

# MODE-ADAPTIVE PUSHOVER ANALYSIS FOR MULTI-STORY RC BUILDINGS

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## SUMMARY

A mode-adaptive pushover (MAP) procedure, which uses a stiffness-dependent lateral force distribution at each loading step without the eigenvalue analysis, is proposed in this paper. 4 and 12 story RC frame buildings and a 6 story RC building with the soft first story are analyzed using MAP procedure to estimate the responses by the Capacity Spectrum Method (CSM). Three kinds of MAP analyses with the lateral force distributions corresponding to the first to third modes of vibration are conducted for each building to consider the higher mode effect. Non-linear response history analyses using several earthquake ground motions are also executed for each building to compare with the predicted maximum responses by CSM. This paper indicates that applying a modal analysis combined CSM with MAP analysis for the first mode and elastic analysis for the second and third modes, the maximum story shears and drifts can be approximately evaluated.

## INTRODUCTION

The Capacity Spectrum Method (CSM) [1] has been adopted as a seismic evaluation method in the Japanese structural design code for buildings, which was revised toward a performance-based structural engineering framework in June 2000 [2]. In CSM, the lateral force resisting capacity of a building is represented by the acceleration-displacement response spectrum (i.e., the capacity spectrum) obtained from pushover analysis. Since the capacity spectrum is what represents the response of the equivalent single degree of freedom (ESDOF) system for the building, how to convert appropriately the building into ESDOF system is a key to improve the accuracy of CSM. In order to construct the appropriate capacity spectrum corresponding to ESDOF system, the use of lateral force distributions proportional to the first mode of vibration of the building is necessary for the non-linear pushover analysis. On the other hand, consideration of the higher mode effect is also an important issue for predicting the earthquake responses of high-rise and medium-height buildings.

A mode-adaptive pushover (MAP) procedure, which uses a stiffness-dependent lateral force distribution proportional to an arbitrary mode of vibration at each loading step where the analyzed building is in elastic or inelastic ranges, is proposed in this paper. Although the similar pushover procedure with the

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eigenvalue analysis at each loading step have already been proposed by Bracci et al. [3], MAP proposed herein adopts the incremental technique without the eigenvalue analysis. 4 and 12 story reinforced concrete (RC) frame buildings and a 6 story RC building with the soft first story are analyzed using MAP procedure to estimate the story responses by CSM. Three kinds of MAP analyses with the lateral force distributions proportional to the first to third modes of vibration are conducted for each building to consider the higher mode effect. Then the responses evaluated by CSM are compared with those by the non-linear response history analysis (RHA) with several earthquake ground motions. The predictability of the higher mode effect in CSM using MAP analysis is also discussed in this paper.

### **MODE-ADOPTIVE PUSHOVER PROCEDURE**

CSM is a method to evaluate the maximum earthquake response of a building using the demand spectrum and the capacity spectrum ( $_{I}S_{a}-_{I}S_{d}$  curve) on the assumption that the multi-story building can reduce to ESDOF system corresponding to the first mode of vibration. Strictly speaking, in pushover analysis used for constructing the capacity spectrum, the lateral forces applied at each story should be proportionate to the first mode of vibration at any loading step where the analyzed building is in elastic or inelastic ranges [4]. Executing the eigenvalue analysis at each loading step in pushover analysis, the lateral force distribution, which changes corresponding to the variation of the mode shape of the building with the inelastic behavior, can be evaluated. However, this method is rather complicated for practical use. In this section, therefore, a pushover procedure using the lateral force distribution proportional to the (equivalent) first mode of vibration for not only elastic range but also inelastic range of the building without the eigenvalue analysis is described.

