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A STUDY OF RESPONSE SPECTRA OF ACTIVELY-CONTROLLED STRUCTURES DUE TO EARTHQUAKE

Takashi MOCHIO[1]

SUMMARY

A newly response spectrum is proposed in order to evaluate simply the structural response of the actively-controlled structure subjected to earthquake. This paper adopts a linear single-degree-of-freedom structure with active mass damper (AMD) system as an analytical model. Also, the earthquake wave is modeled as a stationary Gaussian random process of which power spectral density is equal to the Kanai-Tajimi spectrum. The control design is executed by using linear quadratic Gaussian control strategy against the reconstructed three degree-of-freedom enlarged system to be coupled by soil, structure and AMD.

Finally, the response amplification factor is given by the combination of the obtained statistical response values and the random vibration theory.

Analytical results are compared with numerical simulations, and both show a good agreement. As a result, it seems that the validity of the proposed technique is confirmed.

INTRODUCTION

Recently, the active control technology is often applied to structures due to earthquake [1]. On the structural design for such an actively-controlled structure, the detailed dynamical analysis is executed from a initial design stage, since the design requires not only examination of structural integrity but also estimation of control characteristics against the structure. By the way, the dynamical seismic design of structures is often carried out by using response spectra as functions of damping ratio and natural period of a structure.

For above-mentioned actively-controlled structure, however, the response spectrum analysis is hardly executed, because the control design has several parameters to be selected, and it consequently becomes more difficult against applying the response spectra with only two parameters such as damping ratio and natural period to the design of actively-controlled structure with many design parameters. A newly response spectrum, therefore, is proposed in order to evaluate simply the structural response of the actively-controlled structure subjected to earthquake.

[1] Professor, Kinki University, Wakayama, Japan. Email: mochio@mec.waka.kindai.ac.jp

MATHEMATICAL MODEL TO DERIVE THE RESPONSE SPECTRA OF AN ACTIVELY-CONTROLLED STRUCTURE

Fig. 1 shows the mathematical model to estimate the response spectra of an actively controlled main structure due to earthquake. Although several kinds of active control system have been developed for the aseismic design, this paper adopts one of the simplest systems as shown in Fig.1, that is Active Mass Damper (AMD) method, since the main purpose of this paper is not the development of a new active control system but the proposal of a new response spectrum for structure with active control system.

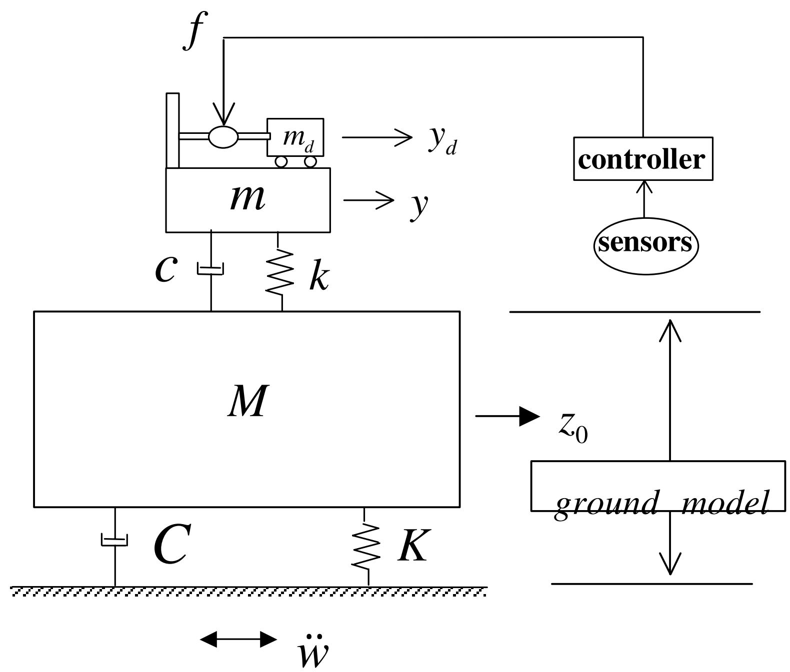

Fig. 1 Mathematical model

In Fig. 1, y, y_d and z_0 describe absolute displacements of main structure, AMD and ground, respectively, and m, c and k mean mass, damping coefficient and stiffness to the structure, respectively, and also the parameters m_d and f show mass of AMD and control force. System equations for the mathematical model shown in Fig. 1 are given as follows:

