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Embedded Foundation with Different Parameters under Dynamic Excitations

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Summary

A general methodology is outlined to study the dynamic behaviour of embedded foundation in homogeneous and layered soil medium. A computational tool is developed to determine the impedance functions of foundation in layered soil medium. Cone frustums are used to model the foundation soil system. Cone frustums are developed based on wave propagation principles and force-equilibrium approach. Various degrees of freedom, such as, horizontal, vertical and rocking are considered for this study. Different parameters, such as, nature of soil medium, embedment depth of foundation, Poisson's ratio of soil medium, depth of top soil layer are considered for the present study.

Key Words: **soil-structure-interaction, embedded foundations, layered soil medium, cone frustums, impedance functions, dynamic excitations.**

INTRODUCTION

There are many parameters affecting the dynamic response of structures, such as: the type of structure, type of foundation, soil characteristics etc. The observations from the earthquake damaged sites show that, the local soil properties, underground and surface topography of soil medium and the foundation geometry have an important effect on the dynamic behaviour of structures. The local soil conditions and the interaction between soil and foundation will affect the dynamic behaviour of a structure in three different ways, such as, soil amplification effect, kinematic interaction effect and inertial interaction effect. The total influence is generally termed as Soil-Structure Interaction (SSI) effects.

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The critical step involved in the substructure approach of SSI analysis is to determine the force-displacement characteristics of the soil. This relationship may be in the form of an impedance (stiffness) function, or, inversely, a compliance (flexibility) function. For the present study, the force-displacement relationship of foundation is expressed with impedance function. The foundation impedance functions depend on the soil configuration, material behaviour, frequency of excitation and type of foundation. In general, for a linear elastic or visco-elastic material with a homogeneous or horizontally stratified soil deposit, the impedance function is both complex valued and frequency dependent. The soil deposits rarely have uniform properties, particularly when considering the variation of shear modulus and Poisson's ratio. In general, the soil media consists of multiple layers with different properties; some times drastic changes in properties can be noticed between two adjacent layers. The impedance functions of foundations embedded in visco-elastic soil medium can be determined either with rigorous approach (Bycroft [1]) or with simple physical model (Wolf and Paranesso [2]).

The analysis becomes more complex if the soil media is inhomogeneous consisting of multiple layers with different properties. Treating the inhomogeneous soil media as a homogeneous one with average properties or as a media with properties varying linearly or parabolically will give rise to unrealistic solutions. The shear and dilatational waves generated by the exciting force propagate through each of the soil layer with differing amplitudes. They will experience reflections at the interfaces of soil layers and will diminish in amplitudes as they travel towards the far field.

Jaya and Prasad [3] have developed a simple physical model (cone frustum model) to determine the dynamic behaviour of the embedded foundation in layered as well as homogeneous soil medium. The developed cone frustum model is capable of incorporating the complexity of layered soil media and the complicated behaviour of propagating waves. The behaviour of embedded foundation in layered soil medium is studied using cone frustum

model and the results are compared with the published results (Jaya and Prasad [3]).This paper presents the influence of various parameters on the impedance functions embedded foundations.

CONE FRUSTUM MODEL

Ehlers [4] has developed the basic cone model for surface foundation under translational motion in homogeneous soil. Later, Meek and Wolf [5] presented a simplified methodology to enable the practitioner to evaluate the dynamic response of base mat on the surface of homogeneous soil. The cone model concept was extended to compute the dynamic response of a footing or base mat on a soil layer resting on rigid rock (Meek and Wolf [6]) and on flexible rock (Wolf and Meek [7]). Meek and Wolf [8] performed the dynamic analysis of embedded footings by idealizing the soil as a truncated cone instead of an elastic half-space. Double cone is introduced to represent the disk in the interior of a homogeneous full-space. Wolf and Meek [8] calculated the dynamic-stiffness coefficients of a disk on the surface of a horizontally stratified site with multiple layers by introducing cone frustum model. Later, the concept of cone frustum model is extended for embedded foundation (Jaya and Prasad [3]). The basic steps for the development of cone frustum model for an embedded foundation are briefly discussed.

Fig.1 shows a rigid cylindrical foundation of radius r_0 embedded to a depth *e* in a layered half-space. The stratified soil consists of multiple layers with varying material properties. For a particular soil layer *m,* $G(m)$, $\nu(m)$, $\rho(m)$ and d_m, are the shear modulus, Poisson's ratio, the mass density and the depth of soil layer, respectively.

The first step in the evaluation of impedance function of the foundation is to develop the stiffness matrix of the free field. The free field is defined as the soil medium without any excavations or rigid foundations. The development of free-field stiffness matrix of the site

demands the discretisation of the cylindrical soil region with rigid disks, where the foundation is to be inserted later (Fig.1). The number of rigid disks existing per soil layer depends on the properties of the corresponding soil layer, foundation geometry and the maximum frequency of excitation. For good accuracy, the soil region is discretised in such a way that the thickness of the soil slices should not exceed one sixth of the shortest wavelength of the propagating waves. For a soil layer *m* with thickness d_m, the thickness of soil slices within the layer, $\Delta e(m)$, is determined with the expression Eq.1. The number of disks, *N(m)*, stacked per layer is determined as per Eq.2 and the total number of disks stacked in the cylindrical soil region, where the foundation is to be inserted is *nd*, (Eq.3).

