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A PROBABILISTIC METHODOLOGY FOR ASSESSING SEISMIC SLOPE DISPLACEMENTS

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SUMMARY

The assessment of the performance of earth structures in geotechnical earthquake engineering is of great importance, because the failure of an earth dam, solid-waste landfill, natural slope or constructed embankment, can result in significant human and financial losses and severe environmental impact. Although it is important to quantify the risk of failure of these systems, most of the research in seismic slope stability has been performed deterministically. The objective of the current research is to offer a probabilistic methodology for the assessment of seismically induced permanent displacements. The proposed approach utilizes a generalized 1-D equivalent-linear fully coupled stick-slip sliding block model to characterize a slope's dynamic response, making use of an earthquake database comprising over 680 recorded ground motions to compute the simulated seismic displacements. A predictive model for seismic displacements is developed, using as predictive variables the yield coefficient of the slope (k_y), the initial one-dimensional fundamental period (T_s), and the spectral acceleration at a degraded period equal to $1.5T_s$. The predictive equation has two branches separately computing the probability of "zero" displacement occurring from the distribution of "nonzero" displacement. Displacements smaller or equal to 0.1 cm are defined as "zero" for practical purposes. Following its validation with 16 case histories of earth dam and solid-waste landfill performance, the proposed model is implemented in a probabilistic framework for the evaluation of the seismic displacement hazard.

INTRODUCTION

General

Evaluating the deformation potential of earth slopes in seismic engineering design involves accounting for the variability in the properties of the earth slope, the strong ground motion, and the aleatory variability in the seismic permanent displacements given the slope properties and the strong ground motion. Hence, the evolution towards probabilistic approaches is necessary to properly account for the aforementioned sources of uncertainty and to explicitly describe the level of hazard adopted in design. In general, research in probabilistic approaches in seismic slope stability has been limited. In recent years, however, several researchers have recognized the importance of advancing the state of practice towards such methods and

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have proposed methodologies to account for risk in engineering design of earth slopes. These methodologies usually involve three steps: (1) establishing a model for prediction of seismic slope displacements, where seismic displacements are conditioned on a number of variables characterizing the properties of the slope and important characteristics of the strong ground motion, (2) computing the joint hazard of the conditioning ground motion variables, and (3) integrating the above two steps to compute the seismic displacement hazard. As a result, the differences in the proposed models are primarily in these areas: (1) the type of "idealized" slope model used for the simulation of seismic displacements, (2) the selection of the conditioning variables, (3) the number of records used as seismic excitations, (4) whether these records constitute real recordings or simulated ground motions, and (5) the type of mathematical model used to generate the relationship for seismic slope displacement estimation.

Previous Studies

Although a number of researchers have explored the problem of seismic slope displacements from a probabilistic perspective, a comprehensive review of their work surpasses the scope limitations of this paper. For the sake of clarity and conciseness, the findings of the research that influenced the current study are summarized herein.

Lin and Whitman [1] first studied the probability of failure of sliding blocks using a rigid block assumption and modeling the strong ground motion as a Gaussian stationary process. Based on an analytical study with ground motion pulses they used the peak ground acceleration, the root mean square acceleration, and the central frequency of the ground motion as the parameters characterizing the ground motion, and the yield acceleration as the parameter characterizing the slope strength to condition displacements upon. In their final methodology, displacements are only conditioned on the peak ground acceleration and the yield acceleration, and the computation of the annual probability of exceeding specified displacement thresholds involves computing the seismic hazard for one scalar parameter, the peak ground acceleration.

Yegian et al. [2], [3] addressed the problem of seismic slope deformations using a rigid block assumption and different conditioning variables. They normalized seismic displacements to the value of the peak ground acceleration (PGA), the number of equivalent cycles (N_{eq}), and the predominant period of the ground motion (T). They proposed a relationship between the normalized displacements and the ratio of the yield to the peak ground acceleration based on simulated displacement data computed from 86 earthquake records. The fact that the seismic displacement is conditioned on more than one parameter that characterizes the strong ground motion (i.e. PGA and N_{eq}) requires the computation of the joint hazard for these parameters, which may increase computational effort. In a subsequent study, Ghahraman and Yegian [4] propose a methodology for seismic displacement computation where a relationship is developed for seismic displacement as a function of magnitude and distance. This relationship can then directly be programmed in a software for seismic hazard analysis to produce annual probabilities of displacements being exceeded. The relationship has been developed by making all primary variables, such as the yield acceleration, the peak ground acceleration, the predominant period of the motion within the dam, and the equivalent number of cycles, functions of magnitude. Consequently, the standard deviation of the random error in this relationship is significant, i.e. on the order of 5 in ln-units, which results in the computation of a rather wide displacement range.

Kim and Sitar [5] addressed the problem of seismic deformations using simulated rather than recorded acceleration-time histories. Their study sheds light into the parameters influencing the observed variability in seismic displacement, postulating that the variability in the strong ground motion rather than that in the slope properties primarily controls the variability in the computed displacement. The authors do not provide with a methodology for seismic slope displacement evaluation.

Recent research at the Pacific Earthquake Engineering Research (PEER) Center developed a probabilistic framework for the computation of probabilities of exceedance of specified thresholds for variables that relate to the performance of engineering structures. The PEER framework, which is described in Deirlein et al. [6], is based on deconvolving the performance of an engineering system in intermediate independent steps by introducing intermediate random variables. The framework is generic to encompass all categories of engineering structures. Its application to seismic slope stability with the goal to compute probabilities of exceeding specified displacement thresholds would read as Equation (1):

$$\lambda(D) = \int_{IM} G(D \mid IM) \cdot \left| d\lambda(IM) \right| \tag{1}$$

where: λ is the annual rate of exceedance, D the seismic displacement, IM an intensity measure that characterizes one or more important aspects of the strong ground motion, G a conditional probability of the seismic displacement exceeding D given the intensity measure, and $|d\lambda(IM)|$ the absolute value of the derivative of the hazard curve for the intensity measure in question.

