



## CAPACITY DESIGN FOR TALL BUILDINGS WITH MIXED SYSTEM

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### SUMMARY

The behavior of shear wall – frame can be assessed by any standard computer programme under elastic domain. However it becomes necessary to examine the behavior under nonlinear domain as well in order to design them suitably for earthquake resistance. To assess the behavior of the dual system's lateral load resistance, it is customary to examine their individual behavior and then ensure compatibility by the interacting forces. This approach does not give the sequence of formation of plastic hinges. In this paper an improved step-wise nonlinear analysis is used to analyze a typical mixed system containing both shear walls and frames. The example taken is a typical irregular building in order to expose its deficiency with respect to ductility performance. Rotational ductility demands of the various components of mixed system are found. The method proposed can be used for incorporating capacity design principles for mixed systems.

### INTRODUCTION

Under the action of the lateral loads, a frame primarily deforms in a shear mode whereas a wall deflects as a cantilever with flexural deformations. A preferable and practical mechanism for a typical shear wall frame is shown in Figure 1a and 1b. In this frame plastic hinges are made to develop before plastification of walls.

Provision of strength of various components by a step-by-step procedure for the desired collapse mechanism to form, becomes important rather than a sophisticated computer analysis.

### IMPROVED STEP-WISE NON-LINEAR ANALYSIS

#### Concept

The lateral loads on the design of reinforced concrete structures are based on the limit state design philosophy. The main problem is the determination of design forces at limit state of collapse. Apparently there are two possibilities:

1. To determine the member forces for the seismic loads from an elastic analysis and magnify them by appropriate partial safety factors.

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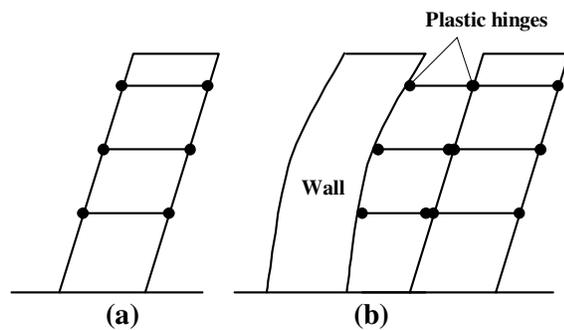
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2. To carryout a detailed non-linear analysis.

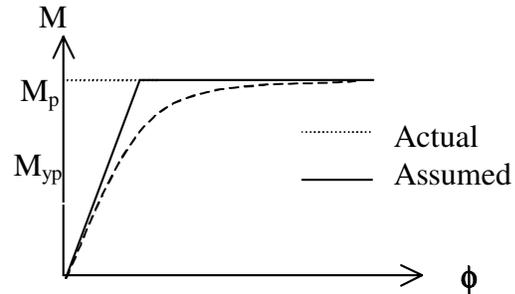
Obviously the first approach is very convenient but nowhere near the true behavior. The design codes however are currently based on the first approach. For the second approach, even though different methods are available is extremely laborious and hence not practiced.

Also for the design of the structure an exact 3D non-linear analysis is not required. However, a non-linear analysis, which will quantify rotation at critical hinges, is required for safety checking.

The calculation of deflections during the entire range of loads up to collapse is based on an assumed moment curvature relationship, which is shown in Figure 2 [1].



**Fig.1 Collapse Mechanism**



**Fig. 2: Moment curvature relationship**

### **Mechanism Method of Analysis**

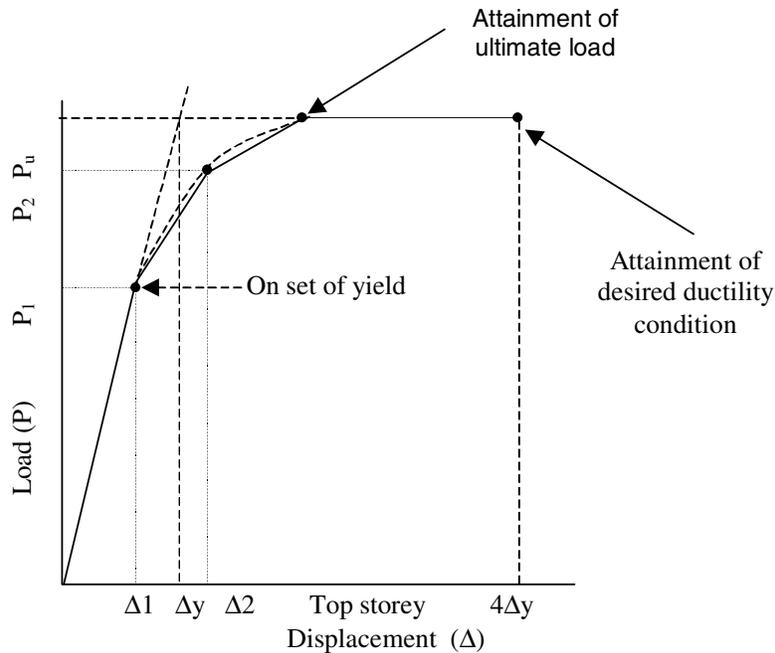
To analyze a structure by the mechanism method in order to obtain a kinematically admissible upper bound solution, the collapse mechanism is assumed first. At each location at which a plastic hinge is assumed, the moment must have a value equal to the known moment capacity of the member at that point  $M_p$ . When a hinge gets formed, the degree of static indeterminacy of the structure reduces by one. The hinge is assumed to rotate with a constant moment.

A plastic hinge is formed whenever a section has become completely plastified and therefore subject to free yielding. A plastic hinge indicates that the member is able to undergo a large amount of rotation or kink at the considered section under a constant moment. However, the mechanism method is an upper bound solution, which does not guarantee the attainment of the postulated mechanism or desired ductility.

### **Step-By-Step Load Increment Method**

Since the mechanism method is capable of giving the  $M_p$  value only, the step-by-step load increment method is used to trace the load displacement path at various load increment stages.

Therefore to obtain the collapse load, stepwise load increment computer analysis method is followed. The linear elastic STAAD analysis package has been used effectively to produce stepped non-linear curve by using assumed linear behaviour between the two adjacent load steps as shown in Figure 3.



