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EXPLICIT PSEUDODYNAMIC TESTING WITHOUT STABILITY LIMITS

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SUMMARY

In performing a pseudodynamic test, an explicit method is preferred over an implicit method since it involves no iteration procedure or extra hardware that is generally needed for an implicit method. However, the integration time step is limited by stability. Therefore, it is promising for the pseudodynamic testing if an explicit method has unconditional stability, which eliminates the restriction on the time step for the test of a multiple degree of freedom system or a substructure test. An explicit pseudodynamic algorithm with unconditional stability has been developed. This pseudodynamic algorithm can be implemented as simply as the very commonly used explicit pseudodynamic algorithms. Its unconditional stability is analytically verified and confirmed with numerical examples. Furthermore, it possesses much better error propagation properties when compared to the Newmark explicit method. Actual pseudodynamic tests attested to the feasibility of the explicit implementation and no stability limits.

INTRODUCTION

After comparing explicit pseudodynamic algorithms Chang [1-5] with implicit pseudodynamic algorithms Chang [6], Shing [7], Thewalt [8], it is found that the implementation of an explicit pseudodynamic algorithm is simpler than that of an implicit pseudodynamic algorithm. This is because that an implicit pseudodynamic algorithm requires some extra hardware Thewalt [8] or becomes more complex Shing [7] since an iteration procedure is often used to yield convergent solutions for each time step. However, an explicit pseudodynamic algorithm can have conditional stability only Bathe [9]. Consequently, the selection of an integration time step may be limited by stability limit for a conditionally stable pseudodynamic algorithm when high frequency modes are present in a multiple degree of freedom test or a substructure test. This

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might engender the use of a small time step and lead to the critical problem of stress relaxation and displacement control. This limitation is overcome by using an unconditionally stable pseudodynamic algorithm since it has no stability limit. In fact, the most promising property of an implicit pseudodynamic algorithm is the possibility of unconditional stability. As a result, the selection of a time step is determined by accuracy consideration only for a pseudodynamic algorithm with unconditional stability. Apparently, it is very promising to integrate the unconditional stability and explicitness of each time step both in a pseudodynamic algorithm.

The pseudodynamic algorithm proposed herein is an unconditionally stable explicit method whose step-by-step test procedure can be implemented the same as the most commonly used explicit pseudodynamic algorithms. Therefore, neither numerical iteration nor additional hardware is required in performing a pseudodynamic test. The formulation and implementation of this explicit pseudodynamic algorithm are presented herein and its error propagation properties are also explored. A series of numerical examples and verification tests are used to illustrate its superior properties in performing a pseudodynamic test.

PROPOSED PSEUDODYNAMIC ALGORITHM

For a multiple degree of freedom system, the general formulation of the proposed explicit algorithm can be expressed as:

$$\begin{aligned} &\mathbf{M}\mathbf{a}_{i+1} + \mathbf{C}\mathbf{v}_{i+1} + \mathbf{r}_{i+1} = \mathbf{f}_{i+1} \\ &\mathbf{d}_{i+1} = \mathbf{d}_i + \beta_1(\Delta t)\mathbf{v}_i + \beta_2(\Delta t)^2\mathbf{a}_i \\ &\mathbf{v}_{i+1} = \mathbf{v}_i + \frac{1}{2}(\Delta t)(\mathbf{a}_i + \mathbf{a}_{i+1}) \end{aligned} \tag{1}$$

where $\mathbf{M}$ and $\mathbf{C}$ are the mass and viscous damping matrices; $\mathbf{r}_{i+1}$ and $\mathbf{f}_{i+1}$ are the restoring force vector and external force vector at the $(i+1)-th$ step, respectively; $\mathbf{d}_{i+1}$, $\mathbf{v}_{i+1}$ and $\mathbf{a}_{i+1}$ are vectors of nodal displacements, velocities and accelerations. The coefficient matrices β_1 and β_2 are defined as:

