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RANDOM OPTIMUM DESIGN FOR FPS-ISOLATED RIGID STRUCTURES

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Abstract: The random optimum problem of the FPS-isolated rigid structure, under Gaussian white noise ground motions, is studied. Based on the analytical solution of the joint probability density function of the steady-state responses of the system, the random optimum mathematical model of the system is configured; the optimization design parameters are calculated.

Key words: friction pendulum system; random vibration; optimum design

1 Introduction

Friction Pendulum System (FPS) is an innovative sliding isolation system consisting of an articulated slider on a concave spherical surface, seeing References [1]~[12] for its configuration and related research progresses in detail. Based on its dynamical equations, the authors of the paper studied non-linear and random vibration performances of the FPS system. The random optimization problems of FPS, under horizontal Gaussian white noise ground actions, will be probed in the paper.

The stochastic differential equations of FPS-isolated rigid structures subjected to horizontal earthquake actions may be written as[11][12]

$$\ddot{X}+\mu\,\mathrm{sgn}(\dot{X})(\dot{X}^2+\frac{g}{r}\cos X)+\frac{g}{r}\sin X=\frac{\ddot{U}_g}{r}(\mu\,\mathrm{sgn}(\dot{X})\sin X-\cos X) \quad (1)$$

Where μ is Coulomb sliding friction coefficient of FPS bearings; g is the acceleration of gravity; X and $\dot{X}$ represent the angle of rotation and angular velocity of slider relative to the concave spherical surface of FPS, respectively; r is the radius of the slider and the concave spherical surface; $\ddot{U}_g$ stands for a zero-mean Gaussian white noise process with the intensity $2\pi S_0$ in the horizontal direction, the constant S_0 is the power spectrum density of $\ddot{U}_g$; $\mathrm{sgn}(\dot{X})$ denotes the Signum function.

For the cases of tiny-amplitude vibration, Equation (1) can be simplified into

$$\ddot{X}+\mu\,\mathrm{sgn}(\dot{X})(\dot{X}^2+\omega^2)+\omega^2 X=-\frac{\ddot{U}_g}{r} \quad (2)$$

where

$$\omega=\sqrt{\frac{g}{r}} \quad (3)$$

2. Linear forecasting for probability density of steady-state responses of the system

Let Equation (2) be replaced by an equivalent system given by

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$$\ddot{X} + \alpha_e \dot{X} + \omega^2 X = -\frac{\ddot{U}_g}{r} \quad (4)$$

where α_e is the equivalent linear parameter. The joint probability density function of the steady-state responses of the system (4) can be expressed as[13]

$$p(x,\dot{x}) = \frac{\alpha_e \omega}{2\pi^2 S_r} \exp\left[-\frac{\alpha_e}{\pi S_r} \left(\frac{\omega^2 x^2}{2} + \frac{\dot{x}^2}{2} \right) \right] \quad (5)$$

where

$$S_r = \frac{S_0}{r^2} \quad (6)$$

Equation (5) shows that the angle of rotation X and angular velocity $\dot{X}$ in the steady state have both the symmetric distribution with zero-mean and they are mutually independent. This leads to

$$E[\dot{X}\ddot{X}] = E[\dot{X}X] = 0 \quad (7)$$

where $E[\cdot]$ is the operator of the expectation.

Multiplying Equation (3) by $\dot{X}$ and using Equation (7), the expected equation of Equation (3) is

$$\mu\, E[\dot{X}(\dot{X}^2 + \omega^2)\,\mathrm{sgn}(\dot{X})] = -\frac{1}{r} E[\dot{X}\ddot{U}_g] \quad (8)$$

and from the equivalent system (4) , one can obtain similarly

$$\alpha_e E[\dot{X}^2] = -\frac{1}{r} E[\dot{X}\ddot{U}_g] \quad (9)$$

Comparing Equation (8) with (9), the parameter α_e becomes

$$\alpha_e = \frac{\mu\, E[\dot{X}(\dot{X}^2 + \omega^2)\,\mathrm{sgn}(\dot{X})]}{E[\dot{X}^2]} \quad (10)$$

