13th World Conference on Earthquake Engineering
Vancouver, B.C., Canada
August 1-6, 2004
Paper No. 2483

STOCHASTIC EVALUATION OF AVAILABILITY FOR SUBSYSTEMS BY MARKOV AND SEMI-MARKOV MODELS

Adrian VULPE [1] , Alexandru CARAUSU [2]

SUMMARY

An approach to the availability evaluation of repairable subsystems and equipment in NPPs, presented in an earlier authors' paper, is further developed. The unavailability state of a subsystem is split into three to five "smaller" ones. The loss-of-availability process can be modeled by means of a semi-Markov process and a Markov renewal process, which generalize Markov jump processes. The criterion to identify the state of the subsystem is expressed in terms of a vector parameter whose components consist of a set of selected relevant parameters, with a specific interval / threshold for each of them.

1. INTRODUCTION

The notion of availability of a system or subsystem admits various interpretations and definitions, depending on the nature of the system in discussion, on the types of situations when it may become unavailable / unfunctional, on the possibilities to repair a system entering a *down* state or to replace its damaged components. Operations of periodic inspections and maintenance must also be taken into account. The concept of availability should not be identified with that of reliability. Yet, when the availability of a system / subsystem / component is analysed some general notions from the theory of reliability and life testing, some of them from the classical reference [1], cannot be avoided but have to be possibly adapted. The most appropriate class of mathematical models to describe and give a ground for the analysis of behavior of systems from the point of view of availability consists of stochastic (or random) processes in time. Since, in many cases, the next state of a system or subsystem depends on its state at a given moment of time only, the Markov-type processes have been accepted as suitable models, e.g. in [2], [3]. We have also approached the problem of availability evaluation of repairable subsystems in NPPs (nuclear power plants) in our paper [4] – a contribution to SMiRT 15 Conference. Here we go further with developing the models presented in that paper. We try to make use of some recent concepts and results presented in several contributions to reference [5], and also to apply the interval reliability method [6] as a tool for identifying the state of some components subjected to a deterioration process.

[1] Department of Structural Mechanics, Technical University of Iasi, Romania, Email: iulianl@go.ro
[2] Department of Mathematics, Technical University of Iasi, Romania, Email: alex.carausu@iasi.astral.ro

The amount / level of deterioration at a moment of time can be represented by a gamma stochastic process characterized by its density function or by its mean and variance. A set of time dependent limit state functions is formulated for a proper definition of change of state for the subsystem. Another section of this paper presents some possibilities to adapt Markov renewal matrices and semi-Markov transition matrices to our models, and gives corresponding methods to estimate a reliability function. Finally, some of the models developed in the previous sections appear to be compatible with the interval availability concept, which allows for the evaluation of accident probabilities in power plant structures and equipment, but also in more general structures. They are adapted to availability / reliability problems in this field, also involving events of type (*k*-out-of-*n*).

2. THE STATE SPACE AND AVAILABILITY OF SUBSYSTEMS

A random process in time may be written as $X(t)$ where X is a random function and the time variable t takes values in an interval of the form $[0,T]$; $T=\infty$ is not excluded. T can be, for instance, the designed lifetime of the system or the operational cycle length of an equipment. For every fixed t, $X(t)$ takes values in a specified set of states, sometimes called the *state space* of the process and denoted by E. This state space for Markov-type processes is usually considered to be finite. Therefore is we may write it as the finite set $\mathsf{E}=\{1, 2,\ldots, s\}$. The number of assumed states depends on the desired complexity of the model. In the simplest case, two states are considered : the *up* state **u** and the *down* state **d**. Conventionally, these states may be represented by 1 and 0, respectively. Hence, $X(t)=1$ if the system is operating at moment t, and $X(t)=0$ otherwise. This model seems to be too simple for describing the state of a subsystem or component in a power plant, for instance. We just proposed, in [4], a partition of E in two subsets of *up* / *down* states :

$$\mathsf{E}=(e_1, e_2, \ldots, e_s)=U\cup D,\ U\cap D=\varnothing. \tag{1}$$

The number of states and their nature depends not only on the desired accuracy of the analysis but rather on the nature of the system itself. Thus, the subsystem / component may not simply be in an *up* / *down* state but also in a state of periodic inspection or maintenance, or also in a state of failure or repair. We are going to give more details in the next section.