$$\Delta e(m) = \frac{1}{3}\frac{\pi \ r_0}{a_0}\left(\frac{c}{c_s}\right) \tag{1}$$

$$N(m) = \frac{3}{\pi}\frac{d_m}{r_0}\frac{c_s}{c}a_0 + 1 \tag{2}$$

$$nd = \sum_{m=1}^{ln} N(m) \tag{3}$$

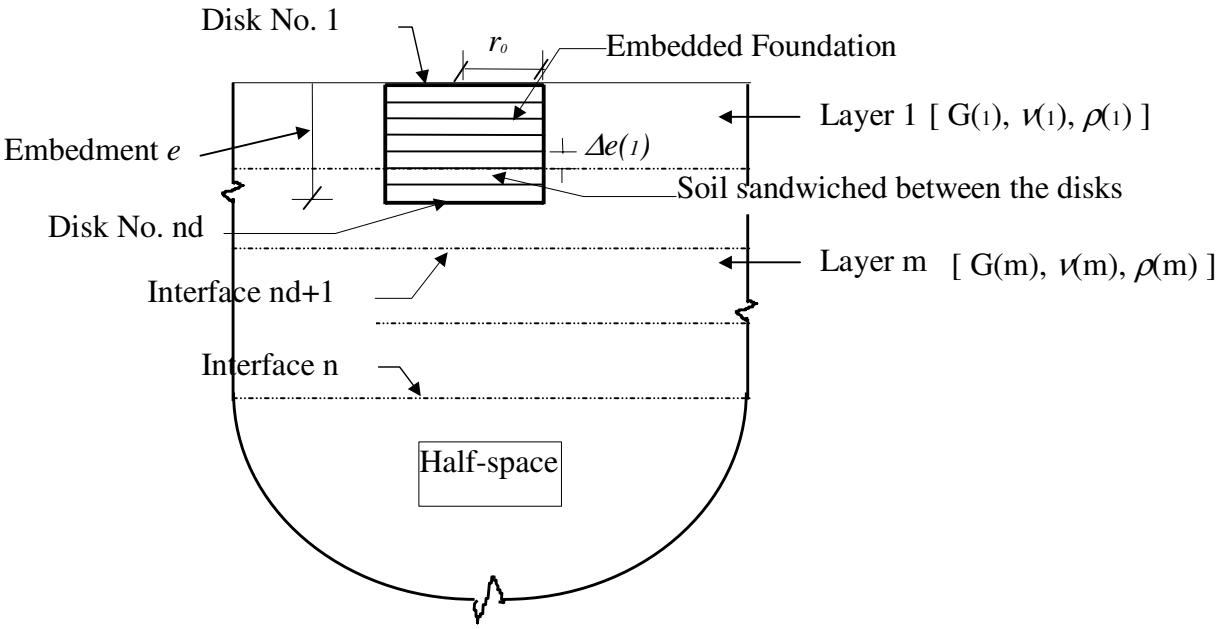

Fig.1. - Embedded cylindrical foundation with radius r_0 and embedment depth e in layered soil medium resting on a half-space

DYNAMIC STIFFNESS COEFFICIENT OF FOUNDATION

The soil layers sandwiched between the rigid disks are treated as cone frustums and the underlying homogeneous half-space as a single cone. The backbone cone for each rigid disk is developed independently [Fig.2a and Fig.2b]. For the rigid disk at the surface level, the backbone cone is developed as shown in Fig. 2a, which starts from the surface disk and extends downwards to infinity. The backbone cones for all other embedded disks are developed as shown in Fig. 2b, which start from the source disk with two cones. One of cones extending upwards to the soil surface and the other extending downwards to infinity. Fig.2c shows the details of a single cone frustum with rigid disks at interfaces. Foundations under translational (horizontal and vertical) and rotational (rocking) modes are treated separately.

