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INVESTIGATION OF DYNAMIC RESPONSE AND MODEL UPDATING OF INSTRUMENTED R/C BRIDGES

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SUMMARY

A finite element model updating methodology is developed as a useful tool for monitoring the condition and assessing the integrity of bridges by utilizing free and ambient vibration measurements generated by wind and traffic. The model updating methodology is based on the availability of an incomplete set of modal frequencies and modeshape components at the measured locations. The modal estimates are obtained from vibration measurements using modal identification techniques applicable to the case of unmeasured (e.g. wind and traffic) excitations. The methodology is applied to a four-span concrete bridge located in Kavala (Greece) using real measured acceleration data obtained from traffic load events.

INTRODUCTION

The need for model updating arises in the process of constructing a theoretical model of a structure for the purpose of predicting structural damage. The location and size of damage can be inferred by monitoring the reduction in stiffness properties of elements or substructures constituting the finite element model of the structure. The general problem of structural model updating involves the selection of the model from a parameterized class of models that provides the best fit to the measured dynamic data as judged by an appropriately selected measure of fit. The parameters involved in the updating are primarily structural stiffness and also mass properties, including boundary conditions as well as fixity conditions at the structural joints.

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In past years, several studies have been devoted in reconciling finite element, models with measured time history or modal data. Each method has its own advantages and shortcomings and there is no acceptable methodology for successfully treating the model-updating problem. A modal-based model updating methodology was recently developed [1] that combines available mode-shape expansion techniques with updating capabilities for predicting both the location and size of errors in the pretest finite element model of a structure. Other model updating methodologies based on various mode-shape expansion can be found in references [2-4]. Applications of these methodologies were focused on structural damage detection and structural health monitoring. These techniques use the modeshape components as unknowns to be determined by the data and have the advantage to avoid the problem of identifying the correspondence between model and measured modes. Moreover, the computation of modal frequencies and modeshapes of the finite element model is avoided.

In this study, a model updating methodology is presented which can be used for determining faithful structural models of multi-span bridges utilizing vibration measurements, as well as determining structural changes that may occur due to degradation or damage over a long period of monitoring. Specifically, a model updating methodology using incomplete modal estimates is presented. A methodology for estimating the modal properties based on free response and ambient vibration (traffic/wind) measurements data is presented. The methodology is applied to a four-span concrete bridge located at Kavala, Greece. Measured data from an array of three reference and three moving sensors are employed to assess the overall performance of the proposed methodology for model updating in the presence of incomplete modal information.

FINITE ELEMENT MODEL UPDATING USING INCOMPLETE MODAL DATA

Consider the following class of linear models:

$$M(\theta)\ddot{x} + C(\theta)\dot{x} + K(\theta)x = f(t) \qquad (1)$$

which is used to model the dynamic behavior of the bridge, where $M(\theta)$ and $K(\theta)$ are respectively the global mass and stiffness matrices which are assembled, using a finite element analysis, from the element (or substructure) mass and stiffness matrices. The class of models has been parameterized using the parameter set θ that may represent mass and stiffness properties at the element or substructure level. The parameterization is chosen such that the pre-test or undamaged finite element model of the structure corresponds to θ=1. Examples of finite element properties that can be included in the parameter set θ are: modulus of elasticity, cross-sectional area, thickness, moment of inertia and mass density of the finite elements comprising the model, as well as spring (translational or rotational) stiffnesses used to model fixity conditions at joints or boundaries.

In general, the system matrices $M(\theta)$ and $K(\theta)$ are nonlinear functions of θ. However, a parameterization which often arises in practical applications is the case for which both $M(\theta)$ and $K(\theta)$ are linear functions of θ, that is,

$$K(\theta) = K_0 + \sum_{i=1}^{p} K_i \theta_i \quad \text{and} \quad M(\theta) = M_0 + \sum_{i=1}^{p} M_i \theta_i \qquad (2)$$

where K_o, K_i, M_o and M_i, are constant matrices independent of θ.