$$\begin{aligned} \beta_1 &= \left[\mathbf{I} + \frac{1}{2}(\Delta t)\mathbf{M}^{-1}\mathbf{C} + \frac{1}{4}(\Delta t)^2\mathbf{M}^{-1}\mathbf{K}_0\right]^{-1}\left[\mathbf{I} + \frac{1}{2}(\Delta t)\mathbf{M}^{-1}\mathbf{C}\right] \\ \beta_2 &= \left(\frac{1}{2}\right)\left[\mathbf{I} + \frac{1}{2}(\Delta t)\mathbf{M}^{-1}\mathbf{C} + \frac{1}{4}(\Delta t)^2\mathbf{M}^{-1}\mathbf{K}_0\right]^{-1} \end{aligned} \tag{2}$$

where $\mathbf{I}$ denotes an identity matrix. It should be mentioned that $\mathbf{K}_0$ in equation (2) is the initial stiffness matrix and is used to determine the coefficient matrices β_1 and β_2, which remain unchanged during a complete pseudodynamic test.

Unlike a conventional pseudodynamic algorithm, it is necessary to determine the initial stiffness matrix $\mathbf{K}_0$ to compute the coefficient matrices β_1 and β_2. The displacement vector $\mathbf{d}_{i+1}$ for the next time step can be obtained from the second line of equation (1) and then using servo hydraulic actuators to impose the computed displacements upon the test specimen. After measuring the restoring forces $\mathbf{r}_{i+1}$ developed by the specimen, the acceleration vector can be expressed in terms of $\mathbf{r}_{i+1}$ and $\mathbf{v}_{i+1}$ by using the first line of equation (1). Then, velocity vector $\mathbf{v}_{i+1}$ can be obtained by substituting this result into the third line of equation (1) and has the following formulation:

$$\mathbf{v}_{i+1} = \left[\mathbf{M} + \frac{1}{2}(\Delta t)\mathbf{C}\right]^{-1} \left\{\mathbf{M}\left[\mathbf{v}_i + \frac{1}{2}(\Delta t)\mathbf{a}_i\right] + \frac{1}{2}(\Delta t)(\mathbf{f}_{i+1} - \mathbf{r}_{i+1})\right\} \tag{3}$$

Finally, the acceleration vector $\mathbf{a}_{i+1}$ can be obtained from the equations of motion and is

$$\mathbf{a}_{i+1} = \mathbf{M}^{-1}(\mathbf{f}_{i+1} - \mathbf{C}\mathbf{v}_{i+1} - \mathbf{r}_{i+1}) \tag{4}$$

This test procedure can be repeated to achieve the desired time history. Apparently, this pseudodynamic implementation is exactly the same as that for the Newmark explicit method.

NUMERICAL PROPERTIES

After the spectral analysis of the proposed pseudodynamic algorithm, it is found that its characteristic equation is exactly the same as that for the constant average acceleration method. This indicates that the numerical properties of the proposed explicit method are exactly the same as that of the constant average acceleration method. Consequently, it is concluded that the algorithm is unconditionally stable and has a second-order accuracy. In addition, it possesses no numerical dissipation and exhibits no overshoot both in displacement and in velocity.

To compare the period distortion of the proposed explicit algorithm with those of the constant average acceleration method and Newmark explicit method, the variations of the absolute relative period error $|(\overline{T} - T)/T|$ with $\Delta t / T$ for the three integration methods are shown in figure 1, where $\overline{T} = 2\pi / \overline{\omega}$ is the computed period and $T = 2\pi / \omega$ is the true period.

