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SHOULD WE RETROFIT STRUCTURES FOR EARTHQUAKE PROTECTION?

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SUMMARY

Under which conditions is it economically convenient to seismic retrofit a structure? This paper present a procedure whose results allow to give a simple answer to the above question, central in earthquake engineering and, in a broader context, in any man-made activity. The procedure uses, as a starting point, the results of a standard reliability analysis conducted on the structure in its present state and after retrofitting. Once these are known, expressed in terms of mean rate of exceedance of specified limit states, it is shown that it is possible to compute whether the upgrading should be made after all and how convenient it is.
The assumptions to make the problem tractable are clearly listed and appear, in authors's viewpoint, quite reasonable. The final results are presented both in diagrams and with a simple formula. The method is finally applied to the case of bridges on an Italian highway stretch [1].

INTRODUCTION

Seismic retrofitting of existing structures is a key issue in any earthquake – prone region. Moderate to strong earthquakes, in fact, claim many lives and cause extensive damage to structures and infrastructures [2, 3] at a regional scale, halting normal life and the economy.
In developed countries, earthquakes may cause severe economic loss also at a national scale; estimates of the monetary damage [4, 5] put the figure for direct costs at some points per cent of the gross domestic product (gdp) and estimates of total costs, sum of direct and indirect costs, indicate [5] values almost double as shown in Table 1.

Table 1: damages expressed in monetary values for some past earthquakes (1)

country	earthquake	year	costs (1995 billion US$)	costs/gdp (%)
Japan	Great Hanshin	1995	110	2.3
California, USA	Northridge	1994	20	2.4
California, USA	Loma Prieta	1989	7.1	0.9

(1): for Northridge and Loma Prieta, the gdp considered is the one of the State of California. In the costs/gdp ratio, costs and gdp are both relative to the year of the earthquake occurrence

To cope with the problem, a variety of technical solution has been implemented for seismic retrofitting, ranging from strengthening of parts of the structure to seismic isolation and active control.

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However a systematic application of retrofitting techniques on a national, or even regional, scale has not been undertaken, because of the high costs involved.

This is true both for developed and developing countries in which seismic risk is a concern. In Italy, where most of the territory is at risk, no special public policy is adopted for earthquake protection of existing structures, apart from temporary and partial eligibility of retrofitting costs for fiscal deduction. Earthquake insurance is moreover used by a negligible minority and so the costs of repairing the earthquake damages have been up to now partially footed by the State (only for the direct part of the costs). This situation is similar to Japan, another country where earthquake insurance is not used (only 3% of home owners in the Kobe area were insured) and different from the US, where about half properties are covered by insurance. The scheme in the US looked like a working one because insurance costs for owners of important properties like industrial plants were a function of the structural risk of failure (considering both *hazard*, earthquake intensity at the site, and *fragility*, the capability of the structure to withstand the action) and so owners were economically motivated to retrofit their facilities. For houses and less important properties, the authors's information is that no or very approximate seismic assessment was made so that insurance costs were more weakly correlated with actual seismic risk. The drawback of the scheme showed up right after the Northridge event: 11.4 billion US$ worth claims for property damage, about 4 billion US$ collected as insurance fees between 1970 and 1994 [5]. Insurers decided then to drop the residential earthquake insurance market.

The drawback is, in the authors's opinion, that earthquake damage is highly random both in place and in time and in magnitude and so, in order to have the economics of investment under risk work, costs and risk must be spread on a large community for a long period of time. Financial resources ought to be used to make preventive retrofitting (isn't it common knowledge that prevention is better than cure?) under a compulsory scheme (retrofit or pay the damage by yourself) fitted to the local conditions and customs.

A second important issue concerns cost-evaluation: in Europe and especially in Italy, unbiased cost – evaluation is more difficult than elsewhere because most civil engineering works are old or ancient [6], often with artistic or historical values, and have not been designed for seismic action, and hence the assessment of their safety is subject to a high degree of uncertainty.

These two issues (Is retrofitting worth the cost? How can costs be computed?) have been dealt with in state-of-the-art literature. Cost – benefit models have been proposed for the evaluation of the profitability of public or private investment in seismic retrofitting [7, 8, 9]. These models permit comparison among alternatives by assigning monetary values to costs and benefits happening in the future and discounting them at the present time accounting for inflation and interest rate. Some of the most advanced models also account for the system behaviour of the networks [10, 11]. Nonetheless, their application to the real world is often problematic because of their complexity and of the subjectivity in the phase of assignment of monetary values to costs and benefits of unhomogeneous nature and of difficult evaluation.

The model presented in this study tries to address both issues. It does not solve the problem of cost – evaluation but, by using an agile representation of the economic problem of retrofitting, identifies the few basic variables which govern the problem. Results readily exploitable are then produced performing parametric studies.

In the model, structural behavior is represented with a fragility curve, relative to a selected limit state, which may be of the serviceability or ultimate type. After structural retrofitting the fragility curve is updated. The seismic action is represented with the classical Cornell's model [12]. The convolution of the structural fragility curve with the probability density function (pdf) of the earthquake intensity, yields the mean rate in time of structural failures. These are Poisson distributed, with a change in the mean rate when seismic upgrading is implemented. We next compute the *Annual Equivalent* cost for the problem thus modeled and minimize it with respect to the time of retrofitting. The option which makes the equivalent cost minimum is defined as the most convenient one. Different options account for design type, its cost and the time of implementation.

It must be certainly highlighted that this study is not the final answer to the problem of seismic retrofitting: first for technical reasons, because the issue of cost estimation is here tackled only in a parametric form and, once one goes down to real world problems, things certainly become more complex and subjective; and above all because the initiative to widespread seismic retrofitting typically comes from governmental decisions through various means (be it legal and/or financial ones) and is therefore a classical political problem, which involves weighting in also such diverse issues like funds availability, global economic conditions, importance of the involved industries, job creation. However we think that application of this procedure can give valuable information to decision makers and may remove a significant amount of uncertainty from the overall decision problem.

