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HORIZONTAL IMPEDANCE FUNCTION OF SINGLE PILE IN SOIL LAYER WITH VARIABLE PROPERTIES

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Abstract: An equivalent method is suggested to analyze the horizontal impedance function of single pile in horizontal soil layer with variable properties. In this approach, three main steps are included. First, a simplified method, which is established on the one-dimensional shear wave equation and modal perturbation technique, is developed to determine the modes of the complicated soil layer with variable properties. Then an equivalent homogeneous soil layer is suggested to instead of the variable-property soil layer. The equivalent physical parameters of the equivalent homogeneous soil layer are determined by weighted integral. Finally, the impedance function of the pile in the equivalent homogeneous soil layer can be considered as the one of the complicated soil layer approximately. Numerical example verifies that reasonable precision can be achieved.

Keywords: Impedance function; single pile; soil layer

1. Introduction

Pile foundation is a popular type of the deep foundation of large-scale structure. The dynamic interaction between soil, pile foundation and structure under seismic excitation is an important problem in the seismic stability research of high-rising buildings and long-span bridges. In the substructure method for soil-structure interaction (SSI) problem, the impedance function or matrix of the pile foundation is usually introduced in the equation of motion of the structure as the consideration of the SSI effect upon the dynamic behavior. Many useful research works have been done for determining the impedance function or matrix of single and group pile foundation[1-4]. However, most of these works are only available to homogeneous half plane (or space) and layered soil. Finite element method (FEM) and boundary element method (BEM) or other numerical methods can be used to solve the impedance function of the pile foundation in more complicated soil layer, but numerous calculation has to be carried out. In the engineering practice, the properties of the soil medium vary with the soil depth for most of soil site. Therefore, it is an interesting work to develop a simplified method with high precision for determining the impedance function of pile foundation for structure design and analysis. As known, time-lag has an important influence on the effectiveness of the structural control. It also requires the simplified method with high precision method to achieve the fast computation for the online analysis if the soil-pile-structure interaction should be considered in the seismic control [5,6]. As a basic work, a simplified method for calculating the horizontal impedance function of pile in horizontal variable-property soil layer is discussed in this paper.

2. Impedance Function of Pile in Homogeneous Soil Layer

The approach suggested by Novak[1] is adopted in this paper to calculate the horizontal impedance function of single pile in homogeneous soil layer. But the soil is assumed as linear viscous-elastic media instead of linear elastic media in the approach. It means that the computing formula of the impedance function of the pile will be same, only the complex viscous-elastic modulus of the soil is used to replace the modulus of elasticity in the equation of motion.

3. Modal Perturbation Technique

The partial differential equation of motion for determining the dynamic behavior of the horizontal soil layer with variable property under the horizontal seismic excitation $\ddot{u}_g(t)$ can be written as following:

$$\rho(y)\frac{\partial^2 u(y,t)}{\partial t^2}+c(y)\frac{\partial u(y,t)}{\partial t}+\frac{\partial}{\partial y}[G(y)\frac{\partial u(y,t)}{\partial y}]=-\rho(y)\ddot{u}_g(t) \tag{1}$$

where $u(y,t)$ is the relative horizontal displacement to the rock surface motion $u_g(t)$; $\rho(y),c(y)$

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and $G(y)$ are the mass density, damping coefficient and shear elastic modulus at vertical location y from the rock surface, respectively. Applying separation of variables, the shear vibration modal characteristics of the variable-property soil layer can obtained from the following equation:

$$\frac{d}{dy}[G(y)\frac{d\varphi_j(y)}{dy}]+\bar{\lambda}_j\,\rho(y)\varphi_j(y)=0 \tag{2}$$

Obviously, it is difficult to solve analytically Eq. (2) to get the eignvalue $\bar{\lambda}_j$ and vibration function $\varphi_j(y)$ of j^{th} shear mode. Let

$$G(y)=G_0+\Delta G(y)\,,\quad \rho(y)=\rho_0+\Delta\rho(y) \tag{3}$$

where G_0 and ρ_0 are arithmetic average of the shear elastic modulus and mass density over the depth H of the soil layer respectively,

$$G_0=\frac{1}{H}\int_0^H G(y)dy\,,\quad \rho_0=\frac{1}{H}\int_0^H \rho(y)dy$$

As known, the j^{th} shear vibration modal parameters λ_j and $\psi_j(y)$ of the homogeneous soil layer with shear elastic modulus G_0, mass density ρ_0 and same depth H can be easily determined from the constant coefficient ordinary differential equation:

$$G_0\frac{d^2\psi_j(y)}{dy^2}+\lambda_j\rho_0\psi_j(y)=0 \tag{4}$$

Based on the modal property λ_j and $\psi_j(y)$ of the homogeneous soil layer, the eigenvalue $\bar{\lambda}_j$ and shear vibration function $\varphi_j(y)$ of the j^{th} mode of the variable-property soil layer can be expressed approximately as

$$\bar{\lambda}=\lambda_j+\Delta\lambda_j \tag{5}$$

$$\varphi_j(y)=\psi_j(y)+\Delta\psi_j(y)=\psi_j(y)+\sum_{\substack{k=1\\k\neq j}}^{m}a_{jk}\psi_j(y) \tag{6}$$

After substituting Eqs. (5) and (6) into Eq. (2) and premultiplying $\psi_j(y)$ on both sides of Eq. (2), an algebraic equation will be obtained by integrating from 0 to H along the layer depth,

$$\begin{aligned}\Delta\lambda_j\sum_{\substack{k=1\\k\neq j}}^{n}(m_i^*\delta_{ik}+\Delta m_{ik})+\sum_{\substack{k=1\\k\neq j}}^{n}[(\lambda_j-\lambda_i)m_i^*\delta_{ik}+\Delta m_{ik}-\Delta k_{ik}]a_{jk}\\+\Delta\lambda_j[\sum_{\substack{k=1\\k\neq j}}^{n}(m_i^*\delta_{ij}+\Delta m_{ij})a_{jk}]+(\lambda_j\Delta m_{ij}-\Delta k_{ij})=0\end{aligned} \tag{7}$$

where $m_i^*=\frac{\rho_0 h}{2}$ is the generalized mass of $\varphi_i(y)$;

$$\Delta m_{ik}=\int_0^H\Delta\rho(y)\psi_i(y)\psi_k(y)dy\,,\quad \Delta k_{ik}=\int_0^H\Delta G(y)\psi_i^{'}(y)\psi_k^{'}(y)dy \tag{8}$$

In Eq. (7), there are n unknown variables, that is $\{a_j\}=\langle a_{j1},a_{j2},\cdots,a_{jj-1},\Delta\lambda_j,a_{jj+1},\cdots a_{jn}\rangle^T$. When circulating i from 1 to n, a nonlinear algebraic equation can be formed to determine the variable vector $\{a_j\}$. Then the j^{th} eigenvalue $\bar{\lambda}_j$ and modal shape function $\varphi_j(y)$ can be obtained immediately from Eq. (5) and (6). If the second order small quantity in Eq.(7) is neglected, a very simple expression can be gotten:

$$\Delta\lambda_j=\frac{\Delta k_j-\lambda_j\Delta m_{jj}}{m_j^*}\,,\quad a_{ji}=\frac{\Delta k_{jj}-\lambda_i\Delta m_{jj}}{(\lambda_j-\lambda_i)m_i^*} \tag{9}$$

4. Equivalent Homogeneous Soil Layer

It had been verified that the dynamic flexibility coefficient of the variable-property soil layer could be computed approximately by mean of the equivalent homogeneous soil layer[7]. This technique will be applied to solve the problem of the impedance function of single pile in the horizontal soil layer with variable properties. Using this equivalent technique, the problem to obtain the impedance function of single pile in variable-property soil layer is displaced by the problem of solving the impedance function of the single pile in the homogeneous soil layer with same depth. It becomes easy to solve the problem, because there have been several theoretical and analytical method to solve the latter problem.

Two basic principles will be used in the equivalent procedure. First, the static equivalent principle is applied to find the equivalent elastic modulus for the variable-property soil layer. And then the dynamic equivalent principle, which requires to keep the same fundamental frequency for the variable-property soil layer and its equivalent homogeneous soil layer, is used to determine the equivalent mass density. Detail of the equivalent method is described as following.

4.1 Equivalent Elastic Modulus

As known, the elastic modulus of soil medium is the key factor to influence the static deformation of the soil layer. Therefore, the first thing to do in the equivalent procedure is to choose the reasonable equivalent formula for determining Young's elastic modulus E_e and shear elastic modulus G_e. Three formulas are listed for trial analysis and comparison.