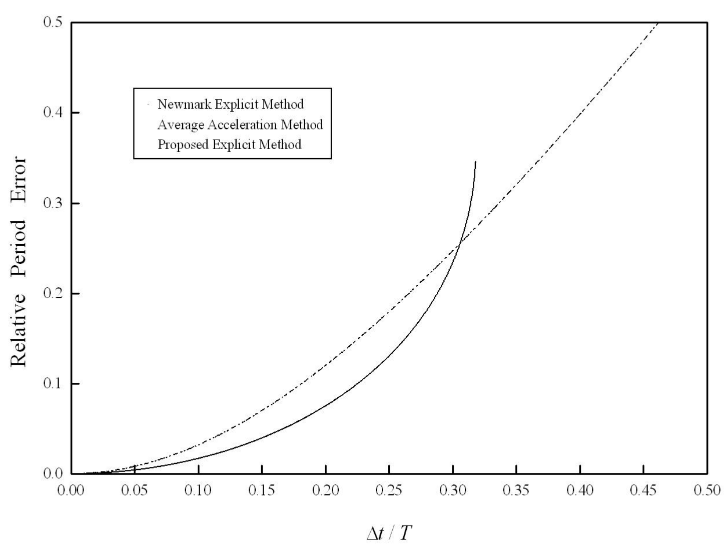

Figure 1. Variations of relative period errors with $\Delta t / T$

It should be mentioned that period elongation is found in the proposed explicit method and the constant average acceleration method while the Newmark explicit method shows period shrinkage. In this figure, the curves for the proposed explicit method and the constant average acceleration method coincide. Thus, they have the same relative period error for any $\Delta t/T$. On the other hand, it is manifested from this figure that the proposed explicit method and constant average acceleration method have a larger absolute relative period error than for the Newmark explicit method as the value of $\Delta t/T$ is smaller than about 0.3 although it is not very significant.

ERROR PROPAGATION PROPERTIES

Error propagation characteristics of the proposed explicit pseudodynamic algorithm can be obtained after the error propagation analysis Shing [10,11], Chang [12]. As a result, the cumulative displacement error for the $n+1$ time step is found to be

$$e_{n+1}^{d} = E_d \sum_{i=0}^{n} \cos\left[\left(n-i+\tfrac{1}{2}\right)\overline{\Omega}\right] e_i^d - E_r \sum_{i=0}^{n} \sin\left[\left(n-i\right)\overline{\Omega}\right] e_{i+1}^{rd} \tag{5}$$

where e_i^d is the displacement error introduced at step i and e_{i+1}^{rd} represents the amount of displacement error corresponding to the restoring force error e_{i+1}^{r} introduced at step $i+1$ Shing [10,11].

$$E_d = \sqrt{1+\tfrac{1}{4}\Omega^2}\,, \qquad E_r = \Omega \tag{6}$$

where E_d is the error amplification factor for the displacement feedback errors while E_r is the error amplification factor for the restoring force feedback errors.

Figures 2 and 3 show the variations of the error amplification factors E_d and E_r, shown in equation (6), with Ω for the proposed explicit method. For comparisons, those for the Newmark explicit method are also plotted.

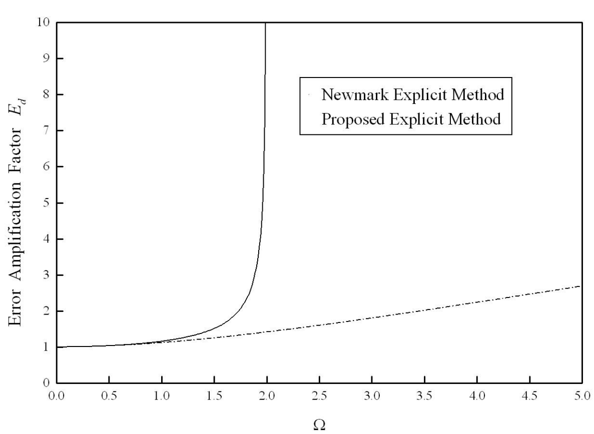

Figure 2. Error amplification factor for displacement feedback error

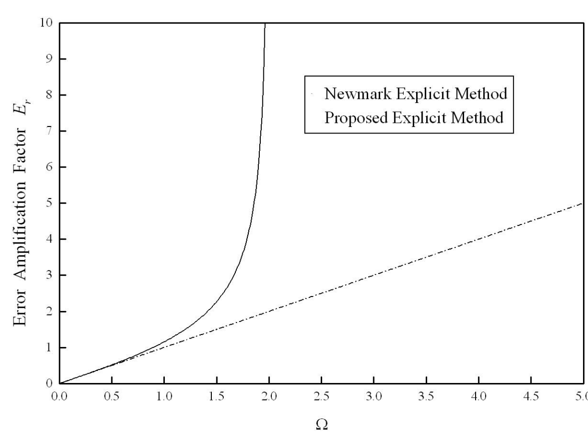

Figure 3. Error amplification factor for restoring force feedback error

In figures 2 and 3, it is apparent that the proposed explicit pseudodynamic algorithm shows smaller error amplification factors for both the displacement and the restoring force feedback errors than those of the

Newmark explicit method, and this phenomenon becomes very significant as $\Omega \geq 1.5$. Furthermore, both error amplification factors, E_d and E_r, approach infinity for the Newmark explicit method as $\Omega \to 2$. However, this adverse characteristic disappears for the proposed explicit pseudodynamic algorithm. In fact, it is manifested from both figures that the error amplification factors E_d and E_r for the proposed explicit pseudodynamic algorithm increases gradually as the value of Ω increases from zero to infinity. As a summary, it is concluded that the proposed explicit pseudodynamic algorithm possesses much better error propagation properties than for the Newmark explicit method, especially in the range of $\Omega \geq 1.5$.

