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CRITERIA FOR PLAN REGULARITY BASED ON SEISMIC RELIABILITY ANALYSIS

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SUMMARY

A reliability analysis has been conducted on a simple non-linear spring model idealizing a one-way asymmetric one-storey building in order to compare the plan irregularity criteria used by two major seismic codes like EC8 and IBC. Limited to the numerical example carried on a sample of structures, it appears that the IBC criterion is more conservative and complete than the EC8 ones.

INTRODUCTION

Damage states resulting from past earthquakes have demonstrated that plan irregularities due to asymmetric distributions of mass, stiffness and/or strength of buildings are a critical issue [1].
Indeed, the dynamic response of plan irregular structures under horizontal earthquake excitations is characterized, in addition to lateral motions, by a significant torsional rotation of the floor slabs. In the elastic range of behaviour such a response is due to the eccentricity between the centre of the masses (*CM*), where the seismic lateral inertial forces are applied, and the centre of stiffness (*CS*), point in plan where the application of a lateral force will produce a simple translation of the floor slab (Figure 1).
This complex interaction between floor rotation and translation, denoted by torsional coupling, can result in significant response amplifications. As a consequence, both forces and displacement demands on vertical resisting elements (frames or walls) of plan-irregular structures can be larger than those they would experience in the presence of structural symmetry.

OBJECTIVES AND METHODS

Advanced seismic codes of practice recognize the problem of plan asymmetry in design phase [2]. Generally they address the identification of potential torsional coupling through regularity criteria, but different approaches, based either on geometrical or mechanical parameters, are subscribed. In the following only the latter are considered.

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Eurocode8 (EC8) [3] uses two criteria: a building is regular if the eccentricity is less than 30% of the torsional radius (square root of ratio of torsional to lateral stiffness), and torsional radius is less than the mass radius of the floor plan.

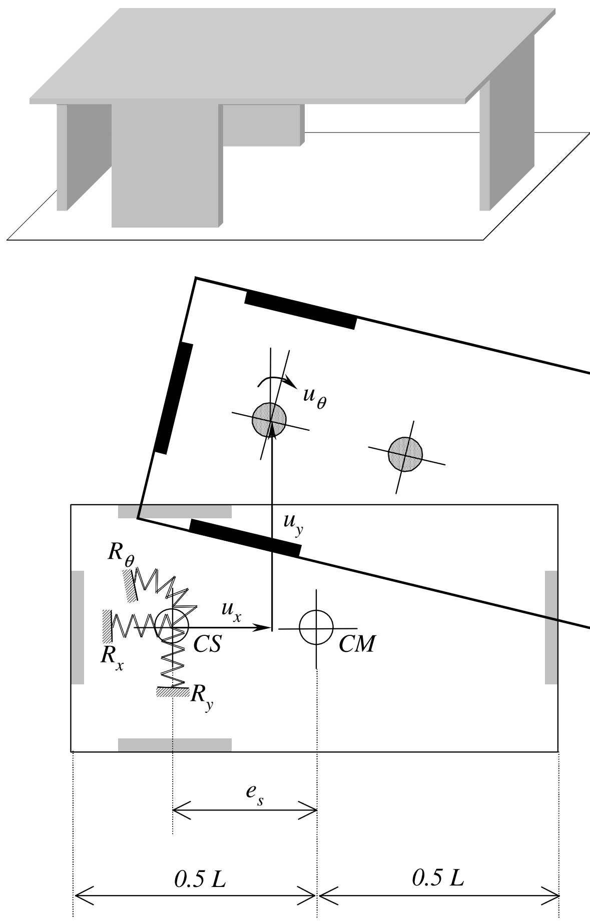

Figure 1. Simplified model of planar asymmetric one-storey building: a rigid deck with three DOF.

