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ACTIVE CONTROL OPTIMIZATION OF THE STRUCTURE BY EARTHQUAKE

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SUMMARY

The aim of the contribution is the numerical analysis of structural response suppression by earthquake. The proposed solution way uses actuators. The closed-loop control is applied.

The discrete model of the structure is built on the base of the finite element method. The nodal dynamic excitation is applied. The time history analysis results the response in all nodes.

The control problem is as an optimization problem defined. The objective of the optimization is the time integral of weighted displacement-velocity function square and weighted control force function square. The problem of objective function minimization is constrained with D'Alembert equilibrium equation, stress inequality constraint and control force inequality constraint. The optimization problem definition enables using of the Lagrange multiplier method effectively. The application of the Lagrange multiplier method leads to the Riccati equation. With help of the determined Riccati matrix is obtained the solution for optimal control. The control low can be modified in dependence in the stress and control force at every time step. The stress constraint at every step controls the Riccati matrix calculation through stiffness matrix. Linear and nonlinear cases are considered. The nonlinear case uses GAP element model for crack. Displacement and control force at every time step result the control energy. The control energy is calculated for all possible actuator positions. Only actuators with the highest integral values of consuming control energy are finally chosen. The procedure of displacements and control forces calculation on the base of above introduced optimization has to be repeated for the finally placed actuators.

Examples illustrate the theoretical solution.

INTRODUCTION

The foundation of control concepts for mechanical systems is connected with radar work during the Second World War. Applications in civil engineering are related to advances in computational technology, mechanical and electrical engineering during last three decades. The modern trend in active control of civil structures is to use response depending systems (closed-loop systems). The measured response

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(output) of the system is used for input – control force to the system. The paper is focused on mathematical and numerical issues of controllability in engineering practice. In our research we calculate control forces, without discussion about technical details of actuators. We take into consideration that control forces are technologically constrained. The optimization of controlled system means the minimization of the structural response in connection with the minimization of control energy by satisfying all structural, material and technological constraints.

MODEL OF THE STRUCTURE

The civil engineering structure is given. The discrete model of the structure is built on the base of the finite element method (FEM) and is characterized with help of stiffness-, mass- and damping matrices. The reduction of FE model to system with low number of degrees of freedom system (Single-Degree-of Freedom: SDOF as well Multiple-Degree-of-Freedom: MDOF) is prepared for the further analysis.

DYNAMIC LOADING

Corrected time history acceleration data are used in this work as input for the dynamic analysis. Accelerograms are chosen from Database of European Strong-Motion Data [2]. The nodal excitation of the structure is applied.

DYNAMIC ANALYSIS

Dynamic analysis and control calculation follow the concept of Hart and Wong [3].
The set of dynamic equilibrium equations is

$$\boldsymbol{M}\ \ddot{\boldsymbol{X}}(t)+\boldsymbol{C}\ \dot{\boldsymbol{X}}(t)+\boldsymbol{K}\ \boldsymbol{X}(t)=\boldsymbol{F}_e(t)+\boldsymbol{D}\boldsymbol{f}_c(t) \tag{1}$$

where

$\boldsymbol{M}$ mass matrix
$\boldsymbol{C}$ damping matrix
$\boldsymbol{K}$ stiffness matrix
$\boldsymbol{F}_e(t)$ external force vector
$\boldsymbol{f}_c(t)$ control force vector
$\boldsymbol{D}$ control force distribution matrix
$\boldsymbol{X}(t),\ \dot{\boldsymbol{X}}(t),\ \ddot{\boldsymbol{X}}(t)$ displacement-, velocity- and acceleration- vector

(1) rewritten for earthquake excitation

$$\boldsymbol{M}\ \ddot{\boldsymbol{X}}(t)+\boldsymbol{C}\ \dot{\boldsymbol{X}}(t)+\boldsymbol{K}\ \boldsymbol{X}(t)=-\boldsymbol{M}\ \{\boldsymbol{I}\}\ a(t)+\boldsymbol{D}\boldsymbol{f}_c(t) \tag{2}$$

where

$a(t)$ earthquake acceleration time history
$\{\boldsymbol{I}\}$ identity vector

Define $\boldsymbol{z}(t)$

$$z(t)=\begin{Bmatrix} \boldsymbol{X}(t) \\ \dot{\boldsymbol{X}}(t) \end{Bmatrix} \tag{3}$$

from (2) follows

$$\dot{z}(t)=\begin{Bmatrix} \dot{\boldsymbol{X}}(t) \\ \ddot{\boldsymbol{X}}(t) \end{Bmatrix}=\begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix}\begin{Bmatrix} \boldsymbol{X}(t) \\ \dot{\boldsymbol{X}}(t) \end{Bmatrix}+\begin{Bmatrix} \boldsymbol{0} \\ -\boldsymbol{a}(t) \end{Bmatrix}+\begin{Bmatrix} \boldsymbol{0} \\ \boldsymbol{M}^{-1}\boldsymbol{D}\boldsymbol{f}_c(t) \end{Bmatrix} \tag{4}$$

shortly written

$$\dot{z}(t)=\boldsymbol{A}z(t)+\boldsymbol{F}(t)+\boldsymbol{B}\boldsymbol{f}_c(t) \tag{5}$$