$$m\ddot{y} + c(\dot{y} - \dot{z}_0) + k(y - z_0) = -f \qquad (1)$$

$$m_d \ddot{y}_d = f \qquad (2)$$

$$f = gu \ , \qquad (3)$$

where u is control signal and g means transformed gain from u to f. Suppose that the effect of soil/structure interaction is negligible small, and then the ground acceleration $\ddot{z}_0$ can be independently set up. The ground acceleration $\ddot{z}_0$ may be frequently modeled as a response absolute acceleration through one-degree-of-freedom system subjected to Gaussian white noise $\ddot{w}$.

This model is well known as an artificial earthquake having so-called Kanai-Tajimi spectrum [2], and adopted in this paper. The governing equation, especially for Fig. 1, is expressed by

$$M\ddot{z}_0 + C\left(\dot{z}_0 - \dot{w}\right) + K\left(z_0 - w\right) = 0 \ , \tag{4}$$

and replacing with $v = z_0 - w$ in Fig. (4) yields

$$\ddot{v} + 2h\Omega\dot{v} + \Omega^2 v = -\ddot{w} \ , \tag{5}$$

where

$$2h\Omega = \left(\frac{C}{M}\right) \quad , \quad \Omega^2 = \left(\frac{K}{M}\right) \quad .$$

CONTROL RULE AND COVARIANCE OF STRUCTURAL RESPONSE

By defining the relative displacements as follows,

$$x = y - z_0 \quad , \quad x_a = y_d - y \tag{6}$$

and coupling those variables with Eqs. (1) (2) (3) and (5), the following extended state equation related to an actively-controlled structure can be obtained:

$$\dot{\boldsymbol{X}} = \boldsymbol{A}\boldsymbol{X} + \boldsymbol{B}u + \boldsymbol{D}\ddot{w} \quad , \tag{7}$$

where

$$\boldsymbol{X} = \begin{bmatrix} x \\ \dot{x} \\ x_a \\ \dot{x}_a \\ v \\ \dot{v} \end{bmatrix} \quad , \quad \boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\omega_0^2 & -2\xi\omega_0 & 0 & 0 & \Omega^2 & 2h\Omega \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \omega_0^2 & 2\xi\omega_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\Omega^2 & -2h\Omega \end{bmatrix},$$

$$\boldsymbol{B} = \begin{bmatrix} 0 \\ -\left(\dfrac{g}{m}\right) \\ 0 \\ \dfrac{g}{m}\left\{1 + \dfrac{1}{\left(\dfrac{m_d}{m}\right)}\right\} \\ 0 \\ 0 \end{bmatrix} \quad , \quad \boldsymbol{D} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad , \quad \begin{aligned} 2\xi\omega_0 &= \left(\frac{c}{m}\right) \\ \omega_0^2 &= \left(\frac{k}{m}\right) \end{aligned} \quad .$$

To achieve the active vibration control, LQG algorithm is employed in this paper, and analysis is carried out under the steady state conditions. Then a feedback gain vector $\boldsymbol{F}$ is obtained as:

$$u = -\boldsymbol{FX} \quad , \quad \boldsymbol{F} = r^{-1}\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{P} \equiv \left[k_1 \quad k_2 \quad k_3 \quad k_4 \quad k_5 \quad k_6 \right] \quad , \tag{8}$$

where $\boldsymbol{P}$ is solution of the Riccati equation

$$\boldsymbol{Q} - r^{-1}\boldsymbol{PBB}^{\boldsymbol{T}}\boldsymbol{P} + \boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{P} + \boldsymbol{PA} = \boldsymbol{0} \quad , \tag{9}$$

and $\boldsymbol{Q}$, r are the weighting parameters in the following performance index:

$$J = E\left[\boldsymbol{X}^{\boldsymbol{T}}\boldsymbol{QX} + ru^2 \right] \quad , \tag{10}$$

where $E\,[\cdot\,]$ represents the expected value related to a random variable.