Considering the multi degree of freedom (MDOF) system corresponding to a *N*-story building, the maximum response displacement proportional to the first mode of vibration at the *i*-th story,  $_{I}\delta_{i}$ , can be given by

$${}_{1}\delta_{i}={}_{1}\beta_{1}u_{i}\cdot{}_{1}S_{d} \tag{1}$$

in which,  $_{i}u_{i}$  = normal mode of the first mode in the *i*-th story

 $_{1}\beta$  = participation factor of the first mode

 $_{I}S_{d}$  = spectral displacement for the first mode

The lateral force applied at the i-th story,  $_{1}P_{i}$ , is given as follows:

$${}_{I}P_{i} = m_{i'I}\beta \cdot {}_{I}u_{i'I}S_{a}$$

$$\tag{2}$$

in which,  $m_i$  = lumped mass in the *i*-th story

 $_{I}S_{a}$  = spectral acceleration for the first mode

Using Eqs.(1) and (2), the following relation can be obtained.

$${}_{I}P_{i} = m_{i} \cdot {}_{I}S_{i} / {}_{I}S_{d}$$

$$\tag{3}$$

On the other hand, the spectral acceleration,  ${}_{I}S_{a}$ , and the spectral displacement,  ${}_{I}S_{d}$ , for ESDOF system are given by the following equations [4].

$${}_{I}S_{a} = \frac{\sum_{j=l}^{N} m_{j} \cdot {}_{I} \delta_{j}^{2}}{\left(\sum_{j=l}^{N} m_{j} \cdot {}_{I} \delta_{j}\right)^{2}} \cdot {}_{I}Q_{B}$$

$$\tag{4}$$

$${}_{I}S_{d} = \frac{\sum_{j=l}^{N} m_{j} \cdot {}_{I}\delta_{j}^{2}}{\sum_{j=l}^{N} P_{j} \cdot {}_{I}\delta_{j}} \cdot {}_{I}S_{a} \qquad \left( \begin{array}{c} = \frac{\sum_{j=l}^{N} m_{j} \cdot {}_{I}\delta_{j}^{2}}{\sum_{j=l}^{N} m_{j} \cdot {}_{I}\delta_{j}} \\ = \frac{\sum_{j=l}^{N} m_{j} \cdot {}_{I}\delta_{j}}{\sum_{j=l}^{N} m_{j} \cdot {}_{I}\delta_{j}} \end{array} \right)$$
(5)

in which, N = number of story  ${}_{I}Q_{B} =$  base shear

Considering equilibrium between the external work by applied lateral force proportional to the first mode at the *i*-th story in MDOF system,  $_{I}P_{i}$ , and the internal work by shear in ESDOF system,  $_{I}Q_{B}$ , the following relation are given:

$$\sum_{j=1}^{N} P_{j \cdot j} \delta_{j} = Q_{B \cdot j} S_{d}$$
(6)

From Eq.(3) to Eq.(6), then, the relation between the lateral force applied,  $_{I}P_{i}$ , and the base shear,  $_{I}Q_{B}$ , is obtained as

$${}_{I}P_{i} = \frac{m_{i} \cdot {}_{I} \delta_{i}}{\sum\limits_{j=I}^{N} m_{j} \cdot {}_{I} \delta_{j}} Q_{B}$$

$$\tag{7}$$

Using Eq.(8) instead of Eq.(7) for lateral forces applied for each story at *k*-step in pushover analysis, the mode-adoptive pushover (MAP) with the incremental base shear,  $d_1Q_B$ , is available.

$${}_{I}P_{i,k} = \frac{m_{i} \cdot {}_{I} \delta_{i,k-I}}{\sum\limits_{j=I}^{N} m_{j} \cdot {}_{I} \delta_{j,k-I}} \cdot \left({}_{I}Q_{B,k-I} + d_{I}Q_{B}\right)$$

$$\tag{8}$$

where the initial value of  ${}_{I}\delta_{i,k-I}$  should be proportionate to the elastic first mode obtained from the eigenvalue analysis.



Fig. 1 Analyzed Buildings

### **OUTLINES OF ANALYZED BUILDINGS**

Buildings analyzed are 4 and 12 story RC frame buildings and a 6 story RC building with the soft first story (hereafter referred to as 4-story frame 12-story frame and 6-story piloti, respectively), as shown in Fig.1. The dimensions of cross section and arrangement of reinforcements for main members in each building are listed in Table 1.

The 6-story piloti has the total height of 18.6m and is composed of frames with 6 spans of 7.2m in the x-direction, while bare frames at the first story and multiple shear walls at the upper story with 1 span of 10.8m in the y-direction. The 4-story and 12-story frames are composed of frames with 3 spans of 6.0m in both directions. The total heights are 14.5m for the 4-story frame and 42.5m for 12-story frame, respectively. The base shear coefficients when the story drift angle of any story attains to 1/100 radian are 0.495 for the 6-story piloti, 0.398 for the 4-story frame and 0.299 for the 12-story frame, respectively.