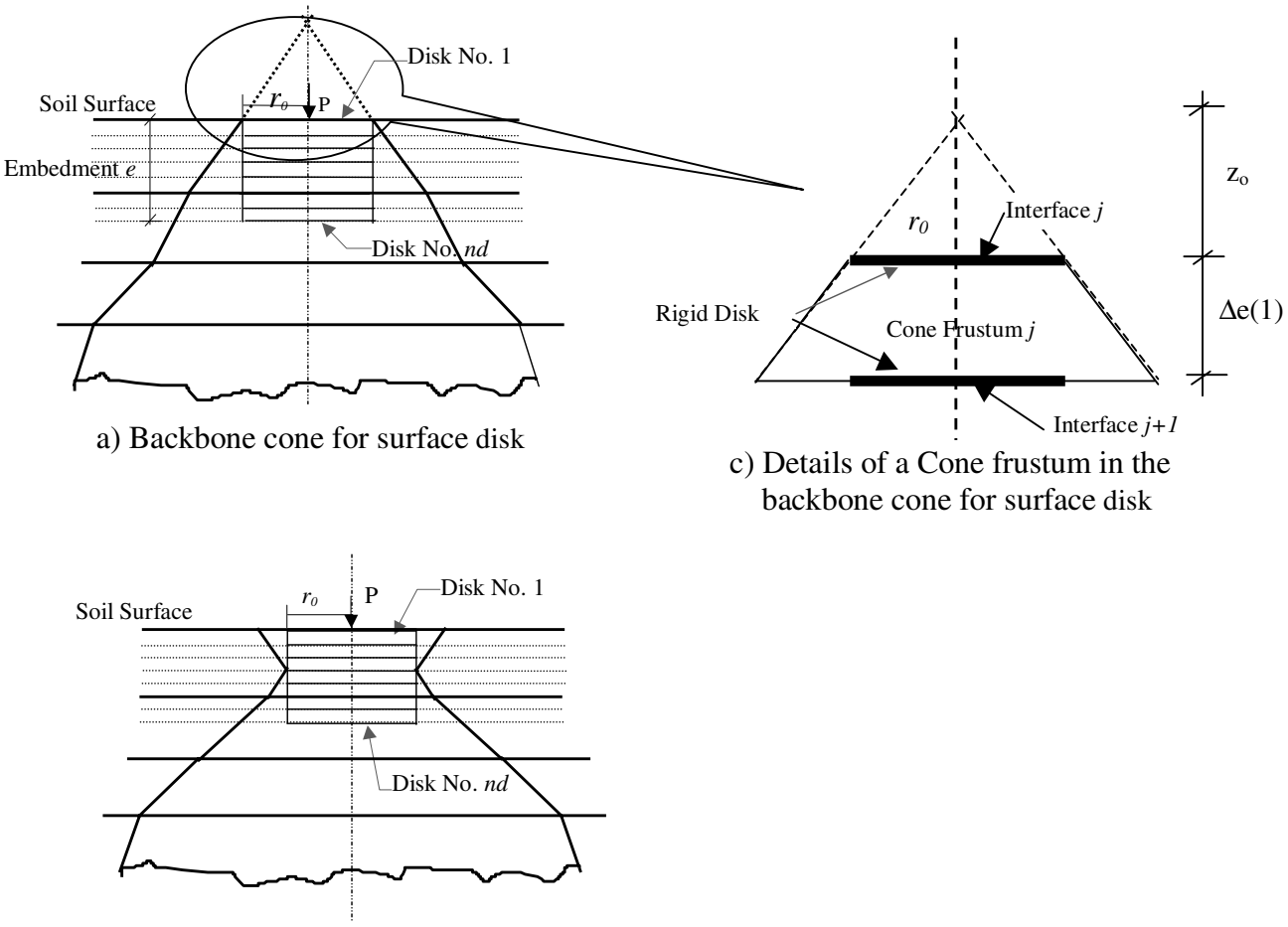

Fig.2 – Backbone cones for surface and embedded disks.

At first, consider the backbone cone for a surface disk, as shown in Fig. 2a. The radii of the cone frustums at the interfaces are determined from the geometry of the backbone cone. In the backbone cone, any cone frustum *j* is bounded by interfaces *j* and *j+1* (Fig. 2c). The force-displacement relationship for the cone frustum *j,* can be written as given in Eq.4 (Wolf [9]). The coefficient matrix in Eq.4 denotes the dynamic-flexibility matrix of the cone frustum, *j*, which connects the displacements and the forces at interfaces *j* and *j+1*.

$$\begin{Bmatrix} u_j(\omega) \\ u_{j+1}(\omega) \end{Bmatrix} = \begin{bmatrix} \dfrac{T_{j,j}(\omega)}{S_j(\omega)} & \dfrac{T_{j,j+1}(\omega)}{S_{j+1}(\omega)} \\ \dfrac{T_{j+1,j}(\omega)}{S_j(\omega)} & \dfrac{T_{j+1,j+1}(\omega)}{S_{j+1}(\omega)} \end{bmatrix} \begin{Bmatrix} P_j(\omega) \\ P_{j+1}(\omega) \end{Bmatrix} \tag{4}$$

where, $P_j(\omega)$ and $u_j(\omega)$ are the force and displacement amplitudes at interface *j* and $P_{j+1}(\omega)$ and $u_{j+1}(\omega)$ are the force and displacement amplitudes at interface *j+1*. The coefficients $S_j(\omega)$ and $S_{j+1}(\omega)$ are the dynamic-stiffness coefficients of the single cones with surface disks of radius r_j and r_{j+1}, respectively. The properties of the soil half-space are same as that of the cone frustum *j,* bounded by the interfaces *j* and *j+1*. The expressions for $S_j(\omega)$ and $S_{j+1}(\omega)$ for various degrees of freedom are given in Jaya and Prasad (3). The transfer functions $T_{j,j}(\omega)$, $T_{j,j+1}(\omega)$, $T_{j+1,j}(\omega)$ and $T_{j+1,j+1}(\omega)$, defined in Eq.4, can be derived from the superposition of the Green's functions defining the displacements at the interfaces of cone frustums (Jaya and Prasad [3]). The inverse of the coefficient matrix in Eq.4 gives the dynamic stiffness matrix of the cone frustum. Similarly, the dynamic –stiffness matrices of all the cone frustums in the backbone cone can be developed independently.

The underlying half-space is modeled as a single cone with disk *n* at the surface, for which the force-displacement relation is written as,

$$P_n(\omega) = S_n(\omega) \;\; u_n(\omega) \tag{5}$$

where, the coefficient $S_n(\omega)$ is the dynamic-stiffness coefficient of the underlying half-space with a rigid disk of radius r_n at the surface. $P_n(\omega)$ and $u_n(\omega)$ are the force and displacement amplitudes at the nth interface. The dynamic-stiffness matrix of the backbone cone is then developed by augmenting the dynamic-stiffness matrices of all the layers and the dynamic-stiffness coefficient of the underlying half-space. The dynamic-equilibrium equation for the backbone cone is then written as,

$$\left[S(\omega)\right]_1 \left\{u(\omega)\right\}_1 = \left\{Q(\omega)\right\}_1 \tag{5}$$

where, $\left[S(\omega)\right]_1$ denotes the dynamic-stiffness matrix associated with the backbone cone representing a rigid disk (disk no.1) on the surface of layered medium, $\left\{u(\omega)\right\}_1$ is the

