



COLUMN INTERACTION EFFECT ON PUSH OVER 3D ANALYSIS OF IRREGULAR STRUCTURES

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SUMMARY

This work presents results of push over analyses to assess the effect of interaction of bending moments and axial loads in columns of low-rise frame buildings with torsion. Translation and rotation push over curves, as well as frame relative displacements, were computed with two type of 3D push over analyses. The first type (*complete* analysis) considers nonlinear interaction between bending moments and axial loads in columns, while the second (*simple* analysis) neglects this interaction and assumes columns of intersecting frames as two independent elements. Results indicate that *simple* analyses can significantly underestimate building lateral displacements at ultimate base shears.

INTRODUCTION

Actual trends in seismic design of buildings seem to be oriented to verify the structural performance at several ground motion intensities [1]. One of these intensities, which is the most important from the structural point of view, is related to the maximum lateral capacity of the structure. Although there are available computational tools to estimate this capacity by using nonlinear dynamic analysis [2], currently it is not practical to use this type of analysis in design offices. In general, this analysis require a large amount of information that demands too much time to prepare, particularly for three dimensional (3D) models. For practical application, simpler procedures are preferred to estimate the lateral capacity of structures. One solution that has been suggested [3] is the use of a static nonlinear (push over) analysis to evaluate the lateral displacement capacity of structures. There are several types of proposals [2, 4] to accomplish this type of analysis. Although similar, push over computer programs do not handle the interaction in columns of intersecting frames (or walls) in the same way. Limiting the discussion to columns for instance, reference [2] allows the user to incorporate multi-spring models that simulate the nonlinear column response and take into account the interaction of bending moments and axial load. On the other hand, simple analysis of reference [3] neglects this interaction.

Before push over analysis can be incorporated effectively in the evaluation of the lateral capacity of buildings within a performance-based design procedure, it is important to assess the importance of column interaction, particularly for buildings susceptible to torsion. This assessment can be useful to determine if

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simple (without interaction) 3D push over analyses can acceptably estimate the main response (or design) parameters of buildings, particularly of those torsionally unbalanced (TU) ones.

This work shows comparisons of the computed structural responses of two TU low-rise frame buildings using two types of 3D push over analyses. The first one, identified here as a “*complete*” push over 3D analysis, which takes into account the interaction of biaxial bending and axial load in columns; and the second one, identified here as a “*simple*” push over 3D analysis, that neglects this interaction. Both analyses, however, use the hypothesis of rigid body motion for slabs.

STRUCTURAL MODELS

In order to study the influence of the interaction of bending moments and axial load in columns, within the context of the push over analysis, two five-story concrete frame building models were used. The model geometry, which is shown in Fig. 1, has three lateral resisting frames along each orthogonal direction. For the frame arrangement shown, and assuming the same column and beam properties for all frames, as indicated in Table 1, the resulting centre of stiffness (CS) is located at coordinates (4.67, 3.83) m.