Member	Story	Cross Section (mm)	Reinforcement	Concrete (N/mm <sup>2</sup> )	Reinforcing Bar
Column	2F - 6F	800 x 700	X:4-D25 Y:2-D25+2-16	24	≥ D19: SD345 <d16: sd295<="" td=""></d16:>
	1F	950 x 950	X:8-D25/Y:6-D25 (8-D13@100)		
Wall	2F - 6F	150 x 10,100	D10@150S		

(a) 6-Story Piloti

(b) 4-Story Fram	е
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Member	Story	Cross Section (mm)	Reinforcement	Concrete (N/mm <sup>2</sup> )	Reinforcing Bar
Column	4F	600 x 600	16-D25 (Outer)	24	D29: SD345 D25: SD345 D16: SD345 D13: SD295
	3F		12-D25 (Inner)		
	2F		16-D25		
	1F		(2-D13@100)		
Beam	4F	400 x 700	8-D25 (2-D13@200)		
	3F				
	2F				
	1F	400 x 750	11-D25 (2-D13@150)		
	FG	450 x 1,500	16-D29 (2-D16@200)		

(	С	) 12-Story	Frame
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Member	Story	Cross Section (mm)	Reinforcement	Concrete (N/mm <sup>2</sup> )	Reinforcing Bar
	9F - 12F	850 x 850	16-D29 (2-D13@100)	24	D35: SD390 D32: SD390 D29: SD390 D25: SD345 D16: SD345 D13: SD295
Column	5F - 8F		16-D32 (2-D13@100)	30	
	2F - 4F		24-D35 (3-D13@100)	36	
	1F		28-D35 (3-D13@100)		
	12F - RF	500 x 800	10-D25 (3-D13@150)	24	
Beam	8F - 11F		11-D29 (4-D13@150)	30	
	5F - 7F		14-D35 (4-D13@150)	36	
	2F - 4F		15-D35 (4-D13@150)		
	FG	500 x 3,000	24-D29 (4-D16@200)		

Both MAP analysis and the non-linear response history analysis (RHA) for above-mentioned three buildings were conducted using an analytical program developed by Gu [5] in which the multi-spring model are applied for columns and shear walls. Analyzed were an intermediate frame in the y-direction for the 6-story piloti and two frames in the x-direction for the 4-story and 12-story frames, as shown in Fig.1.

Two recorded earthquake ground motions, NS component of the 1940 El Centro records (El Centro) and NS component of the 1995 Kobe Marine Observatory records of Japan Meteorological Agency (JMA-Kobe) for which the levels of the maximum velocity were normalized to 50 cm/sec and 75 cm/sec, were selected as input waves for RHA. In RHA, the viscous damping of buildings is assumed to be 3% for the natural period of the first mode in proportion to the transient stiffness of members.

### ANALYTICAL RESULT

#### **Non-linear Pushover Analysis**

The results of pushover analyses for analyzed buildings are shown in Fig.2. In the pushover analyses for each building, two types of lateral force distribution, *Ai* distribution and MAP distribution, were used to investigate the effect on response predictabilities at each story of the buildings. *Ai* distribution is a constant lateral force distribution specified in the Building Standard Law of Japan and generally used for



Fig. 2 Story Shear versus Lateral Displacement Relationships

seismic design, while MAP distribution is mode-adoptive distributions dependent on the inelastic level of the buildings, as mentioned above. In the figure, the vertical axis expresses the story shear and the horizontal axis shows the lateral displacement at each story relative to the bottom of the first story, which is not the story drift. The story responses at the maximum displacement of ESDOF system obtained from RHA for each building are also shown in the figure to compare with the results of the pushover analyses. Therefore, each story response occurred at the same time in RHA. The maximum displacement of ESDOF system can be approximately defined as the maximum value in the time history of the displacement,  $\Delta(t)$ , given by Eq.(9).

$$\Delta(t) = \frac{\sum_{i=1}^{N} m_i \cdot \delta_i^2(t)}{\sum_{i=1}^{N} m_i \cdot \delta_i(t)}$$
(9)

in which,  $\delta_i(t)$  = lateral displacement relative to the bottom of the first story in *i*-th story at the time of t