displacement amplitudes at the locations of all the disks due to unit load at the surface disk (disk no.1) and $\{Q(\omega)\}_1$ is the vector of external load amplitudes at all the disks locations. Similarly, for all other loaded embedded disks, the displacement vectors are developed and represented as, $\{u(\omega)\}_2$, $\{u(\omega)\}_3$, ……,$\{u(\omega)\}_n$. All the displacement vectors are augmented together to form the dynamic-flexibility matrix of the free field, the inverse of which gives the dynamic stiffness matrix of the free field, $\left[S^f(\omega)\right]$. The matrix $\left[S^f(\omega)\right]_{n\times n}$ is discretised in the nodes corresponding to the rigid disks. The force-displacement relationship for the free field is then written as,

$$\{P(\omega)\} = \left[S^f(\omega)\right] \{u(\omega)\} \tag{6}$$

For foundations under horizontal excitations, the same procedure is adopted to evaluate the dynamic-stiffness matrix of the free-field. As the soil mass deforms in shear under horizontal excitations, the shear modulus G of soil and shear wave velocity c_s are considered for the analysis. In the same manner, the force displacement relationship of foundation under rocking motion can also be determined. Then various dynamic-stiffness coefficients of foundation can be determined by considering the kinematic interaction effect as well as extraction of soil mass from the free field and can be expressed as Eq.7, Eq. 8 and Eq.9 (Jaya and Prasad [3]),

$$S(\omega)_h = \{1\}^T \left[S^f(\omega)\right]_h \{1\} + \omega^2 m_a \tag{7}$$

$$S(\omega)_v = \{1\}^T \left[S^f(\omega)\right]_v \{1\} + \omega^2 m_a \tag{8}$$

$$\begin{aligned} S(\omega)_r = {} & \{e\}^T \left[S^f(\omega)\right]_h \{e\} + \omega^2 \frac{m_a e^2}{3} \\ & + \{1\}^T \left[S^f(\omega)\right]_r \{1\} + \omega^2 \frac{m_a r_0^2}{4} \end{aligned} \tag{9}$$

The various inertial quantities involve the mass $m_a = A_0\, e\, \rho$ of the excavated soil cylinder. The rocking dynamic-stiffness coefficient $S(\omega)_r$ (Eq. 9) consists of two separate components. Rigid-body rotation about the centre of the base is viewed as the sum of translation of the disks without rotation (vector $\{e\}$) and a constant rotation of the disks without translation (vector $\{1\}$). The vectors $\{e\}$ and $\{1\}$ can be expressed as,

$$\{1\}_{n\times 1} = [1,1,1,\cdots\cdots,1]^T \tag{10}$$

$$\{e\}_{n\times 1} = [e,\ e-\Delta e, e-2\Delta e, \cdots\cdots, \Delta e, 0]^T \tag{11}$$

The dynamic-stiffness coefficient of foundation for various degrees of freedom can be expressed as function of static-stiffness coefficient, spring coefficient and damping coefficient as,

$$S(\omega) = K_{stat}\left(k(a_0) + ia_0 c(a_0)\right) \tag{12}$$

The results of the present study for the static-stiffness coefficient (K_{sta}) of foundation are matching with the exact solutions. The spring $k(a_0)$ and damping $c(a_0)$ coefficients for various types of foundations embedded in different types of soil medium are evaluated. The cylindrical foundations, which are embedded in homogeneous as well as in layered soil medium, under horizontal, vertical and rocking excitations are considered for the present study. The impedance functions of such foundations are evaluated and the results are then validated with those of the other reported results [Jaya and Prasad (3)]. The important parameters that affect the impedance functions of foundations are identified as the type of soil medium, the amount of embedment and the depth of top soil layer. Foundations embedded in various types of homogeneous soil medium with Poisson's ratios ranging from 0 to 0.45 are analysed. The ratio of embedment depth to radius of foundation considered are 0, 1.0, 1.5 and 2. In the case of layered soil medium, the influence of top soil layer depth on impedance

functions of foundation is investigated. The formulation of the problems and the results are discussed in the following sections.

RESULTS AND DISCUSSION

Type of Soil Medium

Poisson's ratio significantly influences the dilatational wave velocity. In case of vertical vibration, the relative contribution of the dilatational wave is higher than that of the other waves. So, variation of Poisson's ratio affects the vertical response to a considerable extent. The amount of this influence is investigated by considering a cylindrical foundation embedded in various soil half-spaces with Poisson's ratios; 0, 0.25, 0.33 and 0.45, under different excitations.

The normalised impedance functions for foundations under horizontal excitations are shown in Fig. 3. The spring coefficient $k_h(a_0)$ increases with the increase in Poisson's ratio, at higher excitation frequency under consideration. The damping coefficient, $c_h(a_0)$, deceased by 16% for an increase in Poisson's ratio from 0 to 0.45, which remains constant throughout the whole frequency range under considerations.

Consider the foundation under vertical excitations with $e/r_0 = 1$. The normalised impedances are shown in Fig. 4. For the dimensionless frequency of excitation, $a_0 = 1$, the influence of Poisson's ratio of the soil on the spring coefficient $k_v(a_0)$ of the foundation is very little and only 1% increment is noticed due to an increase in Poisson's ratio from 0 to 0.45. But, when a_0 increases to 3, the influence of Poisson's ratio of the soil on the spring coefficient become significant; about 25% increment in the value of $k_v(a_0)$ is noticed. So, it is understood that the influence of Poisson's ratio (ν) on spring coefficient is a frequency dependent phenomenon. Also, it is observed the influence of Poisson's ratio (ν) on damping coefficient $c_v(a_0)$ is significant. There is a decrease of 24.7% in $c_v(a_0)$ value is noticed for an increase in Poisson's ratio from 0 to 0.45. From the Fig. 5, it is observed that the normalised rocking

impedance functions, [$k_r(a_0)$ and $c_r(a_0)$], are not greatly influenced by the change in Poisson's ratio of soil medium.