Another important issue concerns the ways to identify the state of the (sub)system at a certain time moment $t\in[0,T)$. We shall discuss this point in a final section of our paper. Here we only mention the possibility to consider a vector of state parameters

$$\Theta=(\theta_1,\theta_2,\ldots,\theta_m)\in I=\prod_{\ell=1}^{m} I_\ell \subset \mathbf{R}^m. \tag{2}$$

Certainly, not all the components of Θ should be relevant for checking the state of the subsystem ; if – for instance – θ_k is not a relevant parameter for checking whether $X(t)=i\in\mathsf{E}$ or, more precisely, whether i is contained into one of the subsets taken into account (like U or D), then the corresponding interval in Eq. (2) may be taken as $I_k=\mathbf{R}$. Another remark concerns the natural possibility that all (or a part of) the parameter components of Θ are time dependent. For instance, if the system is in a state of maintenance at t and the scheduled maintenance duration is M then the system will be outside this state at a subsequent moment $t+x$ if $x>M$.

The Markov renewal processes and semi-Markov processes are briefly presented in the next section, as being better suited probabilistic models for describing the behavior in time of subsystems as those just suggested. The mathematical theory of Markov-type processes has developed very extensively but we recall the essential definitions, only.

3. SEMI-MARKOV PROCESSES, KERNEL AND TRANSITION MATRICES

A *Markov renewal process* (MRP), defined on a complete probability space, is introduced in [7] as the pair

$$(J, S) = (J_n, S_n)_{n \geq 0},$$

where $(J_n)_{n \geq 0}$ is a Markov chain taking values in a finite set $\mathsf{E} = \{1, 2, \ldots, s\}$ = the state space of the process, and $(S_n)_{n \geq 0}$, $S_n \in \overline{\mathbf{R}}_+ = [0, +\infty]$ is a sequence of jump times, while $J_1, J_2, \ldots, J_n, \ldots$ are the consecutive states to be visited by the MRP. The random variables $X_1, X_2, \ldots, X_n, \ldots$, defined by $X_n = S_n - S_{n-1}$ for $n \geq 1$, are the *sojourn times* in these states taking values in $\overline{\mathbf{R}}_+$. A MRP can be completely determined by its *initial law* and its *semi-Markov kernel*, respectively defined by $\mathsf{P}(J_0 = k) = p(k)$ and

$$\mathsf{P}(J_{n+1} = k, X_{n+1} \leq x \mid J_0, J_1, \ldots, J_n) = Q_{J_n k}(x) \ \text{a.s.}, \tag{3}$$

for all $x \in \overline{\mathbf{R}}_+$ and $1 \leq k \leq s$; a.s. means almost surely. The probabilities

$$p_{ij} = Q_{ij}(\infty) = \lim_{t \to \infty} Q_{ij}(t) \tag{4}$$

are the transition probabilities of the Markov chain $(J_n)_{n \geq 0}$.

A much simpler alternative to the semi-Markov processes are the failure processes described in [8] (Ch. 16 in reference [5]). Two states only are considered : the failure state and the operating state. The times for repairs are ignored, and the times when a failure state occurs are also taken as points $T_1, T_2, \ldots$ on the time axis. Another restriction consists in the hypothesis that the time intervals between two successive failures, that is

$$X_1 = T_1, X_2 = T_2 - T_1, \ldots, X_n = T_n - T_{n-1}, \ldots \tag{5}$$

are independent and identically distributed (abbreviated i.i.d.). A numerical characteristic of such processes is $N(t)$ = the number of failures in interval $[0, t]$. The failure rate of the failure process is defined by

$$r(t) = \lim_{h \to 0} \frac{P(t \leq X < t + h \mid X > t)}{h} = \frac{f(t)}{\overline{F}(t)} = \frac{d}{dt}\left[-\ln \overline{F}(t)\right], \tag{6}$$

where $\overline{F}(t) = 1 - F(t) = P(X > t) = \exp[-\int_0^t r(x)\,dx]$. This later “complementary” *cdf* equals the probability that a continuous operating cycle (without any failure) is longer than t. The conditional probability $\mathsf{P}(X - t \leq y \mid X > t) = [F(t+y) - F(t)] / \overline{F}(t)$ is the residual cycle length ; in [8], this characteristic is called the residual lifetime. We have recalled these notions connected with the failure processes in order to see how some of them can be extended to corresponding characteristics of semi-Markov systems with several states.