The response covariance matrix at steady state concerning Eq. (7), $\boldsymbol{V}$,

$$\boldsymbol{V} = E\left[\boldsymbol{XX}^T \right] = \begin{bmatrix} \sigma_x^2 & \kappa_{x\dot{x}} & \kappa_{xx_a} & \kappa_{x\dot{x}_a} & \kappa_{xv} & \kappa_{x\dot{v}} \\ \kappa_{\dot{x}x} & \sigma_{\dot{x}}^2 & \kappa_{\dot{x}x_a} & \kappa_{\dot{x}\dot{x}_a} & \kappa_{\dot{x}v} & \kappa_{\dot{x}\dot{v}} \\ \kappa_{x_a x} & \kappa_{x_a\dot{x}} & \sigma_{x_a}^2 & \kappa_{x_a\dot{x}_a} & \kappa_{x_a v} & \kappa_{x_a\dot{v}} \\ \kappa_{\dot{x}_a x} & \kappa_{\dot{x}_a\dot{x}} & \kappa_{\dot{x}_a x_a} & \sigma_{\dot{x}_a}^2 & \kappa_{\dot{x}_a v} & \kappa_{\dot{x}_a\dot{v}} \\ \kappa_{vx} & \kappa_{v\dot{x}} & \kappa_{vx_a} & \kappa_{v\dot{x}_a} & \sigma_v^2 & \kappa_{v\dot{v}} \\ \kappa_{\dot{v}x} & \kappa_{\dot{v}\dot{x}} & \kappa_{\dot{v}x_a} & \kappa_{\dot{v}\dot{x}_a} & \kappa_{\dot{v}v} & \sigma_{\dot{v}}^2 \end{bmatrix} \tag{11}$$

satisfies the following Liapunov equation:

$$\left(\boldsymbol{A} - \boldsymbol{BF} \right)\boldsymbol{V} + \boldsymbol{V}\left(\boldsymbol{A} - \boldsymbol{BF} \right)^{\boldsymbol{T}} + \boldsymbol{S} = \boldsymbol{0} \quad , \tag{12}$$

where $\boldsymbol{S}$ is composed of the power spectral intensity ζ for a stationary Gaussian white noise $\ddot{w}(t)$ of which the statistical characteristics may be expressed as follows,

$$E\left[\ddot{w}(t) \right] = 0 \quad , \quad E\left[\ddot{w}(t)\,\ddot{w}(t+\tau) \right] = 2\pi\zeta\delta(\pi) \quad . \tag{13}$$

Then $\boldsymbol{S}$ can be explained in the following form:

$$\boldsymbol{S} = \left(2\pi\zeta \right)\boldsymbol{DD}^T \quad , \tag{14}$$

and consequently the covariance of structural response can be obtained by solving Eq. (12).

DERIVATION OF RESPONSE SPECTRA

Expression for a response spectrum concerning the absolute response acceleration of structure (primary system) is generally classified into the following two patterns:

1) $R_a(T,\xi)$; maximum value of the absolute response acceleration of structure having natural period T and damping ratio ξ

2) $R_r(T,\xi)$; $\left[R_a(T,\xi) \big/ \textit{maximum value of input acceleration} \right]$ (= response factor) ,

where the former may be frequently used to know the absolute response acceleration of structure, the latter being convenient in order to evaluate the response ratio. The latter expression is, in this paper, adopted and also equal to the following term, especially for Fig. 1 :

$$R_r\left(T,\xi\right)=\frac{\left|\ddot{y}\right|_{\max}}{\left|\ddot{z}_0\right|_{\max}} \quad . \tag{15}$$

