

# INELASTIC DISPLACEMENT PATTERNS IN SUPPORT OF DISPLACEMENT-BASED DESIGN FOR MULTI-SPAN BRIDGES

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## SUMMARY

Described in this paper is a study aimed at identifying inelastic displacement patterns for multi-span bridges in support of the direct displacement-based seismic design method. Target displacement profiles for multi-span bridges have a significant impact on the end result of the design; therefore, a recent study was conducted for six different multi-span bridge configurations to identify the possible scenarios for deflection. Three different scenarios were identified, namely: (1) Rigid body translation, (2) Rigid body translation and rotation, and (3) Flexible profile. Those three scenarios were found to be highly dependent on the relative stiffness between superstructure and substructure, bridge regularity and abutment type. The first two scenarios require minimal effort in the direct displacement-based design approach since the target profile and SDOF structure are already determined while the third scenario requires more computations. The goal of the study is to describe a set of common criterion to identify different inelastic displacement scenarios and to develop analytical techniques for calculating inelastic displacement profiles. In order to achieve this objective a large series of nonlinear dynamic time history analysis were conducted on multi-span bridges. Variables considered included superstructure stiffness, substructure stiffness, bridge regularity and symmetry, abutment conditions, column flexural strength, and earthquake time history. Results are presented which provide recommendations for selection of non-linear displacement patterns that are then implemented into the direct displacement-based design approach. An example application of the process is also presented.

# **INTRODUCTION**

With the advent of performance-based design, the need for a fully developed, yet simple, design approach is significant. Such approaches should allow the engineer to control the bridge deflected shape, and hence damage, for a variety of performance limit states and earthquake intensities. One such approach is the direct displacement-based design approach. In DDBD approach, a structure is designed such that a predefined displacement limit is achieved when the structure is subjected to a predefined earthquake that is consistent with that assumed for the design. The design procedure utilizes Jacobsen's approach [1] for equivalent viscous damping and the Gulkan and Sozen [2] substitute structure concept to approximate the displacement of the inelastic system with equivalent elastic system. A nonlinear system which has initial

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stiffness  $K_0$  and secondary stiffness  $rK_0$  could be described as a linear system with effective stiffness  $K_{eff}$  and viscous damping  $\xi_{eff}$  based on the hysteretic energy dissipated as shown in figure 1.



Figure 1. Nonlinear System and Equivalent Elastic System Definition

The effective stiffness is the slope of the line connecting the maximum displacement to the origin which could be related to the nonlinear system initial stiffness as given by equation 1, where  $\mu$  is the displacement ductility. Based of Jacobsen's approach, the hysteretic damping is given by equation 2 where A<sub>1</sub> is the area of the nonlinear system maximum loop and A<sub>2</sub> is the area of the rigid perfectly plastic loop that passes through the maximum displacement. An additional 0%-5% viscous damping could be added to the hysteretic damping to obtain the equivalent viscous damping. For a detailed discussion on the application limitations of the equivalent viscous damping approach, see paper no. 228 in the 13<sup>th</sup> world conference on earthquake engineering.

$$K_{eff} = \frac{r\mu - r + 1}{\mu} K_0 \tag{1}$$

$$\xi_{eff} = \frac{2}{\pi} \frac{A_1}{A_2} \tag{2}$$

The first step in DDBD approach is selecting a target displacement  $(D_t)$  based on the desired level of damage, as well as selection of an earthquake intensity for which the damage level will occur. Utilizing the target displacement, demand earthquake spectra and the equivalent viscous damping, the effective stiffness and base shear are determined. More detailed steps of the design process are discussed in the following section.

For bridge structures, one of the key aspects of the DDBD is the selection of a displacement profile. This step is the starting point in the design process, so the shape selected must be achievable or realistic based on the bridge geometry, stiffness distribution across the bridge, superstructure stiffness and end conditions. Consider a four span bridge with free or integrally built abutments. In these cases, at least 6 main scenarios of the deflected shape are expected, as shown in figure 2. For regular symmetric bridge with fairly rigid superstructure, a rigid translational displacement pattern is expected like the ones shown in figures 2a and 2d. If the bridge has eccentricity between center of mass and center of rigidity, due to asymmetry in the geometry or stiffness, an additional rotation is expected as in figures 2b and 2e. Finally, a symmetric bridge with flexible superstructure is expected to have a displacement pattern as shown in figures 2c and 2f. Obviously, superstructure rigidity is not the only factor that controls the displacement pattern; substructure stiffness and earthquake characteristics are also important factors in defining displacement patterns.

