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A STUDY ON DISPLACEMENT RATIO SPECTRA OF CONSTANT YIELDING STRENGTH FOR SEISMIC DESIGN

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SUMMARY

A through study on the inelastic response spectra can provide significant clues to the seismic design of structures, in particular, when the strong earthquake ground motions drive structures entering its inelastic deformation stage. In this paper, the response spectra of 544 strong-motion records with engineering significance were calculated and a statistical analysis of their inelastic response spectra revealed that the site conditions, intensity of ground motions, yielding strength level and period of structures will give important effects on the inelastic spectra. Finally, an new experimental expression of the inelastic spectra in terms of period, yielding strength and inelastic displacement ratio which permits the estimation of maximum inelastic displacement demands on a structure from maximum elastic displacement demands, was obtained and has been recommended to adopt in seismic design code.

INTRODUCTION

Response spectra theory is one of basic theories in earthquake engineering. It is widely used in most seismic codes of the world. Recently there has been a growing interest in displacement-based seismic design (DBSD). Displacement instead of lateral force is considered as the basic demand parameter for the design, evaluation and rehabilitation of structures in the DBSD. In order to implement the DBSD into the structural engineering practice, it is essential to recommend simplified analysis procedure for evaluation of inelastic displacement of structure under strong ground motions. It is an effective and simple way to evaluate the inelastic displacement of structure by taking into account the relationship (displacement ratio) between the maximum inelastic displacement demands of non-linear SDOF systems and the maximum elastic displacement demands of linear SDOF systems (Jorge R.G. et. al [7]). Thus, the displacement ratio spectra of constant yielding strength that can provide the displacement ratio between the maximum inelastic displacement demands of non-linear SDOF systems and the maximum elastic displacement demands of linear SDOF systems are important.

The displacement ratio spectra of constant yielding strength can provide not only the information on the simplified calculation procedure for inelastic displacement of structure, but also some basic data for

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DBSD. In addition, the displacement ratio spectra of constant yielding strength can provide significant clues to understand the relation between strong ground motion and dynamics characteristics of structure. The ratio spectra of constant yielding strength, which is one of important form of inelastic response spectra, will be investigated in details in this paper.

Many researchers have investigated the constant-ductility inelastic displacement ratio spectra in the past (Veletsos AS et. al [1]; Qi X. and Moehle JP [2]; Miranda E. [3][4][5][6]). The constant-ductility inelastic displacement ratio spectra are very useful in the preliminary design of new structures where the global displacement ductility capacity is known. However, in the evaluation of existing structures with known the lateral strength the main interest is to determine the maximum inelastic displacement of structure under strong ground motions of different intensities. Jorge R.G. et. al [7] concluded the use of constant-ductility inelastic displacement ratio spectra would underestimate the expected value of the maximum deformations of structures with known strength. Hence, inelastic displacement ratios corresponding to systems with equal relative lateral strength (lateral yielding strength relative to the lateral strength required to maintain the system elastic) are particular useful when evaluating the existing structures. Shimazaki K. and Sozen MA. [8] and Whittaker et al.[9] discussed some characteristics of displacement ratio spectra of constant yielding strength, but none of them provided expressions of the spectra. Jorge R.G. and Eduardo Miranda [7] investigated the displacement ratio spectra of constant yielding strength corresponding to firm sites, but the influence of structure damping and hysteretic model on the spectra is not presented.

In China, the study on inelastic spectra began in 1960's (Wang Qianxin [10]). The displacement ratio spectra of constant yielding strength are preliminarily investigated by using 14 strong ground motion records (Wei Chengji, [11]). A bi-linear shape expression of mean displacement ratio spectra of constant yielding strength is gained with the few records. Recently Xiao Mingkui et al. [12] provided valuable information regarding the displacement ratio spectra of constant yielding strength. But it is unreasonable to classify the records only according to the characteristic period of the record rather than the site characteristics of records. As we know, besides the site conditions, many other factors can affect the characteristic period of records, such as magnitude, distance. In addition, the conclusion that limiting period is only the function of site conditions and does not depend on the yielding strength coefficient is unreasonable. The limiting period is the period dividing the short-period spectral region where spectra are strongly period and the long-period spectral region where spectra tend to be constant. Yang Songtao et. al [13] discussed the main characteristics of the normalized displacement spectra. But how to use the normalized displacement spectra in practice is not involved.