Finally, according to the International Building Code (IBC) [4] rules, a building is plan irregular if the maximum storey drift exceeds 1.2 times the average of storey drifts of the two ends of the structure.
Explicit definition of relevant parameters used by this code will be addressed in next section.
In order to evaluate the consistency of the aforementioned measures, a reliability approach has been followed, in particular to compare EC8 and ICB provisions when there is uncertainty in the definition of basic parameters of the problem.

NUMERICAL MODEL

The simple structural model considered for the evaluation of the behaviour of plan irregular structures idealizes a generic one-story plan asymmetric building, supported by lateral force-resisting elements as represented in Figure 1.
It is assumed that the floor diaphragm is rigid in its own plane and that vertical resisting elements are mass-less.
Such system behaves like a two-dimensional rigid body, so it presents only three dynamic degrees of freedom: two translations u_x and u_y along the horizontal directions and a floor rotation u_θ around the vertical axis.

Aiming to simplify the problem the actual plan distribution of stiffness and strength is modelled with two translational and one rotational non-linear springs located in the system centre of stiffness, *CS*.

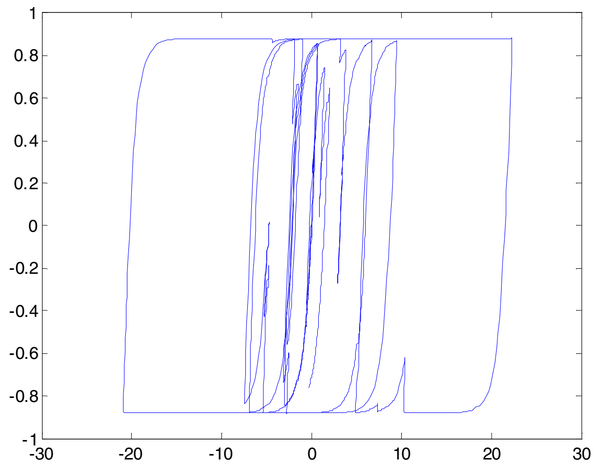

Figure 2. Force-displacement plot for a Bouc-Wen model with K_o=110, α=0.25, A=1, β=0.8, γ=0.34, n=1 and δ=0

As a consequence the location of this point is supposed not to change during the earthquake shaking, despite of the fact that its actual position is expected to vary due to the non-linear response of the single resisting elements.

The restoring forces of those springs have been modelled trough the smooth differential function developed by Bouc [5] and later generalized by Wen [6] and already used in reliability analysis of reinforced concrete framed structures as presented for example by Casciati and Faravelli, [7] or Colangelo et al. [8].

In these constitutive equations the total inelastic response is given by a parallel system of a linear elastic spring and a hysteretic spring controlled by the endochronic variable z:

$$\begin{aligned} R_i &= \alpha_i K_{o,i} u_i + (1-\alpha_i) K_{o,i} z_i \\ \dot{z}_i &= A_i \dot{u}_i - \gamma_i \dot{u}_i |z_i|^{n_i} - \beta_i |\dot{u}_i| |z_i|^{n_i - 1} z_i - \delta_i \dot{u}_i |z_i|^{n_i - 1} \end{aligned} \quad (1).$$

The meaning of the parameters that appear in (1) is explained elsewhere [6], here is just worth to stress that A_i, β_i, γ_i and n_i govern the shape of the hysteretic loop, while the additional parameter δ_i can be used to define asymmetric yielding levels.

A typical layout of the constitutive model is represented in Figure 2.

The equation of motion of the inelastic 3-dof rigid deck, subject to a ground shaking along *y*, can be written, with respect of the centre of stiffness of the system, as follows:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{R}(\mathbf{u}) = -\mathbf{M}\mathbf{a}_g = -\mathbf{M}\begin{Bmatrix} 0 \\ a_y \\ a_y e_s \end{Bmatrix} \quad (2).$$