MODEL OF THE PROBLEM

In the proposed model, the structure is described by its fragility curve $P_f^{(l,z)}(z)$: the function expressing the probability of exceedance of the limit state l vs. an earthquake intensity parameter z, usually the Mercalli intensity or the peak ground acceleration. Methods to establish structural fragility curves are well known [13, 14] and will not be dealt with in this paper.

In the present version of this study, ageing effects [15, 16] are not considered. Ageing effects would cause the structural fragility function to be continuously time-dependent and this would cause sensible increase in complexity of the mathematical passages which follow. On the other hand, any increase in fragility due to age is negligible in most big reinforced concrete infrastructures, like bridges, during their normal economic life, provided that ordinary maintenance operations are routinely carried on and that the structure has been originally well designed and constructed. The effect of ageing might however be included in an improvement of this method while keeping the same solution framework.

The seismic action at the site, following Cornell's method [12], is modeled as a Poisson process with mean rate equal to ν. Earthquake intensity Z, given an event, is distributed according to the Gutenberg - Richter law:

$$F_z(x) = \frac{e^{-\beta x} - e^{-\beta z_{\min}}}{e^{-\beta z_{\max}} - e^{-\beta z_{\min}}} \tag{1}$$

between the minimum and maximum values z_{min} and z_{max}. β is the so – called *severity* parameter for the site. Physically, it is the slope of the minimum error interpolation line in a diagram having the recorded intensities on the abscissa axis and the natural logarithm of the frequency on the ordinate axis. In equation (1), and in the following, $f_X(y)$ and $F_X(y)$ respectively indicate the probability and cumulative distribution functions of the random variable X evaluated at y.

The probability $P_f^{(l,e)}$ of exceedance of the limit state l, henceforth indicated as failure, given an event is equal to:

$$P_f^{(l,e)} = \int_{z_{\min}}^{z_{\max}} P_f^{(l,z)}(x) f_z(x) dx \tag{2}$$

In equation (2), the familiar resistance and action terms may be recognized: the first term, $P_f^{(l,z)}(x)$, is inversely proportional to the structural resistance under the action x, while the second term $f_z(x)$ expresses the likelihood of action x. In short, equation (2). says the probability of failure is obtained summing up the contributions at each value of the action x. The distribution of failures in time is then a homogeneous Poisson process with mean rate equal to:

$$\lambda_l = \nu P_f^{(l,e)} \tag{3}$$

The time τ elapsed between the beginning of exposure of the structure to earthquakes (at time t=0) and the first failure(at time t=τ) is a random variable exponentially distributed:

$$\begin{gathered} F_\tau(t) = P(\tau \le t) = 1 - e^{-\lambda_l t} \\ f_\tau(t) = \lambda_l e^{-\lambda_l t} \end{gathered} \tag{4}$$

Now, let T the time at which retrofitting is implemented. After upgrading, the structural fragility curve $P_f^{(l,z)}(z)$ will have changed and hence the values of the probability of exceedance of the limit state l (equation (2)) and the mean rate of collapses (equation (3)) will have (hopefully) decreased. Let Λ_l the new value for the latter variable. As it was prior to retrofitting, the time to the first failure is exponentially distributed, equation (4), with λ_l substituted by Λ_l and t substituted by $(t-T)$, time elapsed from T.

This paper attempts at answering the following question: is it economically convenient to retrofit the structure at time T? The answer to this question, central in earthquake engineering, will also yield, as will be illustrated in the coming sections, a criterion to discriminate among different retrofitting designs and choose both the most convenient one and the time at which it is best implemented.

SOLUTION SCHEME

Let us first make the following definitions:

- l=particular limit state which we are considering;
- $C_l(t)$=cost (born at time t) to restore the structure to its previous functionality level if the limit state l is exceeded at t;
- $S_l(T)$=cost (born at time T) to upgrade the structure. After upgrading, the mean rate of collapses changes from λ_l to Λ_l;
- L=economic life of the structure after upgrading;
- i_f=money interest rate for the owner of the structure;
- f=inflation rate;
- i^*=inflation-free money interest rate for the owner of the structure=$(i_f-f)/(1+f)$;
- i=inflation-free logarithmic interest rate=$log_e(1+i^*)$.

and the following assumptions:

1. interest and inflation rates are constant in time, i.e. $i_f(t)=i_f$ and $f(t)=f$;
2. the cost to restore the structure to its previous functionality level, $C_l(t)$ is independent of upgrading. Since in the computations that follow we will be using only inflation-free interest rates, and thus we will be moving in an inflation-free environment, because of this assumption we can set $C_l(t<T)=C_l(t\geq T)$;
3. $C_l(t)$ and $S_l(t)$ can be correctly estimated using the prices of today, i.e. $C_l(t)=C_l(t=0)$ and $S_l(t)=S_l(t=0)$;
4. the time required to upgrade the structure at T is negligible i.e. mean rate of failure $(t<T)=\lambda_l$ and mean rate of failure $(t\geq T)=\Lambda_l$;
5. after retrofitting, the structure has a service life equal to L, after which it has no economic value. Before retrofitting, the structure has a service life lower or equal to L, after which it has no economic value;
6. the benefits in time deriving from utilization of the structure are independent of upgrading. Since the cash flow of costs and benefits will be expressed in terms of their *Annual Equivalent* [17], in a procedure to minimize costs, benefits can be canceled out.

From assumptions 2 and 3, it follows that $C_l(t)=C_l(t=0)=C_l$ and $S_l(t)=S_l(t=0)=S_l$. Assumptions 1 and 3 are quite reasonable in developed economies. The remaining assumptions are made in order to keep the problem simple. There are clearly cases that do not respect these assumptions, e.g. long service interruption for upgrading vs. assumption 4, but for many real cases the method should be applicable.