**Fig. 3 : Load displacement curve**

### Procedure

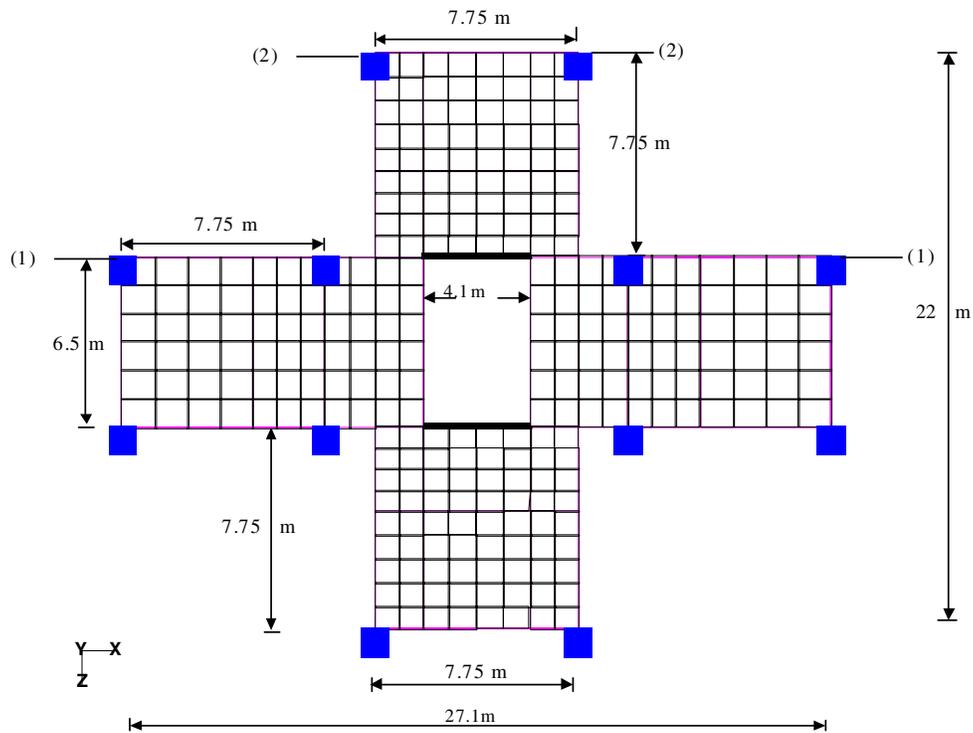
- Step 1: To get the elastic limit load using linear analysis.
- Step 2: Now the axis is assumed to be shifted to point  $P_1 \Delta_1$ , and from there  $P_1 \Delta_1 - P_2 \Delta_2$  is assumed to be linear. The increment in load of any arbitrary value ( $k_1$ ) is applied for seismic load in x direction to find out the load that makes the moment to reach the value  $M_p$  of the critical section. When this value is reached, it is assumed that the section is plastified and hence a hinge is formed in that member at the critical location where the moment is maximum.
- Step 3: Repeating the step 2 sequentially the members of the frames are made to form additional hinges till the collapse load (mechanism condition) is reached. This signifies attainment of ultimate load condition.
- Step 4: For the desired ductility the structure should deflect by  $4\Delta_y$ . The increase in displacement will occur as a mechanism. The rotation at the previously formed hinges can be found using manual method and mechanics approach. At this stage, computer cannot be used. At this mechanism stage, the rotation of the critical hinge is found and recorded. This rotation is the requirement for the critical hinge not to fracture before desired ductility is reached. Hence, this is termed as rotation demand ( $\phi_d$ ).
- Step 5: By providing suitable reinforcement detailing. The demand can be satisfied or supplied ( $\phi_s$ ). Therefore the requisite condition for ductility limit state is  $\phi_s > \phi_d$ .

The stepwise linear analysis can be made either with a plane frame or a space frame idealisation.

## EXAMPLE STRUCTURE AND ANALYSIS

### General

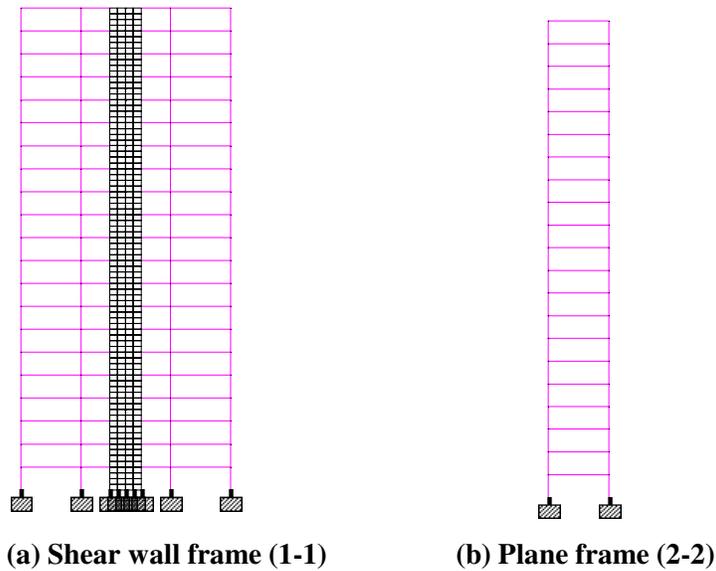
Figure 4 is an example symmetrical twenty-storey reinforced concrete building, which has similar floor plan at each storey up to the twenty storeys. The structure at each floor comprises of totally 2 shear wall frames (marked 1-1) and 2 plane frames (marked 2-2). The shear wall frame and plane frame are analyzed as two individual systems.



**Fig. 4: Structural plan of the building (G+20)**

### Structural Modelling

The structural modeling of the shear wall frame and plane frame of the twenty-storey building is as shown in Figure 5. The column and beam members are modeled as beam elements and the shear wall is modeled as plane stress elements. The dead and live loads on the slab and calculated and applied as member loads. The infills are also considered as member loads.



**Fig. 5: Structural modeling of shear wall frame and plane frame**

### Load cases considered

Following are the load cases and the load combinations considered for the analysis

#### Primary load case

1. Dead Load (DL)
2. Live Load (LL)
3. Seismic Load in X (SL<sub>x</sub>)

#### Load Combinations

4. 1.5 (DL+LL)
5. 1.2 (DL+LL+SL<sub>x</sub>)
6. 1.5 (DL+SL<sub>x</sub>)