where

$$\boldsymbol{A}=\begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix} \qquad \boldsymbol{F}(t)=\begin{Bmatrix} \boldsymbol{0} \\ -\boldsymbol{a}(t) \end{Bmatrix} \qquad \boldsymbol{B}=\begin{Bmatrix} \boldsymbol{0} \\ \boldsymbol{M}^{-1}\boldsymbol{D} \end{Bmatrix} \tag{6}$$

where

a(t) acceleration vector

Numerical solution of (5) using direct integration, see [3] gives

$$z_{k+1}=\boldsymbol{F}_s\, z_k+\boldsymbol{H}_d^{(EQ)}\boldsymbol{a}_k+\boldsymbol{G}\,\boldsymbol{f}_{ck} \tag{7}$$

where

$$\boldsymbol{F}_s=\boldsymbol{e}^{A\Delta t}; \qquad \boldsymbol{H}_d^{(EQ)}=\boldsymbol{A}^{-1}\left(\boldsymbol{e}^{A\Delta t}-\boldsymbol{I}\right)\boldsymbol{H}; \qquad \boldsymbol{G}=\boldsymbol{A}^{-1}\left(\boldsymbol{e}^{A\Delta t}-\boldsymbol{I}\right)\boldsymbol{B} \tag{8}$$

with

$\boldsymbol{H}$ from $$\boldsymbol{F}_k=\begin{Bmatrix} \boldsymbol{0} \\ -\boldsymbol{a}_k \end{Bmatrix}=\boldsymbol{H}\,\boldsymbol{a}_k \tag{9}$$

OPTIMAL CONTROL

The objective function is defined

$$J=\int_0^{t_f}\frac{1}{2}\left[z^T(t)\boldsymbol{Q}\,z(t)+\boldsymbol{f}_c^T(t)\,\boldsymbol{R}\,\boldsymbol{f}_c(t)\right]dt \tag{10}$$

where $\boldsymbol{Q}$ and $\boldsymbol{R}$ are weighting matrices

(10) for discrete time

$$J = \frac{1}{2}\sum_{k=0}^{N}\left(\boldsymbol{z}^T \boldsymbol{Q}\, \boldsymbol{z} + \boldsymbol{f}_{ck}^T\ \boldsymbol{R}\, \boldsymbol{f}_{ck}\right) \tag{11}$$

The Lagrange multiplier method is suitable for the minimization of (11) considering (7).
The Lagrangian

$$L = \sum_{k=0}^{N-1}\left[\frac{1}{2}\left(\boldsymbol{z}_k^T \boldsymbol{Q}\, \boldsymbol{z}_k + \boldsymbol{f}_{ck}^T\ \boldsymbol{R}\, \boldsymbol{f}_{ck}\right) + \boldsymbol{\lambda}_{k+1}^T\left(\boldsymbol{F}_s \boldsymbol{z}_k + \boldsymbol{H}_d^{(EQ)} \boldsymbol{a}_k + \boldsymbol{G}\, \boldsymbol{f}_{ck} - \boldsymbol{z}_{k+1}\right)\right] \tag{12}$$

where

$\boldsymbol{\lambda}_{k+1}$ is the Lagrange multiplier

advantageous form of the Hamiltonian is

$$H_k = \frac{1}{2}\left(\boldsymbol{z}_k^T \boldsymbol{Q}\, \boldsymbol{z}_k + \boldsymbol{f}_{ck}^T\ \boldsymbol{R}\, \boldsymbol{f}_{ck}\right) + \boldsymbol{\lambda}_{k+1}^T\left(\boldsymbol{F}_s \boldsymbol{z}_k + \boldsymbol{H}_d^{(EQ)} \boldsymbol{a}_k + \boldsymbol{G}\, \boldsymbol{f}_{ck}\right) \tag{13}$$

Substitute (13) into (12).
The equation: variation of the Lagrangian is equal zero gives the set of equations. Solving them we obtain expressions for:

■ control force

$$\boldsymbol{f}_{ck} = -\left(\boldsymbol{G}^T \boldsymbol{P}\ \boldsymbol{G} + \boldsymbol{R}\right)^{-1} \boldsymbol{G}^T \boldsymbol{P}\ \boldsymbol{F}_s \boldsymbol{z}_k \tag{14}$$

where

$$\boldsymbol{P} = \boldsymbol{Q} + \boldsymbol{F}_s^T \boldsymbol{P}\left(\boldsymbol{I} + \boldsymbol{G}\ \boldsymbol{R}^{-1} \boldsymbol{G}^T \boldsymbol{P}\right)^{-1} \boldsymbol{F}_s \tag{15}$$

is the Riccati equation with Riccati matrix ***P;***

■ displacement - velocity vector

$$\boldsymbol{z}_{k+1} = \boldsymbol{F}_f \boldsymbol{z}_k + \boldsymbol{H}_d^{(EQ)} \boldsymbol{a}_k \tag{16}$$

with

$$\boldsymbol{F}_f = \left[\boldsymbol{I} - \boldsymbol{G}\left(\boldsymbol{G}^T \boldsymbol{P}\ \boldsymbol{G} + \boldsymbol{R}\right)^{-1} \boldsymbol{G}^T \boldsymbol{P}\right]\boldsymbol{F}_s = \left[\boldsymbol{I} + \boldsymbol{G}\ \boldsymbol{R}^{-1} \boldsymbol{G}^T \boldsymbol{P}\right]^{-1} \boldsymbol{F}_s \tag{17}$$

In case that the stiffness is constant during the whole period of earthquake, $\boldsymbol{P}$, $\boldsymbol{F}_f$ and $\boldsymbol{H}_d^{(EQ)}$ will be calculated only one time. When stresses exceed the value of the limit stress, the damaged part of the structure loses its stiffness. The new stiffness value has to be inputed into (14) and (16) at the same time step.