In the computations that follow, for mathematical convenience, only i, the inflation-free logarithmic interest rate, will be used; notice, however, that for small values of the inflation f and money interest i_f rates, the approximation $i\approx i^*\approx i_f-f$ is correct within a few points per cent. When using i, the familiar formula to bring capital forth in time:

$$K(t)=K(t=0)\left(1+i^*\right)^t \tag{5}$$

in which $K(t)$ is the capital at time t deriving from an investment done at time t=0 with interest rate i^*, can be rewritten as:

$$K(t) = K(t=0)\exp(it) \tag{6}$$

Now, following equation (4), in the range [t; $t+dt$], the expected cost due to exceedance of limit state l may be written as:

$$\begin{aligned} EC_l(t) &= \text{Prob}(\tau \in [t; t+dt])C_l = f_\tau(t)\,dtC_l = \lambda_l e^{-\lambda_l t} C_l dt \quad && t < T \\ EC_l(t) &= \Lambda_l e^{-\Lambda_l (t-T)} C_l dt && t \geq T \end{aligned} \tag{7}$$

Costs deriving from use of the structure may be then summed up in the cash-flow of Figure 1.

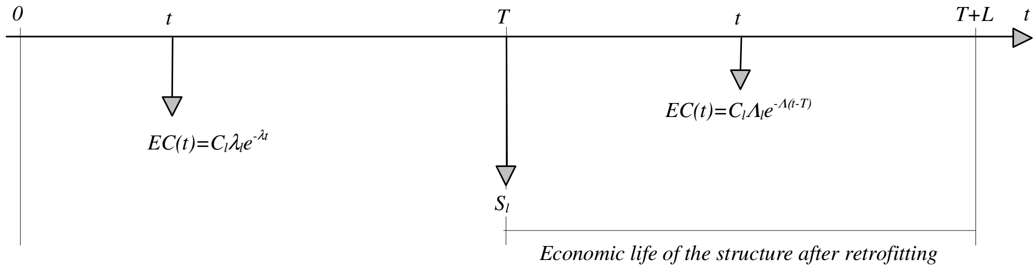

Figure 1: cash-flow deriving from use of the structure exposed to the risk of earthquakes

The Annual Equivalent AE to the cash - flow in Figure 1 can be computed as follows [17]:
(i) compute the present value PV of the expenditures born between 0 and T, at T, and between T and $T+L$:

$$\begin{aligned} PV_{[0;T]} &= \int_0^T [C_l \lambda_l \exp(-\lambda_l t)][\exp(-it)]\,dt \\ PV_T &= S_l \exp(-Ti) \\ PV_{[T;T+L]} &= (\exp(-Ti)) \int_0^L [C_l \Lambda_l \exp(-\Lambda_l t)][\exp(-it)]\,dt \\ PV &= PV_{[0;T]} + PV_T + PV_{[T;T+L]} \end{aligned} \tag{8}$$

(ii) compute, on the period $T+L$, the value of AE, the annual equivalent to PV:

$$AE = PV \frac{[\exp(i) - 1][\exp(i(T+L))]}{\exp[i(T+L)] - 1} \tag{9}$$

(iii) substitute PV of equation (8) in equation (9). After some algebra, it follows:

$$AE = \frac{C_l}{\exp(iT)\exp(iL) - 1} \left\{ \frac{\lambda_l}{\lambda_l + i} [\exp(iT) - \exp(-\lambda_l T)] + \frac{S_l}{C_l} + \frac{\Lambda_l}{\Lambda_l + i} \left[1 - \frac{1}{\exp(L\Lambda_l)\exp(Li)} \right] \right\} \tag{10}$$

Now, we are interested in studying the variability of expression (10) with respect to T. Although the derivative of expression (10) with respect to T has a nasty aspect, and cannot be employed in a simple way to solve the problem of minimization, expression (10) is found to be monotonous in the range $T \in [0;L]$. This means that it is either convenient to do the upgrading at time $T=0$ ($AE(T)$ is a monotonic increasing function) or never to do it ($AE(T)$ is a monotonic decreasing function). This behavior has been proven numerically, allowing large variability to i, L, λ_l, Λ_l with the only (reasonable) constraints that $S_l \leq C_l$ (the cost to upgrade is lower than or equal to the cost of failure) and $\lambda_l \geq \Lambda_l$ (upgrading does not make things worse).

Developing further the property of monotony of $AE(T)$ in equation (10), it is found that, in the range $T \in [0;L]$, the upgrading is convenient if the following condition is met:

$$\begin{array}{llll} R_l + F_l < f_l & \Leftrightarrow \quad AE(T) \text{ monotonically increasing} & \Rightarrow & \text{upgrade at } T = 0 \\ R_l + F_l > f_l & \Leftrightarrow \quad AE(T) \text{ monotonically decreasing} & \Rightarrow & \text{do not upgrade} \end{array} \tag{11}$$

in which:

$$\begin{gathered} R_l = \frac{S_l}{C_l} = \text{costs ratio} \\ F_l = \frac{\Lambda_l}{\Lambda_l + i} i_{\Lambda_l} \qquad f_l = \frac{\lambda_l}{\lambda_l + i} i_{\lambda_l} \\ i_{\Lambda_l} = \frac{\exp\left[L(i+\Lambda_l)\right]-1}{\exp\left[L(i+\Lambda_l)\right]} \qquad i_{\lambda_l} = \frac{\exp\left[L(i+\lambda_l)\right]-1}{\exp\left[L(i+\lambda_l)\right]} \end{gathered} \tag{12}$$

The letter i to indicate i_{λ_l} and i_{Λ_l} in equation (12) has been chosen on purpose since they physically are (rather awkward) interest rates. The numerator of the former, i_{λ_l}, is a difference of capitals: if one invests capital 1 at time t=0, with interest rate $i+\lambda_l$, after L time periods one ends up with a capital equal to $\exp[L(i+\lambda_l)]$. So, the ratio defining i_{λ_l} is a capital difference (value at $t=L$ minus value at $t=0$) divided by a capital (value at $t=L$). The same holds for i_{Λ_l}. Notice also that R_l, F_l, f_l, i_{λ_l} and i_{Λ_l} in equation (12), are all bounded between 0 and 1.

It can further be noticed that the value of the minimum of the annual equivalent AE in equation (10) (be it at T=0 or $T=L$) decreases with decreasing values of (R_l+F_l), the left-hand side of equation (11).