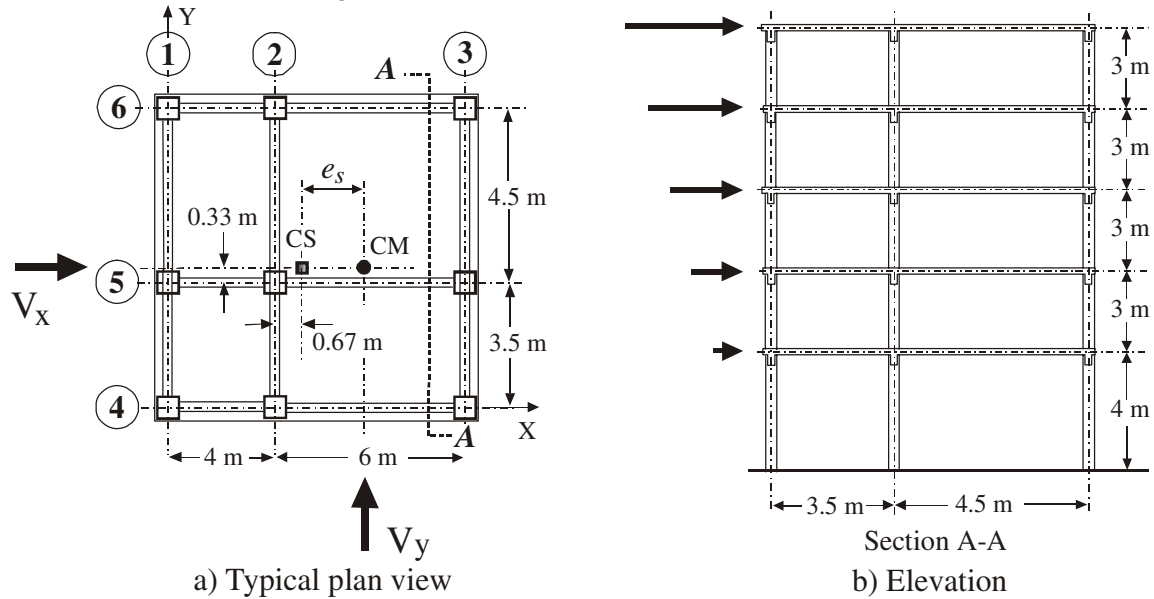


Figure 1. Model geometry

Table 1. Section properties of models (initial modulus of elasticity = $E = 250,000 \text{ kg/cm}^2$)

Columns			Beams of frames 1, 2, and 3			Beams of frames 4, 5, and 6		
Set	Stories	Dimensions [cm]	Set	Levels	Dimensions [cm]	Set	Levels	Dimensions [cm]
1	1	35 x 35	1	1	15 x 45	1	1	20 x 50
2	2 and 3	30 x 30	2	2 and 3	15 x 45	2	2 and 3	20 x 50
3	4 and 5	25 x 25	3	4 and 5	15 x 45	3	4 and 5	20 x 50

Two values of the normalised natural eccentricity ($e = e_s/b$, with $b = 10 \text{ m}$) were considered (0.05 and 0.15). For torsion static design, the Uniform Building Code (UBC-97) [5] recommendations were used for the selection of reinforcement for the section sets indicated in Table 1. These recommendations use the following well-known formulas for computing design eccentricities (e_d)

$$\text{primary eccentricity:} \quad e_{d1} = \alpha e_s + \beta b \quad (1)$$

$$\text{secondary eccentricity:} \quad e_{d2} = \delta e_s - \beta b \quad (2)$$

where α and δ are factors that modify the natural eccentricity e_s to account for dynamic effects in the static design. The factor β , which is related to accidental eccentricity, is assumed equal to zero in this work. Design was carried out with $\alpha = 1.0$, $\delta = 0.0$ and $\beta = 0.0$.

A base shear $V_b = 34,000$ kg was selected for design, which was distributed according to the recommendations of the UBC-97 [5]. By assuming equal distribution of gravity loads for all floors, this led to a triangular distribution of lateral forces as indicated in Fig. 1b. Lateral loads along both orthogonal directions were assumed equal each other for design; however, eccentricity was considered for one direction only, as indicated in Fig. 1a which is typical for all stories. Lateral forces were applied at centres of mass.

PUSH OVER ANALYSES AND OUTPUTS

Types of push over analyses

Two types of push over analyses were considered to assess the importance of interaction in columns. The first type was a “*complete*” 3D push over analysis, which considers a multi-spring model for columns that takes into account the interaction of bending moments and axial loads. To carry out this analysis, the computer program of reference [2] was used. For simplicity however, gravity loads were neglected.

The second type was a “*simple*” 3D push over analysis, which neglects the interaction of bending and axial load in columns. As indicated before, common columns of intersecting frames are considered as separate elements. Moreover, within a frame, bending and axial loads are uncoupled for columns (and beams). For this type of analysis, a special purpose program based on the “*one-component*” model proposed by Giberson [6] was developed. It also was assumed that member axial deformations were equal to zero. The hypothesis of rigid-diaphragm for slab movements was also used. Similarly to the *complete* analysis, for this (*simple*) analysis gravity loads were neglected.

For both types of analyses, two values of natural eccentricity and three magnitudes of orthogonal base shear (V_x) were used, as indicated in Table 2. Base shear V_y , which was the main shear force, was applied at all analysis cases.