ACTUAL PSEUDODYNAMIC TESTS

A series of pseudodynamic tests are performed to confirm the unconditional stability and improved error propagation properties of the proposed explicit algorithm. Several hot-rolled steel beams with the cross section of $\mathrm{H}\,200 \times 200 \times 8 \times 12$ and a length of $3.2m$ were adopted for the tests. The cantilever beam is loaded by 3 static actuators in parallel for simulating a 3-degree of freedom system as shown in figure 4.

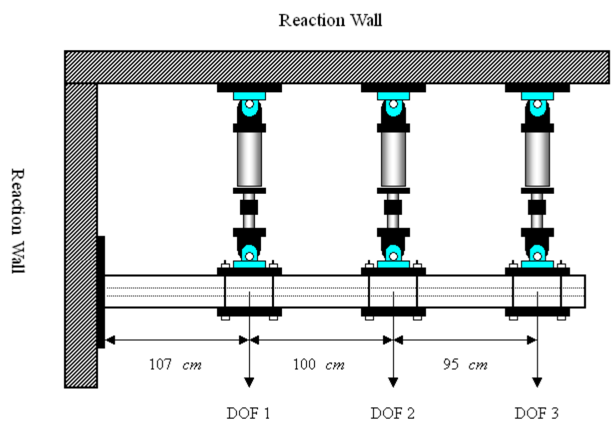

Figure 4. Pseudodynamic test setup

Two systems are considered in this study. One is for the verification tests of unconditional stability and the other is for those of improved error propagation for the proposed explicit pseudodynamic algorithm. Each system is intentionally designed to have a relatively high third mode when compared to the first and second modes since the highest mode leads to instability and severe error propagation for the Newmark explicit method but not for the proposed explicit method. The initial stiffness matrix for each specimen can be experimentally measured and is found to be about

$$\mathbf{K}_0 = \begin{bmatrix} 64.6 & -48.8 & 12.5 \\ -48.8 & 69.3 & -31.2 \\ 12.5 & -31.2 & 20.2 \end{bmatrix} \qquad (7)$$

This matrix is used to compute the coefficient matrices β_1 and β_2 before the pseudodynamic tests.

Unconditional stability

A 3-degree of freedom system is employed to show that the upper stability limit for the Newmark explicit method must be less than 2 while unconditional stability is indicated for the proposed explicit method. The lumped masses for the first, second and third degree of freedom are designated as 405000, 1800 and $58400\,kg$. Thus, the natural frequencies of the system are found to be 5.2, 12.6 and $200\,rad\,/\,sec$.