Inequalities in equation (11) are nice expressions to assess the economic convenience of any retrofitting design. Their use can be summed up in what follows: (i) for the structure to upgrade, first compute the value of λ_l in its present state (ii) compute the value of f_l, equation (12) (iii) different options for structural upgrading are normally available, each one having a different cost S_l and global outcome Λ_l. Only the options having $R_l+F_l<f_l$ are economically convenient; for the remaining ones, the *do nothing* option is preferable. (iv) among the options which are economically convenient the best is the one with the minimum value for (R_l+F_l).

PARAMETRIC STUDY

In the previous section the criterion $R_l+F_l<f_l$, equation (11), was developed to assess economic convenience of the upgrading design. The value of the cost ratio R_l such that $R_l+F_l=f_l$ is therefore the maximum allowable value for a retrofitting to be convenient. Hence we define:

$$R_{l\,\max} : \{R_{l\,\max} + F_l = f_l\} \tag{13}$$

$R_{l\ max}$ is a function of λ_l, Λ_l, i and L and is shown in the following figures for this combination of values: L=50 years and i=0.001 and 0.03; i=0.001 and L=10 and 100 years.

Each figure is drawn for constant values of i and L, and has the values of λ_l and Λ_l on the abscissas and ordinates axes respectively. Level curves are shown for values of $R_{l\ max}$ varying between 0 and 0.9, at 0.1 steps. The values of i chosen, 0.001 and 0.03, should be representative of many situations, ranging from the cost of debt for trusted countries to that for trusted industries, for medium and long term bonds. The range of values for the year to 2002 for selected situations is shown in Table 2 [18].

As an example of the values computed in Table 2, consider a country within the euro area which finances itself by issuing 2 years bonds. The State succeeds in selling its debt at 3.72% annual gross; inflation is worth 2.5%. Since most of the debt is sold to residents, and profits are taxed at about 30%, and the taxes return to the State, $i=log_e(1+i^*)=log_e(1+(i_f-f)/(1+f))=log_e(1+(0.0372(1-0.30)-$

0.025)/(1+0.025)))=0.1014%. Also notice that if the approximate formula for the interest rate were used, $i \approx i^{*} \approx i_{f} - f = 0.0372(1-0.30)-0.025$, the value of 0.1040% would be computed, with a difference of 2.6% with respect to the exact value.

Table 2: values of annual interest rates and inflation in points per cent for selected countries (1)

country	inflation	gross interest rates			inflation free log interest rate *i*		
		government bonds at		corporate bonds	government bonds at		corporate bonds
		2 years	10 years		2 years	10 years	
Euro area	2.50	3.72	4.76	5.63	0.10141	0.80843	1.39606
USA	1.60	2.97	4.93	6.76	0.47035	1.80545	3.03612
Japan	-1.20	0.08	1.53	1.64	1.26324	2.27256	2.34872

(1) the interest rate *i* is computed assuming the issuer (country or company) bears 70% of the gross interest because of taxes remittance (countries) or deduced costs (companies)

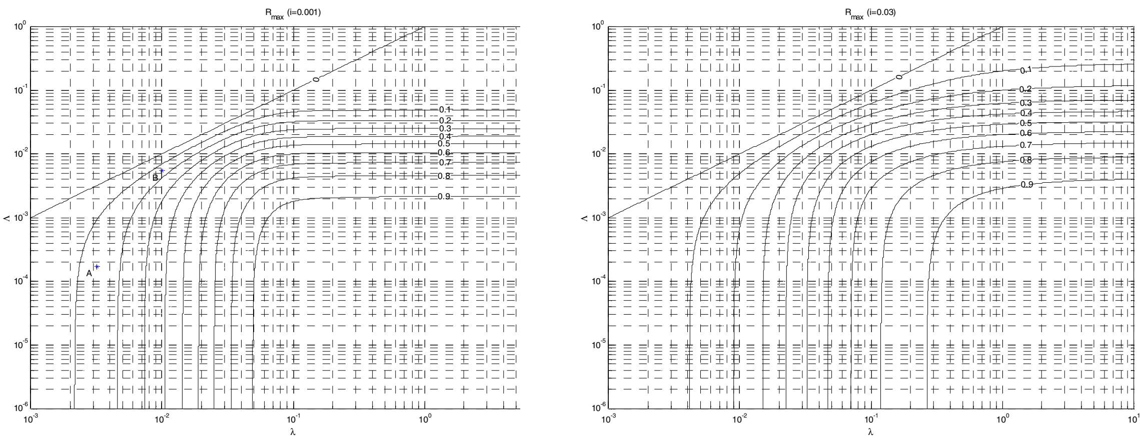

Figure 2: Level curves of $R_{l\,max}(i,\lambda_l,\Lambda_l)$ for L=50 years and (a) i=0.001, (b) i=0.03

Let us examine some interesting characteristics of the diagrams. First, in each diagram, curves at constant $R_{l\,max}$ are similar to circumference sections centred at the bottom-right corner; the circumferences radii are inversely proportional to the values of $R_{l\,max}$.

Notice that, once the value of λ_l is known, the value of $R_{l\,max}$ for the minimum value of Λ_l is the maximum that one can spend to upgrade the structure, independently on the final value for Λ_l; for instance, in Figure 2(a), if the structure *as is* has a mean rate of failures equal to $5 \cdot 10^{-2}$, then the maximum that one can spend is $0.9 \cdot C_l$. Of course, if the final result of retrofitting, as measured by Λ_l, is not good enough, the maximum that one can spend decreases. For instance, if $\Lambda_l=10^{-2}$,then the maximum one can spend is $0.5 \cdot C_l$. In each diagram notice also that, depending on the value of λ_l, ranges of Λ_l more convenient than other ones exist. Consider Figure 2(a) again, and assume that the structure in its present state shows the value of 10^{-2} for λ_l. If $\Lambda_l=\lambda_l$ (point of coordinates $10^{-2};10^{-2}$) from the figure it can be seen that $R_{l\,max}=0$. This is a trivial result: no money should be spent on an upgrading intervention which offers no improvement. If Λ_l decreases to the values of $7 \cdot 10^{-3}$, $4 \cdot 10^{-3}$, $2 \cdot 10^{-3}$, 10^{-4}, R_{l_max} respectively increases to the values of 0.1, 0.2, 0.3 and almost 0.4. Further decrease of Λ_l brings no increase of $R_{l\,max}$: notice that for Λ_l as low as 10^{-6}, $R_{l\,max}$ is still lower than 0.4.

