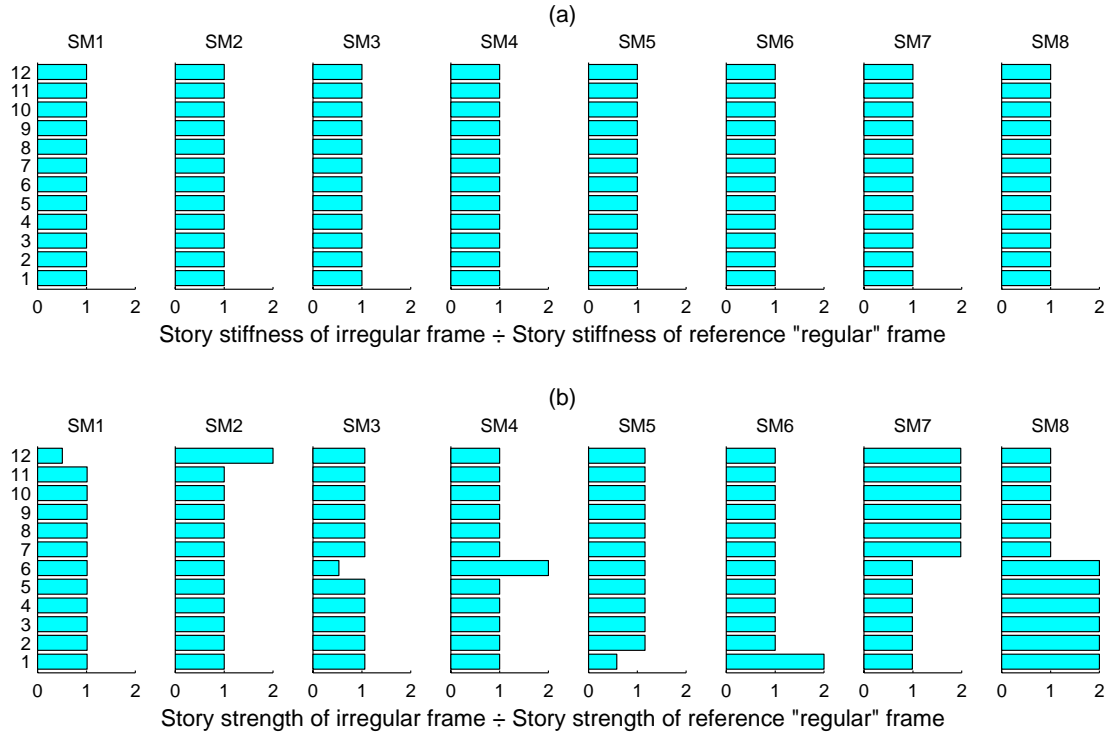
**Figure 2. Ratio of (a) story stiffness and of (b) story strength of stiffness-irregular frames to the corresponding properties of the “regular” frame for modification factor, MF=2.**

### *Strength-Irregular Frames*

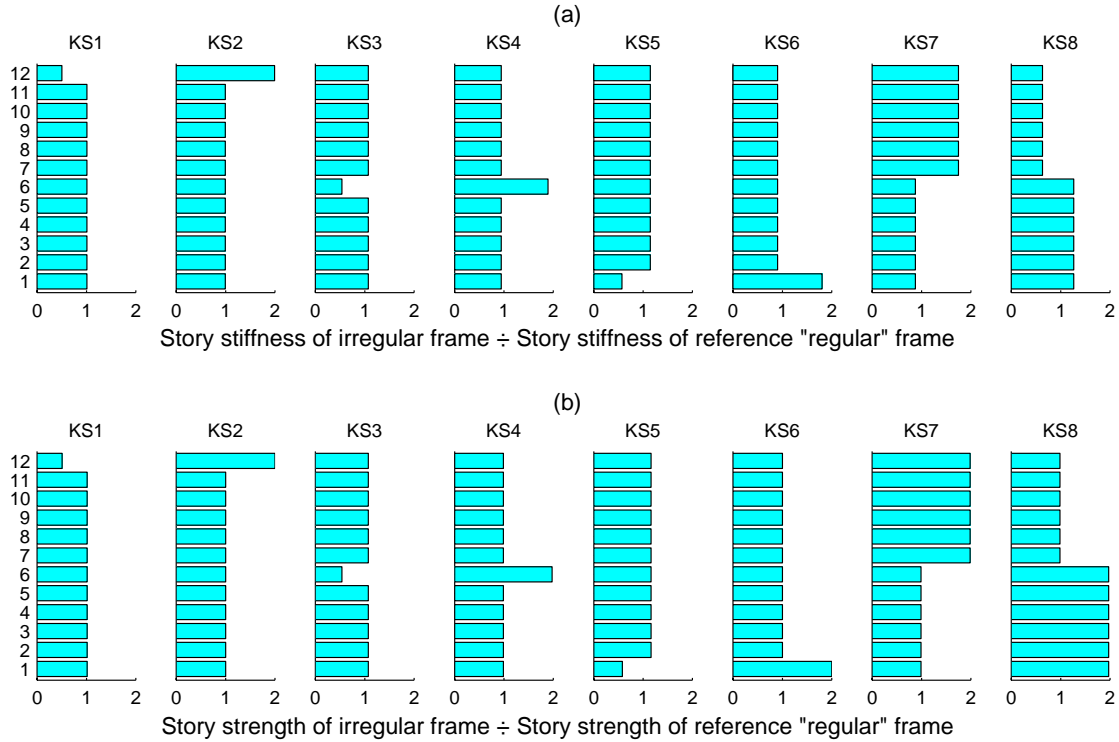
Figure 3 shows the ratio of story stiffness and of story strength of strength-irregular frames (with MF=2) to the corresponding properties of the “regular” frame; SM<sub>j</sub> denotes strength-irregularity case *j*. The story stiffnesses, fundamental period, and damping matrix of strength-irregular frames were kept the same as for the “regular” frame. The strength of a story was modified by changing only the strength of the beam at the top of the story (recall that the columns are assumed to remain elastic). However, in Cases 5 to 8, where strength of the first story was modified (Cases 5-8), the strength of columns in the first story, which were designed to hinge at the base, is also changed. All story strengths are scaled uniformly to obtain an irregular frame with the same yield base shear as the “regular” frame, causing the ratio of the story strengths of irregular and “regular” frames to be different than the modification factor, as shown in Fig. 3.