The displacement ratio spectra of constant yielding strength are investigated in details in this paper. On the results of a statistical study of the ratio of maximum inelastic displacement analysis of SDOF under the strong ground motion, the mean displacement ratio spectra of constant yielding strength corresponding to four classifications of site conditions are gained. The effects of local site conditions, intensity, level of lateral strength, period of vibration are investigated. A new expression of the inelastic spectra is obtained and it can be adopted in seismic code. To be more important, an in-depth investigation of the influence of structure damping and hysteretic model on displacement ratio spectra of constant yielding strength is also performed. From qualitative point, the investigation of the influence of damping and hysteretic model on displacement ratio spectra of constant strength is rare in the past. As the space limits, this part content will be introduced in another paper.

SOME BASIC PROBLEMS ABOUT THE SPECTRA

Earthquake ground motions used in the study

A total of 544 earthquake accelerations recorded in 19 different earthquakes from 1949 to 1994 is used in the study (see table 1). All the earthquake accelerations are recorded on free field stations or in the first floor of low-rise buildings.

The records are divided into four groups according to their local site conditions at the recording station (see table 2). The first group consisted of 70 records on stations located on rock with average shear wave velocities between 760m/s and 1500m/s. The second group consisted of 188 records obtained on station on stiff soil with average shear wave velocities between 360m/s and 760m/s while the third group consisted of 274 records recorded on station on medium soil with average shear wave velocities between 180m/s and 360m/s. The fourth group consisted of 12 records recorded on stations located on soft soil with average shear wave velocities below 180m/s.

Table 1 the strong ground motions used in this paper

Earthquake	Data	Magni- -tude	No. of Record
Western Washington	1949	7.1	2
Kern County	7/21/1952	7.4	3
Parkfield	6/28/1966	6.1	4
San Fernando	2/9/1971	6.6	4
Tangshan	1976	7.8	3
Coyote Lake	8/6/1979	5.8	4
Imperial Valley	10/15/1979	6.5	20
Livermore Valley	1/24/1980	5.8	5
Livermore Valley	1/27/1980	5.8	6
Westmoreland	4/26/1981	5.6	5
Morgan Hill	4/24/1984	6.2	7
Mexico	1985	8.1	5
Palm Springs	7/8/1986	5.9	13
Whittier	10/1/1987	6.0	47
Gengma	1988	4.6	2
Loma Prieta	10/18/1989	6.9	42
Petrolia	4/25/1992	7.1	5
Landers	6/28/1992	7.3	31
Northridge	1/17/1994	6.7	70

Table 2 Classification of site conditions

Site class	Site general description	Shear Wave Velocity Vs (m/sec)	Number of records
A	rock	1500≥ Vs>760	70
B	Stiff soil	760≥ Vs>360	188
C	Medium soil	360≥ Vs≥ 180	274
D	Soft soil	180>Vs	12

Two basic concepts

The inelastic displacement ratio, X_p / X_e, is defined as the maximum lateral inelastic displacement demand, X_p, divided by the maximum lateral elastic displacement demand, X_e, of systems with the same period of vibration when subjected to the same ground motion. X_p is computed in systems with constant yielding constant yielding strength relative to the strength required to maintain the system elastic (i.e. constant strength). Here the relative lateral strength is measured by the yielding strength coefficient C_y, which is defined as

$$C_y = F_y(\mu = \mu_i) \big/ F_y(\mu = 1) \tag{1}$$

where $F_y(\mu = \mu_i)$ is the lateral yielding strength required to maintain the displacement ductility ratio demand, μ equal to a pre-determined target ductility ratio, μ_i ,when subjected to the given ground motion. $F_y(\mu = 1)$ is the lateral yielding strength required to avoid yielding in the system under the same given ground motion.