Embedment depth of foundations

Foundations with embedment depth to radius ratios - (e/r_0) of 0, 1.0, 1.5 and 2.0 are considered for the present study. The impedance functions of foundations under horizontal, vertical and rocking excitations are normalised as given by the Eq. 12. The normalised horizontal impedance functions are shown in Fig. 6. Fig. 7 shows the normalised vertical impedance functions and Fig. 8 shows the normalised rocking impedance functions.

It can be noticed from these results that, the spring coefficients for the horizontal, vertical and rocking excitations are not affected much due to the change in e/r_0 ratio.

In the case of surface foundations with $e/r_0 = 0$, the spring coefficient, $k(a_0)$, and damping coefficient, $c(a_0)$ remains constant throughout the frequency range under consideration for the foundations under horizontal and vertical excitations. For the foundations under rocking motions, the variation of spring as well as damping coefficients are very smooth.

For embedded foundations, the damping coefficients increase with the increase in depth of embedment (e/r_0). For the foundations under horizontal and rocking motions, the influence of e/r_0 ratio to the spring coefficients of foundation is very less. On the other hand, the damping coefficients increase with the increase in embedment depth. About 10 to 15% increment is noticed when the e/r_0 ratio increases from 1.0 to 1.5 as well as from 1.5 to 2.0.

For the foundations under vertical motions with $a_0 = 1$, as the e/r_0 ratio increases from 1 to 2, the damping coefficient increases by 23.8%. While, the spring coefficient decreases with the increase in e/r_0 ratio, but the influence of embedment depth to the spring coefficient is very less. Only 4.76% of reduction in the value of vertical spring coefficient is noticed when the e/r_0 ratio varies from 1 to 2**.**

Depth of top soil layer

The presence of bedrock below the soil layer significantly affects the dynamic behaviour of the embedded foundation. Similarly, the depth of the soil layer overlying the bedrock also influences the impedance functions of foundations. The ratio of depth of layer to the embedment depth of foundation ($\bar{d}/e$) is considered as the influencing parameter. The $\bar{d}/e$ values of 1, 3 and 5 are considered for the present study. The variation of the impedance functions of foundations under horizontal and vertical motions are shown in Fig.9 and Fig.10. When the foundation is resting on the base rock (for $\bar{d}/e = 1$), a smooth variation in the graph for spring and damping coefficients are noticed. As the top layer depth increases, peaks and valleys are noticed in the graphs for the impedance functions, which shows the importance of cut-off frequency in the analysis of foundation in soil layer (Wolf [9]). The resonance frequency of the soil medium, resting on the bottom base rock, is changing with the increase in layer depth.

CONCLUSION

A comprehensive study on the dynamic behaviour of cylindrical foundations embedded in a homogeneous and in layered half-space, under horizontal, vertical and rocking excitations, is carried out. Investigations were carried out to study the effects of embedment depth of foundation, Poisson's ratio of soil medium and the depth of top soil medium, on impedance functions of foundations. From the study, it has been observed that, the foundation imparts maximum spring coefficient values when the embedment depth of foundation is equal to the radius of foundation. Increase in Poisson's ratio of soil makes the soil-foundation system more stiffer. As the top layer depth increases, peaks and valleys are noticed in the graphs for the impedance functions, which shows the importance of cut-off frequency in the analysis of foundation in soil layer. The resonance frequency of the soil medium, resting on the bottom base rock, is changing with the increase in layer depth.

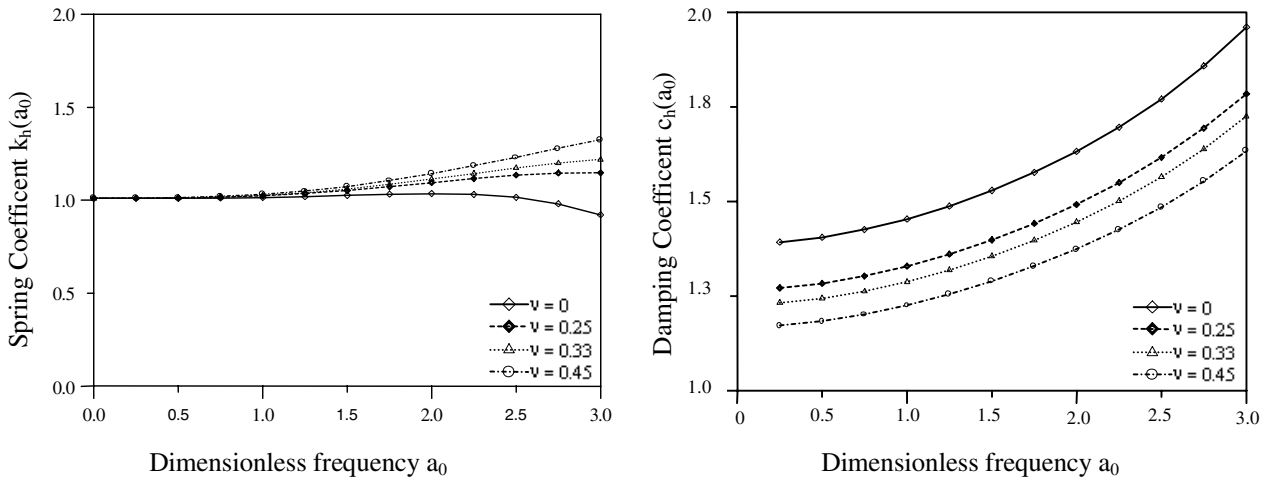

Fig. 3 - The horizontal impedance functions of the cylindrical foundation embedded in soil mediums with different Poisson's ratios