### *Stiffness-and-Strength-Irregular Frames*

Figure 4 shows the ratio of story stiffness and of story strength of stiffness-and-strength-irregular frames (with MF=2) to the corresponding properties of the “regular” frame; KS<sub>j</sub> denotes combined stiffness-and-strength irregularity case *j*. Each frame is designed by modifying the story stiffnesses and damping matrix as described earlier for the stiffness-irregular frame, and the story strengths as described earlier for the strength-irregular frame.



**Figure 3. Ratio of (a) story stiffness and of (b) story strength of strength-irregular frames to the corresponding properties of the “regular” frame for modification factor, MF=2.**



**Figure 4. Ratio of (a) story stiffness and of (b) story strength of combined-stiffness-and-strength-irregular frames to the corresponding properties of the “regular” frame for modification factor, MF=2.**

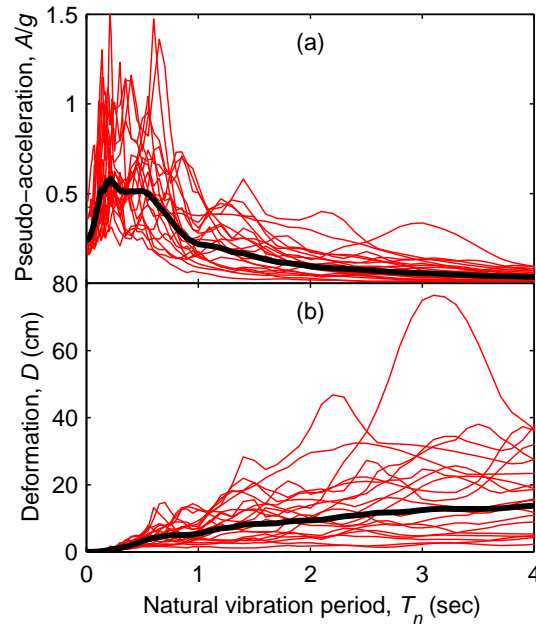
## GROUND MOTIONS AND RESPONSE STATISTICS

### Ground Motions

The seismic excitation for these generic frames is defined by a set of 20 large-magnitude-small-distance records (LMSR) listed in Chintanapakdee [6]. These ground motions were obtained from California earthquakes with magnitudes ranging from 6.6 to 6.9 recorded at distances of 13 to 30 km on firm ground. Their elastic pseudo-acceleration and deformation response spectra and the median spectrum are presented in Fig. 5.

### Response Statistics

The dynamic response of each structural system to each of the 20 ground motions was determined by nonlinear RHA, and MPA (Chopra [2]) without considering P- $\Delta$  effects due to gravity loads; details of both analysis procedures are documented in Chintanapakdee [6]. The “exact” peak value of structural response or demand,  $r$ , determined by nonlinear RHA is denoted by  $r_{NL-RHA}$ , and the approximate value from MPA by  $r_{MPA}$ . From these data for each ground motion, a response ratio was determined from the following equation:  $r_{MPA}^* = r_{MPA} \div r_{NL-RHA}$ . An approximate method is invariably biased in the sense that the median of the response ratio differs from one, underestimates the median response if the ratio is less than one, and provides an overestimate if the ratio exceeds one.