Consideration of earthquake intensity in the study

If the waveforms of two accelerations are identical and only their amplitudes vary in the same scale, the shapes of their elastic response spectra are also the same and the amplitudes of their elastic response spectra vary proportionally with accelerations. However, will the same characteristics occur in the displacement ratio spectra of constant yielding strength?

Let us observe the characteristics of displacement ratio spectra of constant yielding strength for a nonlinear SDOF with lateral yielding strength, F_y, subjected to the two records. Assuming the maximum accelerations of the two records are a_1 and a_2 $(= ka_1, k$ is a constant value), respectively. And if F_{e1} and F_{e2} are the lateral yielding strength required to avoid yielding in the system under the two records with maximum acceleration a_1 and a_2, respectively, F_{e2} equals to kF_{e1}, and the yielding strength coefficient of displacement ratio spectra of constant yielding strength for the record is $C_{y1} = {F_y}/{F_{e1}}$.

If increasing the lateral yielding strength of the nonlinear SDOF k times than the original value F_y, the yielding strength coefficient of displacement ratio spectra of constant yielding strength for the record with maximum acceleration a_2 is $C_{y2} = {kF_y}/{F_{e2}} = {kF_y}/{kF_{e1}} = C_{y1}$. That is to say, as long as the yielding strength coefficients are equivalent, the displacement ratio spectra of constant yielding strength corresponding to the two records are the same.

From above observations, we can draw a more general conclusion: for more records whose waveforms are identical but only amplitudes vary in the same scale, as long as the yielding strength coefficients are equivalent, the displacement ratio spectra of constant yielding strength corresponding to the records are the same. The conclusion is very important for seismic design. Because in seismic design and analysis, the displacement ratio spectra of constant yielding strength under the same record corresponding to different intensity is often analyzed. The significance of the above conclusion is that whether the intensity of a record is high or low, as long as the yielding strength coefficients are equivalent, the displacement ratio spectra of constant yielding strength under the record corresponding to different intensity are the same. In another word, the study of displacement ratio spectra of constant yielding strength corresponding to different intensity may be converted into the study of displacement ratio spectra of constant yielding strength corresponding to different yielding strength coefficients.

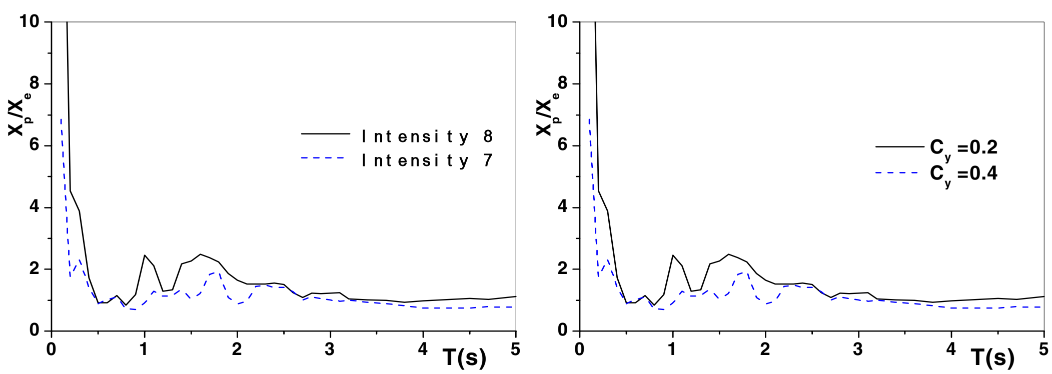


Fig. 1 Consideration of intensity in the displacement ratio spectra of constant yielding strength for (a) different intensity; (b) different yield strength coefficient

Take the 1940 El Centro (NS) record for example. Assuming the maximum acceleration of intensity 7 and 8 are 220gal and 440gal, respectively, and the yielding strength coefficient of displacement ratio spectra of constant yielding strength for the record corresponding to intensity 7 is 0.4, the yielding strength coefficient of displacement ratio spectra of constant yielding strength for the record corresponding to intensity 8 is 0.2. The displacement ratio spectra of constant yielding strength for the 1940 El Centro (NS) record corresponding to intensity 7 and 8 are shown in figure 1(a). The displacement ratio spectra of constant yielding strength for the 1940 El Centro(NS) record for yielding strength coefficient 0.2 and 0.4 corresponding to intensity 7 are shown in figure 1(b). We can see that the two figures are the same.