The results of MAP analysis shown in Fig.2(b) give better agreement with the results of RHA in each story as compared with those of pushover analysis using *Ai* distribution shown in Fig.2(a). The pushover using *Ai* distribution tends to give larger story shear than RHA results in the upper stories. In particular, the tendency is more significant for the 6-story piloti. These results imply that the distribution of applied lateral forces after the building yields is much different from that before the yielding, especially in irregular shaped buildings. In pushover analysis to construct the capacity spectrum, therefore, appropriate lateral force distribution corresponding to the inelastic level at the assumed maximum response and the shape of analyzed buildings. From this point of view, MAP procedure is effective to evaluate the story responses at the maximum displacement response of ESDOF system for a building.

#### **Comparison of Maximum Responses**

The maximum story responses, story drifts and story shears, obtained from RHA are compared with the corresponding story responses by MAP analysis in Figs.3 and 4. Each of the maximum story shears and story drifts occurred at different times in RHA. On the other hand, the corresponding story responses can be obtained from the story shear versus story drift relations of each story in MAP analysis by seeking the loading step on the capacity spectrum where the maximum displacement of ESDOF system obtained from RHA coincides with the displacement of the capacity spectrum. The horizontal axis in both figures shows the ratio of the corresponding story responses by MAP analysis to the maximum story responses by RHA.

As seen in the figures, both the corresponding story shears and drifts obtained based on MAP analysis tend to be smaller than those by RHA. For the 6-story piloti, although good agreement in story drifts are observed as well as the comparison for the story responses at the maximum displacement response of ESDOF system (Fig.2), the corresponding story shears by MAP analysis fall below the maximum story shears by RHA in all stories because the maximum responses occur at the different times from that occurring the maximum displacement response in ESDOF system. In the 4-story frame, the similar results are obtained in the comparison of story shear, while the corresponding story drifts by MAP analysis are smaller in the upper stories and larger in the lower stories than those by RAH. For the story responses, both story shears and story drifts, in the 12-story frame, the results of MAP analysis underestimate those of RHA in both the upper and lower stories. MAP procedure, thus, cannot evaluate appropriately the maximum story responses obtained from RHA due to the lack of consideration of the higher mode effect.

#### **EXAMINATION OF HIGHER MODE EFFECT**

As described in the previous sections, MAP procedure is effective to evaluate the story responses at the maximum displacement response of ESDOF system, but cannot evaluate appropriately the maximum story responses due to the lack of consideration of the higher mode effect. Referring to the literatures [6]



Fig. 4 Comparison of Maximum Story Shear

and [7], in this section, an evaluation method of the maximum story responses including the higher mode effect is examined. The procedure is as follows:

(a) Using Eq.(10) based on Eq.(8), three MAP analyses applying the lateral force distributions proportional to the first, second and third modes of vibration, respectively, are executed to construct the capacity spectra,  ${}_{n}S_{a} - {}_{n}S_{d}$  curves (*n*=1, 2, 3), corresponding to each mode.

$${}_{n}P_{i,k} = \frac{m_{i} \cdot {}_{s} \delta_{i,k-l}}{\sum\limits_{j=l}^{N} m_{j} \cdot {}_{s} \delta_{j,k-l}} \cdot \left({}_{n}Q_{B,k-l} + d_{n}Q_{B}\right)$$
(10)

- (b) As shown in Fig. 5, the performance points of each capacity spectrum for the demand spectrum, which is expressed as the spectral acceleration versus spectral displacement relation for the examined earthquake ground motion, are calculated. Where, hysteretic damping with the yielding of the building should be considered in only the case for the first mode.
- (c) In MAP analysis for the first mode, the demand spectrum is reduced to consider the reduction factor,  $F_h$ , with the equivalent viscous damping, h, associated with the ductility factor,  $\mu$ , at a maximum displacement of ESDOF system. According to the Building Standard Law of Japan [2], the equivalent viscous damping and the reduction factor can be given by