Equation (15) is approximately expressed by adopting the basic study [3] as follows :

$$R_r\left(T,\xi\right)=\frac{\left|\ddot{y}\right|_{\max}}{\left|\ddot{z}_0\right|_{\max}}=\frac{\text{S.D.}\ \textit{of absolute response acceleration of structure}\left(\sigma_{\ddot{y}}\right)}{\text{S.D.}\ \textit{of input acceleration}\left(\sigma_{\ddot{z}_0}\right)} \tag{16}$$

S.D. ; standard deviation

therefore, $R_r\left(T,\xi\right)$ can be approximately given by calculating $\sigma_{\ddot{y}}$ and $\sigma_{\ddot{z}_0}$.

By Eqs. (1),(3) and (6), the following equation:

$$\ddot{y}\left(=\ddot{x}+\ddot{z}_0\right)=-2\xi\omega_0\dot{x}-\omega_0^2x-\left(\frac{g}{m}\right)u \tag{17}$$

is easily derived. Hence, substitution of Eq. (8) into Eq. (17) gives the required square of $\sigma_{\ddot{y}}$ as

$$\begin{aligned}
\sigma_{\ddot{y}}^2 &= \left(\frac{gk_1}{m}-\omega_0^2\right)^2\sigma_x^2+\left(\frac{gk_2}{m}-2\xi\omega_0\right)^2\sigma_{\dot{x}}^2+\left(\frac{gk_3}{m}\right)^2\sigma_{x_a}^2 \\
&\quad +\left(\frac{gk_4}{m}\right)^2\sigma_{\dot{x}_a}^2+\left(\frac{gk_5}{m}\right)^2\sigma_v^2+\left(\frac{gk_6}{m}\right)^2\sigma_{\dot{v}}^2 \\
&\quad +2\left(\frac{gk_1}{m}-\omega_0^2\right)\left(\frac{gk_2}{m}-2\xi\omega_0\right)\kappa_{x\dot{x}}+2\left(\frac{gk_1}{m}-\omega_0^2\right)\frac{gk_3}{m}\kappa_{xx_a}+2\left(\frac{gk_1}{m}-\omega_0^2\right)\frac{gk_4}{m}\kappa_{x\dot{x}_a} \\
&\quad +2\left(\frac{gk_1}{m}-\omega_0^2\right)\frac{gk_5}{m}\kappa_{xv}+2\left(\frac{gk_1}{m}-\omega_0^2\right)\frac{gk_6}{m}\kappa_{x\dot{v}} \\
&\quad +2\left(\frac{gk_2}{m}-2\xi\omega_0\right)\frac{gk_3}{m}\kappa_{\dot{x}x_a}+2\left(\frac{gk_2}{m}-2\xi\omega_0\right)\frac{gk_4}{m}\kappa_{\dot{x}\dot{x}_a} \\
&\quad +2\left(\frac{gk_2}{m}-2\xi\omega_0\right)\frac{gk_5}{m}\kappa_{\dot{x}v}+2\left(\frac{gk_2}{m}-2\xi\omega_0\right)\frac{gk_6}{m}\kappa_{\dot{x}\dot{v}} \\
&\quad +2\left(\frac{g}{m}\right)^2k_3k_4\kappa_{x_a\dot{x}_a}+2\left(\frac{g}{m}\right)^2k_3k_5\kappa_{x_av}+2\left(\frac{g}{m}\right)^2k_3k_6\kappa_{x_a\dot{v}} \\
&\quad +2\left(\frac{g}{m}\right)^2k_4k_5\kappa_{\dot{x}_av}+2\left(\frac{g}{m}\right)^2k_4k_6\kappa_{\dot{x}_a\dot{v}}
\end{aligned}$$

$$+2\left(\frac{g}{m}\right)^2 k_5 k_6 \kappa_{v\dot{v}} \quad . \tag{18}$$

Similarly, Eq. (5) yields

$$\ddot{z}_0\left(=\ddot{v}+\ddot{w}\right)=-2h\Omega\dot{v}-\Omega^2 v \quad , \tag{19}$$

and then, the required square of $\sigma_{\ddot{z}_0}$ can be also given by

$$\sigma_{\ddot{z}_0}^2 = 4h^2\Omega^2\sigma_{\dot{v}}^2 + \Omega^4\sigma_v^2 + 4h\Omega^3\kappa_{v\dot{v}} \quad . \tag{20}$$

Finally, applying Eqs. (18) and (20) to Eq. (16) can approximately produce the $R_r\left(T,\xi\right)$.