Scope of This Study

All previously mentioned approaches provide insight to the problem of seismic slope displacements when cast in a probabilistic framework. However, there are a number of issues remaining to be addressed and improved upon. Most of these methods are based on a rigid block assumption for the earth slope, which is not an "accurate" model for a deformable body oscillating during earthquake loading. Two of the methods recognize the deformability of the earth slope ([4], [5]) but in a decoupled fashion, which has been shown to be overly conservative or unconservative depending on the slope properties (e.g. Rathje and Bray [7]). Recent research (e.g. Rathje and Bray [7]) has shown that more sophisticated coupled models can be used to describe stick-slip sliding and deformable shaking response of an earth mass during an earthquake. In addition, all previous methods are developed based on ground motion simulations or they make use of a limited ground motion database of less than 150 records. Following the well recorded earthquakes of the last decade, the world database of earthquake data has significantly increased, providing an opportunity to better explore the effects of the strong ground motion on the probability of failure on an earth structure. Finally, all existing approaches only address the case of nonzero displacements occurring due to seismic loading. However, situations can arise where a combination of earthquake loading and soil properties will not induce any deformation of an earth slope. There is consequently a need to also model this finite probability as a function of the independent random variables. Considering these issues, this paper has a dual purpose: (1) to propose a predictive equation for the computation of seismically induced permanent displacements, and (2) to outline a methodology for implementing the proposed model in a computation of the seismic displacement hazard. This methodology is developed along the lines of the probabilistic framework adopted by the PEER Center mentioned in the previous paragraph and it excludes cases where the slope materials lose significant strength such as lateral spreading due to liquefaction.

PREDICTIVE MODEL FOR SEISMIC SLOPE DISPLACEMENTS

Mathematical Formulation

In an earthquake event, an earth slope may experience zero or finite permanent displacements depending on the characteristics of the strong ground motion and the slope's mechanical and dynamic properties. As outlined in Travasarou and Bray [8] and Travasarou [9], permanent displacements can be modeled as a mixed random variable, which has a certain probability mass at zero displacement and a probability density for finite displacement values. It can be argued that displacements smaller than 0.1 cm are not of engineering significance and can for all practical purposes be considered as "negligible" or "zero".

Additionally, the regression of displacement as a function of ground motion intensity measure should not be dictated by data at negligible levels of seismic displacement.

For the sake of clarity in the formulation of the relevant equations the values of seismic displacement that are smaller than 0.1 cm can be lumped to d_0= 0.1 cm. The probability density function of seismic displacement is then described by Equation (2) as:

$$f_D(d) = \tilde{p}\delta(d - d_0) + (1 - \tilde{p})\tilde{f}_D(d) \tag{2}$$

where: $f_D(d)$ is the displacement probability density function; $\tilde{p}$ is the probability mass at D = d_0; $\delta(d - d_0)$ is the Dirac delta function, and $\tilde{f}_D(d)$ is the displacement probability density function for D > d_0. Contrary to a continuous random variable, the mixed random variable can take on discrete outcomes with finite probabilities at certain points on the line as well as outcomes over one or more continuous intervals with specified probability densities. Figure 1 illustrates the case of a mixed variable with finite probability mass at D = d_0 and continuous probability density for D > d_0. Using this formulation, the probability of exceedance at small displacements can be smaller than 1, recognizing the possibility of a finite probability of non-failure (Figure 1b).

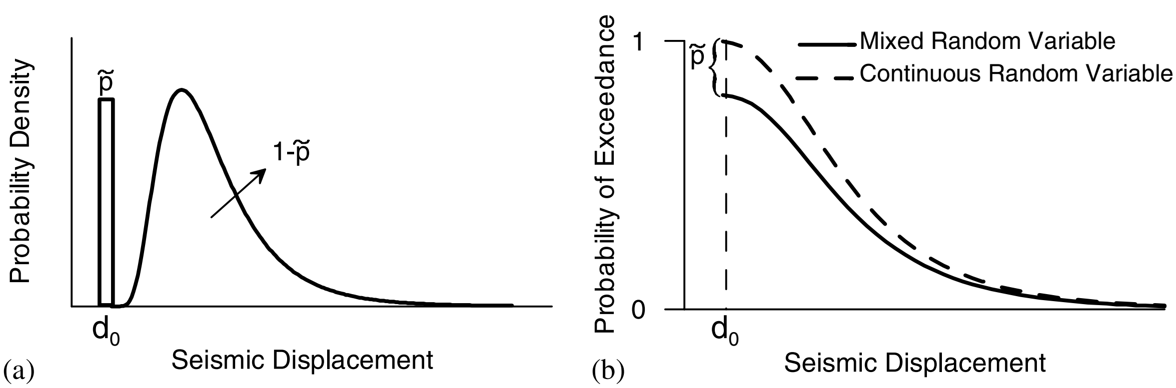


Figure 1: (a) Probability density function and (b) probability of exceedance for a mixed and a continuous random variable.

Simulated Displacement Data

The regression analysis for the development of the predictive model for seismic displacements has been performed on simulated displacement data. The idealized slope model and the ground motion database used in the simulations have already been described by Travasarou and Bray [8] but are repeated here for the sake of clarity and completeness.

Idealized Slope Model

The idealized slope model used in the simulations is a 1-dimensional generalized single degree of freedom equivalent-linear coupled stick-slip model proposed by Rathje and Bray [10]. It represents the slope as a generalized single degree of freedom system with constant shear wave velocity along its depth and responding only at its fundamental mode during dynamic excitation. The slope model is characterized by: (1) its strength as represented by its yield coefficient, k_y, (2) its stiffness, as represented by its initial fundamental period, T_s, and (3) the shear modulus reduction and damping curves relating the reduction of the soil's stiffness and increase of hysteretic material damping with increasing shear strain. Compared to the rigid block model, this coupled model offers a more realistic representation of the dynamic response of an earth structure by accounting for the deformability of the soil mass and by considering the simultaneous occurrence of its dynamic response and periodic sliding episodes. In addition, its validation against 1-D shaking table experiments provides confidence in its use [10].

For the purpose of these simulations the soil was assigned a constant unit weight of 17.6 kN/m^3, a constant hysteretic damping ratio of 10%, and the shear modulus reduction curves for a cohesive soil with a plasticity index of PI=30 as proposed by Vucetic and Dobry [11]. Preliminary sensitivity analysis suggests that these parameters do not have a significant effect on computed displacements. Seismic displacement values were subsequently generated computing the response of the idealized slope with specified values of its fundamental period and yield coefficient to a series of earthquake excitations. The values of the yield coefficient were fixed to 0.02, 0.05, 0.1, 0.2, and 0.3, and the values of the initial fundamental period were fixed to 0, 0.3, 0.5, 0.7, 1.0, and 2.0 seconds. These are realistic values of the yield coefficient and the fundamental period for a wide range of earth slopes.