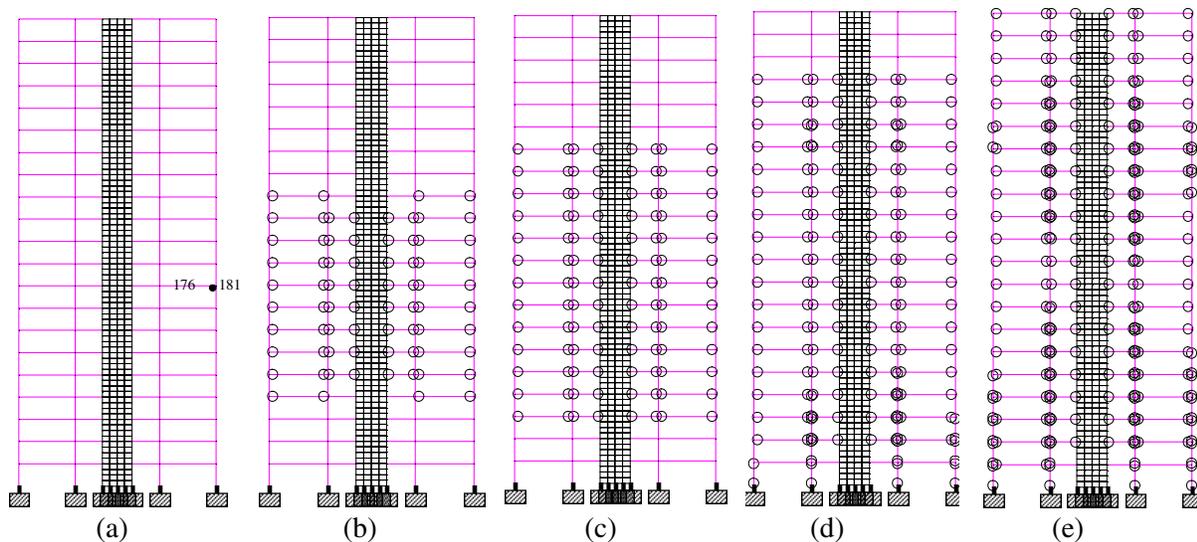
### Non-Linear Analysis Using Step-By-Step Load Increment Method

#### Shear wall frame

The procedure as explained earlier is carried out for the shear wall frame by increasing only the earthquake load (SL<sub>x</sub>). A hinge is introduced when the cumulative moment at the critical section of a member reaches the flexural strength of that member. It is seen that the first hinge is formed in the beam No. 176 at the node No.181 when the earthquake load reaches a load factor of 1.185 (Fig.6a).

The hinges formed at each load increment step and the corresponding condition of the frame at each load increment stage are shown in Figures 6a to 6d. The maximum load reached was with a load factor of 2.16. At this load, the frame-wall system had reached collapse mechanism (Figure 6e). This stage can be considered to be attainment of ultimate load.

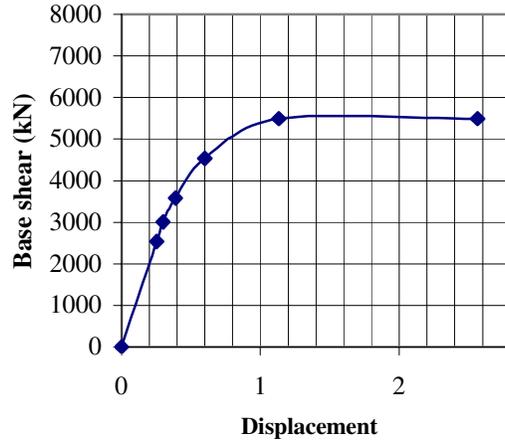
The base shear Vs. displacement values obtained at each load step is given in Figure 7. From Figure 7 it is seen obtained that the displacement at yield is 0.58m. The corresponding M-θ curves for the beam hinge are shown in Figures 8a to 8c.



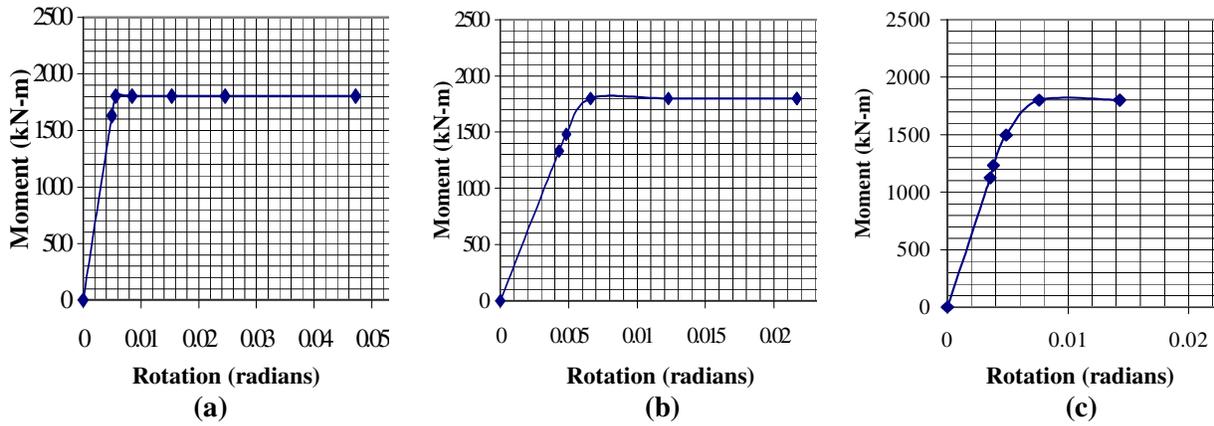
**Fig. 6 : Formation of Hinges in Shear Wall Frame**

Figure 9 shows a typical column hinge. The column hinge is formed at a moment of 4000 kN-m for attainment of ductility condition. The rotation demand ( $\theta_d$ ) of the column hinge is 0.004255 at mechanism stage and 0.005515 at 4Δ<sub>y</sub>.

Figure 17 shows the shear wall hinge. Note that shear wall hinging is assumed based on membrane stress reaching its yield value.