EXAMINATION OF THE PROPOSED METHOD

The response spectrum for an actively-controlled structure, in this paper, is derived from Eq. (16). Hence, the validity of the proposed technique should be examined through the comparison between analytical results and simulations. The following parameters, say ' reference case ' for the later convenience, are prepared for the comparison:

1) *parameters for mass and control*

$$\left(\frac{m_d}{m}\right)=0.02 \quad , \quad \left(\frac{g}{m}\right)=0.98$$

$$r=1 \quad , \quad \boldsymbol{Q}=\begin{bmatrix} qq & & & & & \\ & qq & & & \boldsymbol{0} & \\ & & 1 & & & \\ & & & 1 & & \\ & \boldsymbol{0} & & & 1 & \\ & & & & & 1 \end{bmatrix} \quad ; \quad \left(qq=100\right)$$

2) *parameters for structural dynamics* .

$$\omega_0 = 2\pi \quad , \quad \xi = 0.01$$

3) *parameters for input*

$$\Omega = 5\pi \quad , \quad h = 0.6 \quad , \quad \zeta = 1$$

Table 1 shows the comparison between the analytical values following the definition of right hand side in Eq. (16) and the numerical ones obtained by Monte Carlo simulations according to the definition of left hand side in Eq. (16), as for various combinations of parameters such as ξ , ω_0 , qq (the remains are the same as reference case) . The numerical values are obtained through one hundred simulations by artificial earthquakes with 50-second period as the stationary random process.

Table 1 Comparison of results by analytical definition with those by numerical definition at Eq. (16)

parameters			analytical value	numerical value
ξ	ω_0	qq		
0.01	2π	100	1.14	0.939
0.01	5π	100	2.02	1.84
0.01	5π	1000	1.25	1.21
0.05	2π	100	1.08	0.912
0.05	5π	100	1.92	1.75
0.05	5π	1000	1.24	1.19

As seen in table 1, analytical values are good agreements with numerical ones, and slightly exceed the numerical values in all cases, implying that the analytical estimation may be a kind of safety-sided design. Though this paper evaluates Eq. (16) under the assumption of stationary random process, the reference [3] has examined it, but only for non-active-controlled structure, with respect to some natural earthquakes as non-stationary random process. Eventually, these result in that the proposed method using Eq. (16) is considered as a useful one in order to obtain simply the response spectra for the actively-controlled structure subjected to earthquake.

PARAMETRIC STUDY

Difference of response spectra with/without active control

Response spectra $R_r(T,\xi)$ with/without active control are given in Fig. 2 and 3, respectively, with parameter $\xi = 0.005$, 0.01, 0.05 (the remains are the same as reference case).