The dynamics parameters of system
In seismic response analysis, the dynamic parameters of structure mainly include four, namely: damping, yielding strength coefficient, hysteretic model and structure period.

The elasto-plastic model is assumed in this paper. The displacement ratio spectra for yielding strength coefficient of 0.2, 0.3, 0.4, 0.5, 0.6 with total 39 periods of vibration (from 0.05s to 5s) are investigated. The damping ratio is assumed 0.05.

STATISTICAL RESULTS OF THE MEAN DISPLACEMENT RATIO SPECTRA OF CONSTANT YIELDING STRENGTH

A total of 106,080 inelastic displacement ratios are computed as part of this study (corresponding to 544 ground motions, 39 periods of vibration and 5 levels of relative strength). Mean inelastic displacement ratios are then computed by averaging results for each period, each yielding strength coefficient, and each group of site conditions.

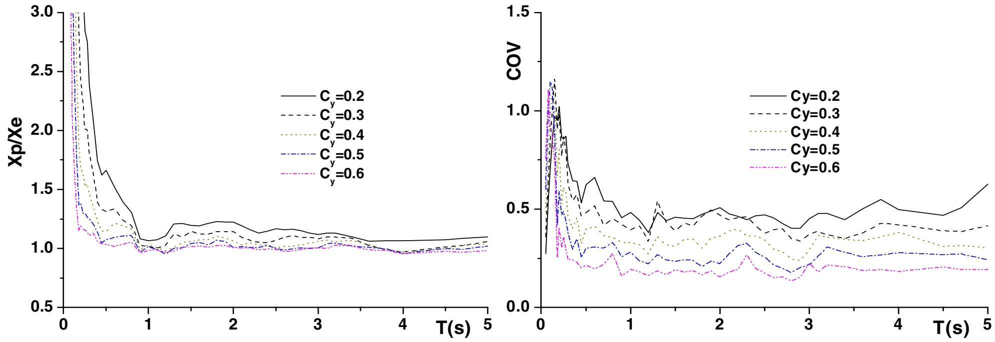
Main characteristics of the mean displacement ratio spectra of constant yielding strength
Figure 2 shows the mean inelastic displacement ratio spectra of constant yielding strength and their COV corresponding to the four groups of site conditions considered here. From the figure, some characteristics of the spectra can be obtained.

(1) Generally, for a given period, displacement ratios decrease with the yielding strength coefficient increasing. And as the yielding strength coefficients increase, the displacement ratios will be closer.

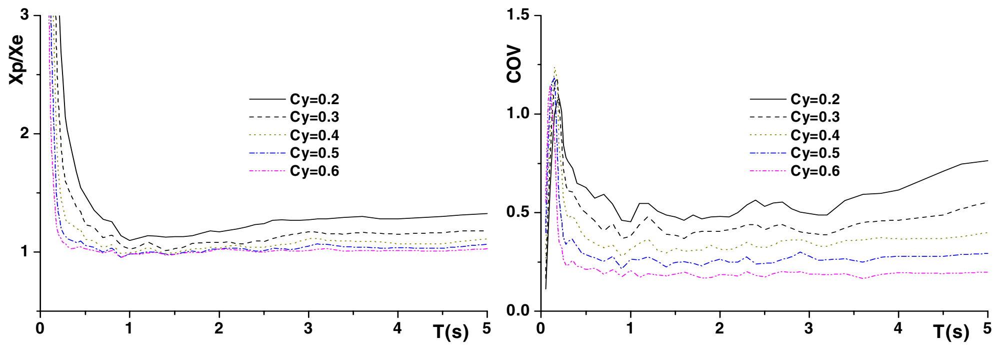
(2) It can be seen that, in general, the spectra exhibit the same trend regardless of the local site conditions. In the short-period region (about less than 1s), inelastic displacement ratios are strongly dependent on the period of vibration and on the yielding strength coefficients. In the long-period region, inelastic displacement ratios tend to be constant. The limiting period depends on the yielding strength coefficient and the local site conditions. The limiting period is the period dividing the short-period spectral region where spectra are strongly period and the long-period spectral region where spectra tend to be constant. In general, as the yielding strength coefficients increase, the limiting period will decrease. The limiting periods are different for different site conditions. This is different from the conclusion that the limiting period is only the function of site conditions and does not depend on the yielding strength coefficient made by Xiao Mingkui (2000).