Ground Motion Database

The ground motion database used to generate the seismic displacement data comprises available records from shallow crustal earthquakes that occurred in active plate margins. These records conform to the following criteria: (1) they correspond to earthquake magnitudes between 5.5 and 7.6, (2) they are recorded at rupture distances R ≤ 100 km, (3) they are recorded on sites B, C or D only, according to the SGS system (Rodriguez-Marek et al. [12]), and (4) frequencies in the range of 0.25-10 Hz have not been filtered out from the recordings. The final set comprises 688 records from 41 earthquakes satisfying the above criteria (Travasarou [9]). The distribution of the simulated displacement data against the independent variables is plotted in Figure 2 for each site class. For the purpose of the regression, all data with displacements smaller or equal to 0.1 cm were later reclassified as "zero" displacement.

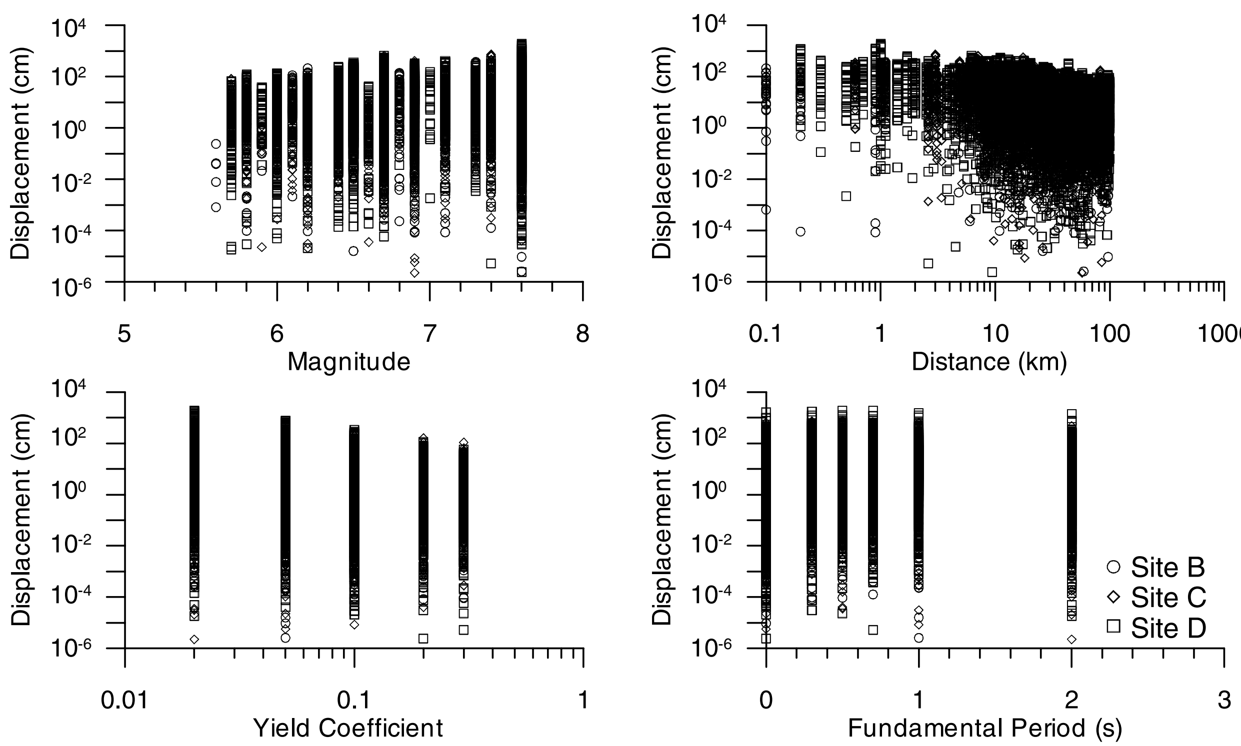


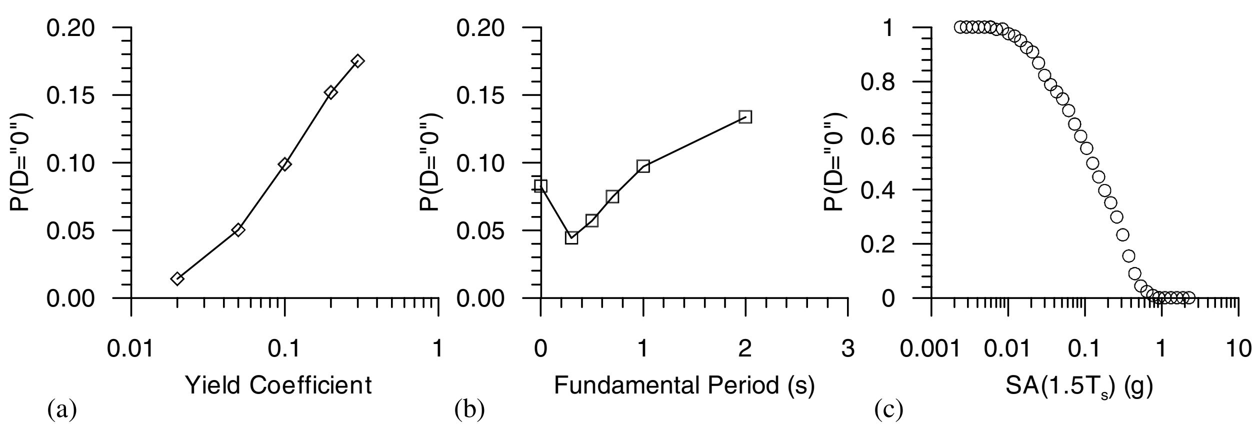
Figure 2: Distribution of simulated displacement data with moment magnitude, rupture distance, yield coefficient, and initial fundamental period.

Selection of Independent Variables

The amount of seismic displacement during earthquake excitation is dependent on the characteristics of the strong ground motion and the properties of the slope. In the majority of the simplified procedures for seismic slope displacement computation, researchers have used the peak ground acceleration as the primary indicator of the severity of the strong ground motion. This has sometimes been supplemented by additional parameters characterizing the duration and frequency content of the ground motion. For example, Yegian et al. [3] used the equivalent number of cycles and the predominant period of the motion, Makdisi and Seed [13] used the duration, and Bray et al [14] used the mean period and the significant duration.