The rigid translation with or without rotation patterns requires minimal effort in the design process, since the deflected shape is easily identified. No iterations are needed to converge to the deflected-shape. On the other hand, the flexible scenario requires iterative procedure until the deflected-shape converges to the assumed one.



Figure 2. Expected Deflected-Shape Scenarios for Continuous Bridges with Free and Integral Abutments, Plan Views

Kowalsky [3] established the concept of effective mode shape as a method to determine the design target-displaced shape. This method is general to any expected displacement pattern but it is time-consuming process especially for the flexible scenarios. The deflected-shape is a function of the effective mode shapes, the mode participation factors and the spectral displacements of the demand time history. Bridge effective mode shapes ( $\phi_i$ ) are evaluated based on secant stiffness as marked by K<sub>eff</sub> on figure 1. This is not known at the design stage but can be assumed in initial iteration. The modal participation factors (P<sub>i</sub>) are computed based on the mode shapes and the lumped mass matrix. The likely displaced shape for each mode is given by equation 3, and then the overall displaced-shape could be computed as the combination of all modes; the square root of the squares sum (SRSS) has been used in equation 4 to determine the overall displacement pattern.

$$D_{i,j} = \Phi_{i,j} P_i S d_i \tag{3}$$

$$D_j = \sqrt{\sum_i D_{i,j}^2} \tag{4}$$

In equations 3 and 4, index 'i' represents the mode number, index 'j' represents the bent number, and  $Sd_i$  is the spectral displacement of mode i. In his paper, Kowalsky designed two bridges one is symmetric and the other is asymmetric for free and constraint abutment cases. For the four designs the effective mode shape method seems to successfully predict the displacement pattern and convergence towards the assumed target profile was achieved after about four iterations.

Assuming any target-deflected shape, as long as it's achievable and compatible with the bridge modes shapes, will eventually converge to the actual shape, but this procedure is time-consuming and negates the idea of having a direct and simple design procedure. If it is possible to determine the criteria for selecting the rigid scenarios (i.e rigid translation with or without rotation) without the need to use the effective mode method, it will be a first step towards time and effort saving displacement-based design procedure. This paper is an extension to Kowalsky's paper [3], in which he proposed a displacement-based design procedure for continuous concrete bridges. Kowalsky's design procedure takes into account the superstructure rigidity or flexibility and its effect on the displacement patterns, based on the superstructure to substructure relative stiffness and various degrees of abutment restraint. A number of inelastic time history analyses were conducted for six different bridge configurations with variable superstructure and substructure stiffnesses, abutment type and earthquake characteristics in order to identify the various displacement patterns. Two design examples have been carried out, utilizing the finding of this study and Kowalsky's design procedure.

# DIRECT DISPLACEMENT-BASED DESIGN REVIEW

Direct Displacement-Based Design aims at designing a structure for prescribed limit states (i.e displacements or drift ratio) under prescribed earthquake intensities, utilizing the elastic response

spectrum and the substitute structure method developed by Gulkan and Sozen [2]. The main steps of the design procedure for MDOF bridges are [3]:

- 1. **Select a Target-Displacement Profile:** In the case of a single column bridge, the target-displacement can be obtained from the drift ratio or strain criterion that defines the desired level of performance of the column. In the case of a MDOF bridge a target displacement profile is required, which depends on the displacement pattern selected and critical column limit state.
- 2. Define the Equivalent SDOF Structure: Based on a research conducted by Calvi and Kingsley [4], an equivalent SDOF structure could be established based on equal work done by the whole bridge and the SDOF structure. The equivalent SDOF structure is described by a system displacement and a system mass as given by equations 5 and 6, respectively:

$$D_{sys} = \frac{\sum m_i D_i^2}{\sum m_i D_i}$$
(5)

$$M_{sys} = \sum \left( \frac{D_i}{D_{sys}} m_i \right) \tag{6}$$

In equations 5 and 6, m<sub>i</sub> is the bent lumped mass and D<sub>i</sub> is the bent target-displacement.