(3) A common and effective way to quantify the dispersion is through the coefficient of variation (COV), which is defined as the ratio of the standard deviation to the mean. It can be seen that, dispersion is particularly high for short-period region and in the long-period region dispersion will be little. The dispersion decreases with the yielding strength coefficient increasing. In addition, the number of records corresponding to soft site conditions is small, so the dispersion of spectra for soft site conditions is much larger than that of the other site conditions.

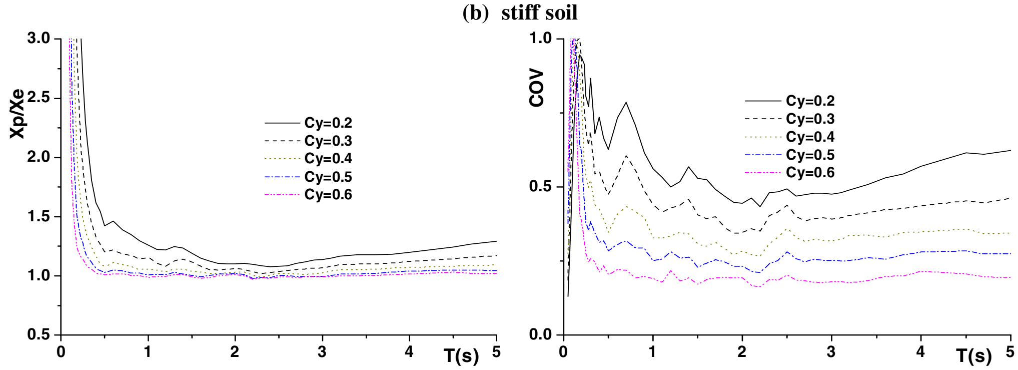
(4) In order to assess the effect of site conditions on the spectra, ratios of mean displacement ratios on each group to mean displacement ratios corresponding to rock site conditions are computed. The results are shown in figure 3. As the number of records corresponding to soft site conditions is small,