Coming back to the MRPs, let us suppose that, for all i and j, $Q_{ij}(\cdot)$ is absolutely continuous with respect to the Lebesgue measure and $q_{ij}(\cdot)$ is the corresponding density. Then, for any two indices i and j $(1 \le i, j \le k)$, the instantaneous transition rate of the semi-Markov kernel is given by

$$\lambda_{ij}(t) = \begin{cases} \dfrac{dQ_{ij}(t)/dt}{1 - \sum_{i \ne j} Q_{ij}(t)} & \text{if } p_{ij} > 0 \text{ and } \sum_{i \ne j} Q_{ij}(t) < 1 \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

The cumulative hazard rate from state i to state j is defined by

$$\Lambda_{ij}(t) = \int_0^t \lambda_{ij}(u)\, du \tag{8}$$

and the total cumulative hazard rate of state i at time t by $\Lambda_i(t) = \sum_{i-1}^{s} \lambda_{ij}(t)$. For $i, j \in \mathsf{E}$ and $t \in [0, \infty)$ the probabilities involved in Eqs. (4) and (5) are given by

$$Q_{ij}(t) = \int_0^t \exp(-\Lambda_i(u))\, \lambda_{ij}(u)\, du. \tag{9}$$

For a uniform division $(v_k)_{0 \le k \le m}$ of interval $[0, T]$, with step $\Delta_m = T/m$ and m a positive integer, the transition rate $\lambda_{ij}(t)$ can be approximated by

$$\lambda^*_{ij}(t) = \sum_{k=0}^{m-1} \lambda_{ijk} \mathbf{1}_{[v_k, v_{k+1})}(t). \tag{10}$$

The maximum likelihood values of the transition rates λ_{ijk} are given by

$$\hat{\lambda}_{ijk} = \begin{cases} d_{ijk} / v_{ik} & \text{if } v_{ik} > 0 \\ 0 & \text{otherwise,} \end{cases} \tag{11}$$

where v_{ik} is the sojourn time in state i on the time interval I_k given, for $N_T > 1$, by

$$v_{ik} = \sum_{\ell=1}^{N_T} \inf(X_{\ell+1}, v_{k+1} - v_k) \mathbf{1}_{\{J_\ell = i,\, X_{\ell+1} \ge v_k\}} + \inf(U_T, v_{k+1} - v_k) \mathbf{1}_{\{J_{N_T} = i,\, U_T \ge v_k\}} \tag{12}$$

and

$$d_{ijk} = \sum_{\ell=1}^{N_T} \mathbf{1}_{\{J_\ell = i,\, J_{\ell+1} = j,\, X_{\ell+1} \in I_k\}} \tag{13}$$

The functions $\mathbf{1}_{[\,,\,)}$ and $\mathbf{1}_{\{\,,\ldots\}}$ are the indicator functions of the interval / set that appears as a subscript.

Availability Estimation by Semi-Markov Models

As we have remarked in the Introduction, the two-state (*up* and *down*) models are inadequate for describing the possible states of a subsystem in a complex system like an industrial facility. As we suggested in Eq. (1), two subsets of states U and D are better suited for describing the evolution of a subsystem during its lifetime or operational cycle. We keep the notation suggesting "up" and "down" states, but a state in U will be – in fact – a state in which the subsystem is available but not necessarily operating ; a state in D will correspond to various situations when the system is unavailable like a state of failure, of repair, of preventive / periodical maintenance or off-service inspection, etc. A more detailed discussion on the possible states in U or D will be presented in the next section. The availability at a certain time t in $[0,T]$ of the subsystem means that it is either operating or (let us say) in a stand-by state allowing it to become immediately operational on demand at time t. We denote the availability function by $A(t)$ and it can be formally defined, for a MRP, by