In equation (2) **M** and **R** are respectively the mass matrix and the restoring forces vector:

$$\mathbf{M} = \begin{bmatrix} m & 0 & 0 \\ 0 & m & e_s m \\ 0 & e_s m & \left(\rho_m{}^2 + e_s{}^2\right) m \end{bmatrix}$$

$$\mathbf{R} = \begin{Bmatrix} R_X \\ R_Y \\ R_\Theta \end{Bmatrix}$$

$$R_\Theta = \rho_k{}^2 R_Y$$

Fundamental importance in the analysis of the deck is on ρ_m, the radius of gyration of the mass, which in the above formulation has to be evaluated with respect to *CM*:

$$\rho_m = \sqrt{\frac{I_m}{m}}$$

that accounts for the mass spread over the deck (having denoted with m the deck translational mass and with $I_m=I_x+I_y$ the polar moment of inertia of the deck around *CM*), and on ρ_k, the radius of gyration of the stiffness, that has to be evaluated with respect to *CS*:

$$\rho_k = \sqrt{\frac{\sum k_{x,i} y_i^{\,2} + \sum k_{y,i} x_i^{\,2}}{\sum k_{y,i}}}$$

that accounts for the torsional stiffness of the deck due to spread of the elastic resisting elements (being $k_{x,i}$ and $k_{y,i}$ the stiffness respectively along x and y of the i-th vertical element, and x_i and y_i its position respect to *CS*).

The damping matrix, $\mathbf{C}$, is constructed to be diagonal in the space of modal displacements, thus:

$$\mathbf{C} = \mathbf{M}\ \mathbf{\Phi} \begin{bmatrix} 2\xi_1\omega_1 & 0 & 0 \\ 0 & 2\xi_2\omega_2 & 0 \\ 0 & 0 & 2\xi_3\omega_3 \end{bmatrix} \mathbf{\Phi}^T\ \mathbf{M}$$

where $\mathbf{\Phi}$ is the matrix of modal shapes and the generic term $2\xi_i\omega_i$ represent the damping of the i-th mode (with ξ damping ratio, ω undamped natural pulse and the eigenvector normalization rule is on kinetic energy such to have $\mathbf{\Phi}^T\ \mathbf{M}\ \mathbf{\Phi} = \mathbf{I}$). In the following the analysis have been carried assuming the typical value of $\xi=0.05$.

Assembling (1) and (2) the dynamic problem can be rewritten as follows:

$$\mathbf{M\ddot{u}} + \mathbf{C\dot{u}} + \mathbf{Ku} + \mathbf{f}(\mathbf{u},\mathbf{\dot{u}},\mathbf{z}) = -\mathbf{M}\begin{Bmatrix} 0 \\ a_y \\ a_y e_s \end{Bmatrix} \tag{3}$$

where the restoring forces have been split in the elastic and hysteretic component, respectively:

$$
\mathbf{K} = \begin{bmatrix} \alpha_x K_{o,x} & 0 & 0 \\ 0 & \alpha_y K_{o,y} & 0 \\ 0 & 0 & \alpha_\theta \rho_k{}^2 K_{o,y} \end{bmatrix}
$$

and

$$
\mathbf{f} = \begin{bmatrix} (1-\alpha_x) K_{o,x} z_x \\ (1-\alpha_y) K_{o,y} z_y \\ (1-\alpha_\theta) \rho_k^2 K_{o,y} z_\theta \end{bmatrix}.
$$

Dynamic equilibrium equation (3) can also be written in the state-space.
Indeed, assuming as main variable:

$$
\mathbf{U}(t) = \begin{bmatrix} \mathbf{u}(t) \\ \dot{\mathbf{u}}(t) \end{bmatrix}
$$

it is possible to obtain the following system of 1-st order non linear dynamic equations:

$$
\dot{\mathbf{U}} = \mathbf{A}\ \mathbf{U} + \mathbf{B}\ \mathbf{f}(\mathbf{U},\mathbf{Z}) + \mathbf{H}\ \mathbf{a}_g \tag{4}
$$

where the matrices **A**, **B** and **H** can be built from the mass, damping and elastic stiffness matrix:

$$
\mathbf{A} = \begin{bmatrix} \mathbf{O}_{3\times3} & \mathbf{I}_{3\times3} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}
$$

$$
\mathbf{B} = \begin{bmatrix} \mathbf{O}_{3\times3} \\ -\mathbf{M}^{-1} \end{bmatrix}
$$

$$
\mathbf{H} = \begin{bmatrix} \mathbf{O}_{3\times3} \\ -\mathbf{I}_{3\times3} \end{bmatrix}
$$

with $\mathbf{O}_{3x3}$ and $\mathbf{I}_{3x3}$ respectively the zero an unitary 3×3 matrices.
Equation (4) can be easily solved, together with the differential equations governing the non linear response of the Bouc-Wen restoring forces, making use of the Runge-Kutta integration technique available in computer mathematical tools like MATLAB® [9].

RELIABILITY ANALISIS

The basic random variables assumed in the reliability analysis are:

- the gyration radius of mass (varying between 0.297 L and 0.408 L) and
- the gyration radius of stiffness (varying between 0.238 L and 0.510 L),

where L is the reference dimension of the floor as indicated in Figure 1.
The prescribed range of variability has been selected considering upper and lower bounds for typical structure configurations.

These random variables have been assumed to be uniformly distributed, since, according with the aims of the analysis, there is no reason to expect some values of the above specified parameters to be more probable then others, and strongly correlated together, in order to take in account the fact that spread of mass and spread of stiff-ness necessarily depend together with floor geometry. Therefore in the analysis a correlation coefficient equal to 0.8 has been arbitrarily chosen for the covariance matrix of these two variables.
The third random variable is the eccentricity that has been constrained to be uniformly distributed in the range from 0.10 to 0.25 times the radius of stiffness in order to have always structures fulfilling the first of the two regularity checks prescribed by the EC8 document.
In this way the selected sample has eccentricities that can be still critical from the point of view of plan irregularity, but that are not recognized so by the EC8.
Assuming as a reference vibration period T_x=T_y=0.6 sec the global stiffness and the strength of the floor have been calculated from EC8 design spectra for soil A assuming a PGA equal to 0.31 g and as behaviour factor q=3.
In order to assess the EC8 requirements, the probability of an irregular plan response, for the examined structure sample, depend only on the check between the mass and stiffness radius and is given by the probability content inside the hatched area above the $\rho_k = \rho_m$ line in Figure 3.
In this case, the probability computation reduces to a geometrical problem: when the two uniform random variables are assumed uncorrelated, the probability is simply given by the area ratio between the red trapeze and the containing rectangle (union of the grey and red trapezes), while the correlation between random variables (represented in figure by the gradient shadow) in the examined structure decreases the probability of an irregular response, resulting as high as 0.33 compared to 0.42 when the two variables are assumed uncorrelated.

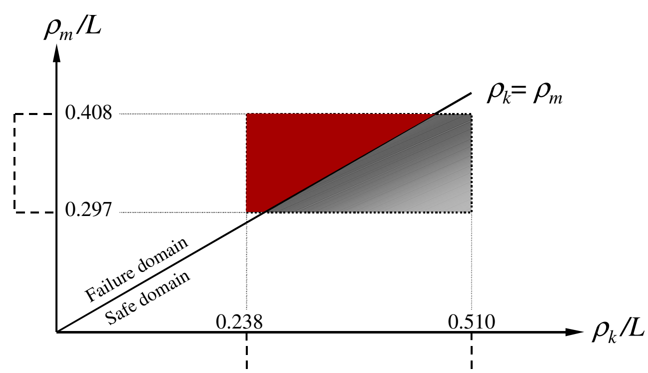

Figure 3. Probability computations according to the second EC8 regularity check. Irregular behaviour is denoted, in standard reliability language, as failure.