**Figure 5. (a) Pseudo-acceleration spectra and (b) deformation spectra of “LMSR” set of ground motions, damping ratio=5%. The median spectrum is shown by a thicker line.**

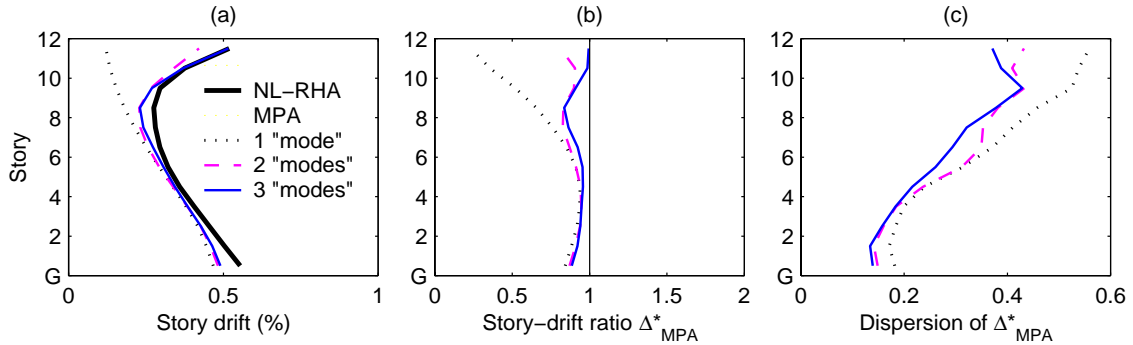
Presented in this paper are the median values,  $\hat{x}$ , defined as the geometric mean, of  $n$  (equals 20) observed values ( $x_i$ ) of  $r_{MPA}$ ,  $r_{NL-RHA}$ , and  $r_{MPA}^*$ ; and the dispersion measure  $\delta$  of  $r_{MPA}^*$ , defined as the standard deviation of logarithm of the  $n$  observed values:

$$\hat{x} = \exp \left[ \frac{\sum_{i=1}^n \ln x_i}{n} \right] \quad \delta = \left[ \frac{\sum_{i=1}^n (\ln x_i - \ln \hat{x})^2}{n-1} \right]^{1/2} \quad (1)$$

For small values, e.g., 0.3 or less, the above dispersion measure is close to the coefficient of variation. This measure will be referred to as “dispersion” in subsequent sections. Equations 1(a) and 1(b) are logical estimators for the median and dispersion, especially if the data are sampled from lognormal distribution, an appropriate distribution for peak earthquake response of structures (Newmark [8]; Shome [9]).

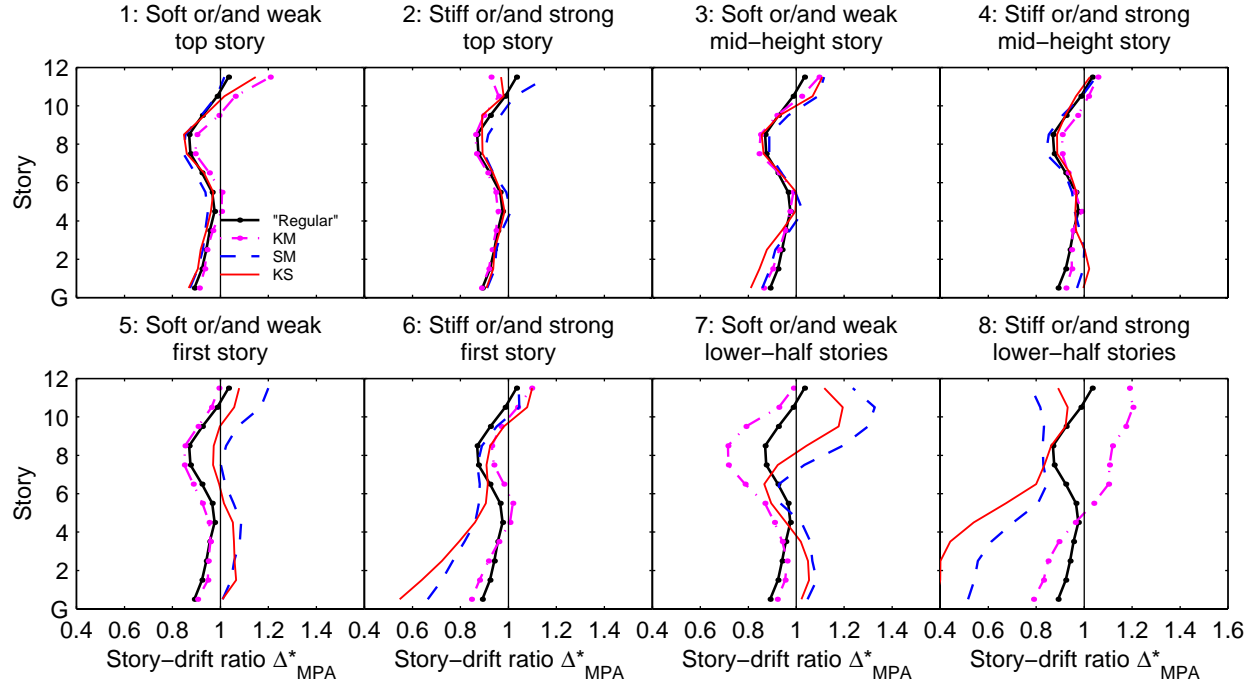
### BIAS AND ACCURACY OF MPA

The median story-drift demands determined by MPA on “regular” frames—including 1, 2 or 3 “modes”—are compared to the results of nonlinear RHA (Fig. 6a). The MPA considering only the first “mode” is inadequate in estimating story drifts demands in upper stories, where contributions of higher modes are known to be significant, even in elastic systems (Chopra [10], Chapter 18). With higher “mode” contributions included, however, MPA follows the heightwise variation of drift demands reasonably well. Examining the median and dispersion of the ratio  $\Delta_{\text{MPA}}^*$  indicates that the bias in MPA (Fig. 6b) and its dispersion (Fig. 6c) decreases as higher “modes” are included; however, even with 3 “modes” included, MPA tends to underestimate the drift demands in all stories of this frame except the top one. This underestimation is similar to a trend found in response spectrum analysis of elastic frames and does not disappear even if all modes are included. In brief, MPA underestimates or overestimates the demand depending on the height, vibration period, and design ductility (Chintanapakdee [4]).