$$A(t) = \mathsf{P}[J_{N_t} \in U] \tag{14}$$

where N_t is a positive integer indirectly defined by N_T of Eqs. (12) and (13). It follows from Eq. (12) that N_t equals the number of states of availability $J_{\ell+1}$ at t provided the preceding state was $J_\ell = i$ and the sojourn time in $J_{\ell+1}$ is $X_{\ell+1}$, at least equal to the left end of the division interval $I_k = [v_k, v_{k+1}] \subset [0,t]$. According to [9], the availability of a semi-Markov system at time t can be expressed in closed form as

$$A(t) = \sum_{i \in E} p(i)\, A_i(t) \tag{15}$$

where $A_i(t)$ is the availability at t given that the system starts in state i at the initial time ; $p(i)$ is the initial law for state i in E. Eq. (15) can be rewritten under the matrix form

$$A(t) = \mathbf{p} \cdot \mathrm{P}(t) \cdot \mathbf{1}_{s_1, s} \tag{16}$$

where $\mathbf{p} = [\,p(1)\ldots p(i)\ldots p(s)]$ is the initial law, $\mathrm{P}(t) = [p_{ij}(t)]_{s-\text{by}-s} = [P(J_{N_t} = j \mid J_0 = i)]_{s-\text{by}-s}$ is the semi-Markov transition matrix, and $\mathbf{1}_{s_1, s} = [1\,1\ldots 1\;0\;0\ldots 0]^{\mathrm{T}}$ with the first s_1 entries = 1 while the next $s - s_1$ are = 0. A more explicit form for Eq. (16) is

$$A(t) = [\,p(1) \cdots p(i) \cdots p(s)\,] \cdot \begin{bmatrix} p_{11}(t) & \cdots & p_{1j}(t) & \cdots & p_{1s}(t) \\ \vdots & & \vdots & & \vdots \\ p_{i1}(t) & \cdots & p_{ij}(t) & \cdots & p_{is}(t) \\ \vdots & & \vdots & & \vdots \\ p_{s1}(t) & \cdots & p_{sj}(t) & \cdots & p_{ss}(t) \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}. \tag{17}$$

It is possible to obtain, from Eqs. (16) – (17), several types of availability (at time t) depending on the initial state of the system. For instance, the availability at t provided the system started in state i is given by

$$A_i(t) = p(i) \cdot \sum_{j=1}^{s_1} p_{ij}(t). \tag{18}$$

A relevant availability measure is the so-called *interval availability* (also considered in our paper [4]). It expresses the probability that the system is available at time $t + x$ provided it has been available at t and on the whole interval $(t, t+x)$. . Formally, we may define it by

$$IA(x,t) = \mathsf{P}(J(\tau) \in U :\; \tau \in [\,t, t+x\,]). \tag{19}$$

If the states of availability are grouped together as above (from 1 to s_1), then the interval availability in Eq. (19) can be equivalently written as

$$IA(x,t) = \mathsf{P}(J(\tau) \le s_1 : \tau \in [t, t+x]). \qquad (20)$$

Certainly, this grouping could be considered as being more or less arbitrary, and it is implicity assumed that the set of states E is − in fact − an ordered s-tuple

$$\mathsf{E} = \{e_1, \dots, e_{s_1}, e_{s_1+1}, \dots, e_s\}.$$

We can also remark that Eqs. (19) & (20) express the probability that the system remains in an available state along the whole interval $[t, t+x]$, what does not exclude the possibility that it changes its state but without going out of U. The evaluation of the interval availability would clearly be of practical interest since it gives a measure of continuous availability over a given time interval. If $J(t) = j \in U$ and the successive jump times to other states of availability, following to t, pass over $t+x$ then it is clear that the system will be still in a state of availability at time $t+x$. However, this is only a sufficient and particular condition for the system to remain available over the whole interval in discussion. We have to look for a general formula to evaluate the probability in Eq. (19). Let us first remark that the interval availability generalizes what is called the reliability of a MRP (in [7]), given by

$$R(t) = \mathsf{P}\big(J(\tau) \in U \ \text{ for all } \ \tau \le t\big). \qquad (21)$$

