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MODELLING OF EARTHQUAKE RESPONSE SPECTRA FOR STRIKE-SLIP EARTHQUAKES IN THE NEAR- AND FAR-FIELD

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SUMMARY

The objective of this paper is to describe the modelling of earthquake response spectra for strike slip earthquakes in the near and far field and assess the statistical significance of its fitting to strong motion data. The present data set consists of strong motion records from two magnitude 6½ events that occurred in the South-Iceland seismic zone in June 2000. This provides us with a data set consisting of roughly 47 records obtained in the two earthquakes, with epicentral distances between 5 and 150 km and hypocentral depth less than 10 km.

The attenuation model of the sdof response spectrum is derived in a closed form based on the Brune spectra for the near- and the far-field, modelling the high frequency tail in both cases by an exponential function. The closed form solution is found to be advantageous in studies on structural reliability and risk.

The main results are that the derived models fits the data fairly well providing residuals comparable to the residuals of models obtained by the widely used regression analysis. Furthermore, the response spectra attenuate with increasing distance more rapidly than R^{-1} where R is the distance to causative fault. The model based on the Brune theory is found to be adequate from an engineering point of view for many applications. It is also considerably more straightforward to apply than models based on the Haskel-Savage theory, although they may give a better fit to the response data, especially in the high frequency range of the response spectrum.

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INTRODUCTION

The earthquake response spectrum is a fundamental quantity in earthquake engineering and in aseismic design of structure and it is one of the key tools applied in design codes (see [1] for the general overview and [10] to explore the example used in this study). It was originally introduced for seismic applications by Biot [2] and Housner [3]. Although the response spectrum is defined for single degree-of-freedom systems, it is used in practise for multi-degree-of-freedom systems applying an appropriate superposition principle. The first practical superposition method was proposed by Biot [4] as early as 1943. Since then number of researchers [5, 6, 7, 8] have refined the response spectrum superposition process to improve the agreement with the true response of real structural systems.

Strictly speaking the response spectrum superposition methods are only applicable to linear elastic structures. However, it has been extended to non-linear structures by suitably modifying the input response spectra for different non-linear effects, for instance by introducing the ductility factor as an additional parameter to describe the system behaviour [9]. Such methods are commonly applied in codified design procedures where the dynamic action, especially for regular structures, is transformed into “statically equivalent loads”. In the design provisions of Eurocode 8 [10], a multi-modal dynamical analysis is required for all but simple regular structures, and in addition full dynamical approach based on time series analysis is an integral and recommended part of the provisions. In this context, a simple method, suitable for engineering applications, that relates time series and linear elastic response spectra could be a helpful tool. The method should preferably include the inherent randomness and scatter encountered in ground motion data.

The objective of this paper is to develop an analytical simplified model that directly relates response spectral ordinates to the ground motion and source parameters. Hence, in the following the response spectrum is expressed formally as:

$$S = S\left(\textit{structural}\ \text{variables}, \textit{source}\ \text{variables}, \textit{site}\ \text{variables}, \textit{wave propagation}\ \text{variables}\right)$$

Here, the primary structural variables are the undamped natural frequency and the critical damping ratio; the source variables include seismic moment, characteristic source dimensions and depth, as well as focal mechanism; site variables include source distance as well as local topography and geology; the wave propagation variables represent the mechanical properties along the path including spectral decay characteristics.

To simplify the presentation only shallow strike slip earthquakes are included. Near-source effects are accounted for but site effects are ignored, assuming rock or firm soil conditions. To enhance the applicability of the presented models a closed form formulas for the near- and the far-field are presented and compared to data.

The present study is based on the so-called Brune model. It was derived for seismic shear waves by considering the effective stress needed to accelerate the sides of a circular causative fault on which a stress pulse is applied instantaneously [11, 12]. This model is commonly used to obtain fault dimensions from spectra of shear waves for small to moderate sized earthquakes [13]. The model describes both near- and far-field displacement-time functions as well as spectral shapes and includes the effect of fractional stress drop. The Brune model has been applied successfully to analyse Icelandic earthquake and strong-motion data [14, 15].