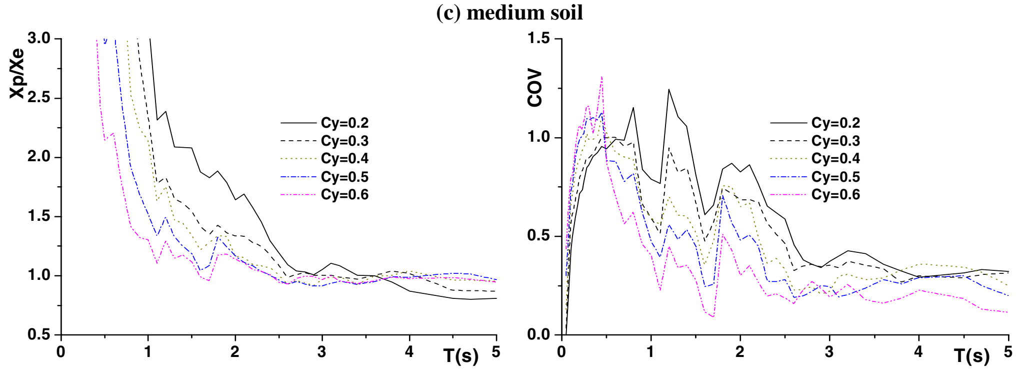
(a) rock



(b) stiff soil



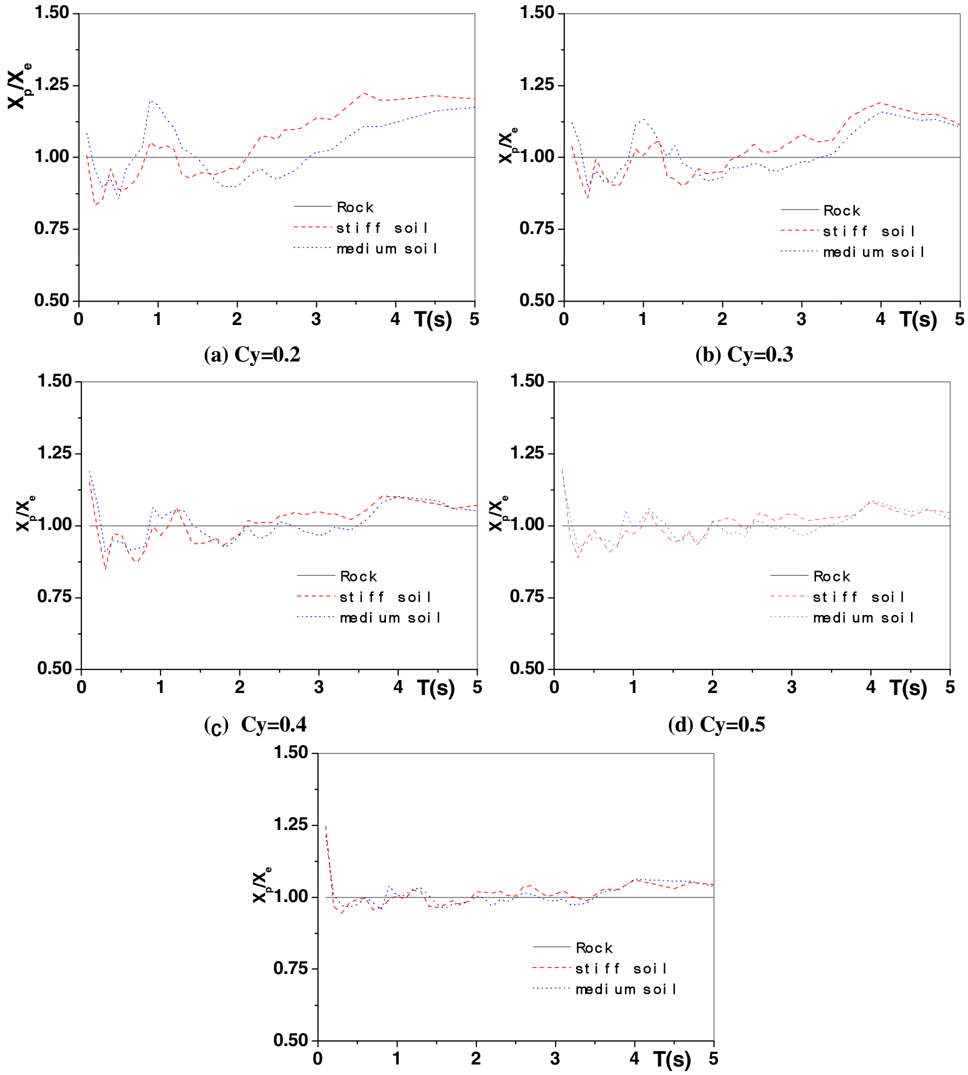
(c) medium soil



(d) soft soil

Fig.2 Mean displacement ratio spectra of constant yielding strength and their COV in different site conditions

the ratio for soft site conditions is not shown in figure 3. The figure shows that, for a given yielding strength coefficient, the difference of the other three site conditions (firm site conditions) spectra is not much large and are typical smaller than 20%. As yielding strength coefficient increasing, the difference will be decreasing. In another word, for different yielding strength coefficient, the effects of site conditions on spectra are different. It is noted, if the spectra corresponding to soft site conditions can be available, the effect of site conditions on the spectra will be large.