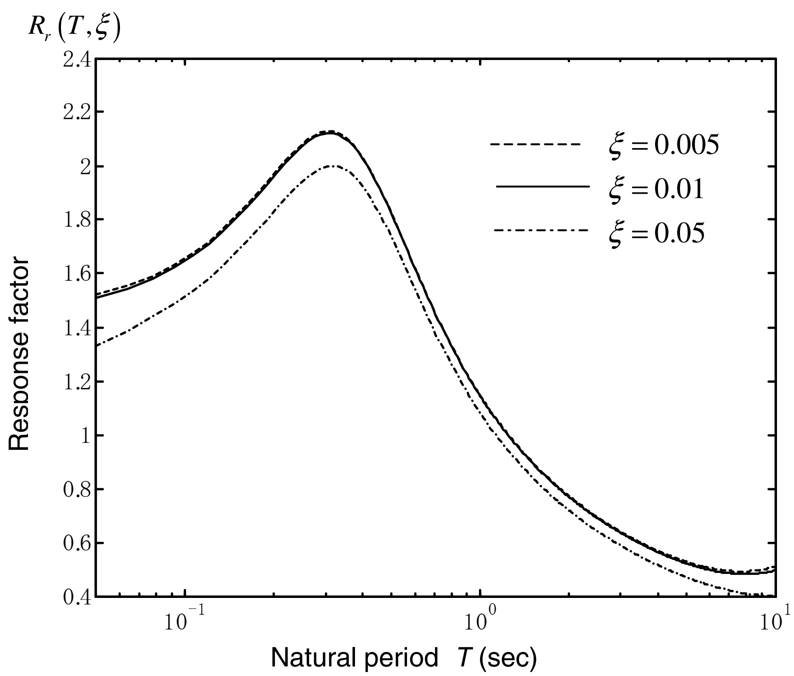

Fig. 2 Response spectrum $R_r(T,\xi)$ with active control

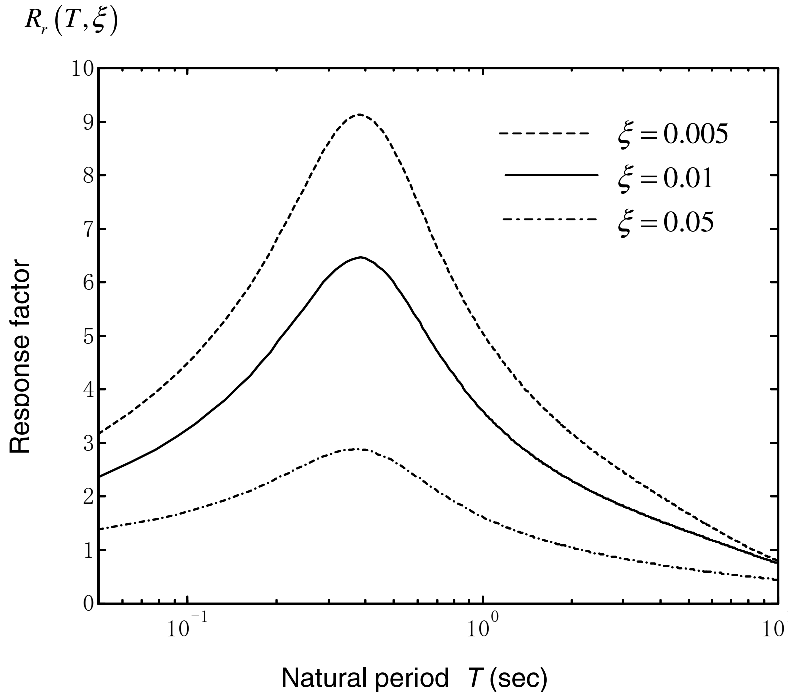

Fig. 3 Response spectrum $R_r(T,\xi)$ without active control

Comparison of Fig. 2 with Fig. 3 shows that the values of response factor with active control become lower than those without active control in any frequency region, and this means the usefulness of active control technique to aseismic design at least relating to the parameters used in this paper. It is noted that response factors at higher frequency region in Fig. 2 tend to 1 which is general characteristics in response spectra without active control, since the control rule adopted in this paper aims to reduce the response level of $x, \dot{x}$ by considering more weighting parameters at Eq. (10), which implies $y \to z_0$ as $x \to 0$.

Response spectra with active control

Response spectra with active control can be simply obtained by the proposed method, concerning the parameters of $\left({m_d}/{m} \right)$, $\left({g}/{m} \right)$ and qq , as shown in Figs. 4,5,6, respectively. Damping ratio ξ used in each figure is 0.01 as a general value. The remains of parameters are the same as reference case.