REVIEW OF BASIC PRINCIPLES

The earthquake response spectrum is defined as the maximum response of an array of single-degree-of-freedom (sdof) damped systems subjected to the same base excitation and expressed as a function of undamped natural frequency, critical damping ratio of the system as well as other parameters required to describe the system. The earthquake response (displacement) spectrum of a linear elastic sdof system can hence be defined formally as (see for instance [16]):

$$S_D(\omega_o,\zeta|source, site, path) = \max_{t\in T}\left(x(t|\,\omega_o,\zeta, source, site, path)\right) \tag{1}$$

where $x(\cdot)$ denotes the response of the system and t is the time, furthermore ω_o refers to the undamped natural frequency of the system and ζ is the critical damping ratio, finally T refers to the duration of excitation. For linear elastic sdof system the response can be expressed as follows (see, for instance, [1] for further details):

$$x(t) = \int_{-\infty}^{\infty} h(t-u)\,\alpha(u)\,du \tag{2}$$

Here, $h(\cdot)$ is the impulse response function of the system and $\alpha(\cdot)$ is a (uni-directional) ground acceleration. It is assumed that the systems considered are lightly damped. Hence, the spectral relations connecting the pseudo-acceleration and pseudo-velocity to the displacement spectrum are applicable. Then we can write [1]: $S_A(\omega_o,\zeta) = \omega_o\, S_V(\omega_o,\zeta) = \omega_o^2\, S_D(\omega_o,\zeta)$. Here S_A and S_V are, respectively, the pseudo-acceleration and pseudo-velocity spectra, which approximate the real acceleration and velocity spectra fairly well for the lightly damped cases.

As a first step towards a closed form solution of Eq.(1) we apply Parseval's theorem to obtain the root mean square response:

$$x_{rms}(t) = \sqrt{\frac{1}{T}\int_{-\infty}^{\infty} x^2(t)\,dt} = \sqrt{\frac{1}{T}\frac{1}{\pi}\int_0^{\infty} \left|X(\omega)\right|^2 d\omega} \tag{3}$$

Here $X(\cdot)$ is the Fourier transform of $x(\cdot)$ expressed as a function of the circular frequency, denoted by ω, and T is the duration of strong shaking. The Fourier spectrum of the response is readily obtained by taking the Fourier transform of Eq.(1), which gives: $X(\omega) = H(\omega)A(\omega)$. Here $H(\cdot)$ is the frequency response function, $H(\omega) = \left[(\omega_o^2 - \omega^2) + i\,2\zeta\omega_o\omega\right]^{-1}$, and $A(\cdot)$ is the Fourier spectrum of the earthquake excitation. Substituting this expression into Eq.(3) gives:

$$x_{rms}(t) = \sqrt{\frac{1}{T}\frac{1}{\pi}\int_0^{\infty} \left|H(\omega)\right|^2 \left|A(\omega)\right|^2 d\omega} \tag{4}$$

The integration gives the following approximate expression for the rms response, which holds for lightly damped systems:

$$x_{rms}(t) = \frac{1}{\omega_o^2}\sqrt{\frac{1}{T\,\pi}\int_0^{\infty} \mathrm{U}(\omega-\omega_p)\left|A(\omega)\right|^2 + \frac{1}{T\,\pi}\left|A(\omega_o)\right|^2\left(\frac{\pi\omega_o}{4\varsigma}-1\right)} \tag{5}$$

where $U(\cdot)$ is the Heavyside step function equal to 1 if $\omega \geq \omega_p$ and zero elsewhere, ω_p is a parameter selected to minimise the total integration error. The solution of the integral under the square root sign is given in the following, respectively, for the near-field and the far-field.