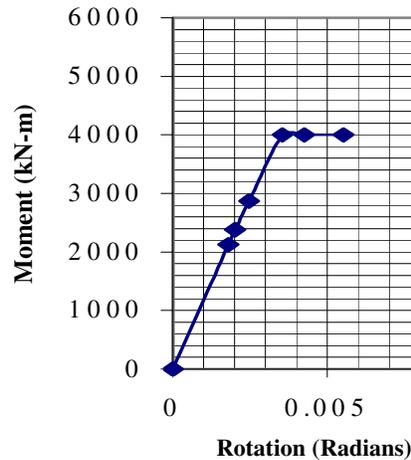


**Fig. 7: Base shear Vs displacement curve for shear wall frame**

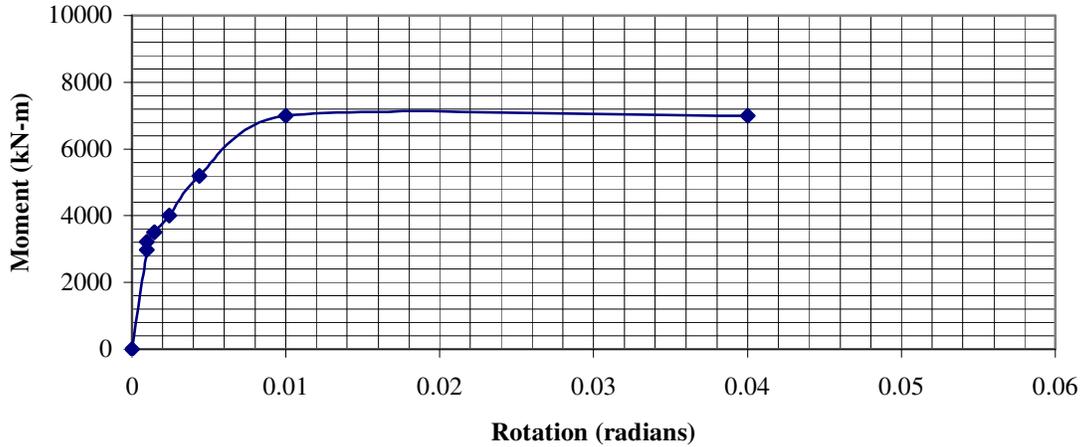


**Fig.8: M-θ curve for hinges formed in shear wall frame**

(a) first load step (Beam No. 176 Node No. 181),  
 (b) second load step (Beam No. 295 Node No. 302) and  
 (c) third load step (Beam No. 364 Node No. 371)



**Fig. 9: M - θ curve for first column hinge formed in shear wall frame at third load step (Column No.6 Node No.9)**

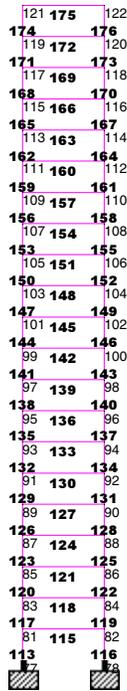


**Fig. 10: M -  $\theta$  curve for first shear wall hinge formed in shear wall frame at fourth load step**

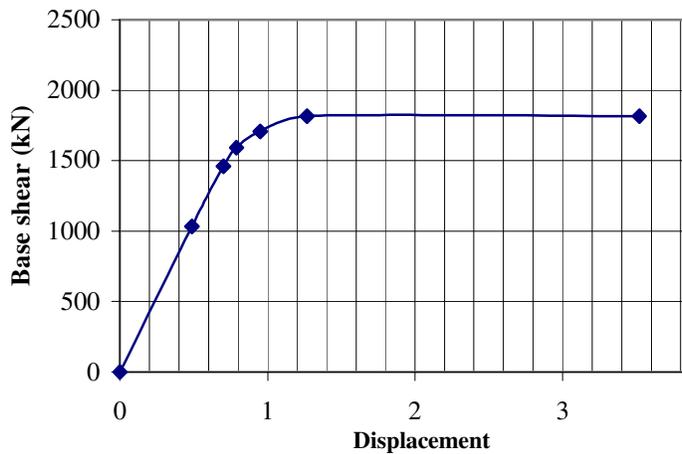
**Plane frame**

A similar analysis was made for the typical frame is shown in Fig.11. The base shear Vs. top storey displacement values obtained at each load step is shown in Figure 12. From Figure 12 the displacement at yield can be obtained and it is 0.88m ( $\Delta_y$ ).

M- $\theta$  values for the critical hinges formed at each load increment step is shown in Figure 24-28. Figure 24 shows the critical hinge formed in plane frame system.  $\theta_d$  of this hinge is 0.036963 radians. The  $\theta_d$  of all other hinges in this system is less than this value. The behaviour of column hinge is shown in Figure 28. Note the significantly large demand of the column hinge.



**Fig. 11: Member numbers and node numbers of the plane frame**



**Fig. 12: Base shear Vs displacement curve for plane frame**

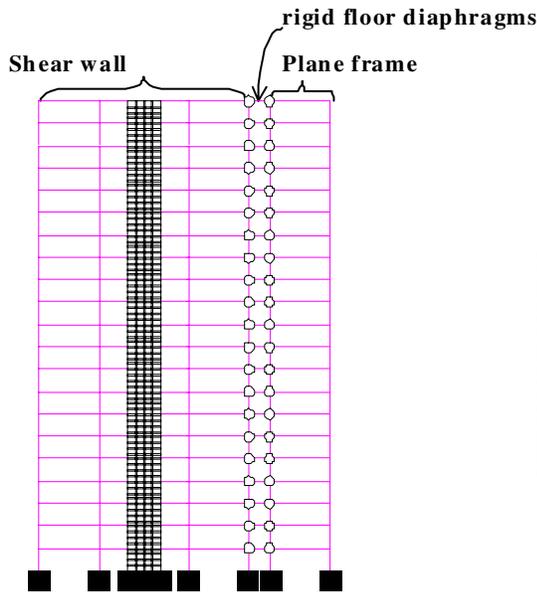
## PLASTIC DEFORMATION OF HINGES AND MEMBERS FOR THE SHEAR WALL FRAME-FRAME (COMPOSITE SYSTEM)

### Frame Details

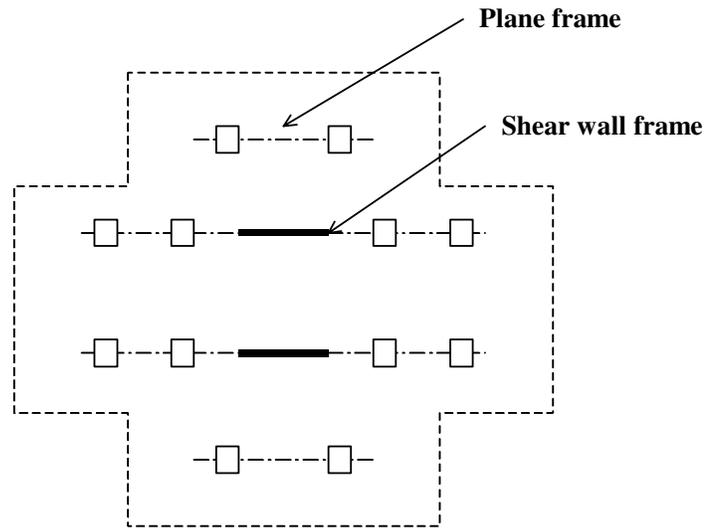
The shear wall frame and the plane frame are assumed to be connected by rigid links to simulate the floor diaphragm effect. The displacement compatibility at every floor is assumed to be assured by means of infinitely rigid floor diaphragms (shown in Figure 13).

### Structural Modelling

Figure 14 shows the diagrammatic representation of the structure taken for the study. It consists of totally two shear wall frames and two plane frames as indicated (by the arrow marks).