**3.** Estimate Level of Equivalent Viscous Damping: Using the chosen target displacement for each column and estimated yield displacements, the ductility levels could be calculated. Then utilizing Jacobsen's approach [1] and assuming a convenient hysteretic model, equivalent hysteretic damping values could be computed for each column. Such relationship between ductility and equivalent hysteretic damping is shown in figure 3 for Takeda hysteretic model [6]; for instance hysteretic damping of %27.5 corresponds to displacement ductility value of 3. An additional 0%-5% viscous damping could be added to obtain the level of equivalent viscous damping. Those damping values need to be combined for the equivalent SDOF structure; according to Kowalsky [3] a weighted average could be found in proportion to the work done by each column as shown in equation 7, where Q<sub>i</sub> is a weighting factor. Shibata and Sozen [2] suggest that the weighting factor be estimated based on flexural strain energy, while in [3], it is proposed that the factor be based on the work done by each DOF.



Figure 3: Equivalent Hysteretic Damping versus Ductility for Modified Takeda Hysteretic Model  $(\alpha = 0.0, \beta = 0.6 \text{ and } r = 0.0)$ 

4. Determine Effective Period of the Equivalent Structure: Utilizing the system target displacement, level of equivalent system damping and elastic response spectra for the chosen seismic demand, the equivalent period of the structure could be determined as shown in figure 4. For a design

displacement of 0.50 m and 10% level of equivalent viscous damping, the equivalent period is estimated to be 3.0 seconds. After the effective period has been determined, the effective stiffness and base shear are computed by equations 8 and 9, respectively.

$$K_{eff} = 4\pi^2 M_{sys} / T_{eff}^2 \tag{8}$$

$$V_B = K_{eff} D_{sys} \tag{9}$$

**5. Structural Analysis**: Distribute the base shear to the masses of the MDOF structure in accordance with the target-deflected shape as given by equation 10 [4]. Perform static structural analysis on the bridge under the inertia loads to get the design base shears for each column. At this stage of the design the columns stiffness are not known, so designers should assume reasonable values, run the analysis and check the displaced shape, if it is not close enough to the assumed target shape, column stiffness should be changed accordingly until convergence is achieved. If the deflected shape differs significantly from the selected one then modify the selected target shape accordingly and start all over from step 1.

$$F_{i} = V_{B}(m_{i}D_{i}) / \sum_{i=1}^{n} (m_{i}D_{i})$$
(10)

In equation 10,  $F_i$  is the bent inertia forces,  $V_B$  is the design base shear and index 'i' refers to the bent number.

6. Design the MDOF Structure: After convergence has been achieved, design the member sections.



Figure 4. Effective Period Evaluation According to DDBD Method

### STUDY PARAMETERS

In order to identify the inelastic displacement patterns of continuous bridges, a series of four span bridge structures were analyzed using inelastic time history analysis for 12 earthquake records. The bridges considered in the study ranges from regular symmetric to irregular asymmetric as shown in figure 5. Each of the bridges was assumed to be with and without abutment restraint. In the case of a restrained abutment, it was assumed that the superstructure is integrally built into the abutment which provides the superstructure with translational stiffness in the transverse direction and no rotational restraint. Abutment stiffness was estimated for yield displacements of 25mm and 60mm, based on CALTRANS memo 5-1 [5]. In the structural model, abutments were modeled as translational springs that follow bilinear with slackness hysteresis as shown in figure 6; a gap of 40mm and a bilinear factor (r) of 5% were assumed.

The inelastic displacement patterns are believed to be highly dependent on the superstructure and substructure stiffnesses; hence the superstructure moment of inertia around the vertical axes was varied between 5m<sup>4</sup> and 500m<sup>4</sup>, although the majority of bridges in practice have a moment of inertia values between 50m<sup>4</sup> and 150m<sup>4</sup>. The pier yield moments (i.e strength) were varied between 2.0MN.m and 24.0 MN.m. All columns were assumed to have a diameter of 1.5m, the same yield curvature and the modified Takeda hysteretic model [6]. Inelastic time history analysis was carried out for all the bridges using

RUAUMOKO [8], a dynamic analysis software package. The total number of inelastic time history analyses conducted in this study was about 5500. Table 1 summarizes all the study parameters.