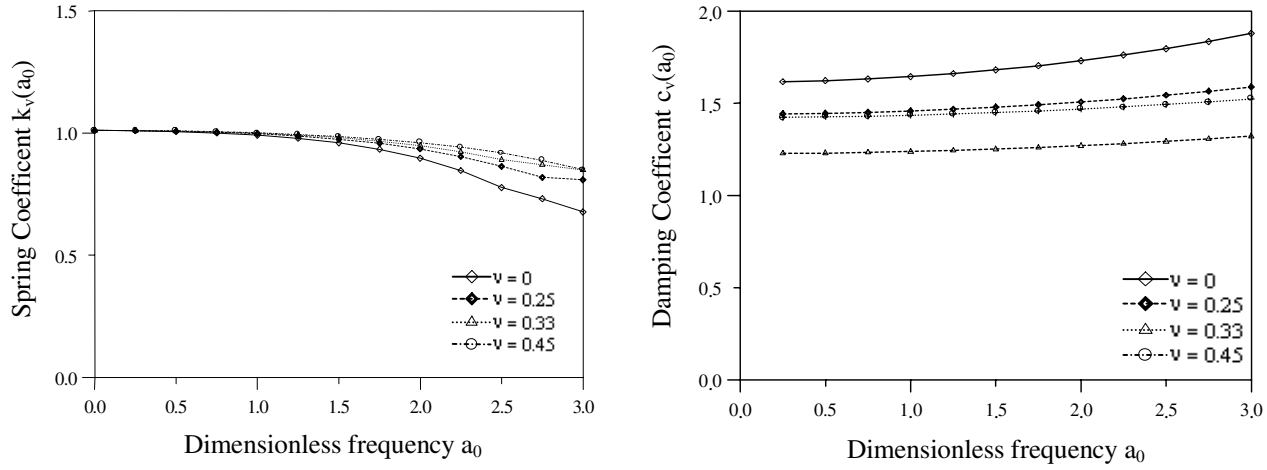

Fig. 4 - The vertical impedance functions of embedded cylindrical foundation with different Poisson's ratios

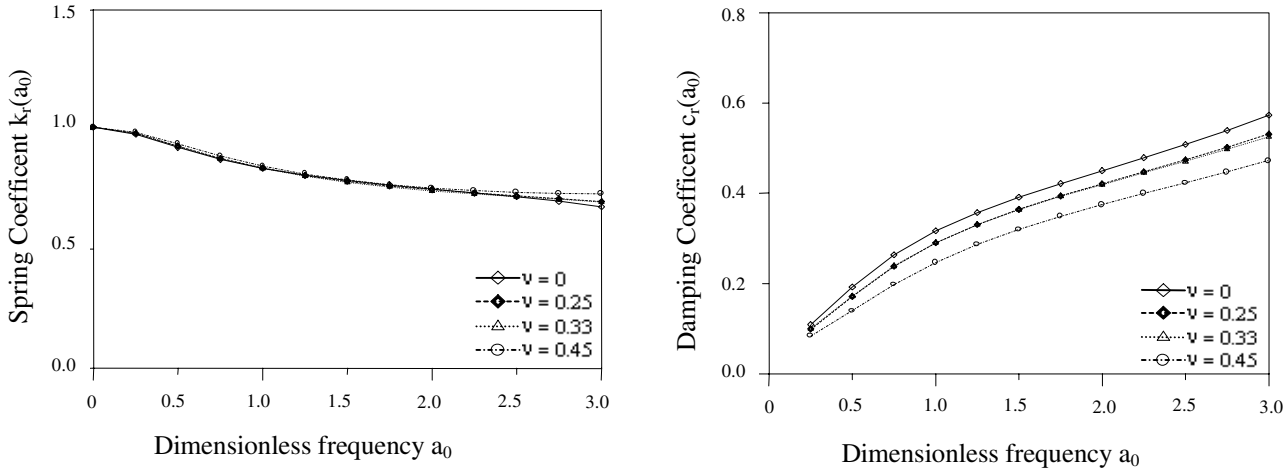

Fig. 5 - The rocking impedance functions of embedded cylindrical foundation with different Poisson's ratios

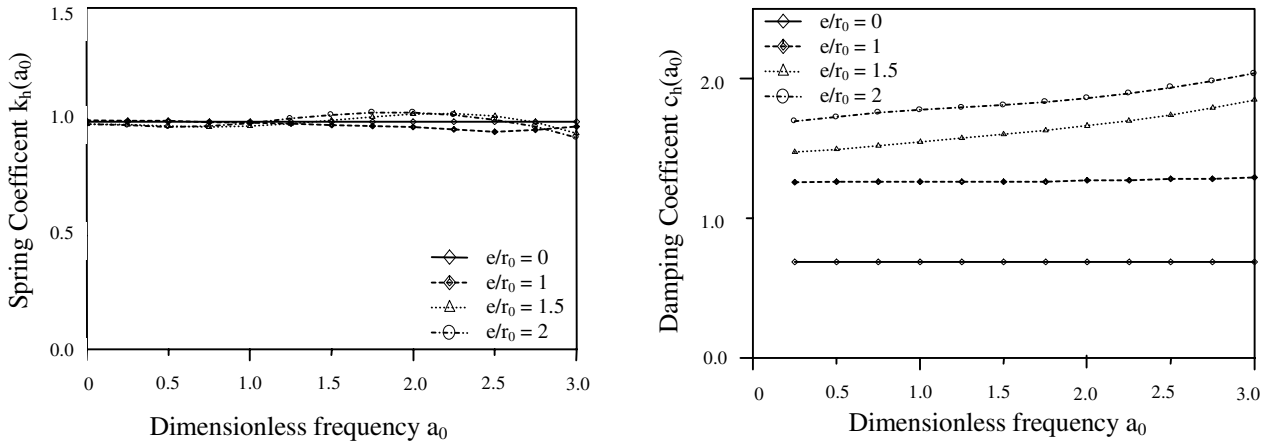

Fig. 6 - The horizontal impedance functions of embedded cylindrical foundation with different e/r_0 ratios