The objective in a modal-based model updating methodology is to estimate the values of the parameter set θ so that the modal data generated by the linear class of models best matches, in some sense, the experimentally obtained modal data. A measure of fit that is explored herein is directly related to the modal dynamic force balance residuals $r(\omega,\phi,\theta)$, defined by $r(\omega,\phi,\theta)=\left(K(\theta)-\omega^2 M(\theta)\right)\phi$. Note that the modal dynamic force balance residuals satisfy the equations $r(\omega_i(\theta),\varphi_i(\theta),\theta)=0$, $i=1,\ldots,m$, where $\omega_i(\theta)$ and $\varphi_i(\theta)$, $i=1,\ldots,m$ are respectively the modal frequencies and mass-normalized mode shapes of the first m modes of the model.

For convenience, let subsets a and o be the sets of measured and unmeasured model degrees of freedom, respectively. The set $[a,o]$ contains the total number of degrees of freedom of the structural model. Each mode shape vector ϕ_i can be partitioned in the form $\phi_i^T=[\phi_{ai},\phi_{oi}]$ where ϕ_{ai} and ϕ_{oi} are the components of the modeshape ϕ_i at the measured and unmeasured degrees of freedom, respectively.

In this study it is assumed that in addition to the model parameters θ, the modeshape components ϕ_i at all DOFs of the model are unknown quantities to be determined by the data. Let $\hat{\omega}_i$ and $\hat{\phi}_{ai}$ be the experimentally obtained frequencies and mode-shapes of the structure at the measured degrees of freedom. The proposed method for model updating searches for the optimal model parameters θ and the modeshape components ϕ_i which minimize an appropriately selected norm of the modal dynamic force balance residuals $r(\omega_i,\phi_i,\theta)$ and a measure of fit between the unknown modeshapes ϕ_i and the measured modeshape components at the measured degrees of freedom. Mathematically, the model-updating problem is stated as: find θ and ϕ_i that minimize the measure of fit [5]

$$J(\theta,\phi_i)=\sum_{i=1}^{m}\left[\frac{1}{\hat{\omega}_i^4}\left\|\left(K(\theta)-\hat{\omega}_i^2 M(\theta)\right)\phi_i\right\|^2+\frac{\left\|\phi_{ai}-\hat{\phi}_{ai}\right\|^2}{\left\|\hat{\phi}_{ai}\right\|^2}\right] \tag{3}$$

where $\|x\|^2 = x^T x$ is the usual Euclidian norm.

The unknown quantities involved in the proposed error measure include the model properties θ, as well as the components of the vector ϕ_i of the contributing modes at both measured and unmeasured model degrees of freedom. The optimal vector ϕ_i resulting from the minimization can be viewed as the expanded modeshape consistent with the measured modal data. An iterative algorithm is used to find the optimal solution to the problem stated above. For this, a computationally efficient finite element model updating methodology has been developed in Matlab programming environment to handle arbitrary, user-defined, parameterization schemes involving parameters associated with physical properties.

MODAL IDENTIFICATION USING VIBRATION MEASUREMENTS

A modal model for the bridge is introduced and a system identification methodology is applied for determining the modal characteristics of the bridge, consisting of modal frequencies, damping ratios and incomplete modeshape components at the measured locations. Both classically damped and non-classically damped modal models are employed.

A time domain output-error approach is used to identify the modal parameters based on measured free vibrations $\hat{X}_j(k\Delta t)$ at $j=1,\ldots,N_{out}$ measured locations, where $k=1,\ldots,N$ is the time index and Δt the discrimination time interval. Using complex mode analysis, the free vibration response $X_j(k\Delta t;\psi)$ of the structure at a measurement location j can be written in the form

$$X_j(k\Delta t;\psi)=\sum_{r=1}^{m} 2\,\mathrm{Re}\{\phi_{jr} e^{\lambda_r k\Delta t}\} \quad (4)$$