# ANALYSIS RESULTS AND DISCUSSION

As discussed earlier, the relative stiffness (RS) of the superstructure to the substructure is the main factor in defining the bridge displacement patterns. In this study, the relative stiffness was used as an index to identify each bridge and its mode of deflection. In order to calculate the RS the deck was modeled as a beam pinned from both ends and its stiffness was computed for a unit displacement at mid-

span for a point loading. The piers were modeled as cantilevers with double curvature and their cracked stiffness was computed for a unit displacement at the free end. The relative stiffness used is the average of the superstructure stiffness to the piers stiffness. The definition of RS is shown in figure 7 and the expression is given by equation 11.

$$RS = Average(Ks/Kc) = 8/n \sum \frac{I_s L_c^3}{I_c L_s^3}$$
(11)

In equation 11, n is number of columns,  $I_s$  is superstructure moment of inertia,  $I_c$  is column moment of inertia,  $L_s$  is superstructure length, and  $L_c$  is column height.



Figure 7. Relative Stiffness (RS) Calculation (a) Deck Modeling for RS Calculation, Plan View (b) Pier Modeling for RS Calculation, Elevation.

In order to identify the displacement patterns, the displacement envelop was determined for each bridge analysis which in most of the cases was close enough to the actual deflected shape. In the case where the superstructure can be assumed to be rigid, all the points on the deck are expected to translate the same amount which results in a theoretical coefficient of variation equal to 0. In the event that the superstructure is rigid with eccentricity between the center of mass and center of rigidity all the points on the deck are expected to have equal rotations which also means a 0 coefficient of variation. In the case of flexible superstructure, the deck is expected to have flexible displacement pattern with coefficient of variation of the displaced shape greater than 0. It is suggested if a bridge structure has coefficient of variation greater than 10% then its displacement pattern should be considered flexible.

The bridge configurations shown in figure 5 are categorized into: (1) Regular symmetric bridges (BR7-7-7, BR7-14-7, and BR14-7-14), and (2) Irregular asymmetric bridges (BR 7-14-14, BR7-14-21 and BR14-7-21). The symmetry term refers to bridge geometry and the regularity terms refer to the symmetry of stiffness distribution across the bridge. Described in the following subsections are the results for each of the previous categories and the various displacement patterns that could be identified.

# Regular Symmetric Bridges (BR7-7-7, BR7-14-7, and BR14-7-14)

For all the bridges in this category, the coefficient of variation of the displacement envelop was plotted against the relative stiffness as shown in figure 8. In this figure, the lower the relative stiffness is, the lower the superstructure stiffness or the higher the substructure stiffness are; each relative stiffness value in the figure has more than one combination and each symbol represents a single earthquake. Clearly, in the case of free abutments, a rigid translational pattern could be identified while for integral abutment bridges, a smaller number of bridges showed that pattern. For example BR7-14-7, in the case of free abutments, with a RS index greater than 2 could be assumed to have a rigid translational displacement pattern, while in the case of integral abutments, a RS index greater than 13 is required to assume such pattern. The abutment stiffnesses and yield displacements did not have any noticeable effect on the displacement pattern; the vast majority of the bridges in figure 8c through 8f had a flexible displacement pattern.





#### Irregular Asymmetric Bridges (BR7-14-14, BR7-14-21, and BR14-7-21)

All bridges in this category had rotation in the displacement pattern due to the asymmetry or irregularity in the stiffness or both. All the coefficients of variation shown in figure 9 are for the rotations of several points on the deck. Similar to the results in the previous category, it is possible to identify a rigid translation and rotation displacement pattern for bridges with free abutments. For example a bridge similar to BR7-14-21 could be assumed to have a translational and rotational displacement pattern if its relative stiffness 'RS' is greater than 10. In the case of integral abutments the results were similar to the regular symmetric bridges, no rigid displacement pattern could be identified and the vast majority of the bridges deflected in a flexible mode pattern.