Calculated control force is compared with the technologically given limit force. There are two possibilities, when the control force exceeds the limit: change the weighting matrix $\boldsymbol{R}$, or replace the calculated value with the limit value.

Displacement-velocity vector and control force at every time step result the control energy. The control energy is calculated for all actuator positions. Only actuators with the highest integral values of consuming control energy are finally chosen. The procedure of displacements and control forces calculation on the base of above introduced optimization is repeated for the finally placed actuators.

Examples illustrate the theoretical solution. The numerical research was realized in MATLAB.

EXAMPLES

Accelerogram is chosen from Database of European Strong-Motion Data [2]. The acceleration time history of Friuli earthquake measured on 6-th May1976 in Tolmezzo-Diga Ambiesta is used. The magnitude of this earthquake was 6,5 Mw.

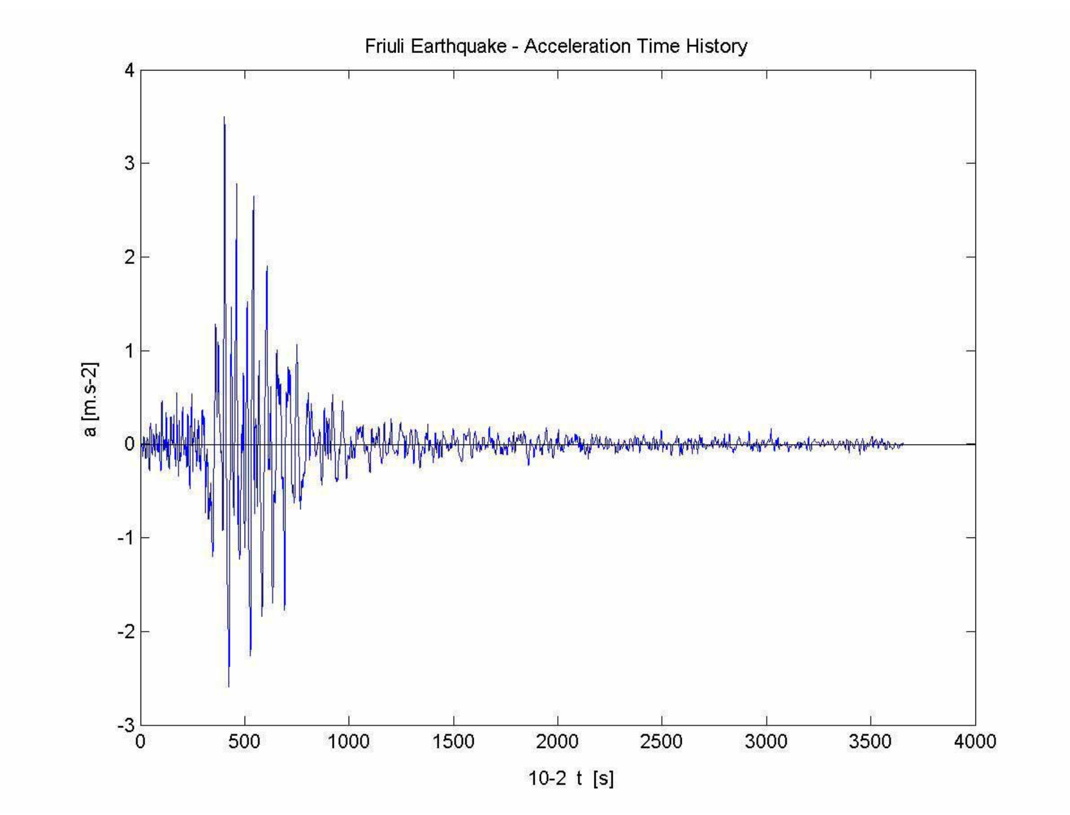

Fig.1 Acceleration time history – Friuli earthquake

1. Example

The example illustrates the case when the stress achieves the limit value and a part of the structure is damaged. The stiffness change at the moment has influence to change of the Riccati matrix and low of the control force calculation. In the presented example the 2/3 stiffness reduction during the earthquake is analyzed. At every step nodal displacements are calculated and compared with the not controlled displacement. The structural model data are m = 175118,1102 kg, k = 27654651,97 N/m, ζ =0,05.

Figures 2, 3 and 4 present the control force and response of the undamaged structure. For comparison figures 5, 6 and 7 present the control force and response when 4,3 s after the earthquake beginning the structure is partially damaged and the stiffness is reduced by 2/3.