The peak response can be obtained applying the random vibration theory as outlined in [16]. Hence, introducing the peak factor, p, the response spectrum for a linear elastic sdof system can be expressed as follows:

$$S_D(\omega_o, \zeta) = \max_{t \in T}(x(t)) = p \cdot x_{rms}(t) \tag{6}$$

It should be noted that the peak factor will generally be a function of the duration and effective frequency and bandwidth of the system, in other words, it depends on the effective number of peaks within the time window considered. Furthermore, the peak factor depends on the probability of exceedance referred to the time window under consideration. However, in the following a median value is used for the peak factor, which is close to the most probable value, corresponding to positive zero crossings:

$$p \cong \sqrt{2 \ln\left(2.8 T f_o / 2\pi\right)} \tag{7}$$

where f_o is the natural frequency of the system. A thorough treatment of the peak factor is given in [16].

STRONG-MOTION RESPONSE MODELLING

Far-field approximation

The acceleration spectrum in the far-field, based on the modified Brune source spectrum, can be expressed as follows, accounting for the free-surface effects and partitioning of the wave energy into two horizontal components, as well as modifying the high frequency part by exponential term [15,17]:

$$\left|A_F(\omega)\right| = \frac{2 C_P R_{\theta\phi} M_o}{4\pi\beta^3\rho R} \frac{\omega^2}{(1+(\omega/\omega_c)^2)} \exp(-\tfrac{1}{2}\kappa\omega) \tag{8}$$

Here, C_P is the partitioning factor, $R_{\theta\phi}$ denotes the radiation pattern, M_o is the seismic moment, β is the shear wave velocity, ρ is the material density of the crust, ω_c is the corner frequency and κ is the so-called spectral decay. The following expression is suggested for the geometrical spreading function [15]:

$$R = \begin{cases} D_2^{1-n} D^n & D_1 < D \leq D_2 \\ D & D_2 < D \leq D_3 \end{cases} \tag{9}$$

where $1 < n \leq 2$ and D is a distance defined as:

$$D = \sqrt{d^2 + h^2} \tag{10}$$

Here, d is the epicentral distance and h is a depth parameter. The parameters D_1, D_2 and D_3 are used to set the limits for the different sectors of the spreading function. The first sector can be thought of as a crude approximation for the near-field. Hence, the quantity D_1 can be approximated by h; D_2 quantifies the size of the sector representing the intermediate field, which is related to the magnitude of the earthquake (as represented by the seismic moment), source dimensions and focal depth and the thickness of the seismogenic zone; while D_3 can be thought of as the distance where cylindrical waves begin to dominate the wave field. This modification of the spreading function is of value especially in the case where the distance to the fault is not known. On the other hand if a reliable measure of the shortest distance to fault trace is known this modification of the spreading function can be omitted.

The corner frequency is given as [11]:

$$\omega_c = \sqrt{\frac{7\pi}{4}}\frac{\beta}{r} \tag{11}$$

where r is the radius of the source.

The spectral decay parameter is related to the quality factor Q through the following equation:

$$\kappa = \frac{R}{\beta Q} \tag{12}$$

The quality factor, Q, is in this context assumed to represent the average scattering and anelastic attenuation over the whole path. Studies of Icelandic strong-motion data indicate that the spectral decay, κ, can be taken as constant [15, 17], at least for moderate epicentral distances, where the seismic wave field is dominated by shear waves and the Brune model is assumed to hold as an engineering approximation. Hence, the quality factor Q varies approximately linearly with increasing distance from the source. This seems consistent with the fact that sites at great distance from the source are receiving shear waves that have penetrated through lower crustal layers with less attenuation than the upper layers.