To compute the displacement hazard in a framework compatible to that outlined in Equation (1) it is desirable to use a single ground motion parameter, which satisfies the requirements for efficiency and sufficiency (Luco and Cornell [15]). That is, it minimizes the variability in the correlation with seismic displacement, and it renders the relationship independent of magnitude and distance, respectively. The spectral acceleration at a degraded period equal to 1.5 times the initial fundamental period of the slope (i.e. $SA(1.5T_s)$), was found to be the optimal parameter satisfying the efficiency criterion (Travasarou [9]), for a number of slopes with initial fundamental periods ranging from 0 – 2.0 seconds. Hence, this degraded fundamental period can be considered a "generic" average degradation for slopes with T_s= 0 - 2.0 s. Although it does not satisfy sufficiency for all ranges of slope properties, it was found to be the most sufficient among the intensity measures examined.

The yield coefficient and the fundamental period were selected to represent the strength and stiffness respectively of the earth slope in the predictive model. The yield coefficient has traditionally been used in similar analysis to represent the onset of sliding. In addition, recent research (Rathje and Bray [7]) has shown that seismic displacements depend on the stiffness of the slope and are larger for slopes closer to resonance conditions compared to very flexible or very stiff slopes. This trend is taken into account by introducing the fundamental period in the predictive equation. This parameter also provides information as to the "starting" point of the degradation. The dependence of the probability of "zero" displacement on the three independent variables is shown in Figure 3. The selection of the functional form was guided by the trends shown in this figure.



(a) (b) (c)

Figure 3. Dependence of the probability of "zero" displacement on the (a) yield coefficient, (b) initial fundamental period, and (c) spectral acceleration at 1.5 times the initial fundamental period.

Functional Form

Probability of "Zero" Displacement – Decision Equation

Compatible with the concept of a mixed random variable, the predictive model for seismic displacements consists of two steps. In the first step, the probability of occurrence of "zero" displacement (i.e. $D \leq 0.1$ cm) is computed as a function of the yield coefficient, k_y, initial fundamental period, T_s, and spectral acceleration at $1.5T_s$ (i.e. $SA(1.5T_s)$). In the second step, the distribution of seismic displacement is computed, given that "nonzero" displacement has occurred. The estimation of the values of the model coefficient was performed using the principle of maximum likelihood. A probit and a truncated regression model were used for the first and second steps respectively, as described in Green [16]. The proposed model for the probability of "zero" displacement is:

$$P(D = "0") = 1 - \Phi\left(c_1 + c_2 \cdot \ln(k_y) + c_3 \cdot \left(\ln(k_y)\right)^2 + c_4 \cdot \ln(k_y) \cdot T_s + c_5 \cdot T_s + c_6 \cdot \ln(SA(1.5T_s))\right) \quad (3)$$

where D is the seismic displacement in cm, Φ is the standard normal cumulative distribution function, and c_1= -1.02, c_2= -4.11, c_3= -0.19, c_4= -0.823, c_5= -2.82, and c_6= 3.27 are the coefficients of the model as determined by the regression. The range of the computed probability is between 0 and 1. An example of the model predictions versus the simulated data is shown in Figure 4 for four cases of slopes with yield acceleration equal to 0.2 and four different values of the initial fundamental period.

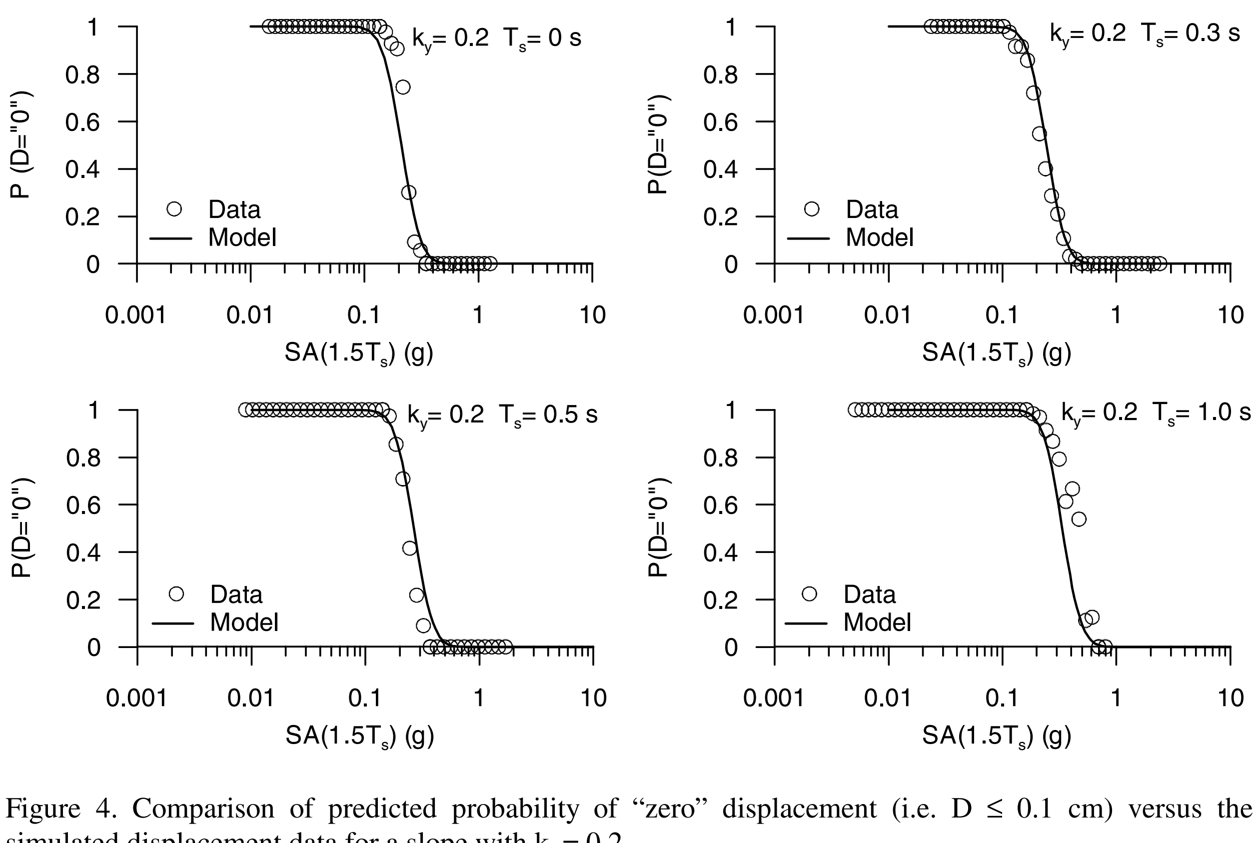


Figure 4. Comparison of predicted probability of "zero" displacement (i.e. $D \leq 0.1$ cm) versus the simulated displacement data for a slope with $k_y = 0.2$.