On the other hand, the assessment of IBC requirements has been conducted carrying out a series of time-history dynamic analysis assuming as limit state function:

$$g(\mathbf{x}) = 1.2 - d_{\max}/d_{\text{ave}} \tag{5}$$

where **x** denotes generically the vector of the above-mentioned random variables, d_{ave} is the displacement of the mass centroid while d_{max} is given by the larger displacement at the two floor ends.
The above function is intended to correspond to IBC regularity check and becomes negative (failure region in standard reliability language) when the structure is sought to be irregular.

The displacement terms appearing in (5) are a result of a time-history dynamic analysis with a prescribed ground acceleration record.
Since the reliability evaluation is carried out using a time-history analysis, this means to face a time-variant random vibration problem [10]. Indeed undergoing the regularity check at *any* time t can be regarded as a “first-passage” failure, and therefore the failure domain of this event is the union of the elementary domains, i.e. the elementary events are arranged in series to give the first excursion, which can be written as:

$$F = \bigcup_{k=1}^{N} F_k = \bigcup_{k=1}^{N} \{ g(t_k) \leq 0 \}$$

The problem can be treated assuming a Poissonian approximation and solved through the computation of the mean crossing rate of excursion: the probability of failure for the first excursion event in time D has the upper bound:

$$P_f \leq \int_0^D \nu(t) dt$$

The problem has been solved following the work by Franchin et al. [11].
The mean crossing rate has been calculated following the steps indicated below:

1. Let $g(t) = g[\mathbf{u}(t,\boldsymbol{\theta})]$ be the time-dependent limit-state function defined in the physical space of the random variables $\boldsymbol{\theta}$ through the time-dependent response process $\mathbf{u}$(t).
2. An out-crossing at time t means that $g(t)>0$ and $g(t+\delta t)<0$; let consider the extra limit state functions defined as follows:
$$g_1(\theta) = -g[u(t,\boldsymbol{\theta})]$$
$$g_2(\theta) = g[u(t+\delta t,\boldsymbol{\theta})] = g[u(t,\boldsymbol{\theta})] + \nabla_u [g[u(t,\boldsymbol{\theta})]]\ \dot{\mathbf{u}}(t,\boldsymbol{\theta})\delta t$$
3. The mean crossing rate is, by definition:
$$\nu(t) = \lim_{\delta t \to 0} \frac{\Pr\{[g_1(\boldsymbol{\theta}) \leq 0] \cap [g_2(\boldsymbol{\theta}) \leq 0]\}}{\delta t}$$
4. The probability at the numerator in the above equation is associated to a parallel event. This computation can be carried out using standard reliability tools for systems like first order approximations (FORM):

$$P_{FORM} = \Phi_2(-\beta_1, -\beta_2, \rho_{12}) = \Phi(-\beta_1) \cdot \Phi(-\beta_2) + \int_0^{\rho_{12}} \phi_2(-\beta_1, -\beta_2, \rho)\ d\rho$$

where ϕ_2 is the bi-variate normal density function, whilst F and F_2 are respectively the mono- and bi- variate normal cumulate function, whilst:

$$\rho_{12} = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{\|\mathbf{n}_1\|\ \|\mathbf{n}_1\|}.$$

The geometrical meaning of symbols is illustrated in Figure 4.