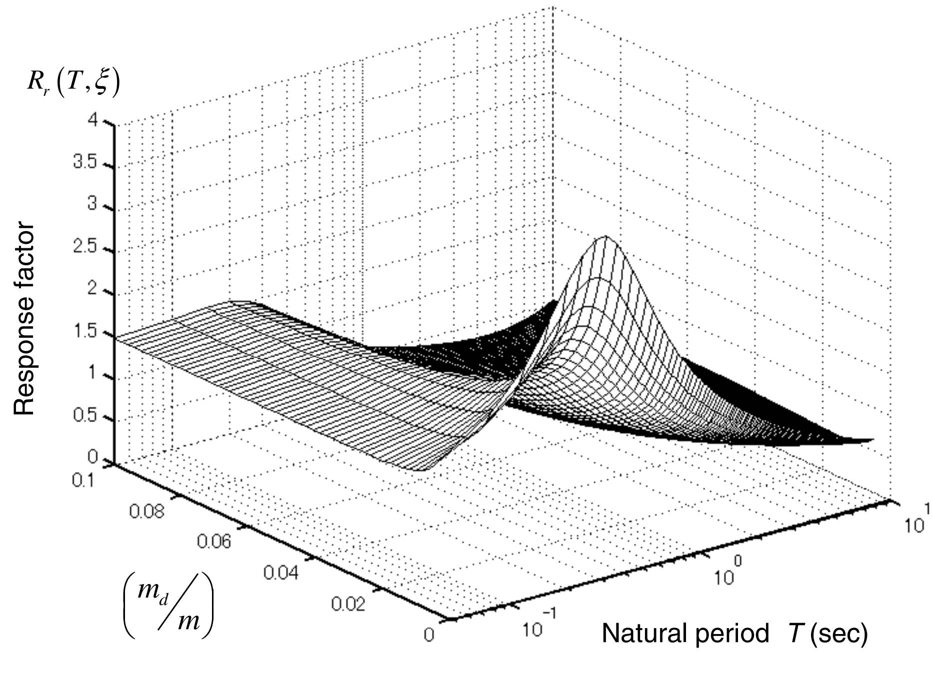

Fig. 4 Response spectrum with active control as function of $\left(m_d/m\right)$

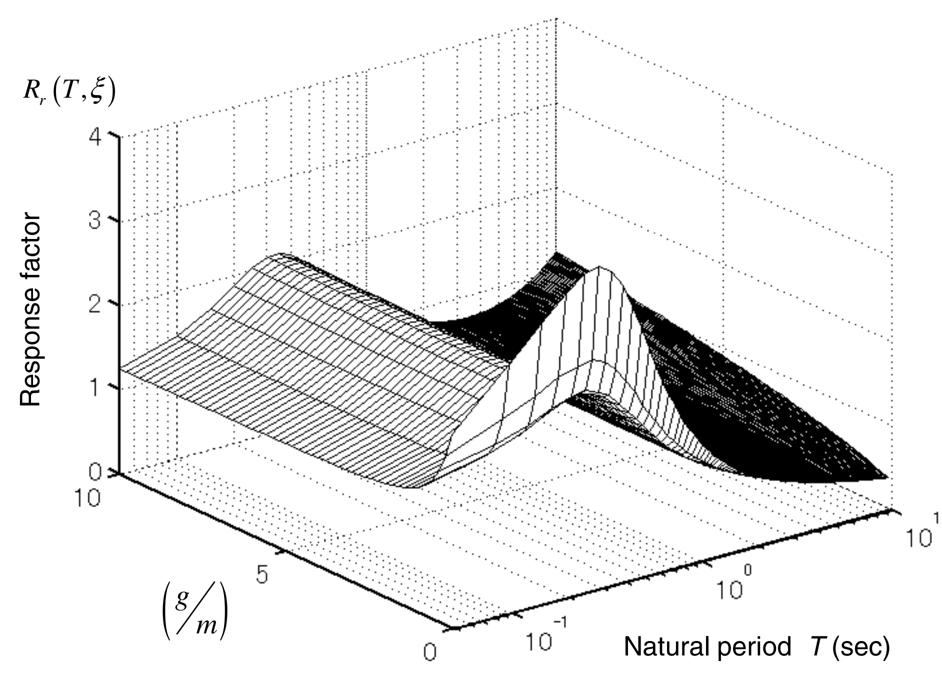

Fig. 5 Response spectrum with active control as function of $\left(g/m\right)$

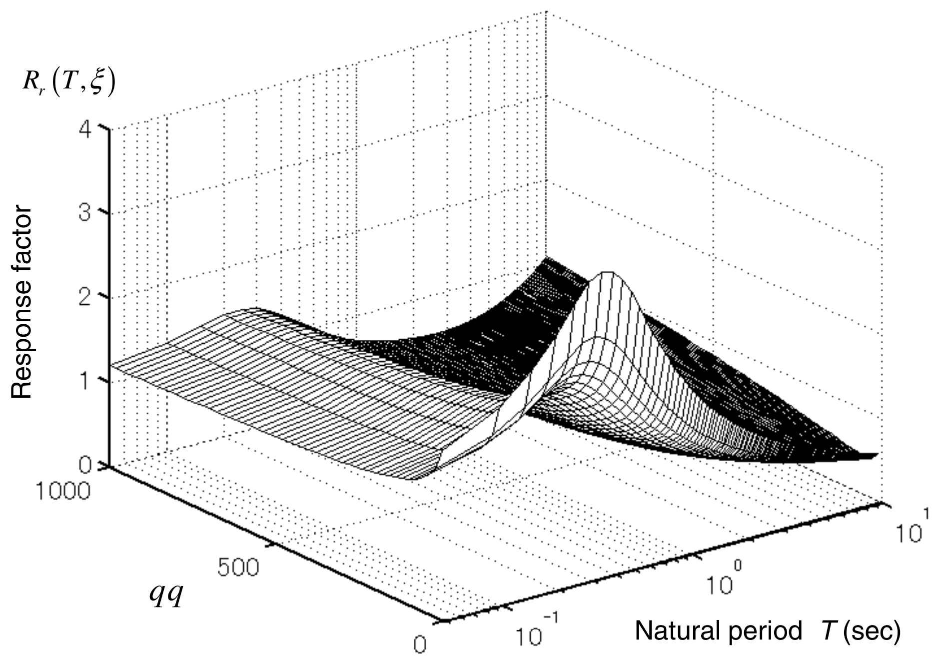

Fig. 6 Response spectrum with active control as function of qq

The parameters of $\left(m_d/m\right)$, $\left(g/m\right)$ and qq are a close relation to the performance of active control. Increase of each parameter, generally speaking, implies the improvement of the performance as for depressing the response level, especially at resonance region, oppositely meaning that the response spectrum with active control moves toward to one without active control as each parameter tends to zero.

By utilizing above mentioned response spectra with active control, one can easily evaluate not only the dynamic structural response of a structure but also the effects of reducing the structural vibration through setting an active control system, simultaneously.

CONCLUSION

The active control technology is often applied to structures due to earthquake. On the structural design for such an actively-controlled structure, the detailed dynamical analysis is executed from a initial design stage, since the design requires not only examination of structural integrity but also estimation of control characteristics against the structure. On the other hand, the dynamical seismic design of structures is also carried out frequently by using response spectra as functions of damping ratio and natural period of a structure. This paper describes a newly response spectrum in order to evaluate simply the structural response of the actively-controlled structure subjected to earthquake, by coupling the concept of response spectra with one of active control.

This paper adopts a linear single-degree-of-freedom structure with active mass damper system as an analytical model. Also, the earthquake wave is modeled as a stationary Gaussian random process with the Kanai-Tajimi spectrum. The control design is executed by using linear quadratic Gaussian control strategy.

Analytical results are compared with numerical simulations, and both show a good agreement. As a result, it seems that the validity of the proposed technique is confirmed.

It is considered that the newly developed technique is useful for a simple estimation at initial seismic design stage for an actively-controlled structure, though it is an approximated method.

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