where $\lambda_r = -\zeta_r\omega_r + i\omega_r\sqrt{1-\zeta_r^2}$, ω_r and ζ_r are respectively the modal frequency and damping ratio of mode r , ϕ_{jr} is the j th component of the r th complex modeshape, and m is the number of contributing modes. Thus, the response depends on the set of modal parameters $\psi=(\theta,\phi)$, where the subset $\theta=\{\omega_r,\zeta_r, r=1,...,m\}$ includes only the modal frequencies and damping ratios, while the subset $\phi=\{\phi_{jr}, j=1,...,N_{out}, r=1,...,m\}$ includes the modeshape components at the corresponding measured locations. In particular, for the case of classically damped modes, it can be easily verified that $\phi_{jr}=\varphi_{jr}(1+ia_r)$, where φ_{jr} is the j th component of the r th real modeshape.
The optimal estimates of the parameters ψ minimize the measure of fit

$$J(\psi)=\sum_{k=1}^{N}\sum_{j=1}^{Nout}[X_j(k\Delta t;\psi)-\hat{X}_j(k\Delta t)]^2 \quad (5)$$

between the measured response $\hat{X}_j(k\Delta t)$ and the response $X_j(k\Delta t;\psi)$ predicted from the modal model. Taking into account that the objective function is quadratic in ϕ, the modeshape components included in ϕ can be obtained in terms of θ by solving an algebraic linear system resulting from the stationarity condition $\partial J(\psi)/\partial\phi=0$. The resulting relation between ϕ and θ is denoted by $\phi(\theta)$. Substituting in (2), the optimal parameter values are finally obtained by minimizing the modified function $J^*(\theta)=J(\theta,\phi(\theta))$ as a function of the model parameters θ, while the optimal modeshape components $\hat{\phi}$ are obtained from the relation $\hat{\phi}=\phi(\hat{\theta})$, where $\hat{\theta}$ are the optimal model parameters obtained by minimizing $J^*(\theta)$. The optimization is carried out using an efficient modal sweep approach in which for each sweep a series of m smaller optimization problems are solved with the rth problem involving the estimation of the parameters in θ associated with the r mode only, while holding the parameter values of the rest of the modes fixed at their latest computed values.
For ambient vibration data generated from traffic and wind loading, the optimal estimates of the modal parameters are obtained by minimizing the measure of fit

$$J(\psi)=\sum_{k=1}^{L} tr[S_{\ddot{Y}}(\omega_k;\psi)-S_{\hat{\ddot{Y}}}(\omega_k)]^2 \quad (6)$$

between the cross power spectral density (CPSD) $S_{\hat{\ddot{Y}}}(\omega)$ of the measured responses and the CPSD $S_{\ddot{Y}}(\omega;\psi)$ of the responses predicted from the modal model assuming a broadband white noise excitation. The CPSD of the response of a modal model of a structure subjected to white noise excitations is $S_{\ddot{Y}}(\omega;\psi)=\Phi\mathrm{H}(\omega)S_f[\overline{\Phi\mathrm{H}(\omega)}]^{\mathrm{T}}$, where $H(\omega)$ is a diagonal matrix with the r diagonal element given by the transfer function $H_{rr}(\omega)=\omega^2(\omega_r^2-\omega^2+2\zeta_r\omega_r\omega i)^{-1}$, Φ is a matrix of the modeshape components at the measured locations, and S_f is constant matrix which depends on the PSD values of the white noise excitation. The elements of S_f are unknown and are included in the modal parameter set ψ. A Matlab code for modal identification in the free and the ambient vibration cases was developed and validated using simulated and real measured acceleration data from a bridge structure.

APPLICATION TO THE MULTI-SPAN BRIDGE IN KAVALA, GREECE

Structural Description and Data Acquisition

The bridge is 180-meter-long, it has four-spans and the deck of each branch of the dual carriageway is 13 meters wide. The deck has no intermediate expansion joint and it is “floating” on laminated elastomeric bearings. The hollow concrete piers are square in plan, with a 40 cm wall thickness. The layout of the structure and the finite element model are presented in Figure 1.