(a) Cy=0.2
(b) Cy=0.3
(c) Cy=0.4
(d) Cy=0.5
(e) Cy=0.6

Fig. 3 Comparison of mean displacement ratio spectra of constant yielding strength for different site conditions

Nonlinear regression analysis of the mean displacement ratio spectra

For displacement-based design and, in general, in earthquake-resistant design it is desirable to have simplified expressions of displacement ratio spectra of constant yielding strength. Many factors can affect displacement ratio spectra of constant yielding strength, such as magnitude, distance, site conditions, damping, yielding strength coefficient, hysteretic model and period of vibration et. al.. In nonlinear regression analysis, yielding strength coefficient, period of vibration, site conditions are considered, which are the most important factors influencing spectra. The proposed hyperbola equation for mean displacement ratio spectra of constant yielding strength is given by:

$$T^{a}(S-b)^{c}=d \tag{2}$$

where S is the displacement ratios, T is the period of vibration, a, b, c and d are constants that depend on the site conditions and yielding strength coefficient. It should be noted that, as the number of records corresponding to soft soil site conditions is small and equation (2) is not so good to regress mean displacement ratio spectra of constant yielding strength for soft soil site conditions, the constant d is given for 1 in the soft soil site conditions spectra regressions.

The constants a , b , c and d are expressed as the a function yielding strength coefficient:

$$F(C_y) = b_1 C_y^3 + b_2 C_y^2 + b_3 C_y + b_4 \quad (3)$$

where $F(C_y)$ are the constants a , b , c and d , b_1 , b_2 , b_3 and b_4 are constants values which are reported in table 3. Figure 4 shows that the simplified curves give an accurate approximation of the mean displacement ratio spectra of constant yielding strength.