As for results for different interest rates *i* is concerned, another important consideration can be made: attempts to decrease the values of Λ_l to lower than 10^{-4}, i.e. one collapse every 10000 years on average, brings about no increase of $R_{l\,max}$. This can be observed in Figure 2 and Figure 3, irrespective of the values of *i* and *L*. We remind that these results are conditioned to the assumptions made.

The global influence of the interest rate on the results is that level curves move towards right and up with increasing *i* levels. This means that, with λ_l and Λ_l fixed, and if λ_l has ordinary values, say lower than or equal to 0.1, $R_{l\,max}$ decreases with increasing values of *i*. If, for instance, $\lambda_l=10^{-1}$ and $\Lambda_l=10^{-3}$, the maximum that one can spend is more than 90% of C_l in the case i=0.001 (Figure 2(a)), but only 75% for i=0.03 (Figure 2(b)). As a corollary, it follows that seismic structural upgrading is most

economically done by States, rather than by smaller – and typically less trusted – organizations, for which the cost of debt is substantially higher.
Increase of the economic life L, has the opposite effect: for increasing values of L level curves move towards left and down. All other things equal, and if λ_l has ordinary values, this means that $R_{l\ max}$ increases with increasing economic life. This results is rather intuitive: the longer the economic life, the more can be spent to achieve a predefined result. In Figure 3 two level curves, with constant i=0.001 and L=10 and 100 years, are shown.

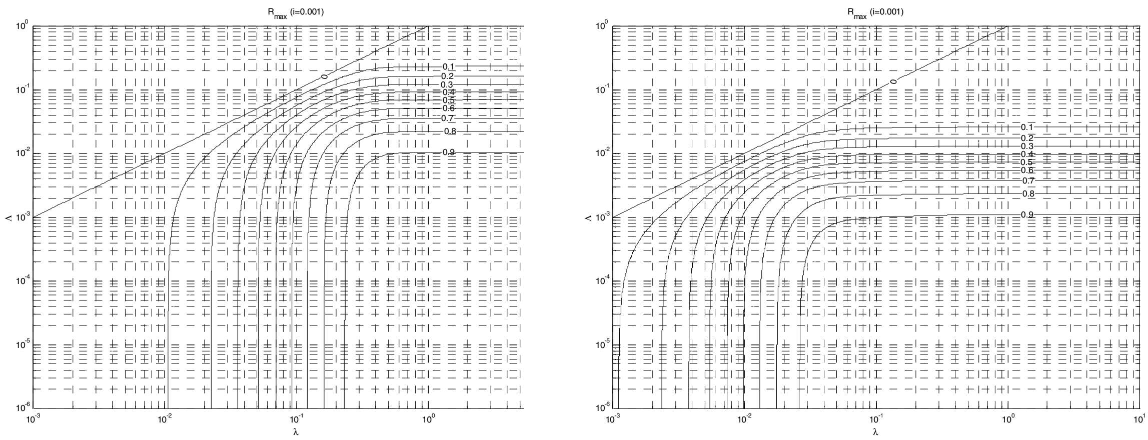

Figure 3: Level curves of $R_{l\ max}(i,\lambda_l,\Lambda_l)$ for i=0.001 and (a) L=10 years (b) L=100 years

Finally, the variability of $AE(T)$, equation (10), is shown in the Figure 4 for selected values of the governing parameters. The situation presented is rather convenient one: $R_{l\ max}$ is in this case respectively equal to 0.57, 0.48, 0.35 for i=0.001, 0.01, 0.03 while the curves shown are relative to the value of $R_l=S_l/C_l$=0.1/1=0.1.
It can be seen that postponing retrofitting can cost more than twice the minimum value.
It must be highlighted that the procedure can be used both for serviceability and for ultimate limit states and, in the next section, an application with the ultimate limit state will be presented.

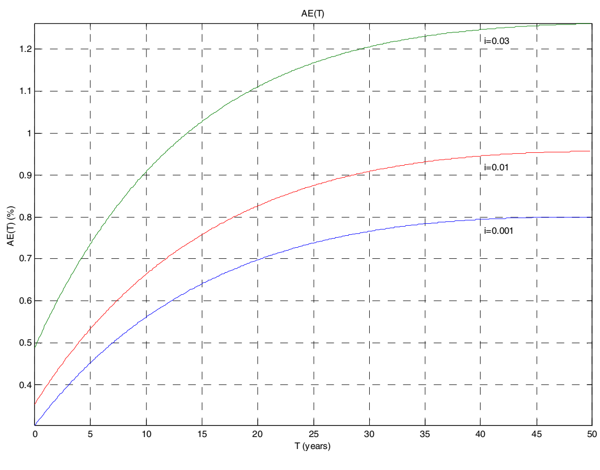

Figure 4: $AE(T)$, eq. (10), for λ_l=0.02, Λ_l=0.001, i=0.001, 0.01, 0.03, L=50 years, C_l=1, S_l=0.1

SIMPLIFIED ASSESSMENT OF THE ECONOMIC CONVENIENCE OF RETROFITTING

The expanded form of equation (13) is:

$$R_{\max} = \frac{S_{\max}}{C} = \left\{ \frac{\lambda}{\lambda + i} \cdot \frac{\exp\left[L(i+\lambda)\right]-1}{\exp\left[L(i+\lambda)\right]} \right\} - \left\{ \frac{\Lambda}{\Lambda + i} \cdot \frac{\exp\left[L(i+\Lambda)\right]-1}{\exp\left[L(i+\Lambda)\right]} \right\} \tag{14}$$

Each of the two terms within braces at the right hand side can be expressed in series. For the former:

$$\frac{\lambda}{\lambda + i} \cdot \frac{\exp\left[L(\lambda+i)\right]-1}{\exp\left[L(\lambda+i)\right]} = \lambda \sum_{k=1}^{\infty} \left[\frac{L^k (\lambda+i)^{k-1}}{k!} \cdot (-1)^{k-1} \right] \tag{15}$$