Nonzero Displacement – Regression Equation

In the case where Equation (3) computes a probability of "zero" displacement (i.e. $D \leq 0.1$ cm) smaller than 1, the amount of the nonzero displacement can be computed using Equation (4).

$$\ln(D) = c_1 + c_{21} \cdot \ln(k_y) + c_{22} \cdot (\ln(k_y))^2 + c_{23} \cdot \ln(k_y) \cdot \ln(SA(1.5T_s)) + c_{31} \cdot \ln(SA(1.5T_s)) + c_{32} \cdot (\ln(SA(1.5T_s)))^2 + c_{41} \cdot T_s + c_{42} \cdot T_s^2 + \varepsilon \tag{4}$$

where: D is the seismic displacement in cm, c_1= -1.64, c_{21}= -3.57, c_{22}= -0.478, c_{23}= 0.825, c_{31}= 3.75, c_{32}= -0.33, c_{41}= 0.872, and c_{42}= -0.082 are the coefficients determined by the regression and ε is a normally-distributed random variable with zero mean and standard deviation $\sigma = 0.78$. When the residuals (i.e. $d_{data} - d_{predicted}$) are plotted versus magnitude there is a magnitude dependence, because $SA(1.5T_s)$ is not fully sufficient with respect to magnitude (or duration). This dependence was addressed by incorporating a magnitude term in the predictive equation. Because the values of the coefficients did not change significantly when the regression was performed, the equation corrected for magnitude remains the same with an added magnitude term, as shown in Equation (5).

$$\ln(D) = c_1 + c_{21} \cdot \ln(k_y) + c_{22} \cdot (\ln(k_y))^2 + c_{23} \cdot \ln(k_y) \cdot \ln(SA(1.5T_s)) + c_{31} \cdot \ln(SA(1.5T_s)) + c_{32} \cdot (\ln(SA(1.5T_s)))^2 + c_{41} \cdot T_s + c_{42} \cdot T_s^2 + c_5 \cdot (M - 6.7) + \varepsilon \tag{5}$$

All coefficients of Equation (5) are identical to those in Equation (4), except now the additional term c_5= 0.30 and ε is a normally-distributed random variable with zero mean and standard deviation $\sigma = 0.77$. Equations (3), (4), and (5) have been developed for values of the yield coefficient between 0.02 and 0.3, fundamental period between 0 and 2.0 s, and $SA(1.5T_s)$ between 0.002 and 2.7 g, and they should be used for cases within these ranges to provide reasonable results. The residuals of Equation (5) are plotted in Figure 5 versus the predictive variables where it can be seen that there remains only a minor dependence on distance, which has not been taken into account in the current model.

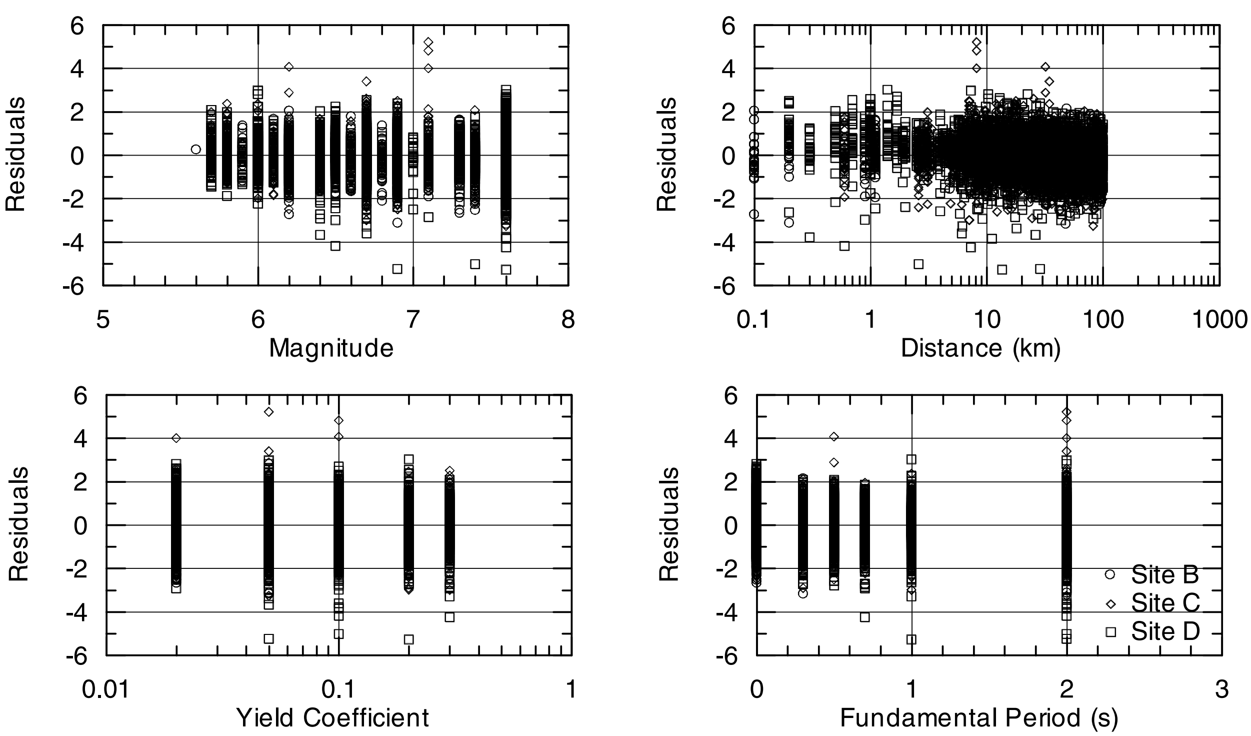


Figure 5. Residuals ($d_{data} - d_{predicted}$) of Equation (5) plotted versus the magnitude, rupture distance, yield coefficient and initial fundamental period.