**Figure 6. (a) Median story-drift demands for the “regular” frame determined by MPA with variable number of “modes” and by nonlinear RHA; (b) Median story-drift ratio  $\Delta_{\text{MPA}}^*$ ; and (c) dispersion of story-drift ratios  $\Delta_{\text{MPA}}^*$ .**

Figure 7 presents the median of the ratio  $\Delta_{\text{MPA}}^*$  of drift demands obtained from MPA (including 4 “modes”) and from nonlinear RHA for all eight cases and three types of irregularity with MF=2 and compares them to results for the “regular” frame. As mentioned earlier, the difference between median and unity represents the bias in the MPA procedure. These results demonstrate that the bias in the MPA procedure does not increase, i.e., its accuracy does not deteriorate, in spite of irregularity in stiffness, strength or both stiffness and strength, provided the irregularity is in the top story or mid-height story (Cases 1-4).

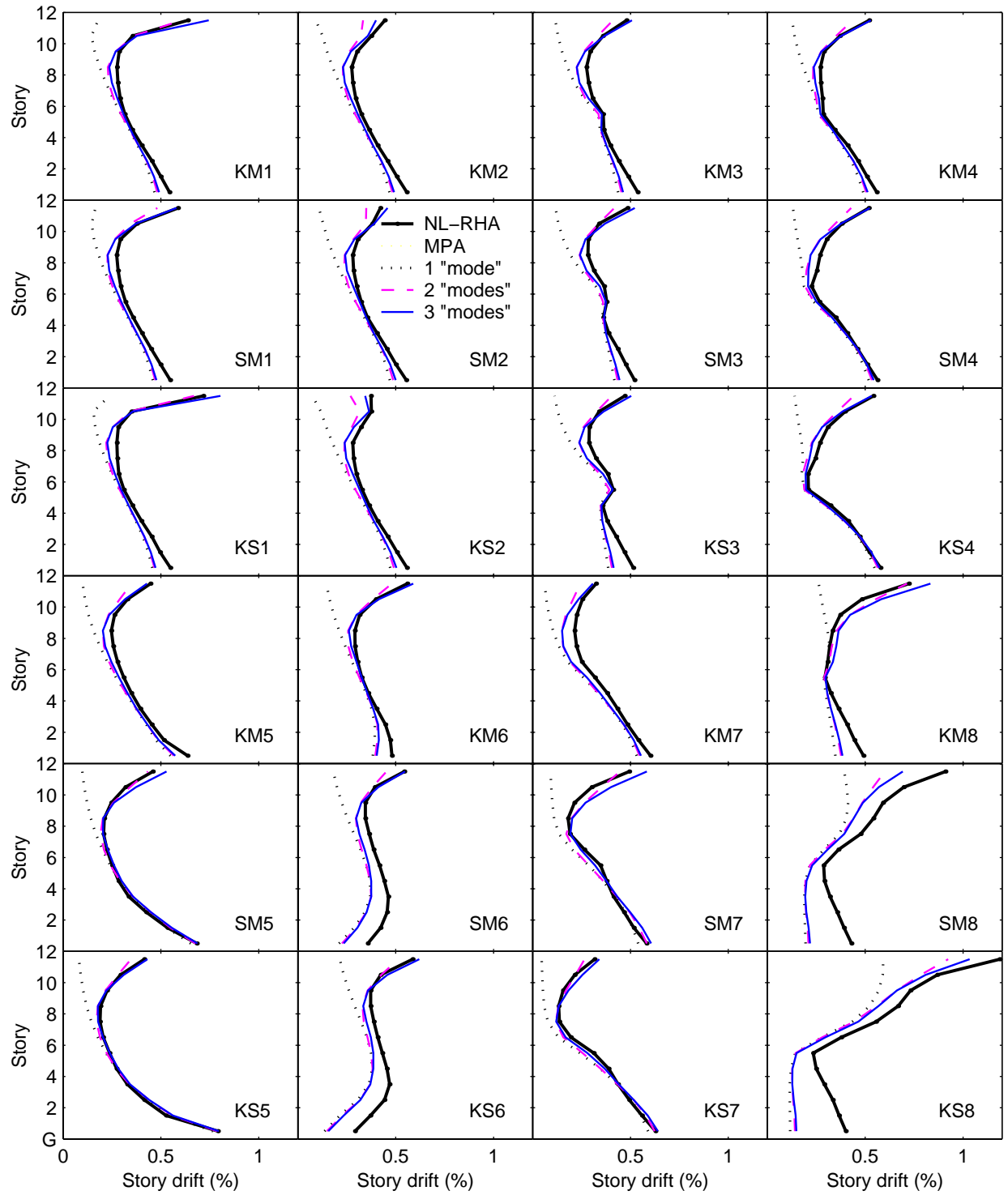


**Figure 7. Median story-drift ratios  $\Delta_{MPA}^*$  for “regular” frame and stiffness-, strength-, and combined-stiffness-and-strength-irregular frames with modification factor, MF=2.**