Owing to

$$E[\dot{X}(\dot{X}^2 + \omega^2)\,\mathrm{sgn}(\dot{X})] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (\dot{x}^3 + \omega^2 \dot{x})\,\mathrm{sgn}(\dot{x})\, p(x,\dot{x})\,\mathrm{d}x\,\mathrm{d}\dot{x} = \sqrt{\frac{2S_r}{\alpha_e}}\left(\frac{2\pi S_r}{\alpha_e} + \omega^2\right) \quad (11)$$

$$E[\dot{X}^2] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \dot{x} p(x,\dot{x})\,\mathrm{d}x\,\mathrm{d}\dot{x} = \frac{\pi S_r}{\alpha_e} \quad (12)$$

and substituting Equations (11) and (12) into Equation (10), one has

$$\alpha_e = \left\{ \sqrt[3]{\frac{q_1}{2} + \sqrt{\left(\frac{q_1}{2}\right)^2 + \left(\frac{q_2}{3}\right)^3}} + \sqrt[3]{\frac{q_1}{2} - \sqrt{\left(\frac{q_1}{2}\right)^2 + \left(\frac{q_2}{3}\right)^3}} + \frac{\mu\omega^2}{3\pi\sqrt{2S_r}} \right\}^2 \quad (13)$$

where

$$\begin{cases} q_1 = \dfrac{2}{27}\left(\dfrac{\mu\omega^2}{\pi\sqrt{2S_r}}\right)^3 + \mu\sqrt{2S_r} \\ q_2 = -\dfrac{\mu^2\omega^4}{6\pi^2 S_r} \end{cases} \quad (14)$$

In Reference [12], the same results as above-mentioned were obtained in the light of the condition that the mean-square value of the difference between Equations (2) and (4) is minimized, and further verified as effective according to Monte-Carlo simulations.

3. Sliding reliability function of the system

The threshold value of angle of rotation X may be defined as

$$\varphi_b = \frac{b}{r} \tag{15}$$

where $b > 0$ represents the threshold value of the horizontal displacement of the FPS slider. In accordance with the bi-barrier problem of Poisson model of the first-passage failure, the sliding reliability function of the slider is given as follows

$$P_s(-\varphi_b, \varphi_b) = P_s(-\frac{b}{r}, \frac{b}{r}) = \exp(-2v_b^+ T) = \exp\left[-\frac{\omega T}{\pi}\exp\left(-\frac{\alpha_e \omega^2}{2\pi S_0} b^2\right)\right] \tag{16}$$

where

$$v_b^+ = \int_0^\infty \dot{x}\, p(\varphi_b, \dot{x}) \mathrm{d}\dot{x} = \frac{\omega}{2\pi}\exp\left(-\frac{\alpha_e \omega^2}{2\pi S_r}\varphi_b^2\right) = \frac{\omega}{2\pi}\exp\left(-\frac{\alpha_e \omega^2}{2\pi S_0} b^2\right)$$

is the expected ratio of the steady-state angle of rotation $X(t)$ sloping the positive barrier $\varphi_b = b/r$; T is the stationary duration of earthquake.

4. Random optimization model and numerical solutions of the system

The sliding friction coefficient μ and the radius of the slider r are chosen as the design variables so as to minimize the root-mean-square (RMS) of horizontal absolute acceleration of the slider, according to

$$\mathrm{RMS} = \sqrt{E[(r\ddot{X} + \ddot{U}_g)^2]} \tag{17}$$

Equation (4) leads to

$$E[(r\ddot{X} + \ddot{U}_g)^2] = E[(-r\alpha_e \dot{X} - r\omega^2 X)^2] = r^2\alpha_e^2 E[\dot{X}^2] + r^2\omega^4 E[X^2] + 2r^2\alpha_e\omega^2 E[X\dot{X}] \tag{18}$$