Substitution of above equation, Eq.(8), into Eq.(5) leads to the following expression after the integration has been carried out:

$$x_{rms}(t) \approx \frac{1}{\omega_o^2}\sqrt{I_F + \frac{1}{\pi T}\left|A_F(\omega_o)\right|^2\left(\frac{\pi\omega_o}{4\zeta} - 1\right)} \tag{13}$$

where

$$I_F = \frac{1}{\pi}\left(\frac{7}{16}\right)^{2/3}\left(\frac{C_P R_{\theta\phi}\,\Delta\sigma^{2/3}}{\beta\rho R}\right)^2 \frac{\Psi}{T\kappa} M_o^{2/3} \tag{14}$$

Here $\Delta\sigma$ is stress drop (see below) and Ψ denotes a dispersion function given as:

$$\Psi = 1 - \frac{1}{2}\lambda\, ci(\lambda)\big(\lambda\cos(\lambda) + 3\sin(\lambda)\big) - \frac{1}{2}\lambda\, si(\lambda)\big(\lambda\sin(\lambda) - 3\cos(\lambda)\big) \tag{15}$$

Here, ci(·) and si(·) represent the cosine and sine integrals, and:

$$\lambda = \kappa\omega_c \tag{16}$$

where ω_c is the corner frequency. The sine and cosine integrals applied in Eq.(14) are given, respectively, as follows:

$$\begin{aligned} si(\lambda) &= -\frac{\pi}{2} + \int_0^{\lambda} \frac{\sin(t)}{t}\,dt \\ ci(\lambda) &= \gamma + \ln(\lambda) + \int_0^{\lambda} \frac{\cos(t)}{t}\,dt \end{aligned} \tag{17}$$

where γ is the Euler constant ($\gamma \approx 0.5772$).

In the above-mentioned studies of Icelandic earthquakes, it is assumed that the effective stress equals the stress drop, i.e. $\sigma = \Delta\sigma$, where $\Delta\sigma$ denotes the stress drop. For a double couple source it can be shown that the stress drop is related to the seismic moment, M_o, through (see, for instance, [13]):

$$\Delta\sigma = \frac{7}{16}\frac{M_o}{r^3} \tag{18}$$

Here, r is the radius of the fault plane, representing a characteristic dimension of the source.

Near-field approximation

The model described in the previous section is not valid in the near-field and can, therefore, not be expected to describe the response accurately close to the fault. To obtain an approximation which is valid for shear waves in the near-fault area the Brune near-field model [11] can be used. Hence, the near-field acceleration spectrum is approximated as follows, after modifying the high frequency part with an exponential term and accounting for the free surface and partitioning of the energy into two horizontal components:

$$|A_N(\omega)| = \frac{7}{8}\frac{C_p M_o}{\rho\beta r^3}\frac{\omega}{\sqrt{\omega^2 + \tau^{-2}}}\exp(-\tfrac{1}{2}\kappa_o\omega) \tag{19}$$

Here, κ_o is the spectral decay of the near-field spectra and τ is the rise time. Otherwise the same notation is used as above.

Substitution of above equation, Eq.(19), into Eq.(5) leads to the following expression after the integration has been carried out:

$$x_{rms}(t) \approx \frac{1}{\omega_o^2}\sqrt{I_N + \frac{1}{\pi T}\left|A_N(\omega_o)\right|^2\left(\frac{\pi\omega_o}{4\zeta} - 1\right)} \tag{20}$$

where

$$I_N = \frac{1}{\pi}\left(\frac{7}{8}\frac{C_p}{\rho\beta r^3}\right)^2 \frac{\Psi_o}{T_o\,\kappa_o} M_o^2 \tag{21}$$

Here, the duration is denoted by T_o and Ψ_o is a dispersion function given as:

$$\Psi_o = 1 - \lambda_o\left(ci(\lambda_o)\sin(\lambda_o) - si(\lambda_o)\cos(\lambda_o)\right) \tag{22}$$

where $\lambda_o = \kappa_o / \tau$.