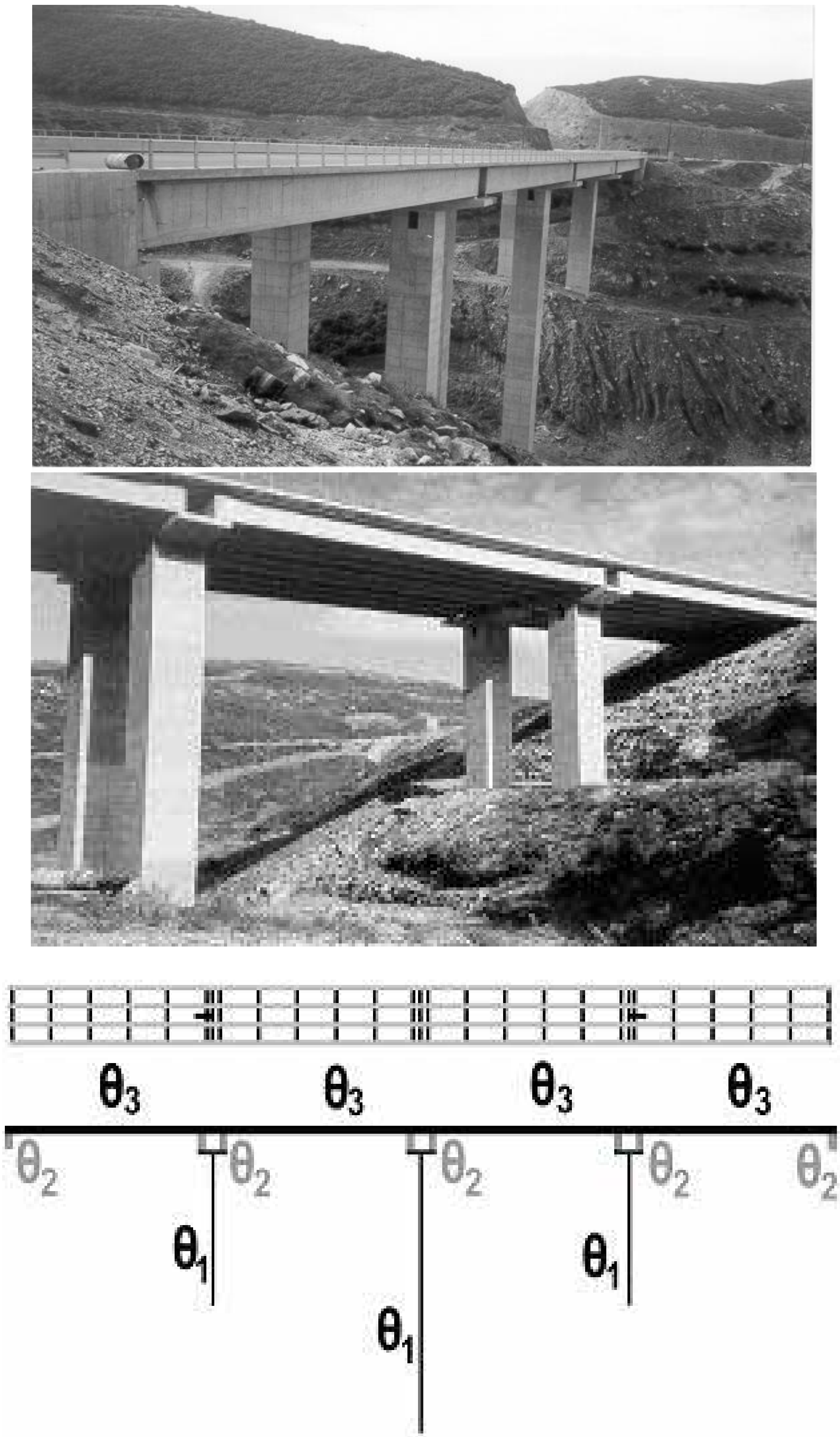

Figure 1. Bridge configuration and finite element model for model updating

The special structural array used in the instrumentation was a mobile one by Kinemetrics Inc. The system consists of a central recording unit (of type K2), which can support up to 12 sensors (uniaxial force balance, ± 2g full scale, accelerometers of type FBA-11). The recording unit has a 19-bit resolution, a sampling rate capacity of up to 200sps and a dynamic range of 108 dB @ 200 sps. The system offers the capability of setting independent triggering threshold (ranging from 0.01% to 100 % full scale) for each sensor, while the user can predetermine the sensors, or combinations of them, that will trigger the system. Recordings are stored in the system's flash memory, and can be retrieved either in site, or through a modem.