**Fig. 13: Combined frame**



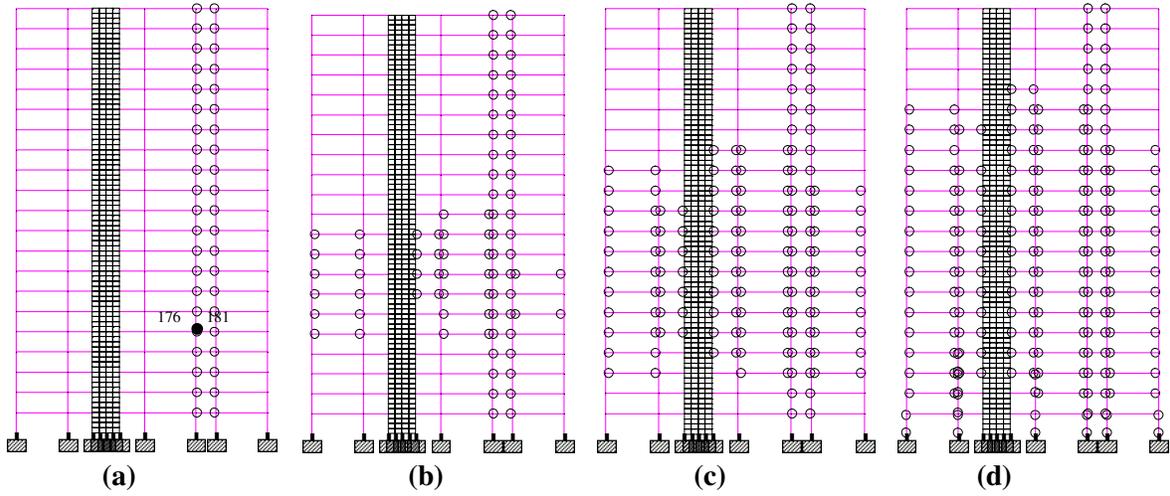
**Fig. 14: Diagrammatic representation of 20 storey building plan considered for combined frame action**

### Non-Linear Analysis Using Step-By-Step Load Increment Method

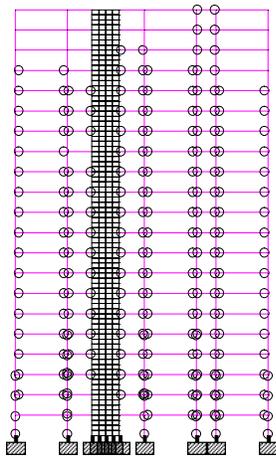
The procedure as explained earlier, is carried out for the combined frame by increasing only the earthquake load (SLx). A hinge is introduced when the cumulative moment of a member reaches the flexural strength. It is seen that the critical hinge is formed in the beam No. 176 at the node No.181 (Figure 15a) when the earthquake load reaches a load factor of 1.185. The stepwise load increment is carried out in 5 stages. At each stage it is assumed that a group of hinges are formed for a particular range of bending moment.

A group of hinges are introduced for various nodes where the bending moment reaches the flexural strength ( $M_p$ ) of the member. The hinges formed at each load increment step is shown in Figures 15b to 15e. At the third load step is first column hinge is formed when the earthquake load factor is 1.33. The shear wall hinge formed at fifth load step when the earthquake load reaches a load factor of 1.98. At this load, the frame-wall system had reached collapse mechanism (Figure 15f).

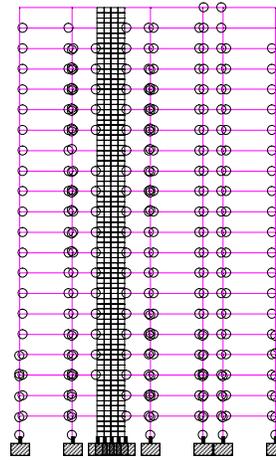
The base shear Vs. top storey displacement is shown in Figure 16. M- $\theta$  values for the critical hinges formed at each load increment step is shown in Figure 17a–17g. Figure 18 shows the comparison of base shear Vs displacement curve for the three system considered.



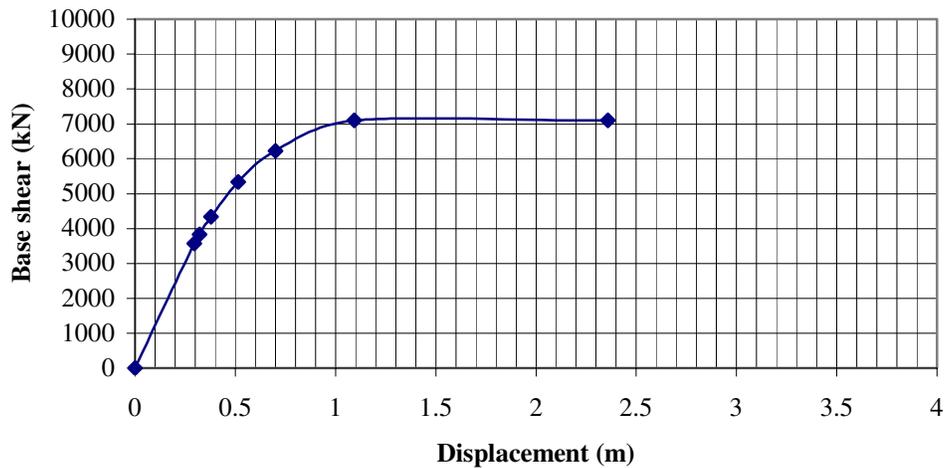
**Fig. 15: Critical hinge (a) and hinges in combined frame at different load steps (b,c and d)**



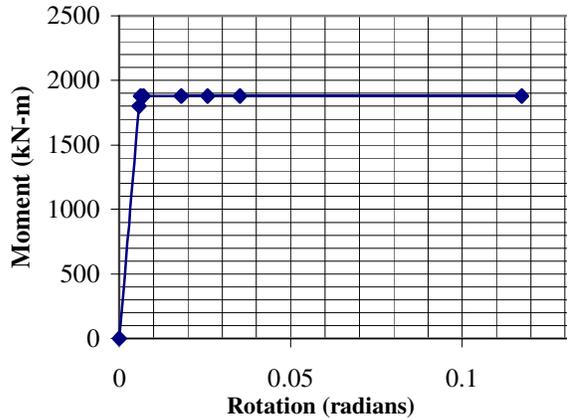
**Fig. 15e: Hinges in combined frame at fourth load step**