Intermediate-field approximation

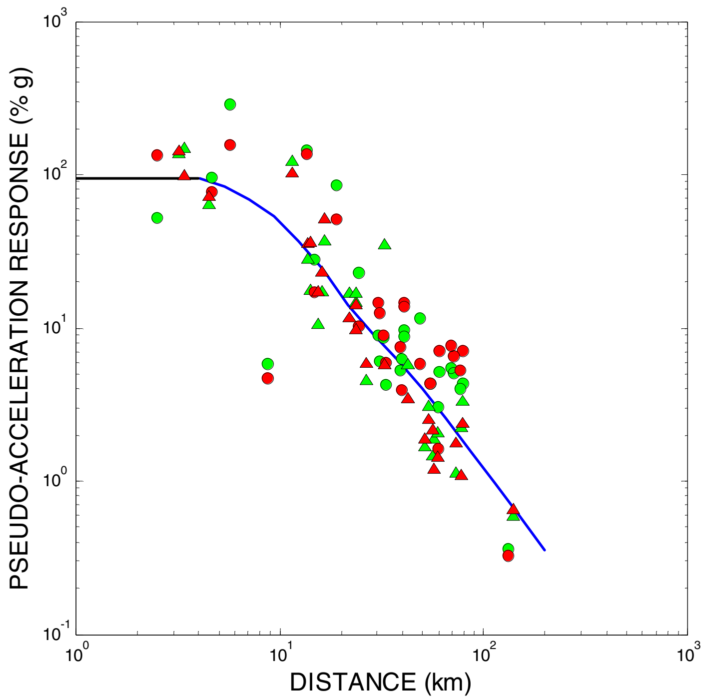
The model described in the two previous sections gives a response approximation that is valid in the far- and near-field respectively. For shallow earthquakes an intermediate-field approximation is required to represent data from stations that do not fall within the far- or near-field conditions. The geometric spreading function given in Eq.(9) can be used for this purpose, by introducing a functional form proportional to R^{-2} which is supported by the analytical solution of the wave equation [21]. Hence the exponent in Eq.(9) is taken as $n = 2$.

The transition between these three fields, i.e. the near-, intermediate- and the far-field, depends on number of parameters. The most important ones being the magnitude of the earthquake as reflected by the seismic moment, the focal depth, the thickness of the seismogenic zone and the size of the causative fault. For shallow strike-slip earthquakes, with a causative fault rupturing to the surface it seems that the size of the near-field stretches out on the surface from the surface trace of the fault equal to about the half of the focal depth, while the transition between the intermediate- and far-field appears to be at a distance about tree times the fault radius. This is, for instance, seen to be a characteristic feature of moderate sized Icelandic earthquakes (see the strong motion modelling data in the following section). This type of grading is of course dependent on the seismic area and must be considered for each case.

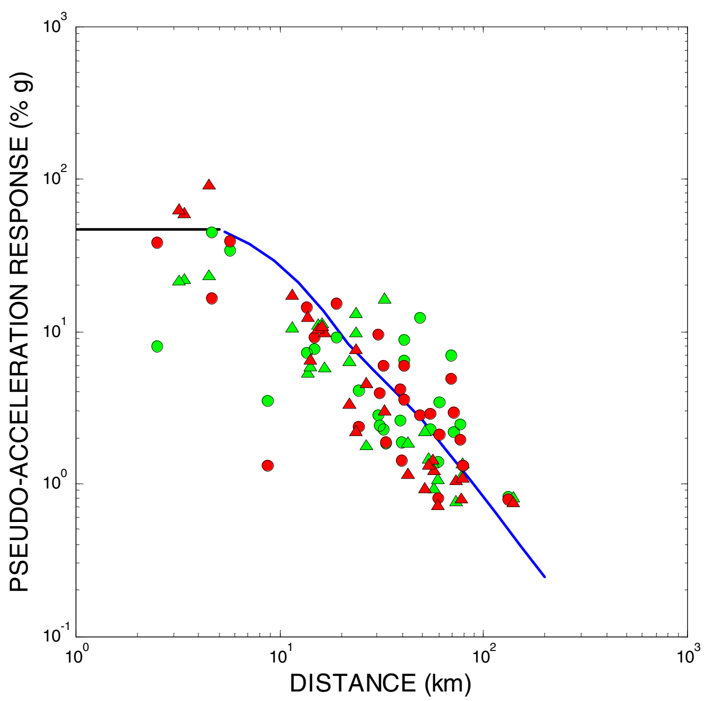
NUMERICAL RESULTS

The models described above have been applied to data obtained in the two magnitude 6½ earthquakes that occurred in South Iceland in June 2000 [18]. Please note that the applied data can be obtained from the ISESD Website [19]. The data used for the strong motion modelling is listed in Table 1.