The MPA procedure is less accurate for structures where the irregularity is in the first story or in the lower half of the frame (Cases 5-8). The bias is significantly larger for (i) the lower stories of an irregular frame with strong or stiff-and-strong first story (Cases SM6 and KS6) compared to the “regular” frame; (ii) the upper stories of frame with a soft, weak, or soft-and-weak lower half (Cases KM7, SM7, KS7); and (iii) the lower stories of a frame with stiff, strong, or stiff-and-strong lower half (Cases KM8, SM8, KS8). Note that the bias in the MPA procedure for frames with a soft, weak, or soft-and-weak first story (Cases KM5, SM5, KS5) is about the same as found for the “regular” frame.

The larger bias in the MPA estimates suggests that the MPA procedure is not appropriate for estimating story drift demands for irregular frames corresponding to Cases 6-8. To better understand the limitations of the MPA procedure, Fig. 8 compares the median story drift demands determined by the MPA including 3 “modes” to the results of nonlinear RHA. The MPA procedure detects the concentration of drift demand in lower stories due to the presence of a weak and/or soft first story (Case 5) and estimates reasonably well the drift demands in all stories. Similarly, the MPA procedure identifies the larger drift demands in lower stories of frames with weak and/or soft lower half (Case 7); the drift estimates are accurate enough for practical application, although they are less accurate compared to the “regular” frame. Although the MPA procedure detects the larger drifts in the upper stories of frames with a stiff and/or strong first story (Case 6), it may underestimate significantly the smaller drift demands in the first few stories of the frame. Although this underestimation by the MPA procedure becomes larger in frames with stiff and/or strong lower half (Case 8), MPA still detects larger drifts in the upper stories of such frames and estimates them to a useful degree of accuracy. The overall impression that emerges is that the MPA procedure is able to identify the stories with the largest drifts, even for Cases 5-8, and hence able to detect critical stories in such frames with the caveat that it may underestimate significantly the smaller drift demands in other stories.

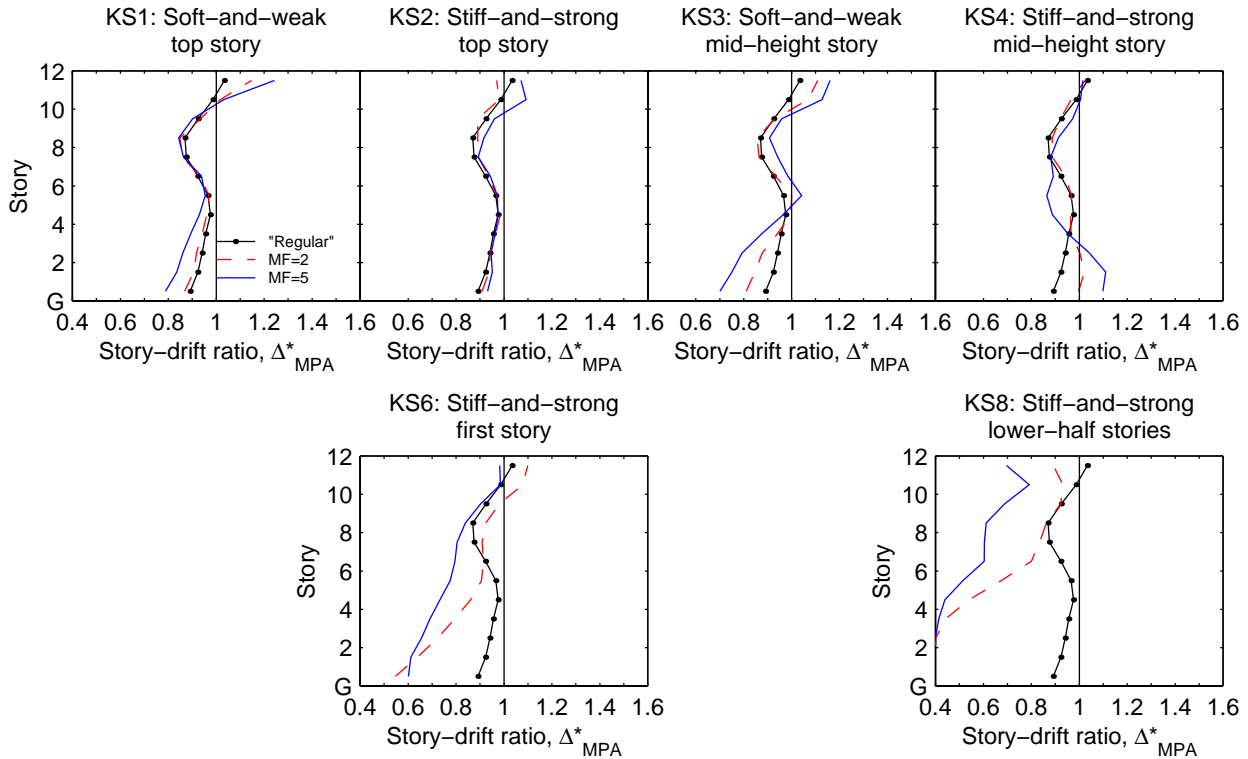




**Figure 8. Median story-drift demands of stiffness-, strength-, and stiffness-and-strength-irregular frames with modification factor, MF=2, determined by MPA including 1, 2, and 3 “modes” and by nonlinear RHA.**

Also presented in Fig. 8 are the drift demands computed by MPA considering only the first “mode” of vibration. Clearly, even for irregular frames, the first “mode” alone is inadequate in estimating the drift demands in the upper stories where the contributions of higher modes are significant.