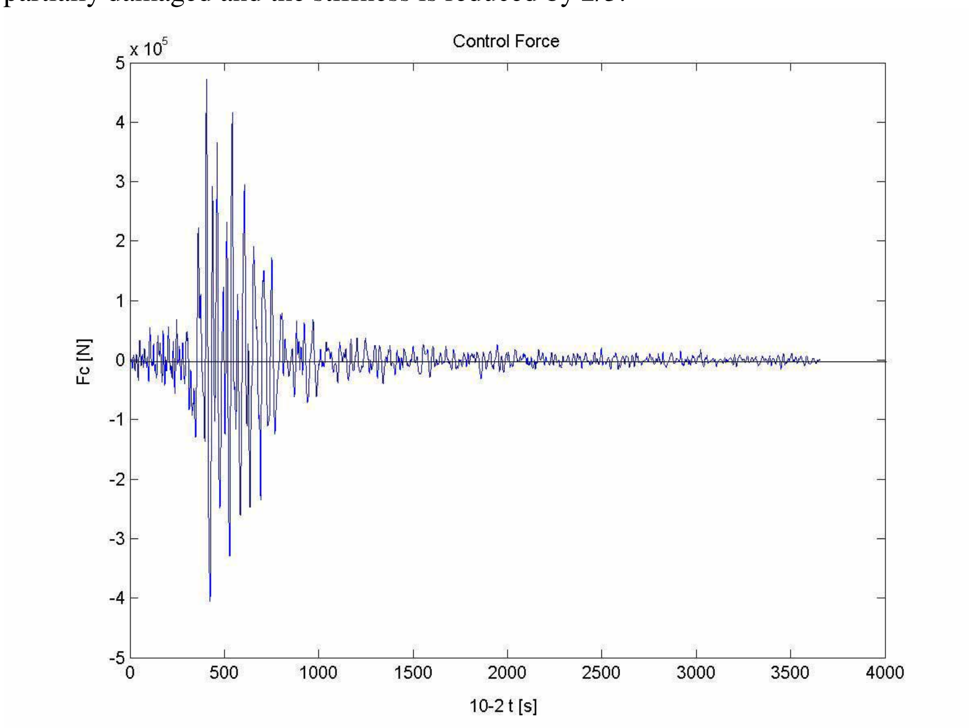

Fig.2 Control force

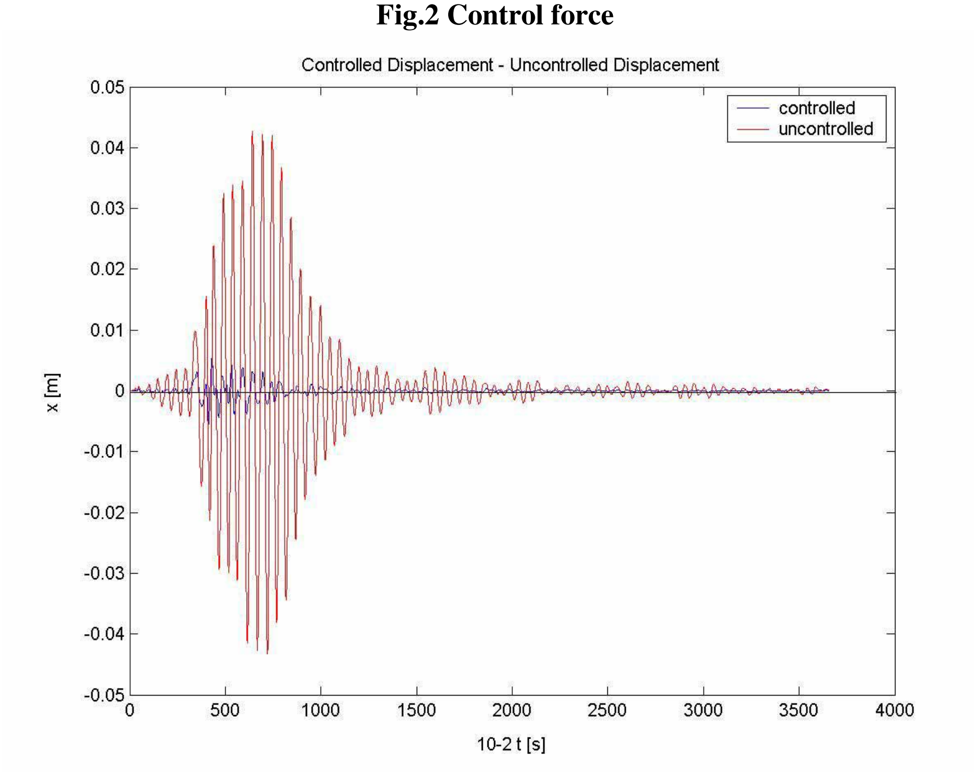

Fig.3 Displacement: controlled – uncontrolled

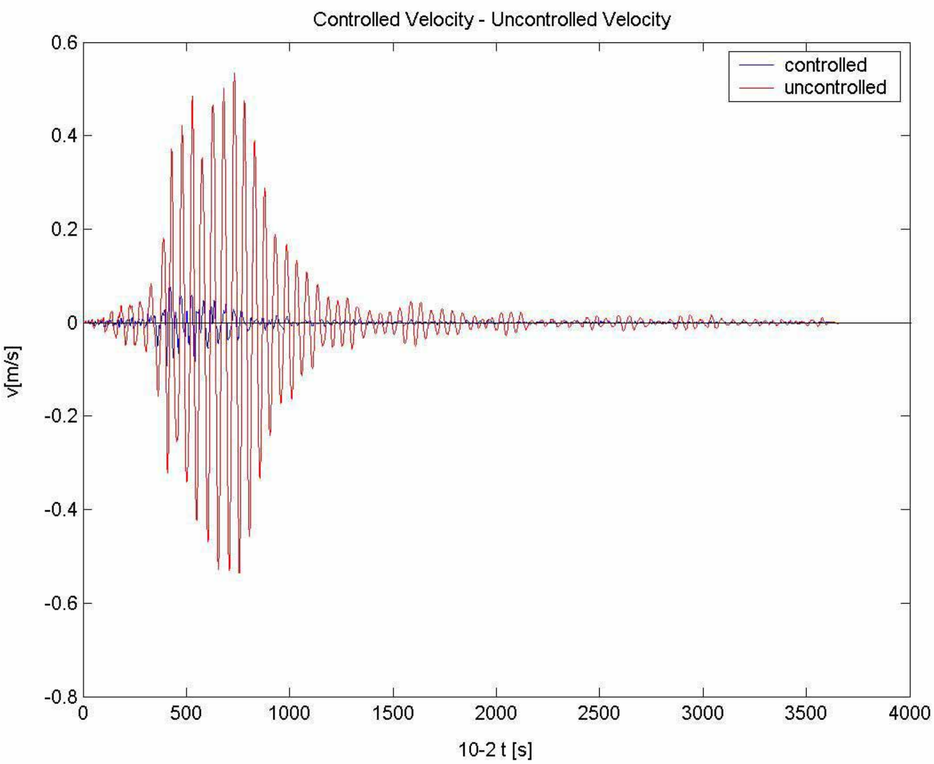