After acquisition, the records are suitably treated in order to eliminate to a great degree the various errors stemming from the whole recording procedure (e.g. elimination of instrument and environmental noise, baseline – offset corrections etc). Further treatment yields the corresponding velocity and displacement time-histories at each sensor position. Especially for the elimination of noise, a filtering procedure is further applied using an Ormsby window filter with terminal (low-pass) and corner (high-pass) frequencies (and corresponding roll-off widths) appropriately selected, so as to get displacement time-histories at the ground sensors acceptable from a physical point of view. Apart from the accumulated experience of the research team on the proper treatment of the recorded data, use is also made of special signal-to-noise methodologies in order to establish the most appropriate filtering limits.

Modal Identification

Modal identification results are presented only for the case of free vibration data generated from traffic excitation by large trucks traveling on the bridge. The free vibration acceleration response that is used in the identification procedure corresponds to the time history segment of the response obtained after the truck(s) leave the bridge deck. One of the objectives is to study the modal characteristics that can be identified from such free vibration data. This analysis is of great importance in the long-term monitoring of the bridge behavior towards service loads, wind pressure and earthquake shocks, and aims at the diagnosis of changes in the dynamic characteristics during the lifetime of the structure.

The recorded data for the structure include several data sets obtained using an array of six accelerometers. In order to be able to get an estimate of the modeshape with the available system of six sensors, three of the sensors were placed at fixed reference positions (two vertical and one transverse) at span C, while the other three were moved along deck of the C and D span of the bridge. The reference and moving positions of the sensors along with the measurement directions are shown in Figure 2. From the set of three sensors used in the reference and the moving positions, two of them monitor the vertical accelerations at the left and right side of the deck, and one of them monitors the deck acceleration in the transverse direction.