A similar expression holds for the latter, substituting Λ for λ. The first order approximation to equation (14) is then:

$$R_{\max} = \frac{S_{\max}}{C} \approx R^{I}_{\max} = (\lambda - \Lambda) L \tag{16}$$

Further, if Λ may be disregarded with respect to λ, equation (16) may be rewritten as:

$$R_{\max} = \frac{S_{\max}}{C} \approx R^{I\_appr}_{\max} = \lambda L \tag{17}$$

Equations (16) and (17) are definitely simple expressions for R_{max} and are certainly suitable for back-of-an-envelope computations (provided one can know or guess the value of λ); but in which range of the parameters can they be used? As a general rule, the smaller the values of i, L, λ and Λ the better the approximation of equation (16) to the real value of R_{max}; the smaller the value of Λ with respect to λ, the better is the approximation of equation (17). The user of equations (16) and (17) should also be aware that the approximations are on the unsafe side, i.e. they tend to overestimate the value of R_{max}.
In order to be more precise on when the approximations can be used, while retaining simplicity, we studied the errors of both equations (16) and (17) with respect to the real value of R_{max} by looking, for selected values of i and L, at the values of the errors:

$$\Delta^{I} = \frac{R^{I}_{\max} - R_{\max}}{R_{\max}} \qquad \Delta^{I\_appr} = \frac{R^{I\_appr}_{\max} - R_{\max}}{R_{\max}} \tag{18}$$

For example, for i=0.001 and L=50 years, we obtained the results shown in Figure 5.
From the following figures, and the similar ones for different values of i and L, not shown here, it is clear that the applicability of equations (16) and (17) strongly depends on the values of i and L since the errors may be either acceptable or not. We have summarized the applicability of the approximations in Table 3. We have considered the values of 10, 50, 100 and 200 years for L and the values of 0.001, 0.01, 0.02 and 0.03 for the inflation-free interest rate i. The four i values are respectively representative of debts for Euro area governments at 2 years ($i\approx0.001$), for Usa and Japan at 2 years and Euro area at 10 years ($i\approx0.01$), for Usa and Japan at 10 years and Euro and Japan companies ($i\approx0.02$), and finally for Usa companies ($i\approx0.03$), see Table 2.
The table shows the maxima of the errors Δ^{I}, equation (18), with the assumption that $\lambda\leq10^{-2}$; it can also be used to estimate the errors Δ^{I-appr}, equation (19), provided that $\Lambda\leq2\cdot10^{-5}$. The *no* symbol indicates that the approximations should not be used. Also notice that the values in Table 3 can be approximated by:

$$\Delta^{I} \approx \Delta^{I-appr} \approx L(0.4 + 75i) \tag{19}$$

Table 3: maxima of errors Δ^{I} and Δ^{I-appr} in points per cent; $\lambda \leq 10^{-2}$ is assumed.

L (years)	*i*≈0.001	*i*≈0.01	*i*≈0.02	*i*≈0.03
10	5	10	15	20
50	25	60	90	*no*
100	50	*no*	*no*	*no*
200	*no*	*no*	*no*	*no*

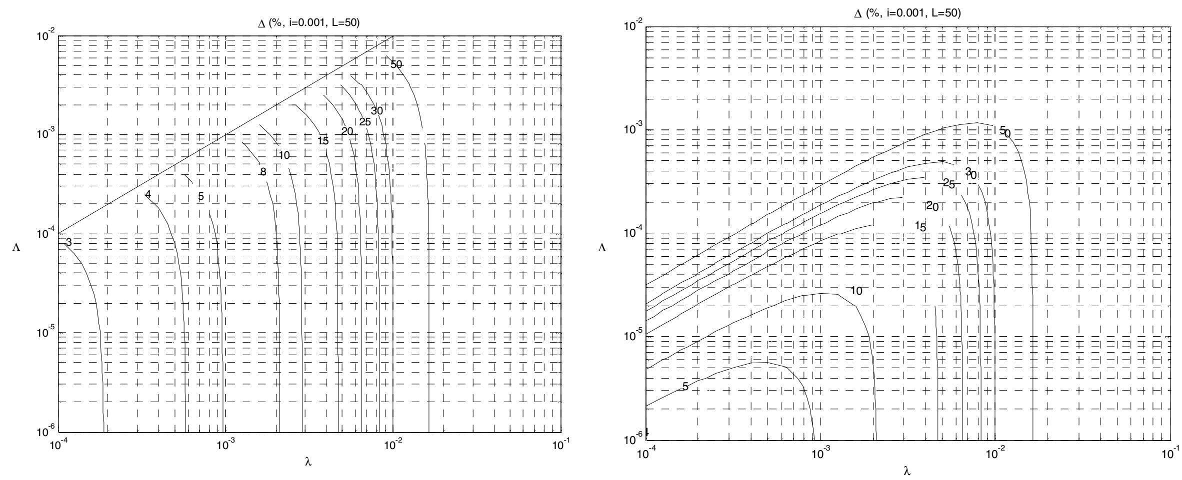

Figure 5: Level curves of (a) $\Delta^{I}(i,L,\lambda_l,\Lambda_l)$, (b) $\Delta^{I\text{-}appr}(i,L,\lambda_l,\Lambda_l)$ in [%] for i=0.001 and L=50 years

As a short example, assume i=0.01, L=50 years, $\lambda=8\cdot10^{-3}$, $\Lambda=5\cdot10^{-6}$. The correct value of R_{max}, equation (14), is 0.26; the value of $R^{I}_{\max}$, equation (16), is 0.39. From Table 3, we read that the maximum error is about 0.6 (0.58 if we use equation (19)) and hence compute a minimum value for R of 0.39/(1+0.6)=0.24. The real value of R_{max} will be in the interval [0.24; 0.39].

Since $\Lambda\leq2\cdot10^{-5}$, we could have made the same computation using $R^{I-appr}_{\max}$, equation (17) which is computed in this case as 0.40; again this is an upper bound for R_{max}, with the lower bound equal to 0.40/(1+0.6)=0.25. As shown, the computation can be very easily made by hand.

Notice that in this case the value of R_{max} is rather close to the lower bound of the intervals; this happens because the value of λ of the example ($\lambda=8\cdot10^{-3}$) is close to the value of $\lambda=10^{-2}$ with which Table 3 has been built.