Fig.4 Velocity: controlled – uncontrolled

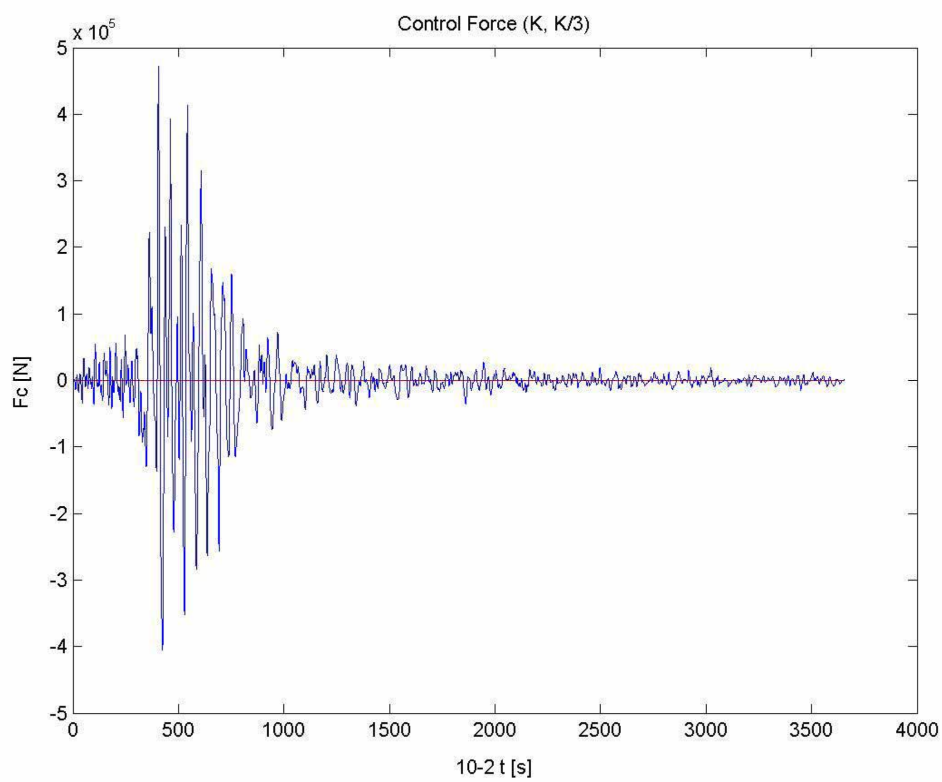

Fig.5 Damage during earthquake – Control force
(*t* = 0-4,3 s …*K, t* = 4,3-36,54 s… 1/3 *K*)