In other words, the reliability in Eq. (21) equals the probability that the system remains available over the whole time interval $[0, t]$. It follows from Eqs. (19) and (21) that $R(t) = IA(t, 0)$. The reliability can be expressed in terms of the reliability conditional on the departure state i, by equations similar to Eqs. (15) and (16) :

$$R(t) = \sum_{i \in U} p(i)\, R_i(t) = \mathbf{p}_U \cdot \mathrm{P}^U(t).\mathbf{1} \qquad (22)$$

where $\mathbf{p}_U$ is the subvector of the (initial) probability law restricted to the *up*-states, $\mathbf{1} = [1\ 1\ \dots\ 1]^{\mathrm{T}}$ and

$$\mathrm{P}^U(t) = [I - Q^U(t)]^{-1} * [I - \mathrm{diag}(Q(t) \cdot \mathbf{1})^U]. \qquad (23)$$

with $*$ = the Stieltjes convolution product. However, it appears that these measures of the time dependent availability of a (sub)system, based on the MRP model, are less adequate for a system which can “visit” (or pass through) states when it is not available, yet such states are not actual failure states. This is the case with states of periodic maintenance / off-service inspections, for instance.

A couple of relevant parameters and functions for the availability of a semi-Markov system are the following: $N_j(t) =$ the number of visits paid to state j in the interval $[0, t]$; $R_{ij}(t) = E[N_j(t) \mid J(0) = i]$, that is the expected number of vistits paid to state j (over the same interval) provided the initial state was i ; $p_{ij}(t) = \mathsf{P}[J(t) = j \mid J(0) = i]$; $G_{ij}(t)$ = the distribution function of the first jump time from i to j. These parameters can be evaluated provided certain assumptions of the distribution / density functions are adopted. Such assumptions will be presented in the next section.

4. AVAILABILITY EVALUATION OF SUBSYSTEMS BY MULTI-STATE SEMI-MARKOV MODELS

We consider subsytems with a larger set of states E, including operating states as well as states of repair / replacement, maintenance, off-service and in-service inspection, stand-by (or ready) states, etc. Certainly, the set of states to be accepted/selected depends on the technical or engineering nature of the subsystem. We extend the set of possible states that we have considered in [4]. Certainly, a larger state space will lead to a more complex model. However, such a larger model will may be rather easily adapted to simpler cases by appropriate choices for the distributions of the corresponding sojourn times. In general, the appropriate stochastic model to formalize the behavior of such subsystems will consist of a semi-Markov system since – for instance – the sojourn times in different states cannot be assumed to be i.i.d. (independent and identically distributed). The set of possible we take into account is given in Table 1.

Table 1. The set of possible states of a subsystem

No. of state	Name / Definition	Type of state : AV : availability UNAV : unavailability	Description
2	Operating	AV	System is operating
2	Stand-by	AV	Ready for operation on demand
3	In-service inspection	AV	Periodic inspection during operation
4	Maintenance	UNAV	Includes off-service inspections
5	Failure	UNAV	Failure due to external events / lack of required input / human error / failure by control system
6	Post-failure repair	UNAV	Repair or replacement after a failure