MODEL VALIDATION

The ability of the proposed model to reliably predict observed performance was examined through comparison with observed displacements measured at 16 earth dams and solid-waste landfills that underwent strong seismic shaking. The observations from these case histories were used solely to validate the model and were not included in the dataset for the development of the predictive equation. The suite of case histories used in the validation is shown in Table 1. Details regarding the pertinent seismological characteristics of the corresponding earthquakes and best estimates of the yield coefficient and fundamental period and the strong ground motion can be found in Travasarou [9]. In all cases, the maximum observed displacement (D_{max}) is the portion of the permanent displacement attributed to stick-slip type movement and distributed deviatoric shear within the deformable mass, and ground movement due to volumetric compression was subtracted from the total observed permanent displacement when appropriate.

The yield coefficient used in the predictive model for these validations was reduced by 10% compared to its best-estimated value. This was decided to implicitly take into account two phenomena: (1) the existence of cycles during seismic shaking that are not large enough to initiate failure, but can still cause distributed deviatoric shear within the failure mass which translates to final permanent deformation (Makdisi and Seed [13]), and (2) the progressive development of the failure surface during earthquake excitation, which leads to a smearing of the strength, resulting in a mobilized average strength along the failure surface less than the peak strength of the soil. The phenomenon of progressive failure was recently demonstrated by Chen [17] who conducted large scale (12x12 inch) direct shear tests and small scale (1-inch diameter) vane shear tests on the same model clay mixture of kaolinite, bentonite and fly ash. The results, confirm the concept of "strength smearing" which occurs in the large scale test compared to the small scale test and results in a smaller peak strength of the soil during failure.

The comparison of the model predictions against the maximum observed horizontal permanent displacement is shown in Table 1. For this comparison, only the best estimate of the yield coefficient, the fundamental period, and the spectral acceleration at 1.5 times the initial fundamental period as represented by their mean values for the first two and the median value for spectral acceleration are considered. Hence, the computed displacement range is due to the variability in the seismic displacement given the value of the slope properties and the seismic load (i.e. $\sigma_{\ln D}$ =0.77 from Equation (5)). In the same table the estimates of the proposed method are compared with the estimates of two of the prevalent state-of-practice methods for seismic displacement prediction, Makdisi and Seed [13], and Bray et al. [14]. For the sake of consistency a yield coefficient reduced by 10% from the yield coefficient computed using the best estimate of the soil strength is used in all cases. Details with respect to the implementation of the two simplified procedures are delineated in Travasarou [9].

The sixth column in Table 1 tabulates the probability of exceeding the maximum observed displacement computed using the model proposed in this paper. The current model offers satisfactory predictions for the first 12 of the 16 case histories. The predicted probability of exceedance of the maximum observed displacements is on the order of 30% to 70%, suggesting that these case histories represent median model estimates. The only notable exceptions are Pacheco Pass Landfill, for which the proposed model predicts a zero probability of exceeding the observed displacement, which is "zero", and La Villita Dam for the S3 event, where a very small displacement of 1 cm was observed and the probability of exceeding it is small. However, these predictions are in agreement with Makdisi and Seed [13] and Bray et al. [14] and accurately model reality. For these 12 case histories, the estimated displacement ranges from the three methods are comparable, with Makdisi and Seed [13] predicting the smallest displacement values, which are also unconservative for some of these cases.

For the remaining four cases, the proposed model predicts either significantly conservative (2 cases) or unconservative (2 cases) values of the seismic displacement. Lexington Dam, and Lopez Canyon Landfill section C-A are the two case histories for which the estimated probability of exceeding the maximum observed displacement is large (i.e. above 80%), and hence the model is overly conservative. For Lexington Dam, the current model predictions are in line with the Bray et al. estimates, whereas Makdisi and Seed is unconservative. Although the predicted range of seismic displacement for Lopez Canyon Landfill section C-A is larger than the observed displacement, the displacements are sufficiently small so that this discrepancy would not have affected design. Among the two case histories where the proposed model offers low seismic displacement estimates, Chiquita Canyon C is underpredicted by all methods. For this case, the liner interface was subjected to significant stain prior to the earthquake, which may have caused high residual stresses, which induced more displacement during the earthquake episode. The case history of OII Landfill, is the only case for which the current method is both unconservative and in disagreement with the other two procedures. The maximum observed displacement of 15 cm is 1.5 standard deviations above the median of the predictive model (5 cm) indicating that even for this case the model may be unconservative but its predictions remain within reasonable range of reality. Overall, the new method can capture observed performance slightly better compared to existing procedures without contradicting their results and hence, it can be used as a predictive tool in practice with confidence.

Table 1. Comparison of the range of computed displacement using three different methods with the maximum observed displacement (Travasarou [9]).

Structure	EQ[1]	D_{max} (cm)[2]	P (D = "0")	Proposed Method[3] Est. Disp (cm)	P(D>D_{max})	Makdisi & Seed [13] D (cm)	Bray et al. [14] D (cm)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
BuenaVista LF	LP	None	0.44	0.5 – 2.6	0.57	0	0 – 0.4
Guadalupe LF	LP	None	0.71	0.2 – 1.0	0.29	0	0
Pacheco Pass LF	LP	None	1.00	-	0.00	0	0
Marina LF	LP	None	0.64	0.3 – 1.4	0.36	0	0
Austrian Dam	LP	48	0	23 – 109	0.52	0 – 11	14 – 167
Lopez Canyon C-B LF	NR	None	0.56	0.3 – 1.5	0.45	0	0 – 0.2
Chiquita Canyon D LF	NR	30	0	5.6 – 26	0.12	0 – 12	1.5 – 35
Sunshine Canyon LF	NR	30	0	17 – 82	0.62	0 – 1	1 – 20
La Villita Dam	S3	1	0.74	0.2 – 1.0	0.04	0	0
La Villita Dam	S4	1.4	0.16	1.0 – 4.6	0.59	0 – 0.6	0
La Villita Dam	S5	4	0.05	2.4 – 11	0.60	0 – 1.3	0
Chabot Dam	SF	Minor	0.60	0.6 – 2.6	0.41	0 - 16	0
Lexington Dam	LP	15	0	23 – 108	0.94	0 – 3	13 – 155
Lopez Canyon C-A LF	NR	None	0.17	1.1 – 5.4	0.83	0	0 – 0.2
Chiquita Canyon C LF	NR	24	0.29	0.7 – 3.2	0.00	0	0 – 1.0
OII Section HH LF	NR	15	0	2.1 – 10	0.06	5 – 38	20 – 200