Fig. 5 Illustration of Modal Capacity Spectrum Method

$$h = 0.25 \left( 1 - \frac{1}{\sqrt{\mu}} \right) + 0.05 \tag{11}$$

$$F_h = \frac{1.5}{1 + 10h} \tag{12}$$

(d) The story shears and drifts for each mode at the *i*-th story,  ${}_{1}Q_{i}$  and  ${}_{1s}\delta_{i}$ ,  ${}_{2}Q_{i}$  and  ${}_{2s}\delta_{i}$  and  ${}_{3}Q_{i}$ and  ${}_{3s}\delta_{i}$ , are obtained by returning the performance points for each mode to the results at the *i*-th story in each Map analysis. Then, applying the square root of sum of square (SRSS) method to the modal responses, the corresponding story shear and drifts at the *i*-th story are given by

$$Q_{i} = \sqrt{\sum_{n=1}^{3} {}_{n} Q_{i}^{2}}$$
(13)

$${}_{s}\delta_{i} = \sqrt{\sum_{n=1}^{3} {}_{ns}\delta_{i}^{2}}$$
(14)

The distributions of the maximum story drift and story shear along the building height for the 12-story frame obtained from RHA for El Centro waves normalized to 50 cm/sec and 75 cm/sec (Elc-50 and Elc-75) and a JMA Kobe wave 75 cm/sec normalized to 75 cm/sec (JMA-75) are compared with those obtained from the above-mentioned modal analysis using MAP, as shown in Figs. 6 and 7. Solid lines and circles in the figures show the results of RHA and MAP, while the triangles express the results of modal analysis using MAP for the first mode and elastic analysis for the second and third modes.

For Elc-50 and JMA-75 inputs, both the distributions of the corresponding story drifts and story shears by the modal analysis using MAP give approximately good agreements with those by RHA. For Elc-75 input, however, the results by the modal analysis using MAP overestimate those by RHA in both the lower and upper stories. In particular, the tendency is significant in the distribution of story drift. It seems to be the reason why the contribution of the second mode in the calculation of the story responses by the modal analysis using MAP for Elc-75 input is larger than that for other inputs.

Figure 8 shows the relations between the capacity spectrum for the second mode of the 12-story frame and the demand spectra for the Elc-50, Elc-75 and JMA-75 inputs. The capacity spectrum has a peak at the displacement of about 15cm because of the yielding at 10th and 11th stories. As seen in both figures,







Fig. 7 Prediction of Story Shear Using Modal Pushover Analysis



Fig. 8 Relation between Capacity Spectrum for the 2nd Mode and Demand Spectrum

the performance point for the Elc-75 input is after the peak though those for the Elc-50 and JMA-75 inputs are before the peak. Moreover, the yielding of the lower stories followed after the upper stories yielded. In the case of Elc-75 input, therefore, it is considered that the contribution of the second mode to the story responses increases through the yielding of the upper stories.

On the other hand, the distributions of story responses by the modal analysis using MAP with elastic analysis approximately agree with those by RHA for all earthquake inputted. Thus, it seems to be appropriate that MAP for the first mode and elastic analysis for the second and third modes uses for the modal analysis to evaluate the maximum story responses of the building, rather than MAP only uses. In other words, the higher mode effect in the maximum responses may be elastic.

# **CONCLUSION REMARKS**

A mode-adaptive pushover (MAP) procedure, which uses a stiffness-dependent lateral force distribution proportional to an arbitrary mode of vibration at each loading step where the analyzed building is in elastic or inelastic ranges, is proposed. For 4 and 12 story RC frame buildings and a 6 story RC building with the soft first story, the predictability of MAP procedure for the maximum story responses is investigated through the comparison with the responses obtained from the non-linear response history analysis (RHA). Main conclusions in this study are summarized as follows:

- 1) MAP analysis with the incremental base shear can be executed using lateral forces applied for each story given by Equation (8).
- 2) MAP analysis for the first mode is effective to evaluate the story responses at the maximum displacement response of the equivalent single degree of freedom system for a building.
- 3) MAP analysis for the first mode cannot evaluate appropriately the maximum story responses of a building due to the lack of consideration of the higher mode effect.
- 4) A modal analysis using MAP procedure can be conducted with lateral forces applied for each story given by Equation (10) and the SRSS method.
- 5) The modal analysis combined MAP for the first mode and elastic analysis for the second and third modes is more appropriate for evaluating the maximum story responses than that with MAP only. This means that the higher mode effect in the responses may be elastic.

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