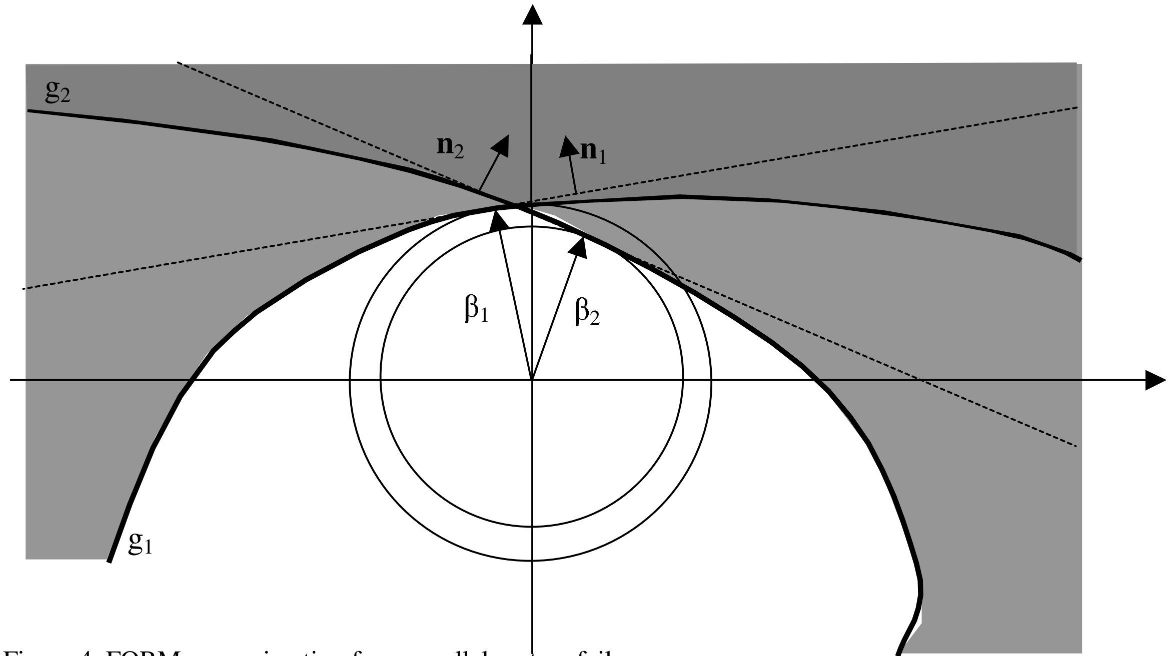

Figure 4. FORM approximation for a parallel system failure.

The analysis has been carried out both from an elastic dynamic analysis, i.e. setting α=1.0 in equation (1), and from a non-linear dynamic analysis, i.e. setting α=0.25. The remaining parameters have been assumed as illustrated in Figure 2.
Elastic dynamic analysis has been conducted for the sake of consistency with the check prescribed by IBC, which relates torsional response with an increase of displacements at the floor edges with respect to floor centroid.
However, under severe ground motion, building structures are expected to experience inelastic behaviour and, therefore, torsional response generally results in an in-crease of ductility demands again at the floor edges with respect to the floor centroid. In a very simplified manner, the parameter d_{max}/d_{ave} that appears in (5), when resulting from a non-linear dynamic analysis, may be assumed to represent a measure of plan variation in ductility demands due to torsional response.
The time-history analysis has been conducted using five records generated as filtered with-noise gaussian processes and scaled to have all a peak acceleration of 0.31 g, thus equal to the design acceleration of our structural sample (see Figure 5). The results, in terms of probability to undergo the IBC regularity check, are summarized in the table below.

Input	**Elastic (α=1.0)**	**Inelastic (α=0.25)**
Earthquake 1	0.74	0.71
Earthquake 2	0.71	0.69
Earthquake 3	0.68	0.64
Earthquake 4	0.65	0.59
Earthquake 5	0.68	0.61