Figure 1 exemplifies how the presented models fit the data. The models given in Figure 1 are indicated by blue and black solid lines, respectively, for the far-field (see Eq.(13)) and the near-field (see Eq.(20)). The data is represented by circles and triangles. The circles indicate earthquake-induced response on 17 June 2000, 15:41, while the triangles refer to the 21 June 2000 event. The colour code, red and green, refers to two horizontal components. In both cases Figure 1(a) and (b) show a reasonable fit of the model to the data. The fit to the data is, however, somewhat better for the flexible system than for the stiff one. This seems especially to be the case in the near-field (see Figure 1(a)). The reason for the relatively high response values in the near-field for the stiff system may be due to site effects. Furthermore, the response recorded at few of the far-field stations on June 17 is augmented due to seismic waves originating from more than one event originating almost simultaneously at epicentres about 60 km apart [20]. In spite of these shortcomings the over-all behaviour of the models seems reasonable and the estimated results appear to be of the right order of magnitude.



(a) Results for undamped natural period of 1/3 s.



(b) Results for undamped natural period of 1 s.

Figure 1: Pseudo-acceleration response. Solid lines represent the far- and intermediate field (blue) and the near-field (black) approximation. The circles represent data from 17 June 2000 (15:41), while the triangles refer to the 21 June 2000 event. The colour code, red and green, refers to the two horizontal components.

Table 1: The data used for the response spectrum given in Figure 1.

Parameter	Symbol	Quantity	Units
moment magnitude	M_w	6.5	
shear wave velocity	β	3.5	km/s
density of rock	ρ	2.8	g/cm^3
average radiation pattern	$R_{\theta\phi}$,	0.63	
partitioning parameter	C_P	$1/\sqrt{2}$	
spectral decay in the far-field	κ	0.04	s
characteristic dimension of the intermediate-field	R_2	25	km
exponent describing attenuation in the intermediate-field	n	2	
depth parameter	h	9	km
spectral decay in the near-field	κ_o	0.02	s
characteristic fault dimension (radius)	r	8.0	km
duration used in near-field model	T_o	$1.5 \cdot r/\beta$	s

Only critical damping ratio equal to 5% is included to facilitate comparison to [10]. Two different undamped natural frequencies are included, i.e.: 3.33 Hz exemplifying stiff structures and 1 Hz indicating the behaviour of flexible structural systems.

DISCUSSION

An analytical simplified model, that directly relates linear response spectral ordinates to the ground motion time series and source parameters, has been developed. The model is found to give a reasonable fit to recorded data from shallow strike slip earthquakes in South Iceland. Furthermore, it is believed to be a useful tool for reliability and risk studies.

The dynamic magnification of the structural system can be obtained by dividing the pseudo-acceleration response by the corresponding peak ground acceleration, i.e. $S_A(\omega_o,\zeta)/PGA$, which then ought to be comparable with the codified value commonly given as 2.5 for structures with short natural period. It is, however, noted that the presented response model in the near-field and for short distances from source, for instance less than 10 km, produces lower magnification values, for commonly observed natural periods, than obtained using the values recommended by Eurocode 8 for type 1 elastic response spectrum [10]. On the other hand the difference in seismic action as represented by S_A might not necessarily be very big unless the same *PGA* values are used. Therefore, a correction of the codified magnification factor could be necessary if the peak ground acceleration is augmented to account for near field effects.

The model needs further development and refinement. A step in that direction is to enlarge the database by including additional data selected from moderate sized shallow strike slip earthquakes world wide, which are found to be statistically comparable to the Icelandic data. This work is underway and will be presented elsewhere.

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