1. LP: Loma Prieta (1989), NR: Northridge (1994), SF: San Francisco (1906), S3, S4 and S5 from Elgamal et al. [18]
2. D_{max}: maximum observed displacement
3. P(D="0") from Equation (3). The range in column (5) corresponds to the median-σ and median+σ predictions from Equation (5)

PROBABILISTIC SEISMIC DISPLACEMENT EVALUATION

Analytical Formulation

The objective of the procedure is to compute the probability of the seismic displacement, D, of an earth slope with given properties in terms of stiffness and strength, exceeding a specified displacement threshold, d in t-years. Conditioned on the slope properties, the probability of the maximum displacement exceeding a specified displacement threshold in a period of t-years is equal to:

$$P(D > d \ in \ t \ years \mid k_y, t_s) = 1 - \exp\left(-\nu \cdot t \cdot P(D > d \ \ for \ random \ sa, \varepsilon \mid k_y, t_s)\right) \tag{6}$$

where P(D>d | x,y) is the conditional probability of the seismic displacement exceeding a specified value, d, conditioned on the random variables x and y, ν is the mean rate of earthquakes, sa is the spectral acceleration at 1.5 times the initial period of the slope for a random earthquake, and ε is a normal random variable with zero mean and standard deviation $\sigma = 0.77$ representing the aleatory variability of the displacements for given k_y, T_s and SA . In all equations presented herein the symbol "sa" is used to represent $SA(1.5T_s)$ for brevity.

Using the total probability formula, for a slope with uncertain k_y and T_s

$$P(D > d \ in \ t \ years) = \int_{K_Y}\int_{T_S} \left[1 - \exp\left(-\nu \cdot t \cdot P(D > d \ \ for \, random \, sa, \varepsilon \mid k_y, t_s)\right)\right] \cdot f(t_s) f(k_y) dt_s dk_y \tag{7}$$

Furthermore, using the total probability by conditioning displacements on $SA(1.5T_s)$

$$P(D > d \ in \, t \ years) = \int_{K_Y}\int_{T_S} \left[1 - \exp\left(-\nu \cdot t \cdot \int_{SA} P(D > d \mid sa, k_y, t_s) f(sa) dsa\right)\right] \cdot f(t_s) f(k_y) dt_s dk_y \tag{8}$$

Assuming that k_y and T_s are statistically independent from event to event, Equation (8) becomes:

$$P(D > d \ in \, t \ years) = 1 - \exp\left(-\nu \cdot t \cdot \int_{SA}\int_{K_Y}\int_{T_S} P(D > d \mid sa, k_y, t_s) f(t_s) f(k_y) f(sa) dt_s \, dk_y \, dsa\right) \tag{9}$$

The probability density function of the spectral acceleration, f(sa), is given in terms of the annual hazard, (H(sa)), as:

$$f(sa) = \frac{1}{\nu} \cdot \frac{1}{1 - H(sa)} \cdot \left|\frac{dH(sa)}{dsa}\right| \approx \frac{1}{\nu} \cdot \left|\frac{dH(sa)}{dsa}\right| \tag{10}$$

Substituting (10) into (9)

$$P(D > d \ in \, t \ years) = 1 - \exp\left(-t \cdot \int_{SA}\int_{K_Y}\int_{T_S} P(D > d \mid sa, k_y, t_s) f(t_s) f(k_y) \left|\frac{dH(sa)}{dsa}\right| dt_s \, dk_y \, dsa\right) \tag{11}$$

Recognizing that seismic displacements are modeled as a mixed random variable, the probability term inside the integral of Equation (11) can be computed as:

$$P(D > d \mid sa, k_y, t_s) = [1 - P(D = "0" \mid sa, k_y, t_s)] \cdot P(D > d \mid sa, k_y, t_s) \tag{12}$$

In Equation (12), $P(D = "0" \mid sa, k_y, t_s)$ is computed directly from Equation (3) and

$$P(D > d \mid sa, k_y, t_s) = 1 - \Phi\left(\frac{\ln(d) - \ln(\hat{d})}{\sigma_{\ln D}}\right) \tag{13}$$

where $\Phi(\bullet)$ is the standard normal cumulative distribution function, $\hat{d}$ is the median displacement computed by Equation (5), and $\sigma_{\ln D}$ is the standard deviation of the natural logarithm of the seismic displacement equal to $\sigma_{\ln D} = 0.77$ (Equation (5)).

Because the yield coefficient, k_y, and fundamental period, T_s, always assume positive values a lognormal distribution is selected to model their variability. The probability density of a lognormal distribution is:

$$f(x) = \frac{1}{\sqrt{2\pi} \cdot \zeta \cdot x} \exp\left[-0.5\left(\frac{\ln(x) - \lambda}{\zeta}\right)^2\right] \tag{14}$$

where λ and ζ are the parameters of the distribution computed in terms of the mean, μ, and the standard deviation, σ, of the random variable as:

$$\zeta = \sqrt{\ln\left(1 + \left(\frac{\sigma}{\mu}\right)^2\right)} \quad \text{and} \quad \lambda = \ln(\mu) - 0.5\zeta^2 . \tag{15}$$

The mean and standard deviation for these variables is usually project specific and can be obtained by performing statistics on the data of the geotechnical investigation program or by expert opinion.

Equation (11) involves computing the derivative of the hazard curve. This can be achieved by fitting a curve to the hazard points and then computing the derivative analytically. Such an analytical form compatible with the notion of extreme value distribution, which characterizes the hazard problem, has been proposed by Cordova et al. [19], as follows:

$$H(sa) = k_0 \cdot sa^{-k} ; \qquad \frac{dH(sa)}{dsa} = -k \cdot k_0 \cdot sa^{(-k-1)} \tag{16}$$

However, there can be cases where a different functional form better describes the hazard. One such curve can be an exponential curve of the form shown in Equation (17).

$$H(sa) = a \cdot \exp(-b \cdot sa) \qquad \frac{dH(sa)}{dsa} = -a \cdot b \cdot \exp(-b \cdot sa) \tag{17}$$

An exponential fit may be a good approximation of the hazard at large spectral acceleration values whereas it may underestimate it at low levels of spectral acceleration. In any case the fitted curves should not be extrapolated beyond the range of the hazard data for use in the displacement calculations.