A couple of comments or explanations might be necessary. Some of the states could be further split into smaller ones, with the risk to get a too complex model. **2** : In some systems a stand-by state could be a cold stand-by or a hot stand-by state ; anyway, the subsystem / equipment is available on demand. **3** : The states of in-service inspections are clearly states of availability. These inspections have deterministic time lengths and also deterministic time intervals between an inspection and the next one. An in-service state could be assimilated to an operating state. However, a difference appears regarding the probability of a jump to a state 4 (of maintenance) or even to a state 6 (of replacement). **4** : In some models, it is also made a distinction between the preventive (or prophylactic) maintenance and corrective maintenance. The former may be periodic, similar to the periodic inspections. An off-service inspection can obviously be assimilated to a maintenance, but the same remark holds on the probabilities of jumps to next states. **5** : As it is specified in the fourth column of Table 1, a state of failure may occur due to an external event like an earthquake, a LOSP accident (loss of off-site power), the lack of required input caused by a failure of an upstream component (in the feedwater system of a NPP, for instance) ; a failure may also occur due to an automatic shutdown by the control channels. Other types of failures could be included in this state and all of them depend on the nature of the system and its subsystems / components. In the case of NPPs, the possible failures and accident sequences are identified by means of the so-called CCF (Common Cause Failure) analysis and fault tress or event trees, and the PSA (Probabilistic Safety Assessment) developed by NUREG in the United States are widely known and applied since decades. Another remark regards the possible distinction between failures which may be revealed or not revealed [9]. Let us also mention that the structure of a complex system (parallel / series systems and combinations thereof) is quite important since this structure may be essential not merely for the whole system but also for the subsystems it consists of. This situation occurs, e.g., when events of the type "*k*-out-of-*n*" are considered : a system consisting of n components (or subsystems) is available if at least k subsystems are available. We give a simple example of a series system consisting of two parallel subsystems in the figure that follows.

Fig. 1 A series system consisting of two parallel subsystems

A typical event for the system in Fig. 1 to remain in an *up* state consists in the availability of two among the subsystems (components) **1** , **2** , **3** and the availability of two components among **4** , **5**. Then the event that the system is in an *up* state can be expressed in terms of a Boolean expression. If we denote by $[\mathbf{i}]$ the event that component **i** is up, while $[\bar{\mathbf{j}}]$ stands for the component **j** in a down state, then the event we consider can be expressed by

$$[\text{System is up}] = ([\mathbf{1}] \cap [\mathbf{2}] \cap [\bar{\mathbf{3}}] \cap [\mathbf{4}] \cap [\bar{\mathbf{5}}]) \cup ([\mathbf{1}] \cap [\bar{\mathbf{2}}] \cap [\mathbf{3}] \cap [\mathbf{4}] \cap [\bar{\mathbf{5}}]) \cup \ldots$$

with a total number of six join events in this union.

Before seeing how some of the formulas connected with the semi-Markov processes of the previous section can be applied, let us sketch a transition diagram corresponding to a subsystem with the state space $E = \{1, 2, 3, 4, 5, 6\}$ described in the previous table.

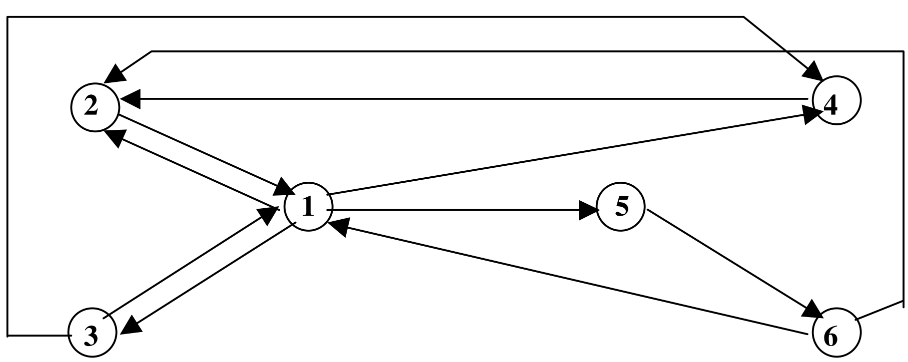

Fig. 2 The transition diagram for a subsystem with the states in Table 1

In reference [7] it is presented a simple application in terms of a 3-states semi-Markov system. The sojourn times in a certain state are conditional on the nature of the next state to be visited. This dependence is expressed in terms of the distributions assumed for the sojourn times. We also admit similar assumptions for the 6-states system in what follows. We recall that the time variable is assumed to take values in the interval $[0, T]$ with $T =$ the designed lifetime or operational cycle length of the system.