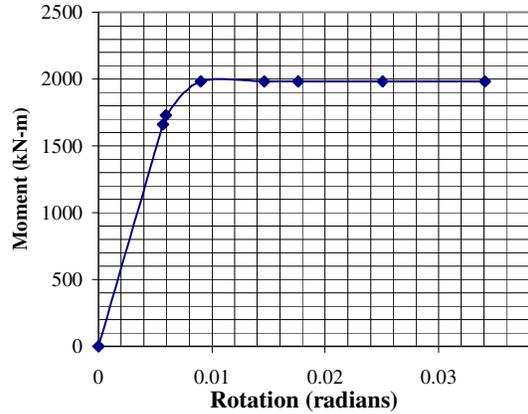
**Fig. 15f: Hinges at collapse in combined frame at fifth load step (mechanism stage)**



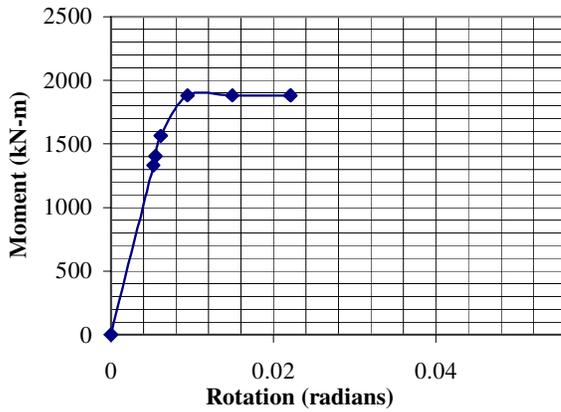
**Fig. 16: Base shear vs displacement curve for combined frame**



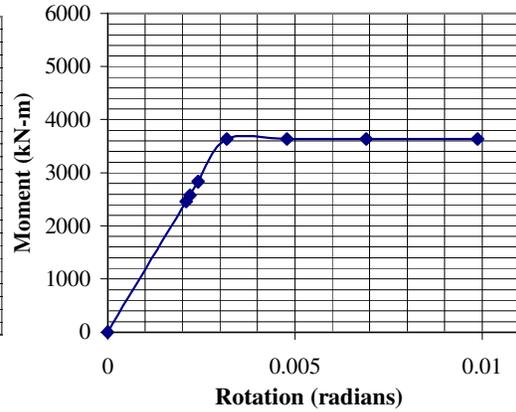
**Fig. 17a: M -  $\theta$  curve for first hinge formed in combined frame at first load step (Beam No. 176 Node No. 181)**



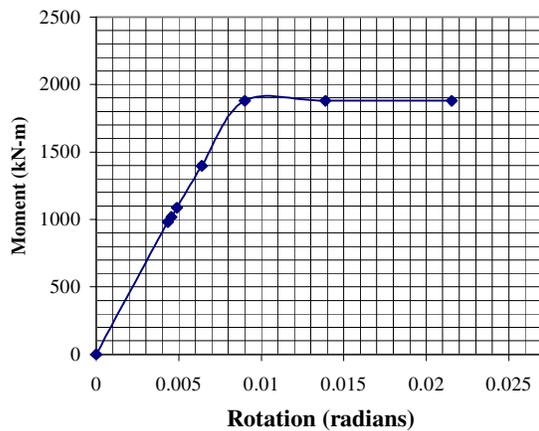
**Fig. 17b: M -  $\theta$  curve for first hinge formed in combined frame at second load step (Beam No. 524 Node No. 529)**



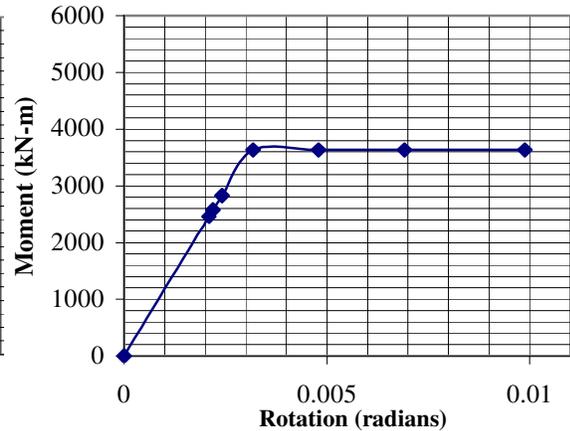
**Fig. 17c: M -  $\theta$  curve for first hinge formed in combined frame at third load step (Beam No. 267 Node No. 275)**



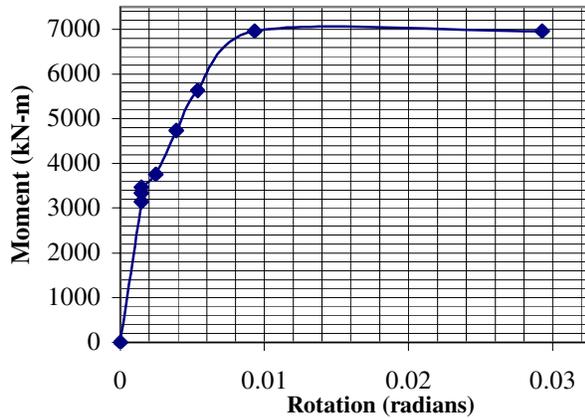
**Fig. 17d: M -  $\theta$  curve for first column hinge formed in combined frame at third load step (Column No. 6 Node No. 9)**



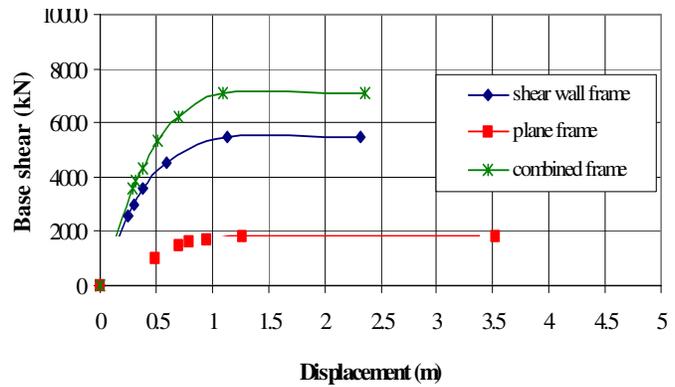
**Fig. 17e: M -  $\theta$  curve for first hinge formed in combined frame at fourth load step (Beam No. 551 Node No. 547)**



**Fig. 17f: M -  $\theta$  curve for first column hinge formed in combined frame (Beam No. 30 Node No.38)**



**Fig. 17f: M -  $\theta$  curve for hinge formed at shear wall in combined frame at fifth load step**



**Fig. 18: Comparison of base shear Vs displacement curve**

### Total Displacement

To obtain the collapse load of the total structure, the load in the Base shear Vs top storey displacement curve of the combined frame system has to be multiplied by 2 since the total building consists of two such combined systems [2,3,4].