As discussed earlier, identifying whether the bridge has a rigid translation shape (with or without rotation) simplifies the design procedure, without any need for iterations. Table 2 identifies the inelastic displacement patterns for continuous bridges with free abutments based on the results obtained from the

inelastic time history analysis. If the relative stiffness index of a bridge is greater or equal than the value given in the table, the bridge undergoes rigid translation, with or without rotation as specified in the table; otherwise it has a flexible displacement pattern.

Bridge Configuration	Abutment Condition	Deflected-Shape Mode	RS Index ≥
BR7-7-7	Free	Т	1.0
BR7-14-7	Free	Т	2.0
BR14-7-14	Free	Т	3.0
BR7-14-14	Free	TR	6.0
BR7-14-21	Free	TR	10.0
BR14-7-21	Free	TR	12.0

**Table 2. Inelastic Displacement Pattern Identification** 

T = Rigid Translation, TR = Rigid Translation with Rotation.



Figure 9. Coefficient of Variation for the Rotation Envelops of Irregular Asymmetric Bridges

#### **DESIGN EXAMPLES**

Described in this section, are two design examples to demonstrate the direct displacement-based design approach for continuous bridges. The first example utilizes the results given in table 2 to identify the displacement pattern and design the bridge for a rigid translational deflected shape. The second example utilizes the same results to design the bridge for a flexible displacement pattern. Both bridges are to be designed in the transverse direction.

The two bridges, BR11-22-11 and BR6-12-6, have the configurations shown in figure 10; the first bridge has abutments free to move in the transverse direction while the second has abutments restrained against transverse displacement. All columns are estimated to have a diameter of 2.0m; their heights are measured to the centerline of the deck, and have a monolithic connection to the superstructure. Superstructure mass is 200KN/m and its moment of inertia about the vertical axes is  $100m^4$  for BR11-22-11 and  $50m^4$  for BR6-12-6. Material properties are assumed as follows: reinforcement yield stress = 455MPa and concrete modulus of elasticity = 33.7GPa. Both structures are to be designed for a drift ratio of 4.0% under the damage-control limit state, represented by the design response spectra shown in figure 11.



Figure11. Design Response Spectra. (a) Acceleration Spectrum (b) Displacement Spectra.

#### Example 1: Regular Symmetric Bridge without Abutment Restraint, (BR11-22-11)

The first bridge considered, BR11-22-11, is without abutment restraint and to be designed to sustain 4% drift under the design spectra shown in figure 11. The design steps of this bridge are presented in details as follows:

# Step One: The first step in the design is to select target displacement profile.

In order to compute the bridge relative stiffness, the column secant stiffness needs to be estimated. A moment of inertia = 50% Ig = 0.393m<sup>4</sup> is assumed to account for cracked section behavior. Utilizing equation 11, the relative stiffness 'RS' =  $(8/3)(100/140^3*0.393)(11^3 + 22^3 + 11^3) = 3.3$ ; since this bridge is similar to BR7-14-7 and based on the information given in table 2, a rigid translational pattern is assumed.

Step Two: In this step, the equivalent SDOF structure needs to be defined.

Since all columns will have the same displacement, then the target displacement profile is governed by the displacement of the critical column which in this case the shortest.  $D_{max} = 0.04 * 11 = 0.44$ m. Since the design profile is a straight-line with displacements of  $D_1 = D_2 = D_3 = 0.44$ m, the equivalent SDOF structure has a system displacement of 0.44m and a system mass = 200\*140 = 28,000 KN/g.