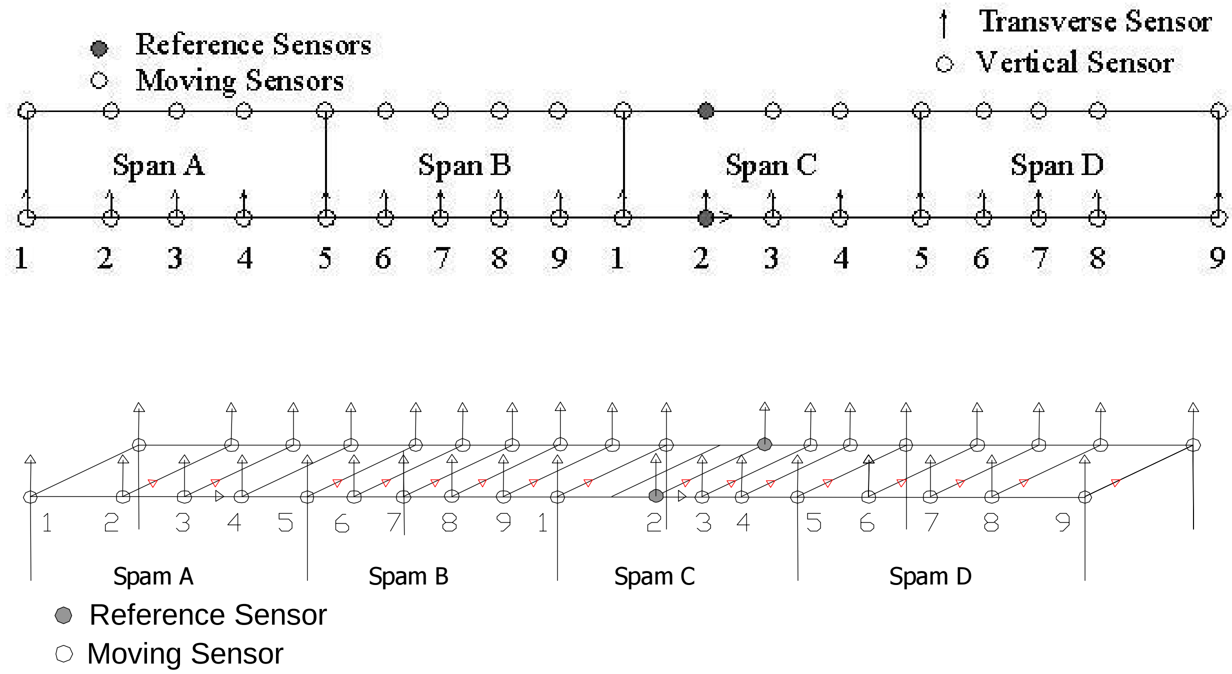

Figure 2. Reference and moving sensor locations on the Kavala bridge deck

Three modes, one transverse, one bending and one longitudinal mode were excited and identified in almost all traffic excitation events. Three additional modes were also excited and identified in a very small fraction of the total number of excitation events and moving sensor locations so that a complete characterization of the modeshape of each of these additional modes was not possible. The latter modes are not considered in the present work. The values of the former three identified modal frequencies and modal damping ratios for the identified modes are presented in Table 1. Results are obtained for the case of non classically damped modal models. The values shown in Table 1 correspond to the mean and the coefficient of variation of the values obtained by processing several traffic excitation events. The results indicate a very good estimate for the modal frequencies with scatter of the order of 1% about the mean. The modal damping ratio estimates appear to have much greater scatter compared to the modal frequency estimates. The estimated modeshapes are shown in Figure 3 for the transverse and bending modes.

Table 1. Optimal Estimates of Modal Frequencies and Damping Ratios

Mode	Frequency (Hz)	Damping ratio %
Transverse	0.8200	1.31
Longitudinal	1.9200	3.87
Bending	3.5400	1.53