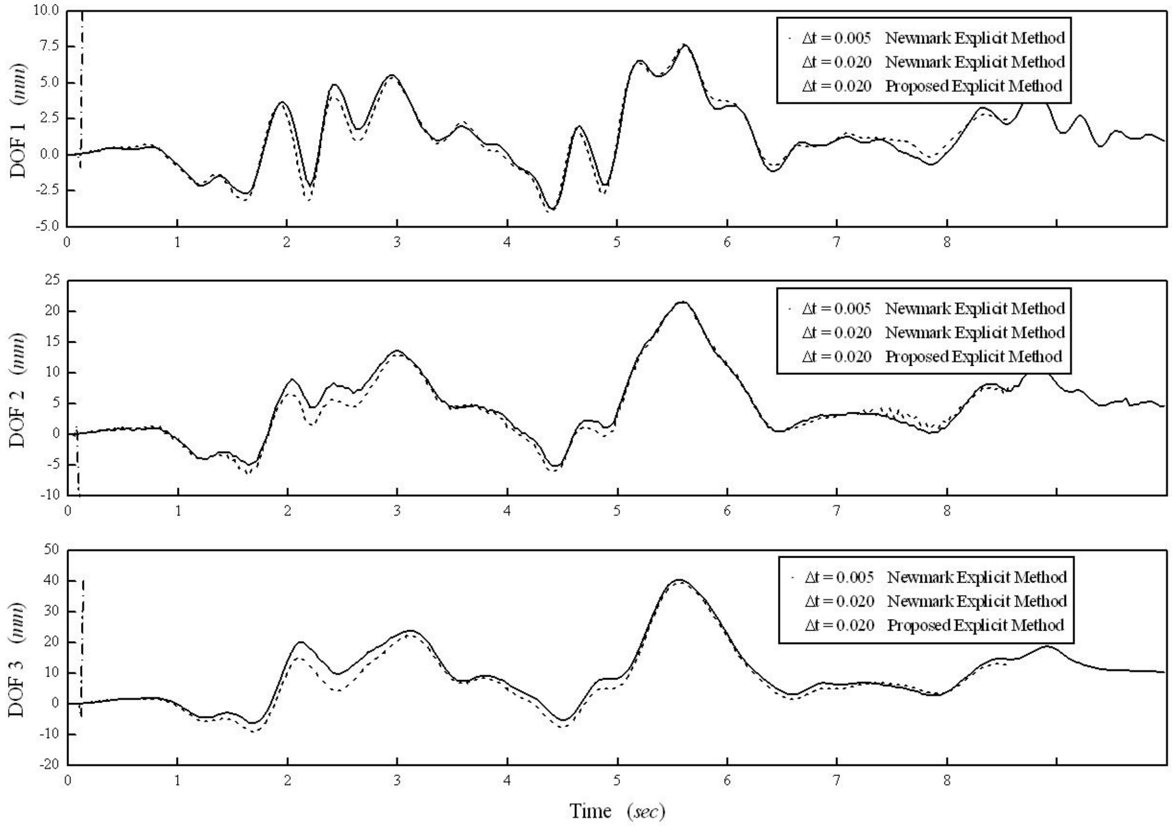

Figure 5. Pseudodynamic responses to 0.04g El Centro earthquake

The system is subjected to 1940 El Centro earthquake with a peak ground acceleration of $0.04\,g$. Both the proposed explicit method and the Newmark explicit method are used to perform pseudodynamic tests. Pseudodynamic results are shown in figure 5, where the responses obtained from the Newmark explicit method with $\Delta t = 0.005\,sec$ are considered as "c**orrect**" solutions. For this time step, the values of Ω for all the three modes are found to be 0.026, 0.063 and 1.0, and thus very accurate solutions can be achieved if the Newmark explicit method is employed. It is manifested from figure 5 that instability occurs in the early responses if using the Newmark explicit method with a time step of $\Delta t = 0.02\,sec$ while the proposed explicit method still gives very reliable results. This is because the value of Ω for the third mode is equal to 4, which is larger than the upper stability limit 2 for the Newmark explicit method. Consequently, instability occurs. On the other hand, it is indicative that the proposed explicit method is unconditionally stable since acceptable results can still be achieved for the value of $\Omega = 4$.

Improved error propagation

Error propagation analysis reveals that the proposed explicit method has much better error propagation than for the Newmark explicit method, especially for the value of Ω approaching the upper stability limit

2. This result will be confirmed by the pseudodynamic testing of a 3-degree of freedom system, whose natural frequency of the third mode is intentionally chosen to be about $100\,rad/sec$ so that the value of Ω tends to the upper stability limit if an integration time step of $\Delta t = 0.02\,sec$ is applied. This can be achieved by assigning the lumped masses for the first to third degree of freedom to be 1.6×10^5, 8×10^3 and $5.5\times10^4\,kg$. Thus, the natural frequencies of the structural system are found to be 6.3, 15.9 and $94.3\,rad/sec$. The test results are plotted in figure 6.