Figure 9 presents the median of the drift ratio  $\Delta_{MPA}^*$  for stiffness-and-strength-irregular frame cases KS1-KS4, KS6, and KS8, and two values of the modification factor, MF=2 and 5 and for the reference “regular” frame. Note that the bias in the MPA procedure for irregular frames with MF=5 is not much larger than when MF=2 or the “regular” frame indicating that even when this irregularity is extreme the MPA procedure works well for frames with a soft or/and weak story or with a stiff or/and strong story at mid-height or at the top of the frame. The bias in MPA procedure for frames with a stiff-and-strong first story (Case KS6) or with stiff-and-strong lower half of the frame (Case KS8), which was shown to be unacceptably large when MF=2 (Fig. 7), is even worse when MF=5. However, the MPA procedure is able to detect the critical stories in such frames even when the irregularity is extreme, MF=5. Frames with a soft-and-weak first story (Case KS5) or with soft-and-weak lower half (Case KS7) are not included because the second-“mode” pushover analysis is unable to reach the target displacement when MF=5; this difficulty arises because the frame is unrealistically weak in lower stories. This difficulty in MPA procedure is explained in Appendix.

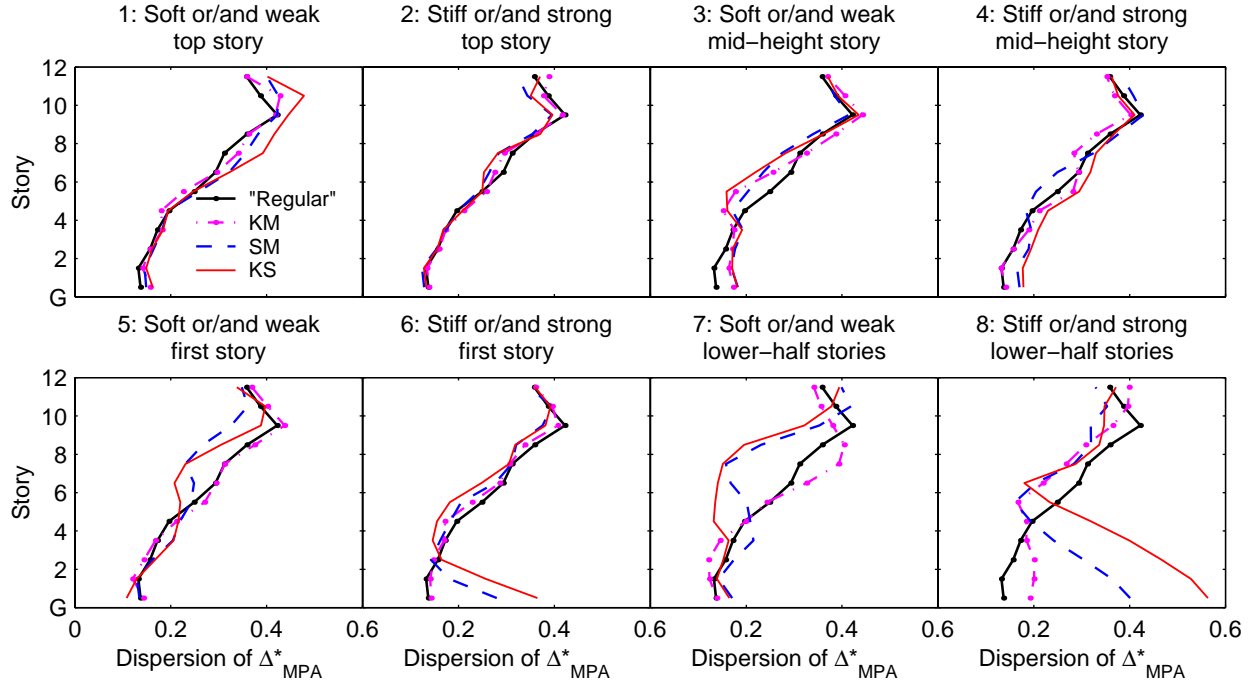


**Figure 9. Median story-drift ratios  $\Delta_{MPA}^*$  for “regular” frame and combined-stiffness-and-strength-irregular frames with modification factor, MF=2 and 5.**

### DISPERSION OF MPA

Figure 10 presents the dispersion of  $\Delta_{MPA}^*$  for all 8 cases and three types of irregularity with MF=2 and compares them to the “regular” frame. The dispersion of  $\Delta_{MPA}^*$  for irregular frames is similar to that found for the “regular” frame, except that it is much larger for the lower stories of Cases 6 and 8 of strength and

stiffness-and-strength-irregular frames, for which the bias in MPA is also large (Fig. 7). Except for these two cases, the MPA procedure should be similarly reliable in estimating the seismic demands of irregular and “regular” frames due to an individual ground motion, although pushover analysis procedures may be inappropriate for such applications.