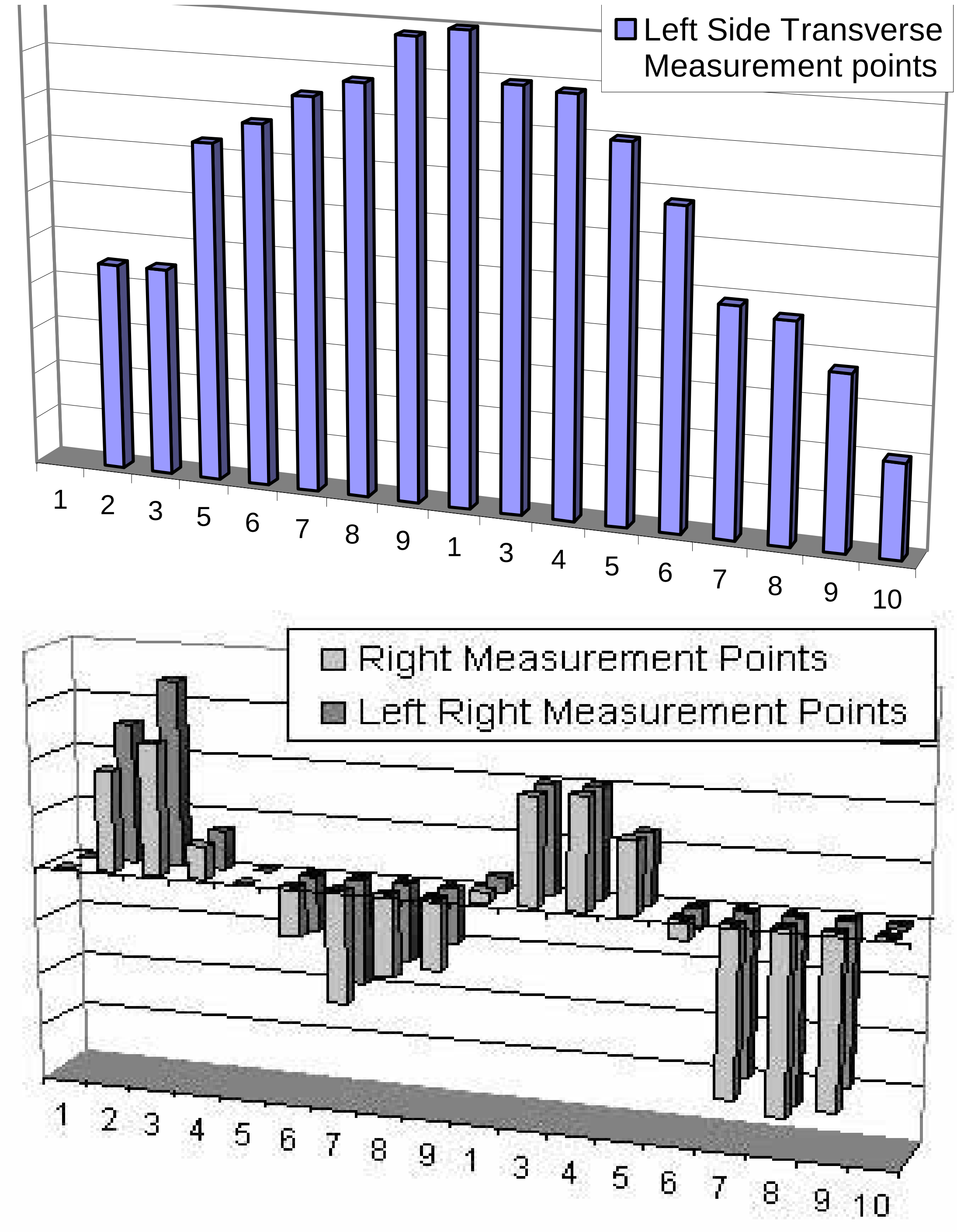

Figure 3. Identified modeshapes for transverse and bending modes

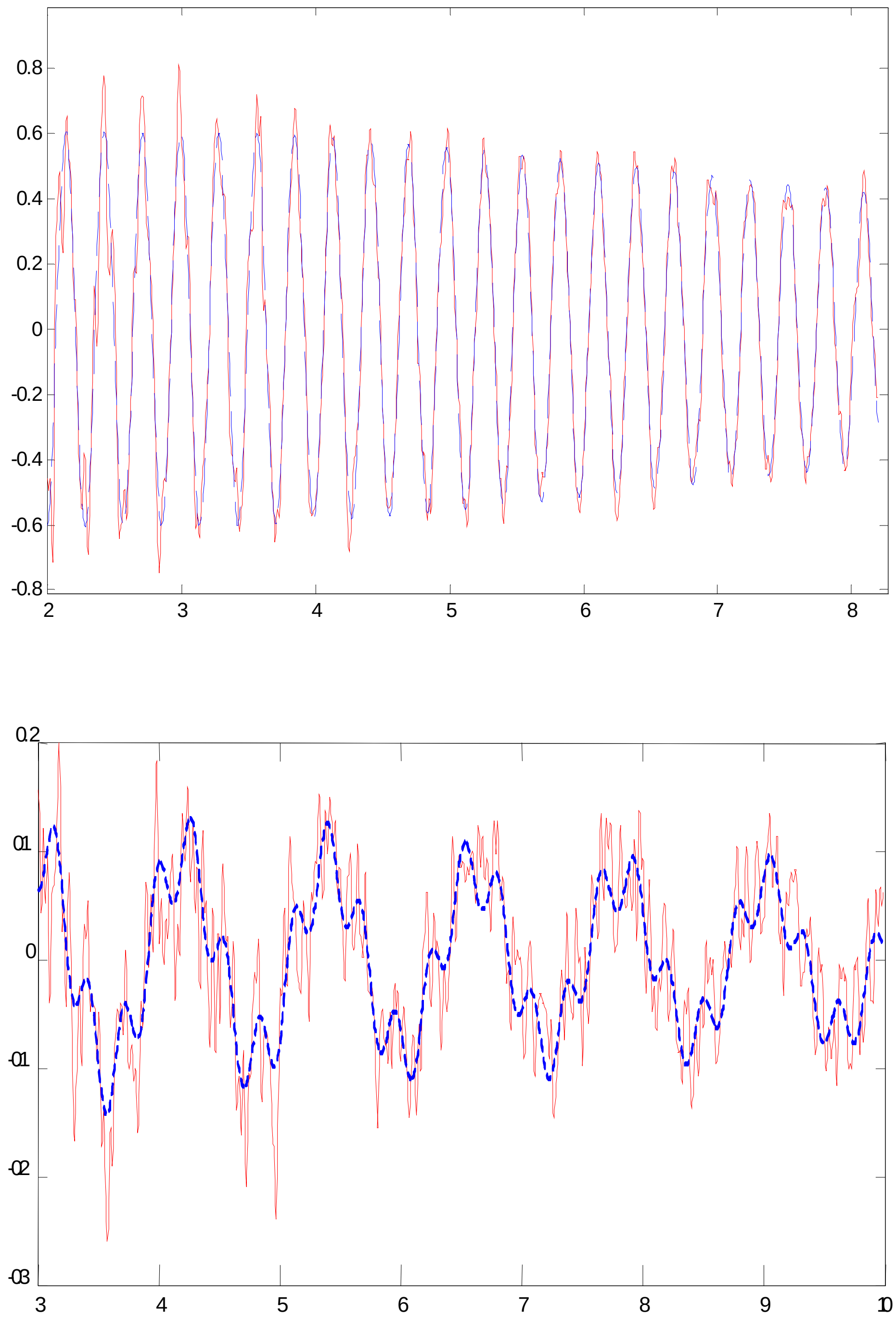

Figure 4: Comparison between measured acceleration and predicted acceleration from the non-classically damped modal model (a) Vertical direction at position 4, (b) Transverse direction at position 2

Finite Element Model Updating

A three dimensional, 900 degree-of-freedom, finite element model for the bridge (Figure 1) consisting of beam elements was developed using a finite element program written in Matlab. The parameters of the beam elements were chosen so that the pre-test finite element model corresponds to the finite element model used during the design phases of the bridge. This “design” model was constructed to simulate the behavior of the bridge during strong earthquake shaking. The design model was updated based on the measured modal data. For this, the finite element model was parameterized by introducing three parameters θ_1, θ_2 and θ_3. The first parameter accounts for the stiffness of the three piers, the second parameter accounts for the stiffness of the elastomeric bearings and the third parameter accounts for the stiffness of the deck. The parameterization is schematically shown in Figure 1.

The parameterization was such that the finite element model used in the design phase corresponds to parameter values $\theta_1 = \theta_2 = \theta_3 = 1$. The values of these parameters were updating by minimizing the measure of fit (3) using all three modes, the transverse, the longitudinal and the bending mode identified from the field tests. Using those 3 modes, there was a unique optimal solution. Optimal estimates of the model parameters are given in Table 2. Sensitivity studies showed that the transverse mode is insensitive to changes in the stiffness values θ_3 of the deck, while the vertical mode is insensitive to the stiffness values θ_1 and θ_2 of the piers and the bearings.

Table 2. Optimal Estimates of the Finite Element Model Parameters Using Three Modes

Parameters	Optimal Estimates
θ_1	0.63
θ_2	34.83
θ_3	1.53

It should be noted that the stiffness value of the elastomeric bearings seems to be higher than the stiffness value used in the design finite element model. This is due to the fact that in the design phase, the stiffness of the elastomeric bearings used for defining the design finite element model was reduced considerably from its nominal value in order to account for the observed stiffness reduction expected during strong earthquake shaking. Also, based on the identified results, the stiffness of the deck is approximately 55% higher than the stiffness used in the design finite element model. This may be due to the fact that the small amplitude vibrations used to identify the modes and update the finite element model were not strong enough to excite the mechanisms upon which the design finite element was based.

CONCLUSIONS

A finite element model updating methodology was presented which is based on incomplete modal data information obtained by analyzing free and ambient vibration measurements generated by wind and traffic loads. The proposed model updating methodology can be integrated with a monitoring system to continually monitor structural response in order to determine faithful structural models for bridges, as well as update the information about the integrity and hence the reliability of structures after severe loading events. This information is of great importance because of its high potential for practical applications related to improving structural safety during operation and reducing structural maintenance and rehabilitation costs.

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