Substituting Equations (7), (12) and

$$E[X^2] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x^2 p(x, \dot{x}) \mathrm{d}x \mathrm{d}\dot{x} = \frac{\pi S_r}{\alpha_e \omega^2} \tag{19}$$

into Equation (18), gives

$$E[(r\ddot{X} + \ddot{U}_g)^2] = \pi S_0 (\alpha_e + \frac{\omega^2}{\alpha_e})$$

such that

$$\mathrm{RMS} = \sqrt{\pi S_0 (\alpha_e + \frac{\omega^2}{\alpha_e})} \tag{20}$$

The constrains in the optimization can be represented as

① $P_s(-\varphi_b, \varphi_b) \ge \overline{P}_s$. Substituting Equation (16) into this equation, gets

$$\overline{P}_s - \exp\left[-\frac{\omega T}{\pi}\exp\left(-\frac{\alpha_e \omega^2}{2\pi S_0}b^2\right)\right] \le 0 \qquad (21)$$

where $\overline{P}_s$ is the inferior limit of $P_s(-\varphi_b, \varphi_b)$.

② $\sqrt{E[X^2]} \le 0.3\varphi_b$. On the basis of Equations (15) and (19), yields

$$\sqrt{\frac{\pi S_0}{\alpha_e \omega^2}} - 0.3b \le 0 \qquad (22)$$

③ $0 < \mu^l \le \mu \le \mu^u$. μ^l and μ^u are the inferior and superior limit of the friction coefficient μ, respectively.

④ $0 < r^l \le r \le r^u$. r^l and r^u are the inferior and superior limit of the radius of the slide r, respectively.

Some calculated results are represented in Table 1 in line with

$T = 25\,\text{s}$； $\overline{P}_s = 0.99$； $\mu^l = 0.01,\ \mu^u = 0.4$； $r^l = 50\,\text{cm},\ r^u = 600\,\text{cm}$

Table 1 Optimum values of RMS and design variables

		S_0 (cm^2/s^3)			
		100	200	300	400
$b = 10\,\text{cm}$	RMS(cm/s^2)	46.6681	74.4553	98.7367	124.4887
	μ^*	0.0978	0.1556	0.2186	0.2915
	r^* (cm)	92.9074	57.3389	50.0000	50.0000
$b = 15\,\text{cm}$	RMS(cm/s^2)	40.5201	64.6592	84.9807	103.1616
	μ^*	0.0849	0.1353	0.1774	0.2148
	r^* (cm)	163.5485	100.8542	76.0369	62.2367
$b = 20\,\text{cm}$	RMS(cm/s^2)	36.6504	58.4928	76.8826	93.3361
	μ^*	0.0768	0.1224	0.1607	0.1947
	r^* (cm)	244.4349	150.6432	113.5346	92.9073
$b = 25\,\text{cm}$	RMS(cm/s^2)	33.9022	54.1132	71.1308	86.3573
	μ^*	0.0711	0.1133	0.1487	0.1803
	r^* (cm)	333.9578	205.7124	154.9959	126.8113
$b = 30\,\text{cm}$	RMS(cm/s^2)	31.8085	50.7766	66.7486	81.0402
	μ^*	0.0667	0.1064	0.1396	0.1693
	r^* (cm)	431.0586	265.4104	199.9274	163.5495

Table 1 shows that the optimum value of the absolute horizontal acceleration of the slider decreases with the additional width b of the base increasing and increases with the power spectrum intensity S_0 increasing; for smaller additional width b, larger friction coefficient and smaller radius of a FPS bearing should be selected to synthetically control the sliding displacement of the system, especially under strong earthquake actions.

5. Concluding Remarks

The random optimum problem of FPS-isolated rigid structures, under Gaussian white noise ground motions, is studied in the paper. Based on the analytical solutions of the joint probability density function of the steady-state responses and the sliding reliability function of the system, the random optimum mathematical model of the system is configured by selecting the root-mean-square of the horizontal absolute acceleration of FPS slider as the objective function and the optimization design parameters are calculated. The numerical simulation results can provide useful suggestions for the practical design of FPS systems.

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