**Figure 10. Dispersion of story-drift ratios  $\Delta_{MPA}^*$  for “regular” frame and stiffness-, strength-, and combined-stiffness-and-strength-irregular frames with modification factor, MF=2.**

Figure 11 shows the dispersion of  $\Delta_{MPA}^*$  of the combined-stiffness-and-strength irregular frame Cases KS1-KS4 for two values of the modification factor (MF=2 and 5) and for the “regular” frame. The dispersion for MF=5 is roughly similar to that for MF=2 in Cases KS2-4, but larger in Case KS1, implying that the MPA procedure is similarly reliable in estimating the seismic demands due to an individual ground motion even for highly irregular frames, provided that the irregular story is at mid-height or upper part of the frame.

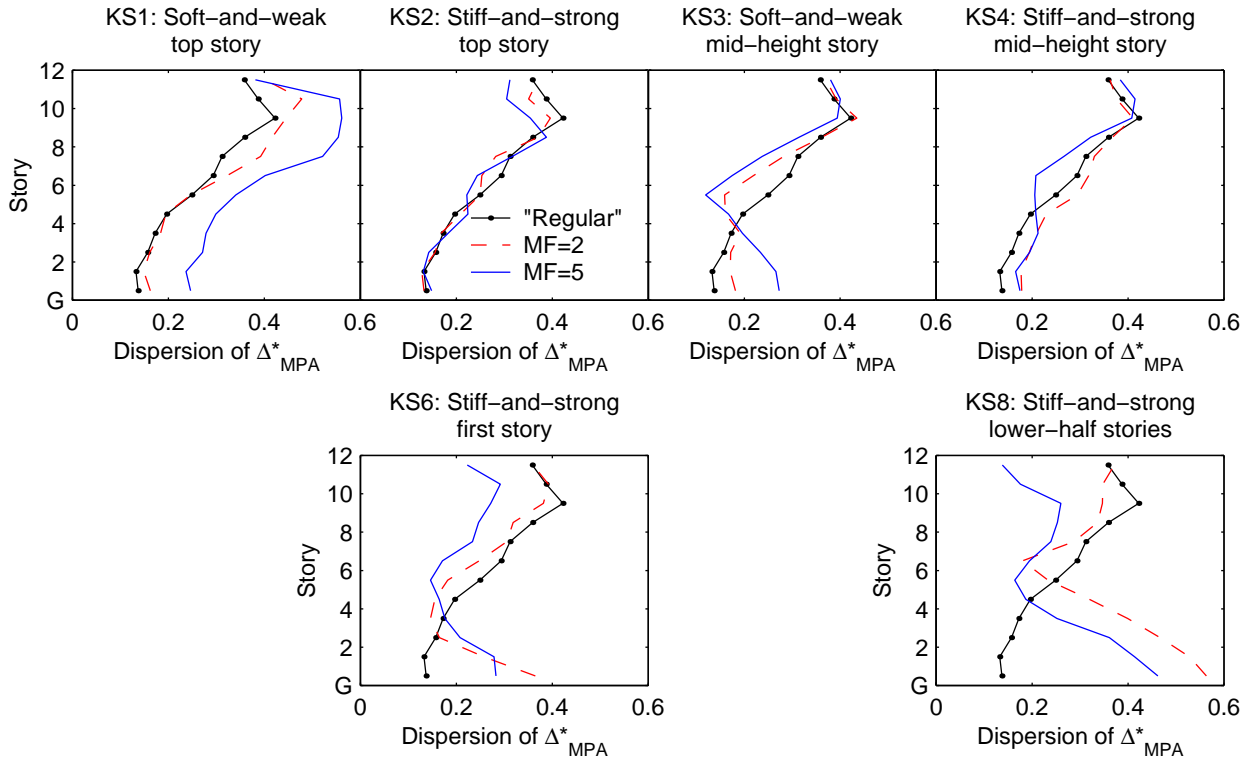
## CONCLUSIONS

This investigation of the accuracy of the MPA procedure in estimating seismic demands for vertically irregular frames has led to the following conclusions:

1. The bias in the MPA procedure does not increase, i.e., its accuracy does not deteriorate, in spite of irregularity in stiffness, strength, or stiffness and strength provided the irregularity is in the top story or mid-height story (Cases 1-4).
2. The MPA procedure can be more biased, i.e. less accurate, relative to the “regular” frame in estimating the seismic demands of frames with strong or stiff-and-strong first story; soft, weak, or soft-and-weak lower half; stiff, strong, or stiff-and-strong lower half. In contrast, the bias in the MPA procedure for frames with soft, weak or soft-and-weak first story is about the same as for the “regular” frame.