(1)
$$G_e = \frac{1}{\alpha}\int_0^H G(y) f'(y) f(y) dy \tag{10}$$

where α is a constant. It is obtained from the condition that when $G(y)$ is equal to G_e, two sides of Eq. (10) must be same, that is, $\alpha = \int_0^H f'(y) f(y) dy$

(2)
$$G_e = \frac{1}{\alpha}\int_0^H f(y) G(y) dy \tag{11}$$

where $\alpha = \int_0^H f(y) dy$

(3)
$$G_e = \frac{1}{\alpha}\frac{\left[\int_0^H G(y) f(y) dy\right]^2}{\int_0^H G(y) f^2(y) dy} \tag{12}$$

where $\alpha = [\int_0^H f(y)dy]^2 \Big/ \int_0^H f^2(y)dy$

In above formulas, G_e is the equivalent shear modulus; $f(y)$ is a weighted function and is assigned as the first modal function $\phi_1(y)$ of the variable-property soil layer that can be determined by the perturbation method in above section. If $G(y)$ is displaced by $E(y)$ in Eqs. (10)-(12), the equivalent Young's elastic modulus E_e can be obtained.

4.2 Equivalent Poisson's Ratio

The equivalent Poisson's ratio can be determined from equivalent Young's elastic modulus and shear modulus under the isotropy assumption of the soil medium:

$$\mu_e = \frac{E_e}{2G_e} - 1 \tag{13}$$

4.3 Equivalent Damping Ratio

If damping form of soil medium is assumed as a hysteretic damping, the complex shear modulus may be expressed as:

$$G^*(y) = G(y)[1 + i\zeta(y)] \tag{14}$$

in which, $\zeta(y)$ is defined as the equivalent hysteretic damping factor; $i = \sqrt{-1}$. For the homogeneous soil layer, the complex shear modulus can be expressed correspondingly as:

$$G_e^* = G_e(1 + i\zeta_e) \tag{15}$$

in which ζ_e is defined as the equivalent hysteretic damping factor. Due to both of the damping force and the elastic resistance are proportional to the displacement amplitude, ζ_e can be determined by using similar equations shown in Eqs. (10) to (12), except using $\zeta(y)$ in place of $G(y)$.

4.4 Equivalent Mass Density

According to the second equivalent principle, the equivalent mass density of the equivalent soil layer is determined by:

$$\rho_e = \left(\pi \big/ 2h\bar{\omega}_1\right)^2 G_e \tag{16}$$

in which $\overline{\omega}_1$ is the fundamental frequency of the variable-property soil layer and can be calculated by modal perturbation technique mentioned last section in this paper.

5. Numerical Example

A horizontal soil layer with 9 groups of variable properties shown in table 1 is taken as the numerical example. In the table, Δz, E, V, ζ and μ are the thickness, Young's elastic modulus, shear wave velocity, hysteretic damping factor and Poisson's ratio of each soil layer. The first layer is located at the top of the variable-property soil layer.

Table1 The parameters of variable-property soil layer

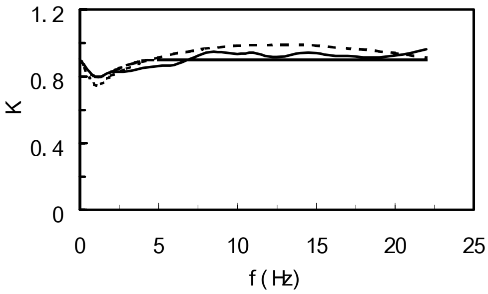
Layer number	ΔZ (m)	E (MPa)	V (m/s)	ζ	μ
1	0.42	58	112	0.04	0.25
2	0.42	87	135	0.04	0.25
3	0.43	116	159	0.04	0.25
4	2.99	132	165	0.04	0.25
5	7.53	86	130	0.07	0.49
6	10.72	200	184	0.03	0.45
7	5.0	357	257	0.02	0.35
8	2.0	273	232	0.02	0.30
9	20.0	480	300	0.02	0.30

The properties of the equivalent homogeneous soil layer, decided by using three basic Eqs. (10) to (12) respectively, are listed in Table 2 ,.