# Step Three: Estimate the level of equivalent viscous damping.

Columns yield curvature [9]:  $\phi_y = 2.25\epsilon_y$ /Diameter = 2.25\*0.00228/2 = 0.002565/m

The effective column heights for yield displacement including strain penetration of  $0.022F_yd_b=0.41m$  are 11.41m and 22.41m. Hence the yield displacements are:

 $D_{y1} = D_{y3} = 0.002565 * 11.41^2/6 = 0.0557 m$ 

 $D_{y2} = 0.002565 * 22.41^2/6 = 0.2147m$ 

Using the target and yield displacements, displacement ductilities are:

 $\mu_1=\mu_3=0.44/0.0557=7.91$ 

 $\mu_2 = 0.44 / 0.2147 = 2.05$ 

Using the ductility values into figure 3 and additional 2% viscous damping, the following equivalent viscous damping values could be obtained:

 $\xi_1 = \xi_3 = 0.02 + 0.36 = 38\%$ 

 $\xi_2 = 0.02 + 0.21 = 23\%$ 

The system equivalent damping is computed using equation 7. Kowalsky [3] suggested using 1/L as the weighting factor Q.

 $\xi_{sys} = (2*0.0909*38\% + 0.0455*23\%)/(2*0.0909 + 0.0455) = 32.5\%$ 

Step Four: Effective period, Effective Stiffness and Base Shear.

From the displacement response spectra, figure 11, for a design displacement of 0.44m and viscous damping of %32.5, the *Effective Period* is  $T_{eff} = 3.0$  seconds.

*Effective Stiffness*:  $K_{eff} = 4\pi^2 m_{sys}/T^2_{eff} = 4\pi^2 (28/9.805)/3^2 = 12.5 MN/m.$ 

**Base Shear**:  $V_B = K_{eff} D_{sys} = 12.5*0.44 = 5.5 MN$ .

#### Step Five: Base shear distribution.

Since the structure has free abutments and a rigid translation deflected shape, there is no need for structural analysis to distribute the base shear. It is recommended to distribute the base shear based on a weighting factor of 1/L, which will result in equal column moment demand [3]:

 $V_{B1} = V_{B3} = 5.5*0.0909/(2*0.0909 + 0.0455) = 2.2MN$  $V_{B2} = 5.5*0.0455/(2*0.0909 + 0.0455) = 1.1MN$ 

Step Six: Design the structure members for the design base shears obtained in the previous step.

#### Example 2: Regular Symmetric Bridge with Abutment Restraint, (BR6-12-6)

The second bridge, BR6-12-6, will be designed for fully restrained abutments against transverse displacement. The design steps are as follows:

#### Step one: Select target displacement profile.

Since the bridge abutments are restrained against translations, it is reasonable to assume a flexible displacement pattern. The effective mode method, suggested by Kowalsky [3] and discussed earlier, will be used to estimate the displacement pattern.

Assume the column stiffness is 50% of the uncracked section stiffness. Solving the eigenvalue problem, the mode shapes are obtained. Applying the effective mode procedure, the following

normalized effective mode shape is obtained as well as the target displacement profile as given in table 3.

Table 5. Target Displacement Profile						
Member	Normalized Effective Mode Shape	Target Displacement Profile (m)				
Pier 1	0.491	0.491*0.48=0.236				
Pier 2	1.0	12*0.04 = 0.480				
Pier 3	0.491	0.491*0.48=0.236				

#### Table 2 Ta t Dianl

# Step Two: Define Equivalent SDOF Structure.

Utilizing equation 5 and 6, the system displacement and system mass are calculated as follows:

Member	D <sub>i</sub> (m)	m <sub>i</sub> (N/g)	m <sub>i</sub> D <sub>i</sub>	m <sub>i</sub> D <sup>2</sup> i			
Pier 1	0.236	1*10 <sup>7</sup>	2.36*10 <sup>6</sup>	5.56*10 <sup>5</sup>			
Pier 2	0.480	1*10 <sup>7</sup>	4.80*10 <sup>6</sup>	2.30*10 <sup>6</sup>			
Pier 3	0.236	1*10 <sup>7</sup>	2.36*10 <sup>6</sup>	5.56*10 <sup>5</sup>			
		Total	9.52*10 <sup>6</sup>	3.42*10 <sup>6</sup>			

System Displacement:  $D_{sys} = 3.42*10^{6}/9.52*10^{6} = 0.359$ m.

System Mass:  $M_{sys} = 9.52 \times 10^6 / 0.359 = 2.65 \times 10^7 N/g$ .

# Step Three: Equivalent Viscous Damping.

Utilizing the yield curvature and strain penetration values obtained in the first example, the following table is constructed to evaluate the equivalent viscous damping of each pier.