## COMPARISON OF DUCTILITY REQUIREMENTS OF MEMBERS OF MULTISTOREY FRAMES

### General

To satisfy the demand it becomes necessary to check the allowable rotation of the critical hinges. The rotational ductility demand of the member at the attainment of mechanism condition and prescribed ductility condition is evaluated and checked [5].

### Allowable Rotation

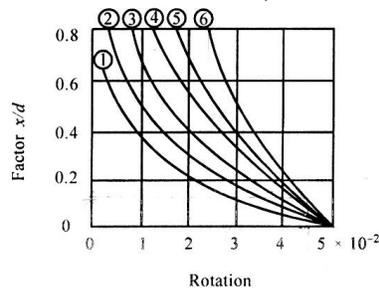
The allowable rotation at the plastic hinge depends on many factors and some of them are

1. Neutral axis depth factor at the section ( $x/d$  ratio)
2. Type of steel (yield strength and ultimate tensile strength ratio)
3. Reinforcement index ( $A_s f_y / b_d f_c$ )
4. Span depth ratio

The major factor considered is  $x/d$  ratio.

There are number of methods available for calculating the allowable rotational capacity ( $\theta_s$ ) of the plastic hinges at the beam or column members. The methods adopted are

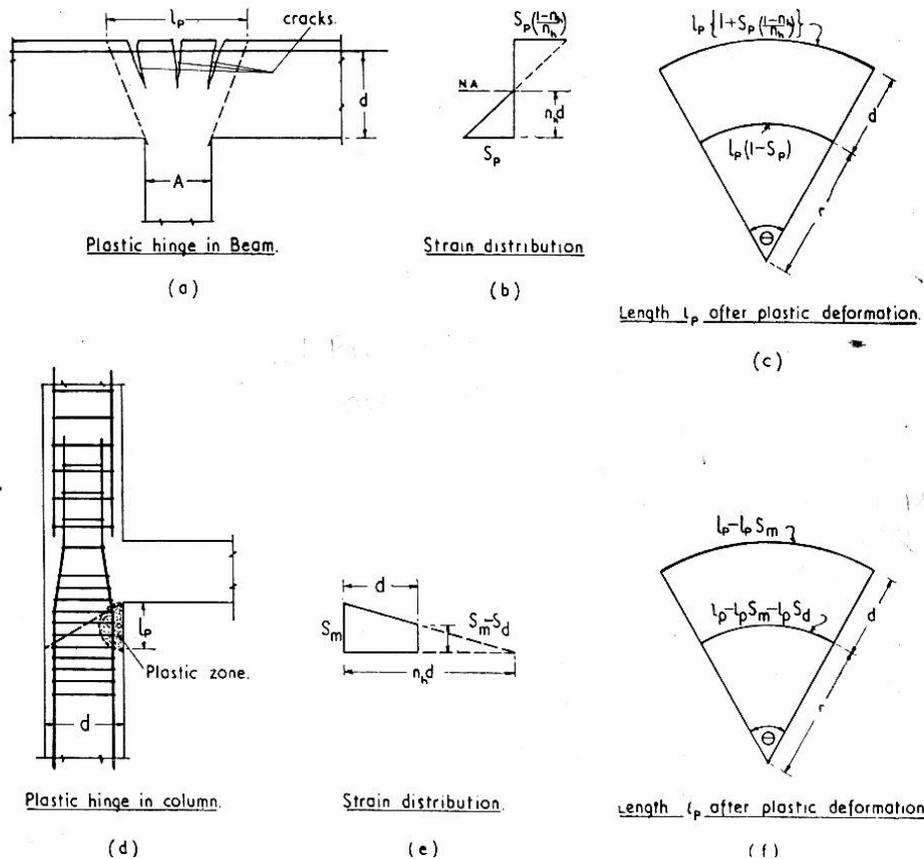
1. Using the model code CEB-FIP recommendations (shown in Fig.19)



**Fig. 19: Plastic rotation capacity of reinforced concrete members.**

2. A.L.L. Baker's method [6]

A.L.L Baker suggests that permissible rotation ( $\theta_s$ ) can be calculated for beam and column hinges using the following expression and details shown in Figure 20.



**Fig. 20: Details of plastic hinges**

Permissible rotation for

$$\text{Beam hinge} : \frac{S_p l_p}{n_h d} \qquad \text{Column hinge} : \frac{S_d l_p}{d}$$

where,

$S_p$  = the average strain of the concrete at the compressive edge which occurs under increasing load between the yielding of the steel and the crushing of concrete.

$S_d$  = difference of strain across the section at failure of the column due to plasticity.

$l_p$  = length of plastic hinge.

Normally the safe limiting values  $S_p$  or  $S_d$  is assumed not to exceed be 0.001 for unbound concrete and 0.01 for concrete with closely spaced binders.

$$\begin{aligned} l_p &= (d / 2) \quad \text{to } d, \text{ where} \\ n_h &= x/d \\ d &= \text{depth of the section} \\ x &= \text{depth of neutral axis} \end{aligned}$$

The beam – hinge calculations of permissible rotation should be based on the value of  $n_h$  after yielding of the steel but prior to crushing of the concrete.

### Application to the Present Problem

(i) *By CEB-FIP recommendations*

a. *Beam hinge*

In the case of shear wall frame the critical member that plastifies first is beam member 176 and the critical hinge has formed at node 181(Figure 6a). The rotational demand ( $\theta_d$ )at this node is 0.0246 radians at mechanism condition and 0.047235 radians at ductility condition. To meet these demands ( $\theta_d$ ) the allowable rotation ( $\theta_s$ ) has to be calculated and checked.

The  $x/d$  ratio of the beam member 181 is 0.466. Referring to Figure 47 the allowable rotation is 0.005 radians for no extra binders which is not satisfying the demand ( $\theta_d$ ) either at mechanism or at ductility condition. Therefore to satisfy these demand extra binders are required. From the curves it is seen that the rotation achievable is 0.03 radians for an extra binder ratio of 1.5%. This satisfies the mechanism condition. For the ductility condition even higher percentage of extra binders are required. The curves given by CEB-FIP model code [7,8] is not adequate to determine the percentage binders.

b. *Column hinge*

There is no explicit recommendation given for column hinge by CEB-FIP recommendation.