EXAMPLE: BRIDGES ON A16 NAPLES – CANOSA HIGHWAY, RETROFIT FOR THE ULTIMATE LIMIT STATE

In [1], the results of the risk analysis on the highway bridges belonging to the network of Società Autostrade S.p.A. were published. The methods presented in this paper are applied to some of these infrastructure, those of simple structural conception: single and framed piers bridges, belonging to the A16 Naples – Canosa stretch.

The hazard values obtained applying the Cornell's method [12] (for two different return periods, 50 and 500 years), for points along the highway lie in the range [8.5; 9.6] *IMM* (Modified Mercalli scale) for 500 years return period and [6.5; 7.6] *IMM* for 50 years return period. The first return period is the value ordinarily chosen to check the serviceability limit state; the second is relative to the ultimate limit state. The maxima of both curves are found at about halfway the highway stretch, when it passes by the region of Irpinia which is seismically very active. With the 500 years return period earthquake, the bridges were checked for the ultimate limit state; risks were very high: 14 of the 50 bridges examined had values above 0.5 and 3 above 0.9. The limit states considered were flexural and shear ultimate capacities of piers, for which non linear analyses were conducted. The earthquake action is assumed to occur in the transverse direction with respect to the bridge, which in many cases is the most severe load condition.

Departing from these data, performing the convolution of hazard and fragility curve for each bridge we have computed the value of λ_u (equation (2)) for each of the 50 bridges (notice that in the expression of λ the suffix l has been changed to u because we are considering the ultimate limit state). The values for λ_u are, as expected, very high and are shown in Figure 6, highlighted by square symbols.

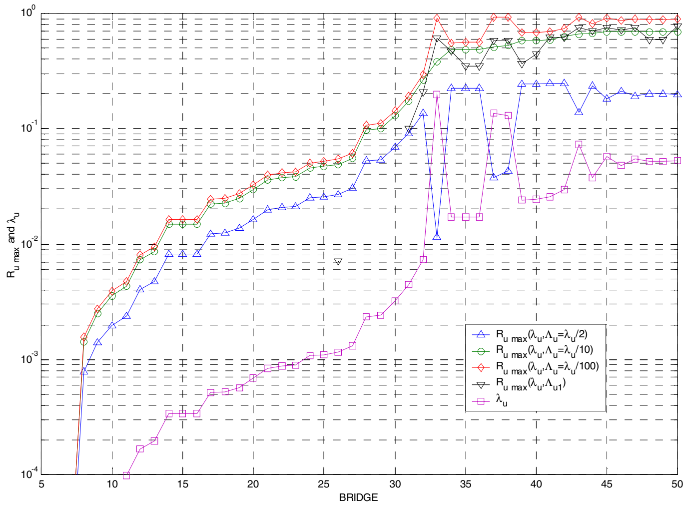

Figure 6: λ_u and $R_{u\_max}(\lambda_u,\Lambda_u)$ computed with different assumptions on Λ_u

As a first application of the procedure, in the same Figure 6 we show the values of $R_{u\_max}(\lambda_u,\Lambda_u)$ (equation (13)) computed with i=0.001 (an appropriate value since resources for upgrading would likely come from bonds guaranteed by the State), L=50 years and four different assumptions for the values of Λ_u of each bridge:

$$(i)\ \ \Lambda_u = \frac{\lambda_u}{2}\quad (ii)\ \ \Lambda_u = \frac{\lambda_u}{10}\quad (iii)\ \ \Lambda_u = \frac{\lambda_u}{100}\quad (iv)\ \ \Lambda_u = \Lambda_{u\ 1} = \sum_{j=1}^{N} \min(\lambda_j^{pier}, 10^{-3}) \qquad (20)$$

where N is the number of piers in each bridge, λ_j^{pier} is the mean annual rate of failures for the j-th pier of the bridge.

Assumption (iv) has the following rationale: design guidelines and codes specify earthquake design actions for retrofitting in terms of their return periods. For the ultimate limit state, the value of 1000 years is selected by the Applied Technology Council [20]. The latter value, if we regard the structure as deterministic, is the inverse of the mean annual rate of collapses for each pier after retrofitting (unless the bridge, in the state before upgrading, as measured by λ_j^{pier}, over performs this code requirement). For a bridge composed of N piers, assuming that pier failure is a Poisson process and that pier failures are independent events, the mean annual rate of failures is then equal to expression (iv) in equation (20).

All four assumptions are indeed crude but can help in visualizing the order of magnitude which one can expect for $R_{u\_max}(\lambda_u,\Lambda_u)$ in this example.

From Figure 6 one can see that the upgrading is likely to be convenient for many bridges. Broadly speaking, a good generalized design criterion would be to decrease by ten the value of λ_u (curve with circle symbols). In this case, in fact, the values of R_{u_max} would be sufficiently high from the 27th bridge on, with values equal or higher than 0.1. Such interventions, provided that pier foundations are strong enough, often require just jacketing the base of the bridge piers which in turn, provided that yard conditions are normal, is rather inexpensive.

For bridge number 30 in Figure 6, the viaduct 'Lenze Penze di Valle', in reinforced concrete, a complete example safety assessment before and after retrofitting has also been conducted. The structural scheme of the viaduct is shown in Figure 7.