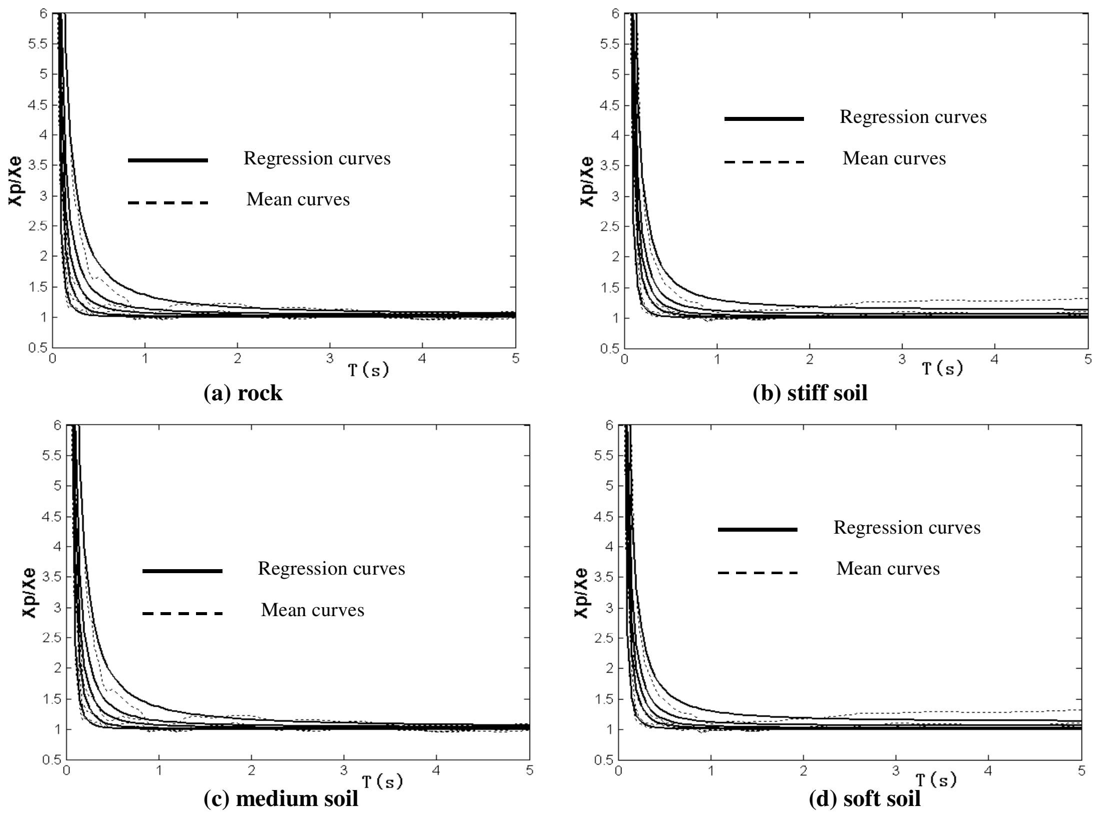


Fig.4 Comparison of regression curve with the mean curve (Note: the curves from top to bottom are the Curves corresponding to different yielding strength coefficient (0.2, 0.3, 0.4, 0.5) ,respectively)

CONCLUSIONS

The purpose of study on mean displacement ratio spectra of constant yielding strength is to assess the inelastic displacement ratios that permit the estimation of maximum inelastic displacements from maximum elastic displacements for existing structures built on rock, stiff soil, medium soil soft soil site conditions, whose lateral strength is known. A statistical study has been performed of inelastic displacement ratios computed for SDOF systems with an elasto-plastic hysteretic behavior with different levels of lateral strengths relative to the strength required to maintain the system elastic when subjected to 544 earthquake ground motions. A new experimental expression of the inelastic spectra was obtained. The following conclusions are drawn from the results of this investigation.

(1) The inelastic displacement ratio spectra of constant yielding strength can provide significant clues to evaluate the inelastic displacement of existing structures, develop the displacement-based seismic design and understand the relations between strong ground motion and dynamics characteristics of structures.

(2) Whether the intensity of a record is high or low, as long as the yielding strength coefficients are equivalent, the displacement ratio spectra of constant yielding strength under the record corresponding to different intensity are same. In another word, the study of displacement ratio spectra of constant yielding strength corresponding to different intensity may be converted into the study of displacement ratio spectra of constant yielding strength corresponding to different yielding strength coefficient.

Table 3 Parameters in the regression curves of displacement ratio spectra of constant yielding strength

Site conditions	Constants	b_1	b_2	b_3	b_4
Rock	*a*	21.660	-23.477	7.531	-0.373
	b	2.770	-3.491	1.289	0.888
	c	7.852	-7.821	1.841	0.156
	d	-30.682	35.449	-13.158	2.203
Stiff Soil	*a*	15.023	-15.158	4.205	0.067
	b	-1.477	2.819	-1.785	1.382
	c	2.501	-1.347	-0.565	0.420
	d	-22.860	25.122	-8.669	1.540
Medium Soil	*a*	36.014	-39.693	12.969	-0.898
	b	5.811	-7.691	3.147	0.622
	c	11.180	-11.064	2.637	0.120
	d	-51.863	59.886	-21.605	3.061
Soft Soil	*a*	29.929	-35.102	12.607	-1.202
	b	32.837	-38.817	13.925	-1.298
	c	10.964	-13.454	4.444	0.770

(3) Generally, for a given period, displacement ratio decreases with the yielding strength coefficient increasing. And as the yielding strength coefficient increase, the displacement ratio will be closer.

(4) In general, the spectra exhibit the same trend regardless of the local site conditions. In the short-period region (about less than 1s), inelastic displacement ratios are strongly dependent on the period of vibration and on the yielding strength coefficient. In the long-period region, inelastic displacement ratios tend to be constant. The limiting period depends on the yielding strength coefficient and the local site conditions. The limiting period is the period divides the short-period spectral region where spectra are strongly period and the long-period spectral region where spectra tend to be constant. As the yielding strength coefficient increases, the limiting period will be decreasing. The limiting period will be different for different site conditions.

(5) The effect of site conditions on the inelastic displacement ratio spectra of constant yielding strength is evident.

(6) In nonlinear regression analysis of the inelastic displacement ratio spectra of constant yielding strength yielding strength coefficient, period of vibration, site conditions are considered, which are the most important factors influencing spectra.

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