(ii) *By A.L.L Baker's method*

a. *Beam hinge*

$$\theta_s = \frac{S_p l_p}{n_h d}$$

$$S_p = 0.0035 \text{ (without extra binders)}$$

$$l_p = 0.5d$$

$$n_h = x/d = 0.4667$$

$$\therefore (\theta_s) = 0.0037 \text{ radians}$$

If we increase the  $S_p$  value by increasing the binder percentage ( $\theta_s$ ) can be made to the desired value to meet the demand ( $\theta_d$ ) of 0.0246 or 0.047235 radians.

b. *Column hinge*

The rotational demand of the column hinge is 0.004255 radians at mechanism condition and 0.005515 radians at ductility condition.

$$(\theta_s) = \frac{S_d l_p}{d}$$

$$S_d = 0.01 \text{ (for closely spaced binders)}$$

$$l_p = 0.5d$$

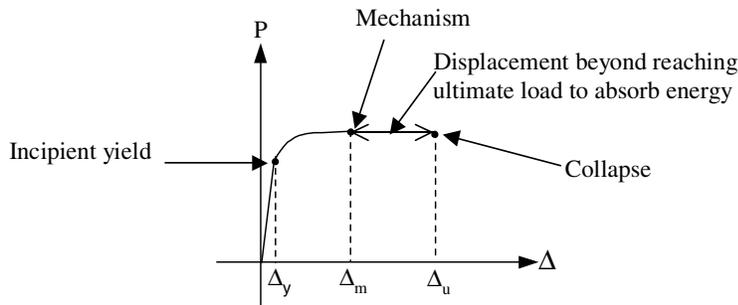
$$\therefore (\theta_s) = 0.005 \text{ radians}$$

This satisfies the demand at mechanism condition and need to increase the  $S_d$  value to fully satisfies the rotation at ductility condition. The lateral binding ratio required to increase the  $S_d$  value can be obtained from the curves available from the tests conducted by A.L.L. Baker. These curves are not explicit but require further experimental data.

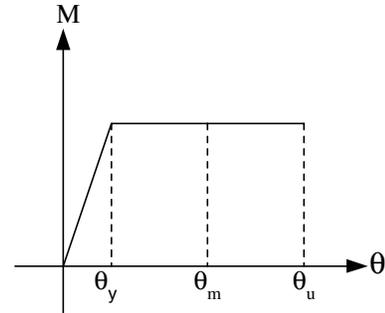
### Ductility Requirements

The ductility requirements of the multistorey building are calculated based on a specified displacement of the top storey. The ability of plastic hinges to rotate and supply this ductility requirement will ensure attainment of desired energy absorption during earthquake.

- a. Displacement ductility factor: The displacement ductility factor is the ratio of the maximum displacement to the yield displacement. The displacement ductility can also be called as structure ductility. Figure 21 shows the typical load – displacement curve for the system indicating the displacement at yield stage ( $\Delta_y$ ) mechanism stage ( $\Delta_m$ ) at incipient ultimate load and specified ductility stage ( $\Delta_u$ ) just before collapse.
- b. Rotational ductility factor : The rotational ductility factor is defined as the ratio of maximum rotation at plastic hinge to the rotation there at yield. The rotation ductility can also be termed as member ductility Figure 22 shows the typical moment - rotation curve for the member indicating the rotation at yield stage ( $\theta_y$ ) mechanism stage ( $\theta_m$ ) and at incipient collapse stage ( $\theta_u$ ).



**Fig. 21 : Shows typical load - displacement curve for a system**



**Fig. 22: Shows typical moment – rotation curve for a system**

*To find the displacement ductility limit at mechanism condition*

In the case of shear wall frame the displacement at the top storey when yield is reached at node 181 is 0.58 m (Figure 12). Therefore the displacement ductility demand for the mechanism to form is  $\Delta_m = 1.132$  m.

The ductility required at this stage of formation of mechanism is  $\mu_m = 1.132/0.58 = 1.95$ . Similarly the displacement ductility of the three systems at the incipient formation mechanism are given in Table 1.

**Table 1: Displacement ductility factor of the system at mechanism condition**

Sl.No.	Systems considered	Displacement ductility factor
1.	Shear wall frame	1.95
2.	Plane frame	1.44
3.	Combined frame	1.826

*To find the rotational ductility at mechanism condition*

In the case of shear wall frame the rotation  $\theta$  at yield of the beam node 181 is 0.005 radians and the rotation of the node when ultimate capacity of the system reached is 0.0246 radians.

Therefore rotational ductility  $\mu_{\theta_m} = 0.0246/0.005 = 4.92$ . Similarly, the rotational ductility for the other members of the systems considered are shown in Table 2.

## CONCLUSION

The proposed analysis [9] gives a realistic method of estimating the ductility demand of the components of the mixed systems.

The combined system analysis gives lesser ductility demand compared to individual system analysis for the beams whereas the ductility demand for the column is more. Hence the columns are more vulnerable than that predicted by individual system analysis.

**Table 2: Rotational ductility of factor of the system at mechanism condition**

Sl.No.	Systems considered	System component	Rotational ductility factor
1.	Shear wall frame	Beam (critical)	4.92
		Column (critical)	2.35
		Shear wall	4.5
2.	Plane frame	Beam (critical)	3.38
		Column (critical)	2.51
3.	Combined system containing both plane frame and shear wall frame	Beam (critical)	4.49
		Column (critical)	3.3
		Shear wall	3.67

Note that the combined analysis gives more realistic results.

## REFERENCES

- 1 Kurt H.Gerstle (1967), 'Basic Structural Design', McGraw Hill Book Company, New York.
- 2 Tom Paulay (2002) 'The displacement capacity of reinforced concrete coupled walls', Earthquake Engineering Research Institute, New Zealand.
- 3 Tom Paulay (2002), 'A displacement focused seismic design of mixed building systems', Earthquake Engineering Research Institute, New Zealand.
- 4 Park R and Paulay T. (1975), 'Reinforced concrete structures', Wiley-Inter Science Publications, John Wiley and Sons, New York.
- 5 Ramakrishnan and Arthur, (1984), 'Ultimate strength design for structural concrete', Wheeler Books, Fourth Revision.
- 6 Baker A.L.L. (1956), 'The ultimate load they applied to the design of reinforced and prestressed concrete frame', Concrete Publications Limited, London.
- 7 Varghese P.C. (1996), 'Limit State Design of Reinforced Concrete', Prentice-Hall of India Limited, New Delhi.
- 8 Varghese P.C. (2001), 'Advanced Reinforced Concrete Design', Prentice-Hall of India Limited, New Delhi.
- 9 Umamageshwari M. (2003), 'Structural behaviour of lateral load resisting systems, a thesis submitted to Anna University, Chennai, for the Degree of Master of Science (By Research) in Civil Engineering.