The initial probability law, i.e., the vector of the probabilities of the initial (or departure) states is

$$\mathbf{p}(0) = [1\ 0\ 0\ 0\ 0\ 0]. \tag{24}$$

The transition matrix is here considered as constant, meaning that the transition probabilities do not depend on time. We take this matrix, corresponding to the transition diagram, as given below :

$$\mathbf{P} = \begin{bmatrix} 0.55 & 0.15 & 0.15 & 0.10 & 0.05 & 0 \\ 0.60 & 0.40 & 0 & 0 & 0 & 0 \\ 0.55 & 0 & 0.30 & 0.15 & 0 & 0 \\ 0 & 0.60 & 0 & 0.40 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0.65 & 0.15 & 0 & 0 & 0 & 0.20 \end{bmatrix}. \tag{25}$$

Certainly, this is only an illustrative example. For instance, the entries on the main diagonal give the probability that the system remains in the respective state, but this obviously depends on the time passing since that state was entered and the expected sojourn time, specific to the same state. The matrix in (25) should be taken as giving only plausible transition probabilities at a moment when the system changes its state. A realistic model should operate with time dependent transition probabilities, and also with specific rates like repair rates, maintenance rates, etc.

In our opinion, one of the most interesting approaches is that due to A. Csenki [6] since his model operates with the probabilities that a system keeps or changes its state over a time interval of length x. This notion of interval reliability is quite naturally suitable for using semi-Markov models since the sojourn time (in a certain state) is just the essential issue that makes the difference between these models and other (simpler) ones. It remains as an objective for us to go further along this line of using notions like interval reliability and availability and semi-Markov systems for the stochastic evaluation of subsystem availability.

5. CONCLUDING REMARKS

We have continued our research of [4], giving more details on the mathematical nature of the semi-Markov systems. We have also discussed possibilities to consider systems with a larger variety of possible states. Certainly, the availability evaluation for such systems would imply rather sophisticated and complex mathematical (stochastic) models. The semi-Markov models provide the important advantage that they take into account the sojourn times in the possible states, while the renewal models assume that any failing component is immediately replaced by a new one. It still remains to go further with the application of semi-Markov models and interval reliability in the stochastic evaluation of the availability of subsystems and systems.

The impact of earthquake motions on the reliability and availability of (sub)systems has not been explicitly taken into account in this paper. However, the seismic effect can be expressed in terms of certain parameter components of the vector of Eq. (2). On another hand, the relevance of the Markov-type systems in the reliability assessment of structures subjected to earthquake actions is taken into account in the other paper we have submitted to 13WCEE, that is Paper 2278.

REFERENCES

1. Lin YK. “Application of Markov process theory to nonlinear random vibration problems.” Chapter 4 in Stochastic Methods in Structural Dynamics, Dordrecht: Martinus Nijhoff Publishers, 1987, 134-153.
2. Elsayed AE, Ribeiro JD. “A reliability approach for optimal design of structural components.” Transactions of the 13th SMiRT International Conference, Porto Alegre, Brasil, 1995, Vol. IV, 545-552.
3. Choi SY, Yang J-E, Jin Y. “Development of Markov model of emergency Diesel generator for dynamic reliability analysis.” Transactions of the 15th SMiRT International Conference, Seoul, Korea, 1999, Paper M/05/2, X-491-498.
4. Carausu A, Vulpe A. “Markovian and semi-Markovian models for availability evaluation of repairable subsystems & equipment in NPP’s.” Transactions of the 15th SMiRT International Conference, Seoul, Korea, 1999, Paper M/05/4, X-499-506.
5. Ionescu DC and Limnios N (Eds.). “Statistical and Probabilistic Models in Reliability.”, Boston-Basel-Berlin: Birkhäuser, 1999.
6. Csenki A. “An integral equation approach to the interval reliability of systems modelled by finite semi-Markov models.” Reliability Engineering and System Safety 1995: **47**, 37-45.
7. Ouhbi B, Limnios N. “Estimation of kernel, availability and reliability of semi-Markov systems.” Chapter 8 in Ref. # 5: “Statistical and Probabilistic Models in Reliability.”, Boston-Basel-Berlin: Birkhäuser, 1999, 113-124.
8. Oprisan Gh. “On the failure rate.” Chapter 16 in Ref. # 5: “Statistical and Probabilistic Models in Reliability.”, Boston-Basel-Berlin: Birkhäuser, 1999, 235-242.
9. Rao SS. “Reliability-based design.” New York: McGraw-Hill, 1992.