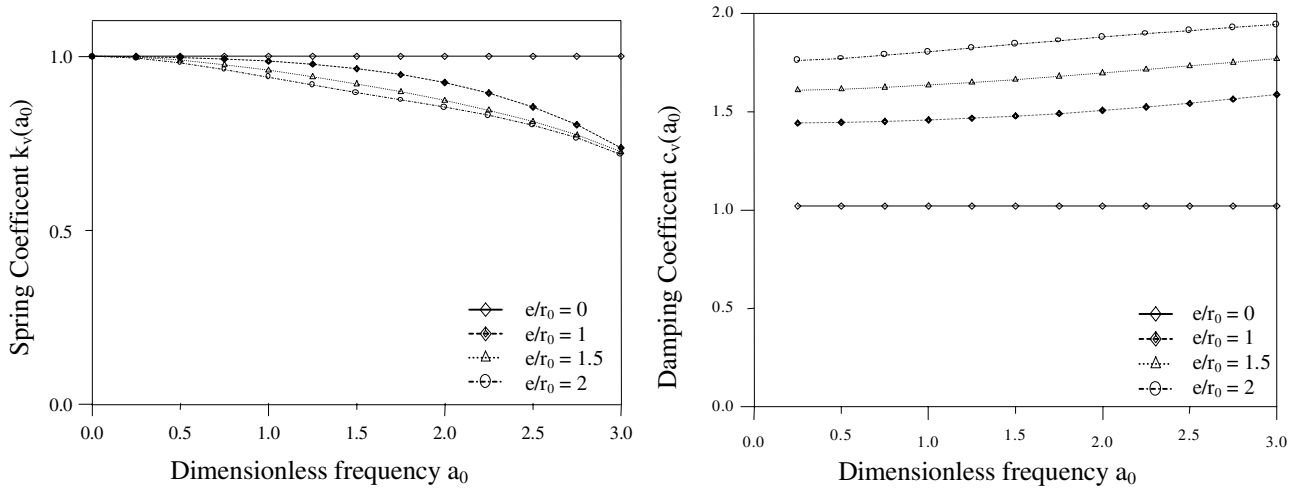

Fig. 7 - The vertical impedance functions of embedded cylindrical foundation with different e/r_0 ratios

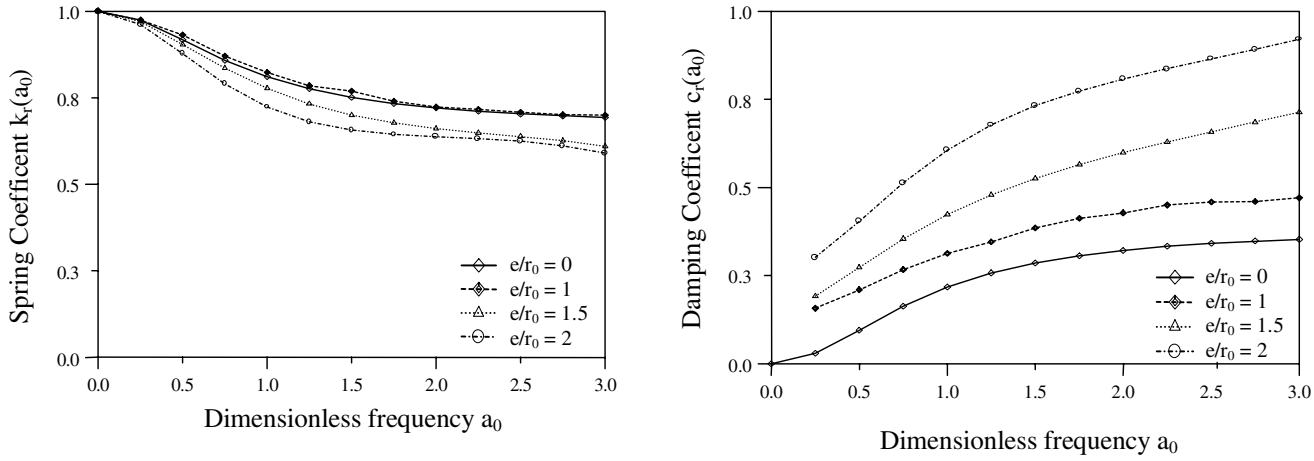

Fig. 8 - The rocking impedance functions of embedded cylindrical foundation with different e/r_0 ratios