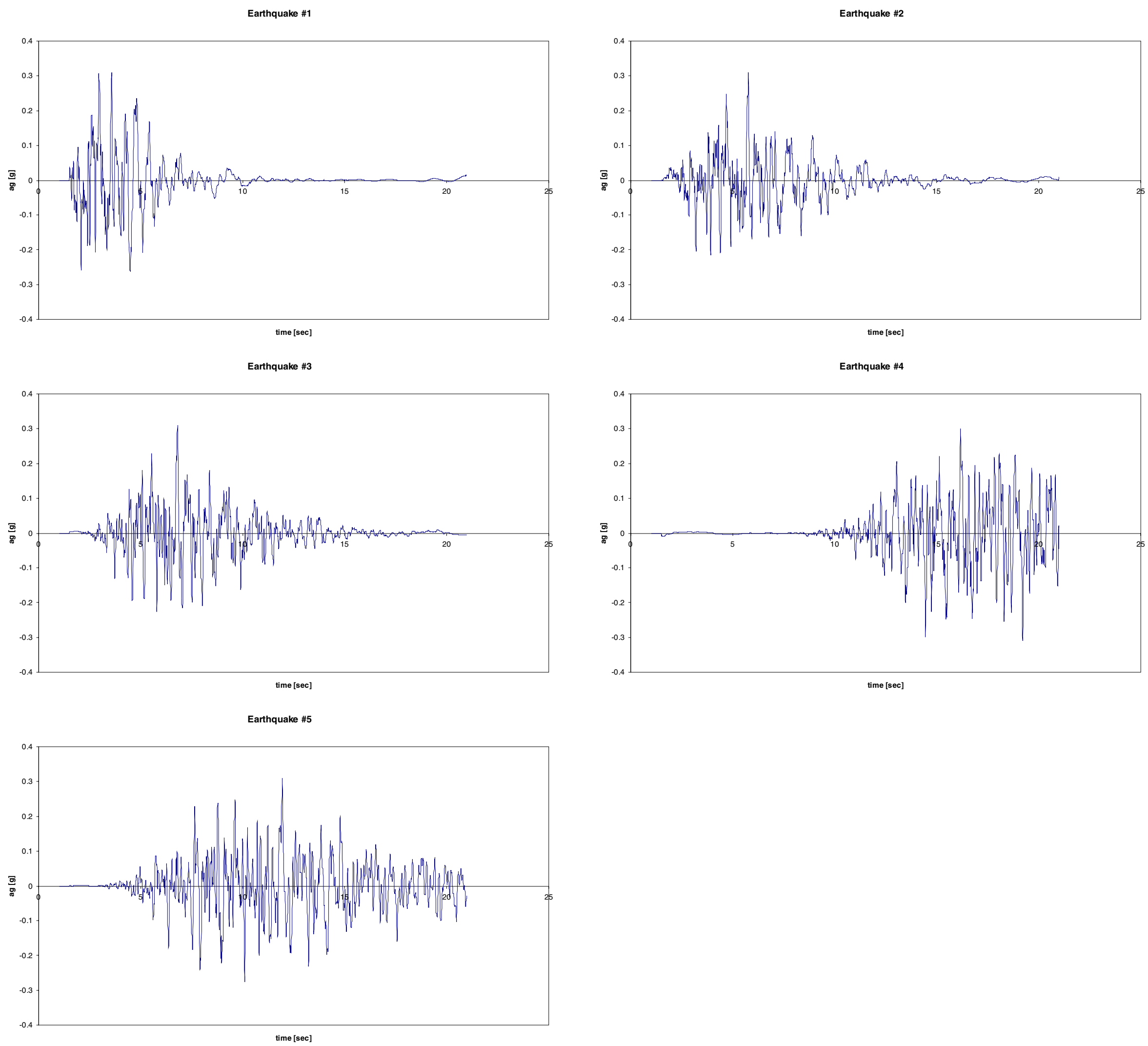

Figure 5. Records used for the dynamic analysis.

CONCLUSIONS

The mechanical criteria subscribed by IBC and EC8 to identify plan irregularity are substantially different. Indeed the former is based on straightforward assessment of building deformation demands under earthquake loading; the latter synthesizes the check through simple mechanical parameters whose evaluation appears problematic for multi-storey buildings.

A reliability analysis has been carried out in order to compare the two approaches for a selected sample of one-way plan asymmetric structures.

Although the limitations of the selected sample do not permit to carry out general conclusions on the discrepancies between the ways the two codes address the problem of plan irregularity and their relation with damage state in the buildings, it emerges that the IBC criterion is more conservative and complete than the EC8 ones for which inequalities produce a pass/not-pass scheme on the parameters expected to influence the problem rather that the problem effects.

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