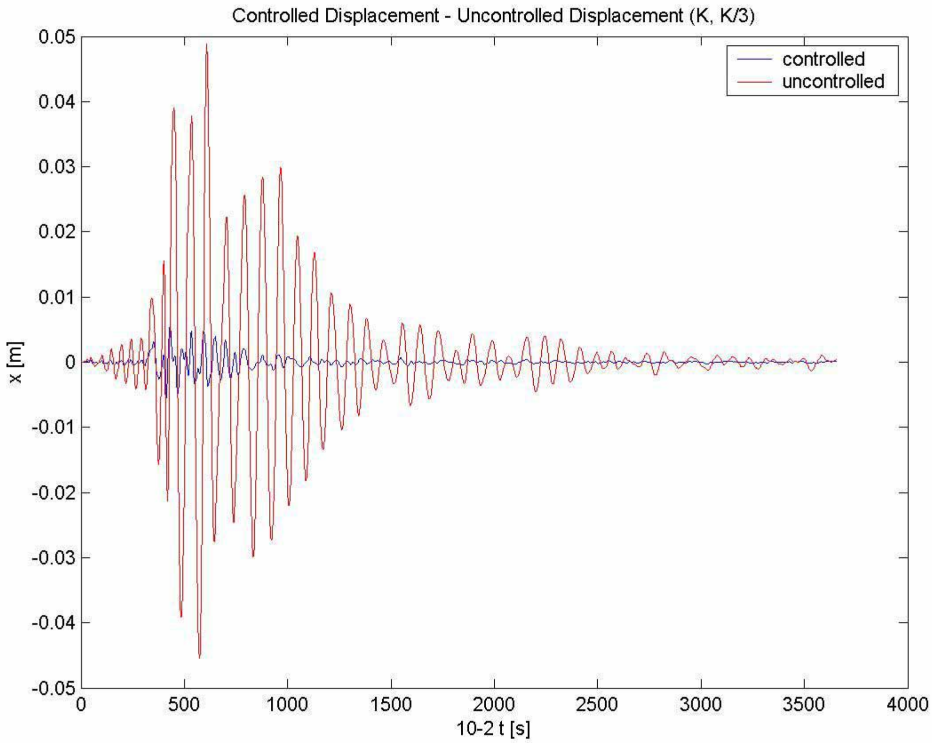

Fig.6 Damage during earthquake – Displacement: controlled – uncontrolled
(*t* = 0-4,3 s …*K*, *t* = 4,3-36,54 s… 1/3 *K*)

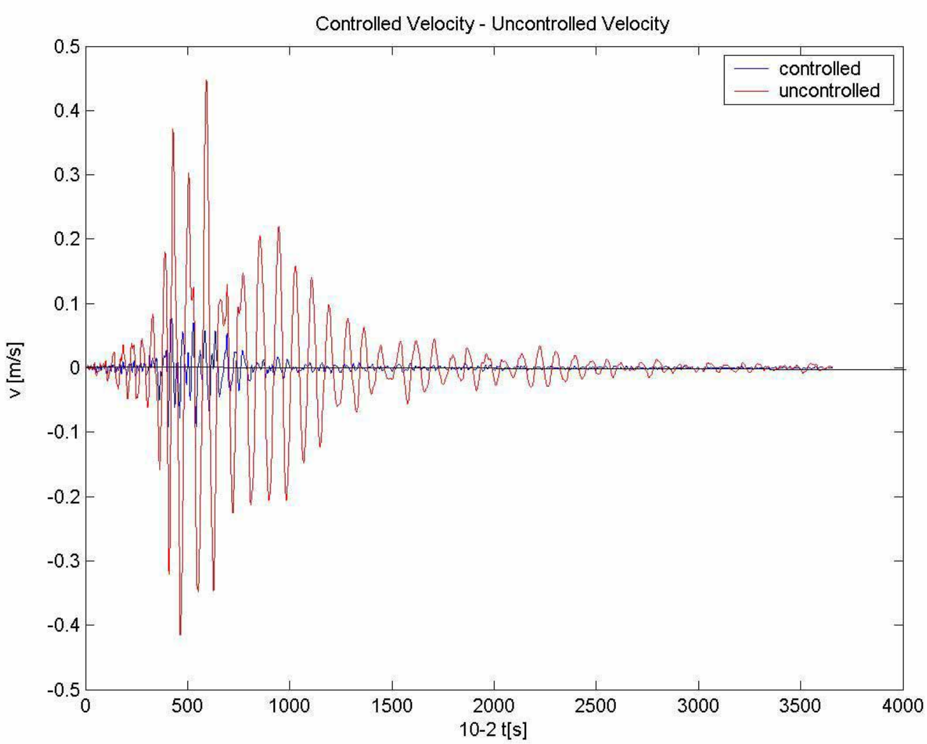

Fig.7 Damage during earthquake – Velocity: controlled – uncontrolled
(*t* = 0-4,3 s …*K*, *t* = 4,3-36,54 s… 1/3 *K*)

In following figures crack originates at the same time as in the upper case. When the structure moves to the position when the crack is opened, than just 1/3 of stiffness resists and by moving to the opposite direction the full stiffness resists. Figures 8, 9 and 10 present control force and response of the nonlinear model of the damaged structure.