3. In spite of the larger bias in estimating drift demands for some stories in Cases 6-8, the MPA procedure identifies the stories with largest drift demands and estimates them well, detecting the critical stories in such frames.
4. The dispersion of  $\Delta_{MPA}^*$  for irregular frames is similar to the “regular” frame, except in the lower stories of the frames with a strong first story (Cases SM6 and KS6) or with strong lower half (Cases SM8 and KS8), for which the bias in MPA is also large.
5. The MPA procedure provides usefully accurate seismic demands also for irregular frames, except for those with a strong first story or strong lower half. The seismic demands for such irregular frames should be determined by nonlinear RHA.

These conclusions were based on an investigation of frames designed according to the strong-column-weak-beam philosophy, pervasive and preferable in seismic design. These conclusions may not be valid if columns are also expected to yield. The conclusions are also limited to frames with 12 or fewer stories.



**Figure 11. Dispersion of story-drift ratios  $\Delta_{MPA}^*$  for “regular” frame and combined-stiffness-and-strength-irregular frames with modification factor, MF=2 and 5.**

#### ACKNOWLEDGMENTS

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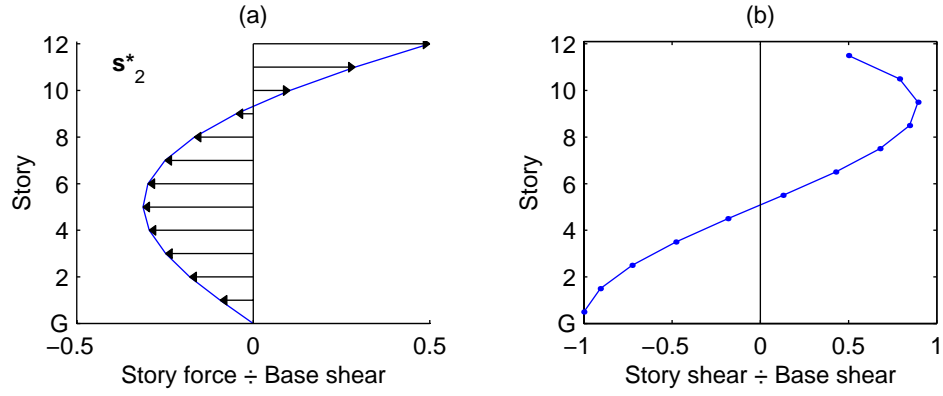
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## APPENDIX

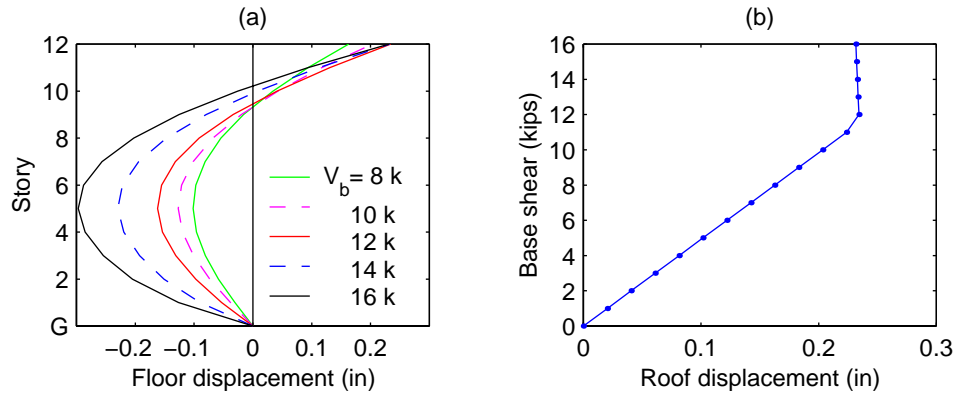
The second-"mode" pushover analysis is unable to reach the target displacement for frames with a weak first story (Cases SM5 and KS5) or weak lower half (Cases SM7 and KS7) when MF=5. This difficulty arises because the frame is unrealistically weak in lower stories.

The force distribution  $\mathbf{s}_n^*$  (Chopra [2]) for higher "modes" ( $n > 1$ ) consists of forces in both forward and backward directions (e.g. Fig. A.1a); thus, the story shear does not increase monotonically from top to bottom (e.g. Fig.A.1b). At some locations along the height of the frame, the story shear, which equals to the summation of forces in all stories above, is in the opposite direction to the target roof displacement. When yielding occurs in such story, the large plastic deformation in this story can cause the roof displacement to decrease while the applied forces with invariant distribution are increasing. This is demonstrated in Fig. A.2a, which shows the deflected shape of the irregular frame with a weak first story (Case SM5) under the second-"mode" force distribution  $\mathbf{s}_2^*$  just before and after yielding, which occurs at the first story. Figure A.2b shows the pushover curve of this case and it is seen that the target roof displacement can not be reached because as the base shear increases the roof displacement becomes decreasing after the yielding occurs.

Although MPA estimate for such irregular frames can not be obtained because of this difficulty, MPA can detect the deficiency, which is the very weak first story or lower half, in the structure.



**Figure A.1** (a) Force distribution  $s_2^*$  and (b) the corresponding story shear of the irregular frame with a weak first story (Case SM5).



**Figure A.2** (a) Deflected shape and (b) pushover curve of pushover analysis of the irregular frame with a weak first story (Case SM5) under  $s_2^*$  force distribution.