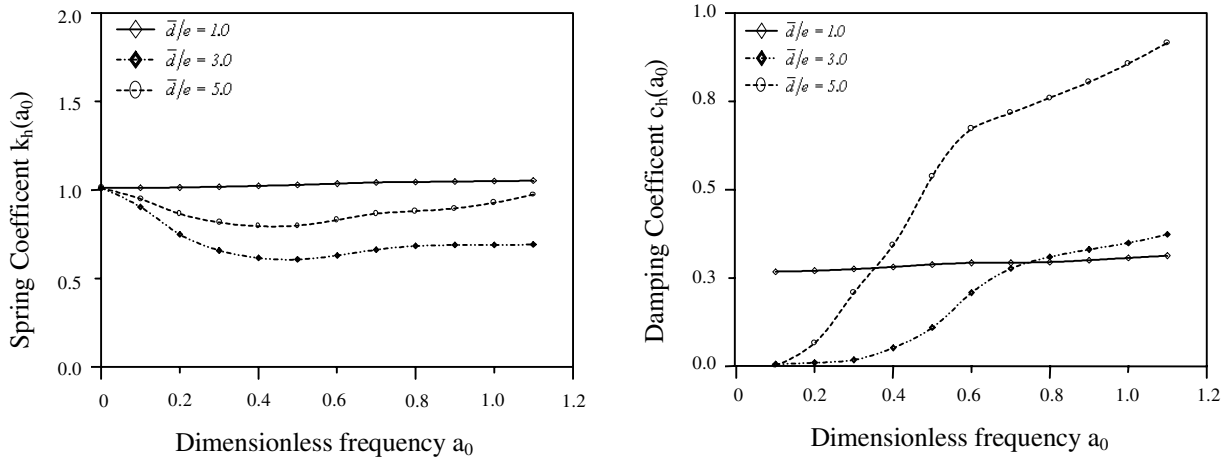

Fig. 9 - The horizontal impedance functions of embedded cylindrical foundation with various $\bar{d}/e$ ratios

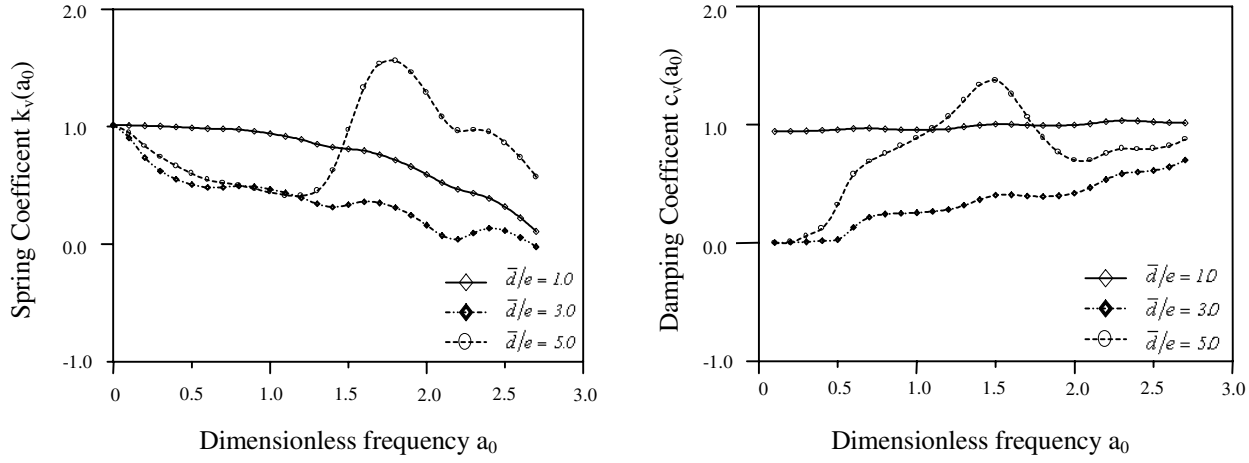

Fig. 10 - The Vertical impedance functions of embedded cylindrical foundation with various $\bar{d}/e$ ratios

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NOTATIONS

Notations		Description
$\Delta e(m)$	-	Thickness of soil sandwiched between rigid disks
r_0	-	Radius of foundation
r_j	-	Radius of cone frustum at jth interface
a_0	-	Dimensionless frequency $\left(\frac{\omega r_0}{c_s}\right)$
ω	-	Frequency of excitation
c	-	Appropriate wave velocity
c_s	-	Shear wave velocity
$N(m)$	-	Number of rigid disks in layer *m*
nd	-	Total number of disks
d_m	-	Depth of layer *m*
$u_j(\omega)$	-	Displacements at interface *j*

$P_j(\omega)$	-	Force amplitudes at interface *j*
$S_j(\omega)$	-	Transnational dynamic stiffness matrix of rigid disk at interface *j*
$T_{j,j}(\omega)$	-	Transfer function describes the displacement at interface *j* due to force at *j* under translational motions.
$\left[S(\omega)\right]_1$	-	Translational dynamic-stiffness matrix associated with the backbone cone representing a rigid disk (disk no.1)
$\{u(\omega)\}_1$	-	Displacement amplitudes at the locations of all embedded disks due to unit load at disk no.1
$\{Q(\omega)\}_1$	-	Vector of external load amplitudes
$\left[S^f(\omega)\right]$	-	Dynamic stiffness matrix of the free-field
e	-	Embedment depth of foundation
m_a	-	Mass of soil to be excavated to impose the foundation
A_0	-	Cross-sectional area of foundation
ρ	-	Mass density of soil
$S(\omega)_h$	-	Dynamic stiffness coefficient of foundation under horizontal excitations
$S(\omega)_v$	-	Dynamic stiffness coefficient of foundation under vertical excitations
$S(\omega)_r$	-	Dynamic stiffness coefficient of foundation under rocking excitations
G(m)	-	Shear modulus of soil medium at j^{th} interface
ρ(m)	-	Mass density of soil medium at j^{th} interface
ν(m)	-	Poisson's ratio of soil medium at j^{th} interface