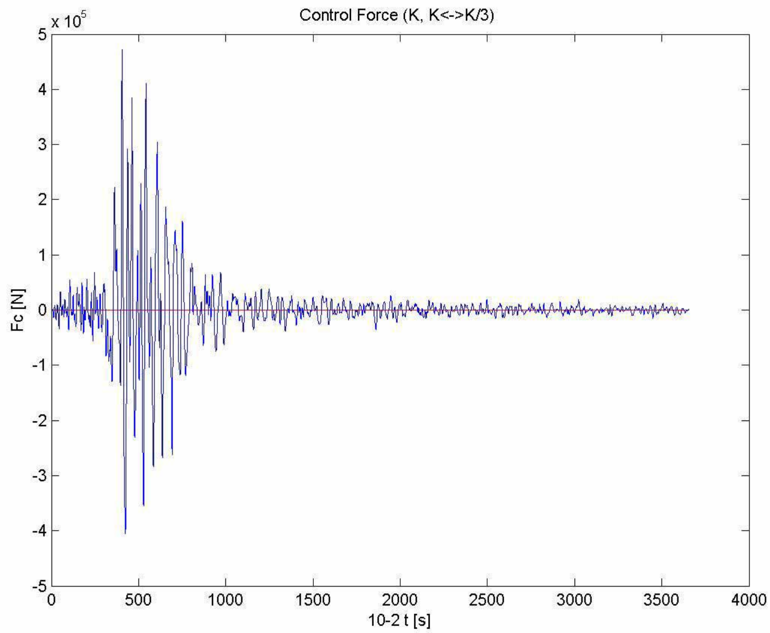

Fig.8 Damage during earthquake – Control force
(t = 0-4,3 s …K, t = 4,3-36,54 s… $K \leftrightarrow K/3$)

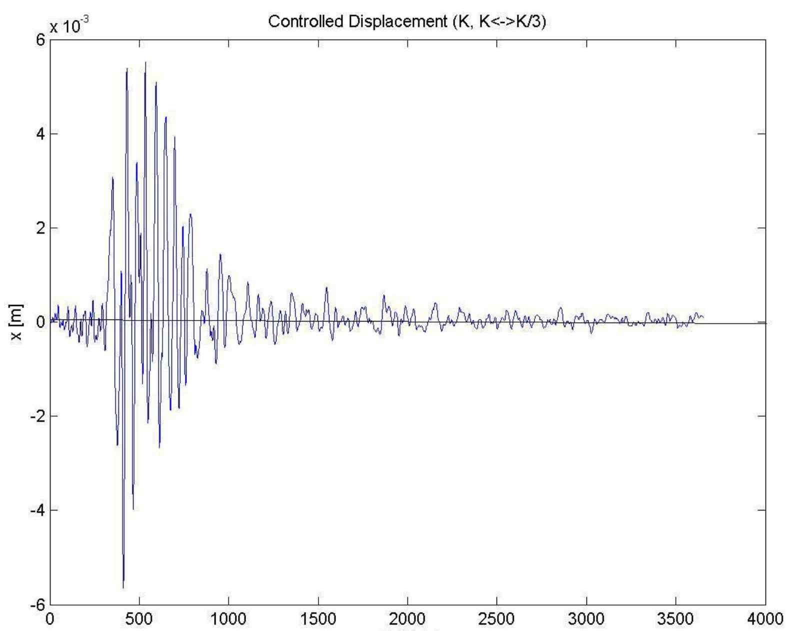

Fig.9 Damage during earthquake – Displacement controlled
(t = 0-4,3 s …K, t = 4,3-36,54 s… $K \leftrightarrow K/3$)

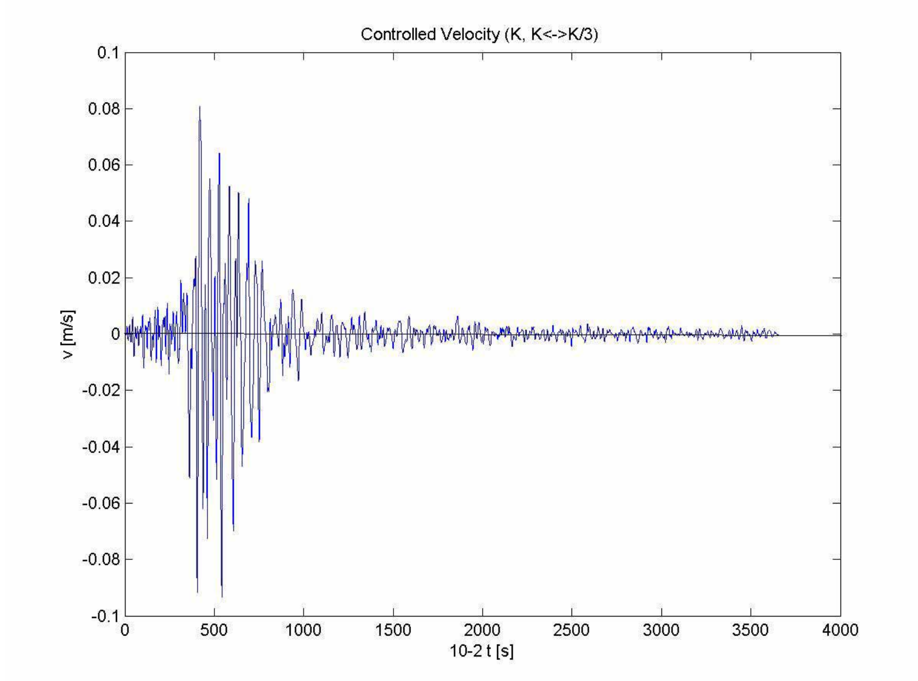

Fig.10 Damage during earthquake – Velocity controlled
(t = 0-4,3 s …K, t = 4,3-36,54 s… $K \leftrightarrow K/3$)

2. Example

The example illustrates MDOF system. Actuators are firstly simultaneously placed in all possible positions. Secondly the number of actuators is reduced. Actuators with highest amount of control energy are kept. 2DOF system is given: $\boldsymbol{K}$ = [2,2124e8 -1,1062e8; -1,1062e8 1,1062e8],
$\boldsymbol{C}$ = [5,9475e4 -1,9683e5; -1,9683e5 3,9650e4], $\boldsymbol{M}$ = [1,7512e5 0; 0 1,7512e5] - SI units.