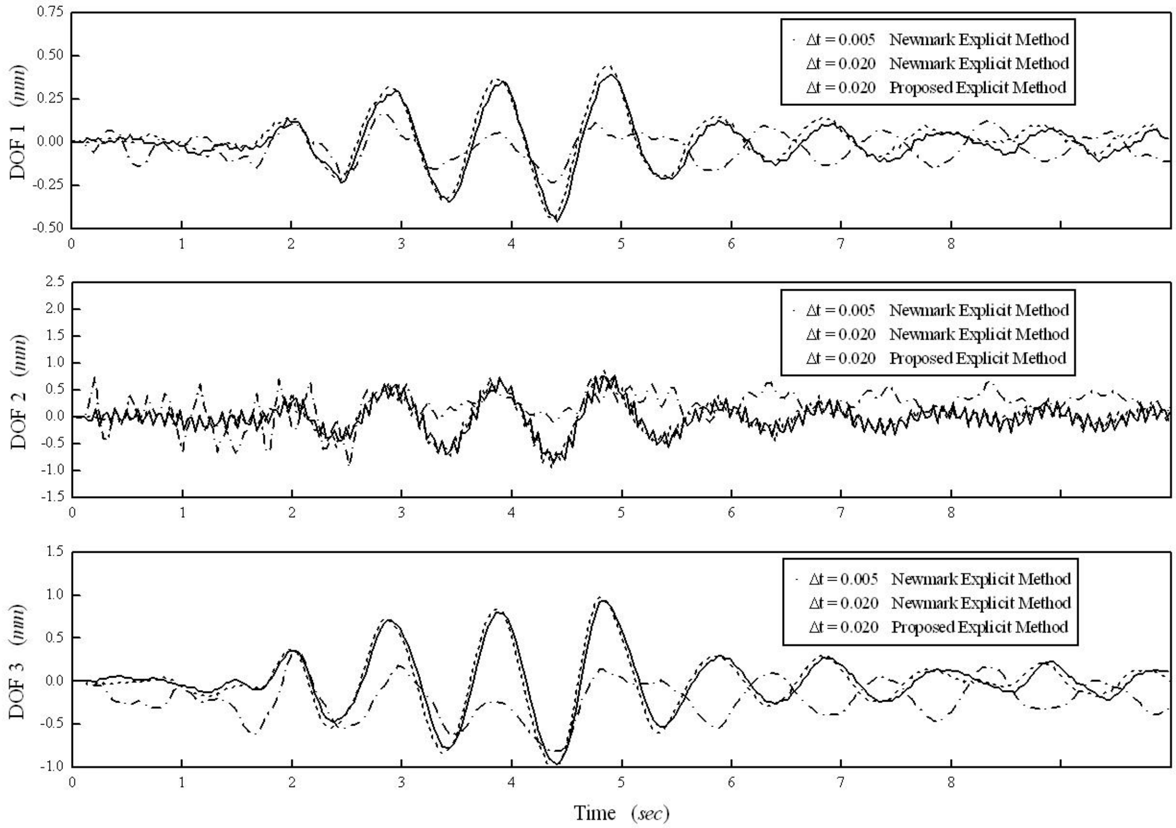

Figure 6. Pseudodynamic responses to 0.015g El Centro earthquake

Pseudodynamic results obtained from the Newmark explicit method with $\Delta t = 0.005\,sec$ are considered as “c**orrect**” solutions for comparison. For this time step, the values of Ω for all the three modes are 0.03, 0.08 and 0.47. Thus, very accurate solutions are obtained. However, the results obtained from the Newmark explicit method using $\Delta t = 0.02\,sec$ significantly deviate from the correction solutions while the proposed explicit method still gives reliable test results. Apparently, this is because that the Newmark explicit method introduces much more severe error propagation than for the proposed explicit method in the third mode response. In fact, it is found that for the use of $\Delta t = 0.02\,sec$, error amplification factors for the third modes are found to be $E_d = 3.02$ and $E_r = 5.69$ for the Newmark explicit method while for the proposed explicit method they are $E_d = 1.38$ and $E_r = 1.89$.

CONCLUSIONS

This paper presents the feasibility and the superiority of using an unconditionally stable explicit method to perform a pseudodynamic test. This algorithm has exactly the same numerical characteristics as those for the constant average acceleration method since they possess exactly the same characteristic equation. Due to the explicitness of each time step in computation, its pseudodynamic implementation can be as simple as an explicit pseudodynamic algorithm. This explicit pseudodynamic algorithm has much better error propagation properties when compared to the Newmark explicit method. All the improved error propagation characteristics are numerically and/or experimentally verified herein.

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