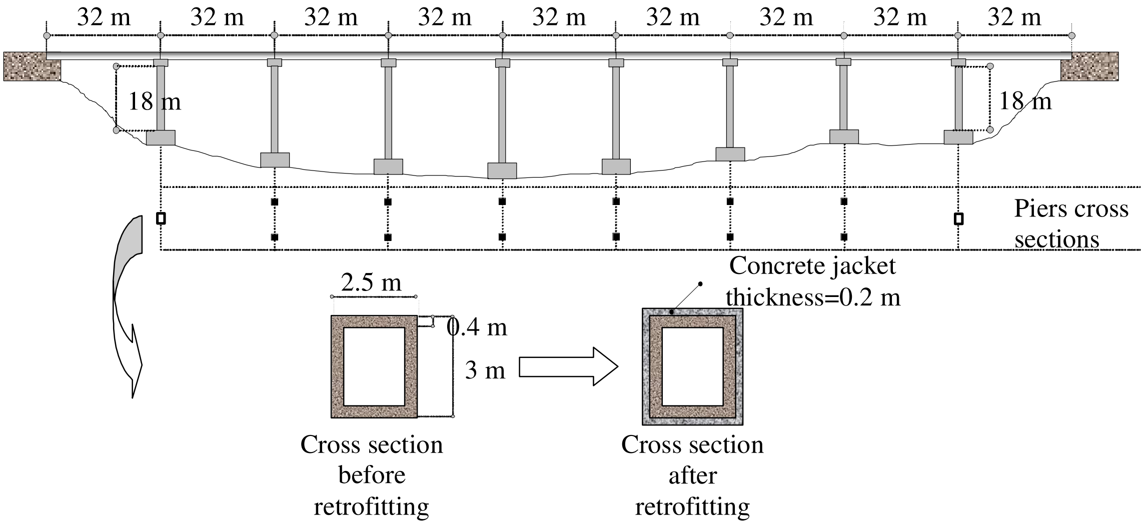

Figure 7: structural scheme of the 'Lenze Penze di Valle' viaduct

The bridge is composed of eight piers plus abutments, with heights varying between 18 and 27 m. The structural scheme is isostatic since slabs are simply supported. The two extreme piers are single columns with the same height (18m) and cross section (rectangular and hollow, 3.0×2.5×0.4 m^3); the remaining piers are portal frames and are much more ductile than the extreme ones, which fail in bending. Figure 7 shows also the cross section of the extreme piers; the material is reinforced concrete f_{ck} 30 MPa with steel rebars classified in Italy as Feb38k (f_{yk}=380 MPa), for a total of 150 cm^2.

The safety assessment of the extreme piers, which govern the overall structural fragility, has been conducted with the methods of structural reliability. For the sake of conciseness, we repeat here only the conceptual steps used in the computations; interested readers can find the details of the computations in [1].

The hazard assessment has given the following values for the hazard curves (equation (1)): z_{min}=5 *IMM*; z_{max}=12 *IMM*; β=1.123; ν=0.361 earthquakes/year.

The fragility curve has been obtained by comparing the available and required ductilities with varying peak ground acceleration (*pga*); the available ductility has been found to have a lognormal distribution with mean equal to 3.39 and coefficient of variation equal to 0.25.

For each value of the *pga*, the required ductility has been modeled with a *Type 1*, *Largest Value*, distribution. The mean value of the distribution has been obtained with the equal displacement assumption (the elastic period of the bridge is 1.5 s) using the EC8, medium soil response spectrum [19]; its coefficient of variation has been found equal to 0.30, following [14].

Convolution of hazard and fragility (equations (2) and (3)) has provided the value of the mean annual rate of failure λ_u of the bridge in its present state. This is equal to $3.2 \cdot 10^{-3}$ (see Figure 6, bridge 30).

The assumed upgrading consists of a concrete jacket at the base, with thickness equal to 0.20 m and with 100 cm^2 steel rebars. This causes the mean value of the available ductility to increase to about 5 and, hence, the mean annual rate of failures Λ_u decreases to $1.7 \cdot 10^{-4}$. These values are shown in Figure 2(a) (point *A*). For the viaduct R_{u_max} is equal to about 0.15. Considering that this value has been obtained with concrete jacketing on two piers only and that, on the other hand, failure of the bridge would be a major cost, convenience of retrofitting is almost assured.

CONCLUSIONS

The method presented allows to give clear answers in a simple way to the question of whether seismic retrofitting is convenient. Input to the method are the pre and post intervention mean annual rates of

exceedance of limit states, λ_l and Λ_l; rather reasonable assumptions are then made, to derive the final results, i.e. the relationship, expressed in an algebraic formula, between the main problem variables: λ_l, Λ_l, interest rate and maximum amount of money to spend for retrofitting to be profitable.
The approximating formulas presented allow a fast estimation of the economic convenience of seismic structural retrofitting. The range of applicability of the formulas is such that it should be useful for many a structure. We have also shown a table and a formula which can be alternatively used to compute the maxima of the estimation errors so as to have an upper and lower bound for the maximum amount of money which should be allocated for seismic structural upgrading.
We sum up here the main conclusion of this paper: when we are wondering whether seismic structural retrofitting of a structure is too expensive or not, the answer is that we should do it only if the cost of upgrading S is lower than the cost of failure C multiplied by the structure economic life L and the pre-upgrading mean annual rate of failures λ, i.e.:

$$S < S_{\max} \approx CL\lambda \tag{21}$$

For instance, if C=10 million €, L=50 years, λ=0.004 (the return period of failures is 250 years), the retrofitting is efficient and the owner of the structure is a European public institution, a first estimate of S_{max} is (10 million €)×(50 years)×(0.004 failures/year)=2 million €. A better estimate of S_{max} can be obtained using Table 3 or equation (19) which give the maximum percentage error in the first estimation. Using the latter, we compute the error as $L\cdot(0.4+75\cdot i)=50\cdot(0.4+75\cdot 0.001)\approx 24$. The real (non approximated) value of S_{max} lies within [2/(1+0.24)=1.6; 2] million €.
We think that this simple formulas will be useful for decision making and will help to introduce speedy economic reasoning in the rational choice between competing designs.
The procedure should also apply to problems different from earthquake structural upgrading, as far as they can be cast in a form similar to the assumed one: an undesired event, distributed in time as a constant Poisson process, with mean rate which decreases at a discrete instant because of man-made investment. Such diverse problems as flooding protection and hurricane prevention should fall in this category.
The procedure is then applied to an example case: bridges on an Italian highway stretch, considering ultimate limit state. For many of the highway bridges, upgrading is likely to be highly convenient.
We would finally like to highlight that the current state of knowledge allows to assess the safety of existing structures and the economic convenience of upgrading in a precise way. The major obstacle to the widespread use of this knowledge is model complexity. Studies aiming at giving simple answers to the problem of the economic convenience of retrofitting are therefore much needed and should be central in bridging the gap between research and application.

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