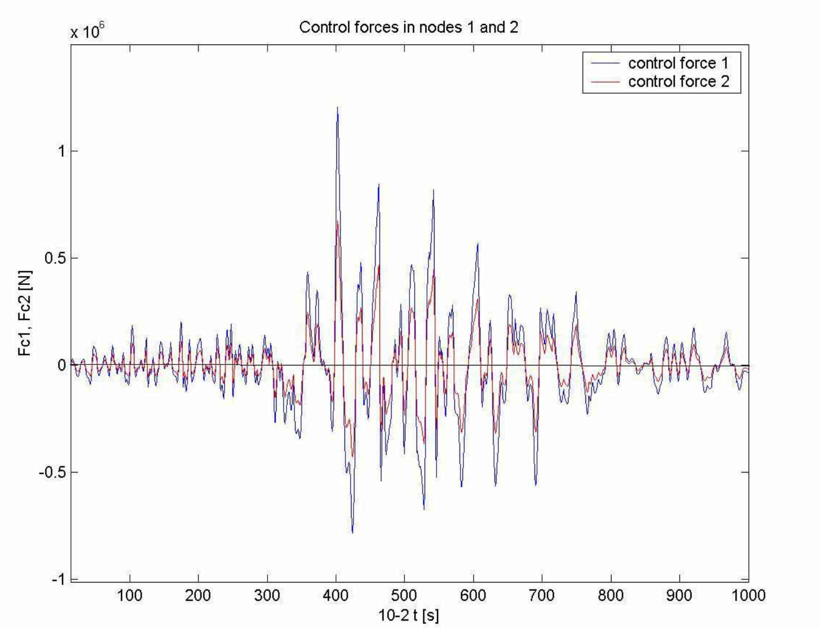

Fig.11 2DOF - optimal control forces in nodes 1 and 2

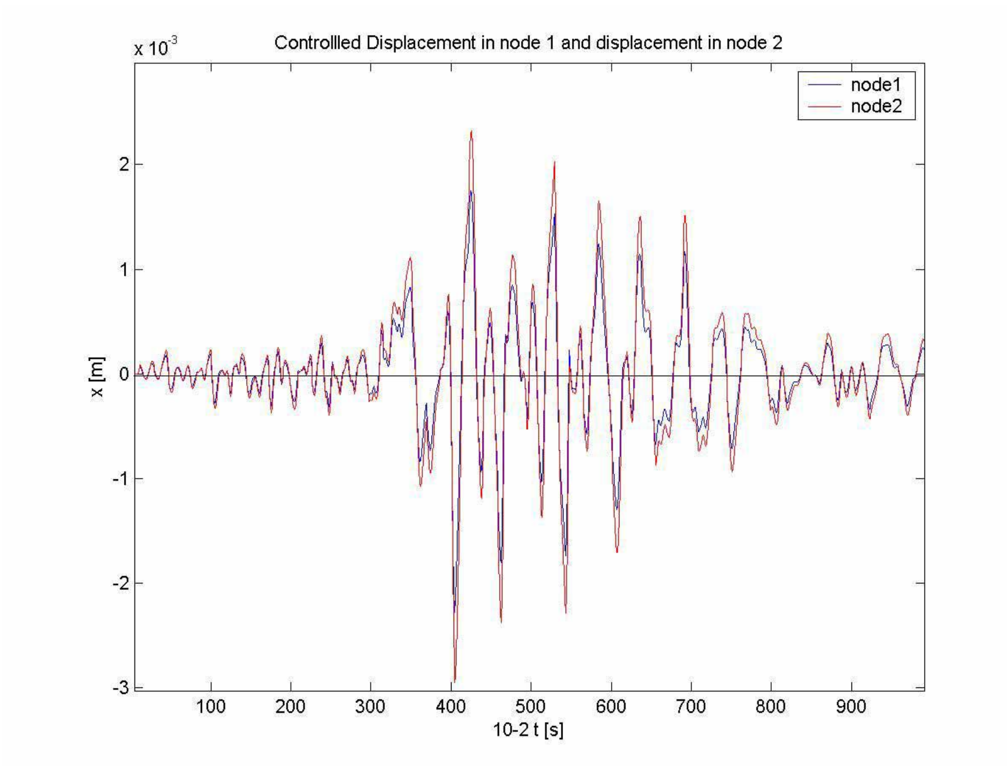

Fig.12 2DOF – Displacements in nodes 1 and 2

The control energy index (calculated from control force and displacement multiplication time integral) for control force *F1* is *en*1= 8.82 and the control energy index for control force *F2* is *en*2= 5.94. The difference between both values is not very high and the number of originally placed actuators is low. The placement of actuators is in this case acceptable.

CONCLUSION

The approach of closed-loop control calculation with help of Riccati matrix is extended with stiffness matrix changes, when the stress limit is overflowed. Linear and nonlinear cases are analyzed. The nonlinear case uses GAP element model for crack.

Calculation of optimal control forces in actuators placed simultaneously in all possible positions leads to control energy analysis. The reduced number of actuators in MDOF system on the base of highest control energy needs repeated calculation of optimal control force values.

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