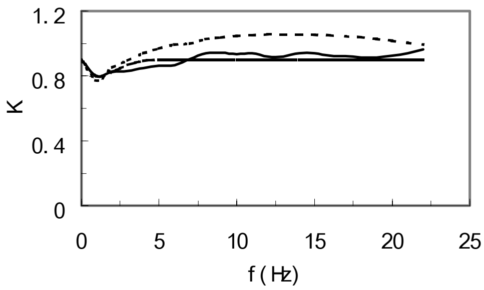
Table 2 Properties of the equivalent homogeneous soil layer

Equivalent basic equation	$E_e(MPa)$	$V_e(m/s)$	ζ_e	μ_e	$G_e(MPa)$
Eq.(10)	214	261	0.039	0.36	79
Eq.(11)	207	261	0.041	0.36	76
Eq.(12)	191	261	0.045	0.38	69

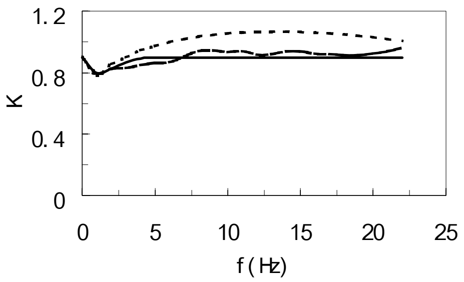
The pile, 34 meters in length and 1.4 meters in diameter, is a steel pipe filled with concrete. The horizontal impedance function of the pile in these equivalent homogeneous soil layers can be easily obtained by using the method suggested by M. Novak[1]. These solutions are taken as the approximate horizontal impedance function of the pile in the variable-property soil layer. The real and imaginary part of the impedance functions gotten from using different equivalent basic equation, that is Eq. (10), (11) or (12), are shown in Figure 1 and 2.



a. Equation (10)

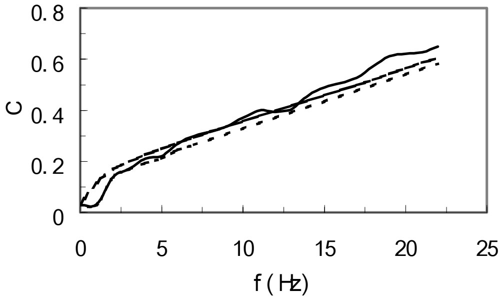


b. Equation (11)

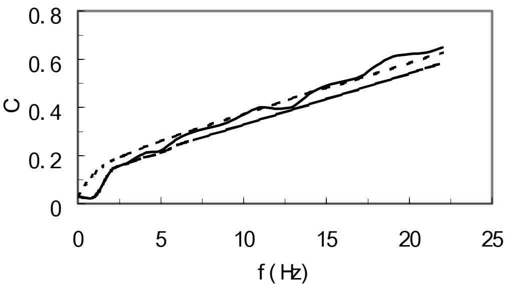


c. Equation (12)

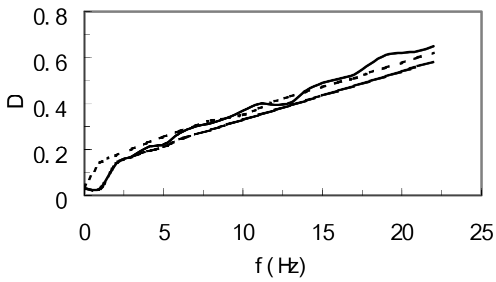
Figure 1 Real part of impedance function



a. Equation (10)



b. Equation (11)



c. Equation (12)

Figure 2 Imaginary part of impedance function

Otherwise, the horizontal impedance function had been calculated before this research by finite element method (FEM) and the simplified method. These results are taken for the comparison. All of comparisons of the current results and previous results are shown in the Figures 1 and 2. The solid line and the dashed line represent the results obtained by the FEM and approximate method suggested by G.Gazetas and R. Dobry[4], respectively. The dot lines show the results obtained by this paper using basic Eq. (10), (11) and (12) respectively.

From the results, it can be shown that, the choice of equivalent equation for elastic modulus has a little influence on the imaginary part of the impedance function but more important effect on the real part of the approximate results. From these comparisons, the best precision is achieved in the case shown in Figure 1a and Figure 2a. Because the equivalent equation (10) describes the reasonable relation that the works by the internal shear stress in both soil layers are equal to. So, the equivalent equation (10) is recommended to use in the equivalent procedure.

Comparing with the approximate method developed by G. Gazetas and R. Dobry, the approximate method suggested by this paper is more easily to get the numerical results. Because, it is not necessary to calculate the static stiffness and deflection line of the pile by FEM to determine the radiation and material damping of soil medium.

6. Conclusion

A simplified method for determining the horizontal impedance function of single pile in the horizontal soil layer with variable properties is suggested in this paper. In this method, the static and dynamic equivalent principles and the modal perturbation technique are applied. The numerical results show the method has good precision.

7. Acknowledgments

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