Tuble et Equivalent viscous Dumping							
Member	Height (m)	фу	D <sub>y</sub> (m)	$\mu_{D}$	ξ <sub>hyst</sub>	ξ <sub>eff</sub>	
Pier 1	6.41		0.0176	13.66	38.3%	40.3%	
Pier 2	12.41	0.002565	0.0658	5.37	33.7%	35.7%	
Pier 3	6.41		0.0176	13.66	38.3%	40.3%	

Table 5. Equivalent Viscous Damping

The system damping is computed based on equation 7. Since at this stage of the design we do not know the inertia forces carried by the abutments due to elastic bending of the superstructure, we make an assumption that 30% of the total shear is carried by the abutments with a damping value of 5%:

 $\xi_{sys} = (0.3*5\% + 2*0.28*40.3\% + 0.14*35.7\%) = 29.07\%$ 

# Step Four: Effective period, Effective Stiffness and Base Shear.

From the displacement response spectra, figure 11, for a design displacement of 0.359m and viscous damping of %29.07, the *Effective Period* is  $T_{eff} = 2.11$  seconds.

*Effective Stiffness*:  $K_{eff} = 4\pi^2 m_{svs}/T_{eff}^2 = 4\pi^2 (26.5/9.805)/2.11^2 = 23.96 MN/m.$ 

**Base Shear**:  $V_B = K_{eff} D_{svs} = 23.96*0.359 = 8.60MN$ .

# Step Five: Base shear distribution (Structural Analysis).

Distribute the base shear, in accordance with equation 8, to the masses on the top of the columns as inertia forces:

Hence; *Pier 1 & Pier 3*:  $F_1 = F_3 = 8.60*0.248 = 2.13$ MN.

*Pier 2*:  $F_2 = 8.60*0.504 = 4.34$ MN.

At this stage, the pier stiffnesses are not known, so we assume the shear carried by pier 2 as 1.50MN, therefore, Pier 1 and 3 carry 3.0MN each.

*Pier Stiffnesses*:  $K_1 = K_3 = 3.0/0.236 = 12.7 MN/m$ ;  $K_2 = 1.5/0.48 = 3.125 MN/m$ .

Analyzing the structure statically under the inertia forces and assumed stiffnesses; the results of the analysis were as follows:

 $D_1 = D_3 = 0.284$ m;  $D_2 = 0.41$ m. Abutments shear = 50% of total shear

Pier 2 displacement is 20% higher than the target value, which indicates that the stiffness is too low. A second iteration with 20% increase in all Piers assumed stiffnesses yield the following results:

 $D_1 = D_3 = 0.260m$ ;  $D_2 = 0.378m$ . Abutments shear = 45% of total shear.

At this stage, the displacement shape differs from the assumed target profile and the abutments carry 45% of the total shear instead of the assumed value of 30%. A new displacement target profile is established based on the results of the second iteration as follows:

 $D_1 = D_3 = 6*0.04 = 0.24m; D_2 = 0.24*(0.378/0.26) = 0.348m$ 

The same previous steps could be carried out to obtain the final design. A summary of the complete design steps are shown in table 6.

Table 6	. Design Exa	mple 2. Symm	etric Regular Bi	ridge (BR6-12-6	) with Restrained	Abutments

Item	Abut. 1	Pier 1	Pier 2	Pier 3	Abut. 2		
Mass (MN/g)	5	10	10	10	5		
Assumed K <sub>eff</sub> (MN/m)		368	46	368			
Displacement Pattern (m)	0	0.099	0.202	0.099	0		
Target Displacement Profile (m)	0	0.236	0.480	0.236	0		
System Displacement (m)		0.359					
System Mass (MN/g)			26.5				
Yield Displacement (m)		0.0176	0.0658	0.0176			
Displacement Ductility		13.42	7.29	13.42			
Damping (%)	5	40.3	35.7	40.3	5		
System Damping (%)			29.07				
Effective Period (sec)			2.11				
Effective Stiffness (MN/m)			24.0				
Base Shear (MN)		8.60					
Inertia Forces (MN)	0	2.13	4.34	2.13	0		
Structural Analysis (First Iteration)							
Assumed Pier Shear (MN)		3	1.5	3			
Analysis Displacements (m)	0	0.284	0.410	0.284	0		
	Second Ite	eration	P		1		
Assumed Pier Shear (MN)		3.61	1.81	3.61			
Analysis Displacements (m)	0	0.260	0.378	0.260	0		
New Tai	rget Displa	cement Pr	ofile				
Target Displacement Profile (m)	0	0.240	0.348	0.240	0		
System Displacement (m)	0.285						
System Mass (MN/g)	29.01						
Yield Displacement (m)		0.0176	0.0658	0.0176			
Displacement Ductility		13.66	5.29	13.66			
Damping (%)	5	40.33	35.50	40.33	5		
System Damping (%)	23.58						
Effective Period (sec)	1.52						
Effective Stiffness (MN/m)	50.54						