Methodology

Based on the analytical formulation described in the previous paragraph, the methodology for the computation of seismic displacement hazard can be outlined as follows:

Seismic Hazard Evaluation

1. Compute the seismic hazard in terms of the spectral acceleration at 1.5 the initial period of the slope ($SA(1.5T_s)$) using hazard software or using interpolated values from the USGS website (<http//geohazards.cr.usgs.gov>).
2. Fit an analytical curve of the form shown in Equations (16) or (17) to the hazard points.

3. Compute the derivative of the hazard curve analytically using Equation (16) or (17).

Slope Characterization

1. Estimate the mean value of the yield coefficient k_y, and initial fundamental period T_s, and their respective coefficient of variation (c.o.v.). The standard deviation, σ, is computed as the product of the mean, μ and the coefficient of variation, c.o.v.
2. In the case where uncertainty in these variables is not considered, a delta function can be assumed for their probability density function, the integrals over k_y and T_s in Equation (11) drop out, and the probabilities are computed at the mean values of k_y and T_s.
3. Compute the parameters ζ and λ for the two distributions using Equation (15).

Seismic Displacement Hazard Evaluation

1. The displacement hazard can be computed by Equation (11), approximating the three-fold integral numerically with a summation over k_y and T_s and $SA(1.5T_s)$ at discrete values of k_y, T_s and $SA(1.5T_s)$.
2. The probability term inside the integral is computed from Equation (12) using Equations (3) and (5). The magnitude, M, inserted in Equation (5) is the magnitude of the modal event controlling the hazard for the specified level of the spectral acceleration. This may be different at each spectral acceleration level and is determined by disaggregating the hazard at each spectral acceleration level.

Discussion

Computing the seismic displacement hazard using Equation (11) involves two approximations. The first is the one stated in Equation (10), which is acceptable for $H(sa) \leq \sim 0.05$. This approximation does not significantly affect the computed seismic displacement hazard, since hazard levels of $H(sa) > 0.05$ correspond to small spectral acceleration values which are unlikely to cause large displacements. However, one could avoid this approximation by using the exact formula of Equation (10) when substituting into Equation (9). The second is the assumption that k_y and T_s are statistically independent from event to event, whereas in reality they are practically invariant. This approximation may result in an error on the order of 5 –20% in the displacement hazard calculations, with the larger values corresponding to larger coefficients of variation of the slope's properties (i.e. c.o.v.k_y ~ 0.5 and c.o.v.T_s ~ 0.3) and the smaller numbers to lower coefficients of variations (i.e. c.o.v.k_y ~ 0.2 and c.o.v.T_s ~ 0.1). In any case, the exact formula for computing the displacement hazard is Equation (8), substituting f(sa) with the term in the right-hand side of the equal sign of Equation (10) and making use of Equation (12) to compute the conditional probability of displacement exceedance.

CONCLUSIONS

The gradual incorporation of probabilistic methods in engineering practice has been the incentive to revisit the problem of seismic slope stability and propose a methodology for seismic slope displacement estimation that can be used in a probabilistic framework. The outcome is a relationship for seismic slope displacement prediction that is a function of the intensity of the earthquake load as represented by the spectral acceleration at a degraded period of the slope ($1.5T_s$), the strength of the slope as represented by the value of the yield coefficient, k_y, and its stiffness as represented by the initial fundamental period, T_s. The predictive equation is derived from simulated displacement data, which are based on a simplified analytical model proposed by Rathje and Bray [10]. This is an equivalent-linear generalized single degree of freedom model, which couples the dynamic response of the earth slope with the stick-slip sliding episodes. Due to the fact that the analytical model only captures the stick-slip type of deformation, the proposed method is only applicable to predict permanent displacements that occur as a combination of sliding along a failure surface and distributed deviatoric shear, and a separate analysis is recommended for the prediction of displacements resulting from volumetric straining. Additionally, this method is not appropriate for cases where the earth materials loose significant strength, such as with liquefaction.

There are several novelties introduced by the proposed method. Firstly, it recognizes that seismic displacement is a mixed variable, which can acquire "zero" or finite values for given combinations of the earthquake load and slope properties. Hence, separate equations are developed for the prediction of the probability of "zero" displacement and the displacement distribution once nonzero displacement occurs. Due to their engineering insignificance displacements smaller or equal to 0.1 cm have been defined as "zero" displacements for the purpose of this method. Secondly, the method benefits from the large amount of recently available earthquake data and thus better characterizes the variability in the strong ground motion compared to existing methods. Thirdly, the proposed equations are based on an analytical "idealized" slope model that accounts for the deformability of the earth slope rather than falsely considering the slope to respond dynamically as a rigid body.

To validate the applicability of the proposed method as a tool to be used in design, the model estimates have been compared against observed displacements for 16 case histories of earth dams and municipal solid-waste landfills that experienced strong earthquake shaking. The method has also been compared with those of Makdisi and Seed [13] and Bray et al [14]. The favorable comparison with the observed displacement data and the reasonable agreement with the predicted displacement ranges of these two established state-of-practice simplified methods validate the accuracy of the proposed model. To account for the effects of progressive failure and distributed deviatoric shear, it is recommended that the proposed relationships be used with a yield coefficient reduced by 10% compared to the yield coefficient computed by a pseudostatic slope stability analysis using the best estimate of the shear strength of the soil.

Recent research has pointed out the greater damage potential of near-fault forward directivity motions compared to "ordinary" motions. Although such motions have been included in the development of the predictive equation, their enhanced damaging potential has not been separated in the proposed model. In addition, the current model does not differentiate displacements in slopes located on soil sites from those located on rock sites. The analysis has showed that the median model predictions are compatible with displacement on SGS C sites (i.e. weathered soft rock and shallow stiff soil with $H < 60$ m). Displacements on slopes located on "rock" (i.e. SGS site B, with $V_s \geq 760$ m/s) are ~20% smaller on average than the median, and displacements on slopes located on soil sites (i.e. SGS site D, deep soil with $H > 60$ m) are ~ 5% larger than the median. The current proposed method is straightforward in its implementation and can be used in a deterministic manner, much in the same way that strong ground motion is computed based on deterministic earthquake scenarios, or it can alternatively be implemented in a probabilistic framework for displacement hazard evaluation. In the second approach, the variability in all key variables identified in this study is taken into account.

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