Table 6. Continue							
Item	Abut. 1	Pier 1	Pier 2	Pier 3	Abut. 2		
Base Shear (MN)			14.42				
Inertia Forces (MN)	0	4.18	6.06	4.18	0		
Structura	al Analysis	(First Itera	ntion)				
Assumed Pier Shear (MN)		4	2	4			
Analysis Displacements (m)	0	0.392	0.563	0.392	0		
	Second Ite	eration					
Assumed Pier Shear (MN)		6.54	3.27	6.54			
Analysis Displacements (m)	0	0.294	0.426	0.294	0		
	Third Iter	ation					
Assumed Pier Shear (MN)		8	4	8			
Analysis Displacements (m)	0	0.257	0.374	0.257	0		
Fourth Iteration							
Assumed Pier Shear (MN)		8.55	4.28	8.55			
Analysis Displacements (m)	0	0.245	0.358	0.245	0		
Fifth Iteration							
Assumed Pier Shear (MN)		8.78	4.39	8.78			
Analysis Displacements (m)	0	0.240	0.351	0.240	0		
Convergence Achieved							
Member Design Force (MN)         1.71         4.40         2.22         4.40         1.71							

# **Time History Analysis and Design Verification**

In order to verify the accuracy of the direct displacement-based design, inelastic time history analysis was applied to the designed bridges with the program RUAUMOKO [7]. Three artificially generated earthquakes were used in the analysis, which were generated to match the design spectrum using the computer program SIMQKE [8]. The 'actual' deflected shapes from the inelastic analysis were compared to the design target profile as shown in figure 12a. Two of the analyses showed a good agreement with the design profile of BR11-12-11; however, the third analysis exceeded the target profile with a small percentage, but it still matches reasonably well, given the scatter in the generated earthquakes. It is clearly noted that the three deflected shapes have a rigid translational displacement pattern as assumed.

Similarly, figure 12b shows the time history analysis displacements and BR6-12-6 target displacement profile, the analysis shows a good agreement for two of the analyses and reasonably acceptable for the third one. Again, due to the scatter of the artificial earthquake and due to the fact that those earthquakes would never match the actual design spectrum, the design has been verified. By comparing the two design examples, it is clear the identifying the displacement pattern in the first example, saved more time and effort than the second design example.



Figure 12. Time History Analysis Results for Regular Symmetric Bridges with and without Abutment Restraint

#### CONCLUSIONS

Three different inelastic displacement patterns could be identified based on extensive inelastic time history analyses of six continuous bridges with various superstructure to substructure stiffness rations, free or integrally built abutments and 12 earthquake records,. It was concluded that a bridge with free abutments could have one of three displacement patterns based on its relative stiffness ratio; those patterns are: (1) Rigid Translation, (2) Rigid translation with rotation, and (3) Flexible mode. It is also very unlikely that a bridge with restrained abutments will deflect in a rigid manner and it should be designed for a flexible displacement pattern. Detailed direct displacement-based design examples for two regular symmetric continuous bridges were demonstrated. The design was verified through inelastic time history analysis utilizing artificially generated earthquake to match the design spectrum.

Additional analysis will be carried out in the future for more bridge configurations, abutment conditions and variable span lengths. The goal will be to identify the inelastic displacement patterns and to estimate the amount of shear force carried by the abutments in the case of a bridge with restrained abutments.

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