Table 2. Cases considered for comparison of push over analysis types

Case No.	$e = e_s/b$	Orthogonal base shear
1	0.05	$V_x = 0.0$
2		$V_x = 0.30 V_y$
3		$V_x = V_y$
4	0.15	$V_x = 0.0$
5		$V_x = 0.30 V_y$
6		$V_x = V_y$

Outputs

Push over analysis results can be given in a wide variety of ways; however, for studying the column interaction effect within the push over context, some specific outputs are selected here. The first selected output, which is the more used, is a graphical relationship between the base shear V_b and the lateral displacement u_n of the building top centre of mass (CM). This curve is also known as the (*translation*) *push over curve*. It is interesting observe that the CM displacement is commonly preferred to report push

over results of TU buildings [7, 8]. The *capacity curve* [9], which is obtained by normalising the push over curve, is not used in this study because basically it gives the same information than the push over curve. Results in this work are referred to both V_y amplitude and displacements along the Y axis.

The second output to be used for the study of column interaction is the rotation of the building top slab. The resulting curve is identified here as the *rotation push over curve*. This parameter is important to assess the building torsion stiffness variations caused by column interaction. The third output is the story relative displacements of frames 1 and 3. Although both previous outputs can give an idea of these displacements, it is desirable to have these distributions at hand to directly observe the column interaction effect at the more critical frames. This third output can also be used to assess the damage increase in the building. The fourth output considered in this work is the set of patterns of nonlinear distribution at each frame. Usually, this can be observed by drawing small circles at each member end with a size (radius) proportional to its ductility demand. It is important to realise that this pattern is useful for design purposes also. By an appropriate “accommodation” of circles of about the same size all over the building (preferable at beam ends), the designer can improve the dissipation-energy mechanism at ultimate lateral loads.

Finally, the fifth output is the value of the plastic energy dissipated at each frame, which can be obtained from the summation of the plastic energy dissipated at element ends. As indicated in Fig. 2, plastic energy at each member end can be estimated with the relationship between bending moment and plastic rotation. This output gives the designer (along with the pattern of nonlinear distribution) the information to accommodate the inelastic behaviour among frames. This parameter, however, is not computed by all push over programs.

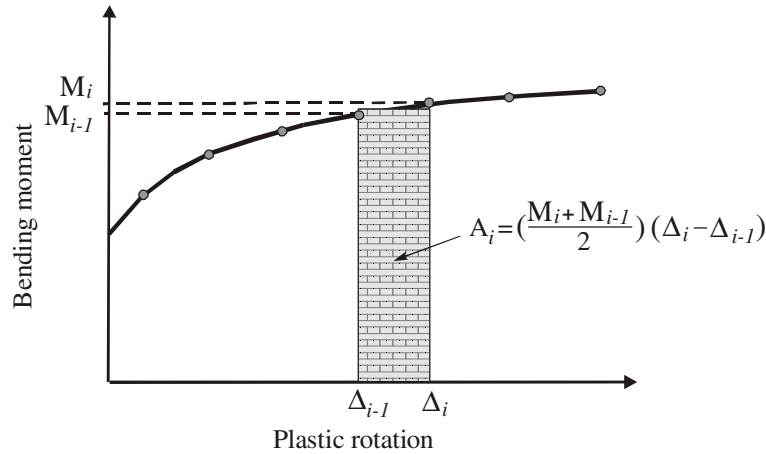


Figure 2. Plastic energy computation for a load increment

RESULTS

In this section, results computed with two types of analyses are studied: 1) a “*complete*” analysis [2] that includes a model that takes into account the interaction of bending moments and axial load in columns and 2) a “*simple*” program that does not take into account the column interaction. It is assumed that column interaction effect can be assessed by comparing outputs of both programs. These comparisons are presented in the following paragraphs for each case indicated in Table 1.

Case 1: $e = 0.05$ and $V_x = 0.0$

Fig. 3 shows translation and rotation push over curves as well as story relative displacements for frames 1 and 3 of *Case 1* (see Table 1). Fig. 3a shows the translation push over curve computed with and without column interaction. It can be observed that for this case, the distance between both curves is small, which seems indicate that if push over analysis is just required to compute the structure *capacity diagram* [9], the use of a simple program (without column interaction) seems to be acceptable.

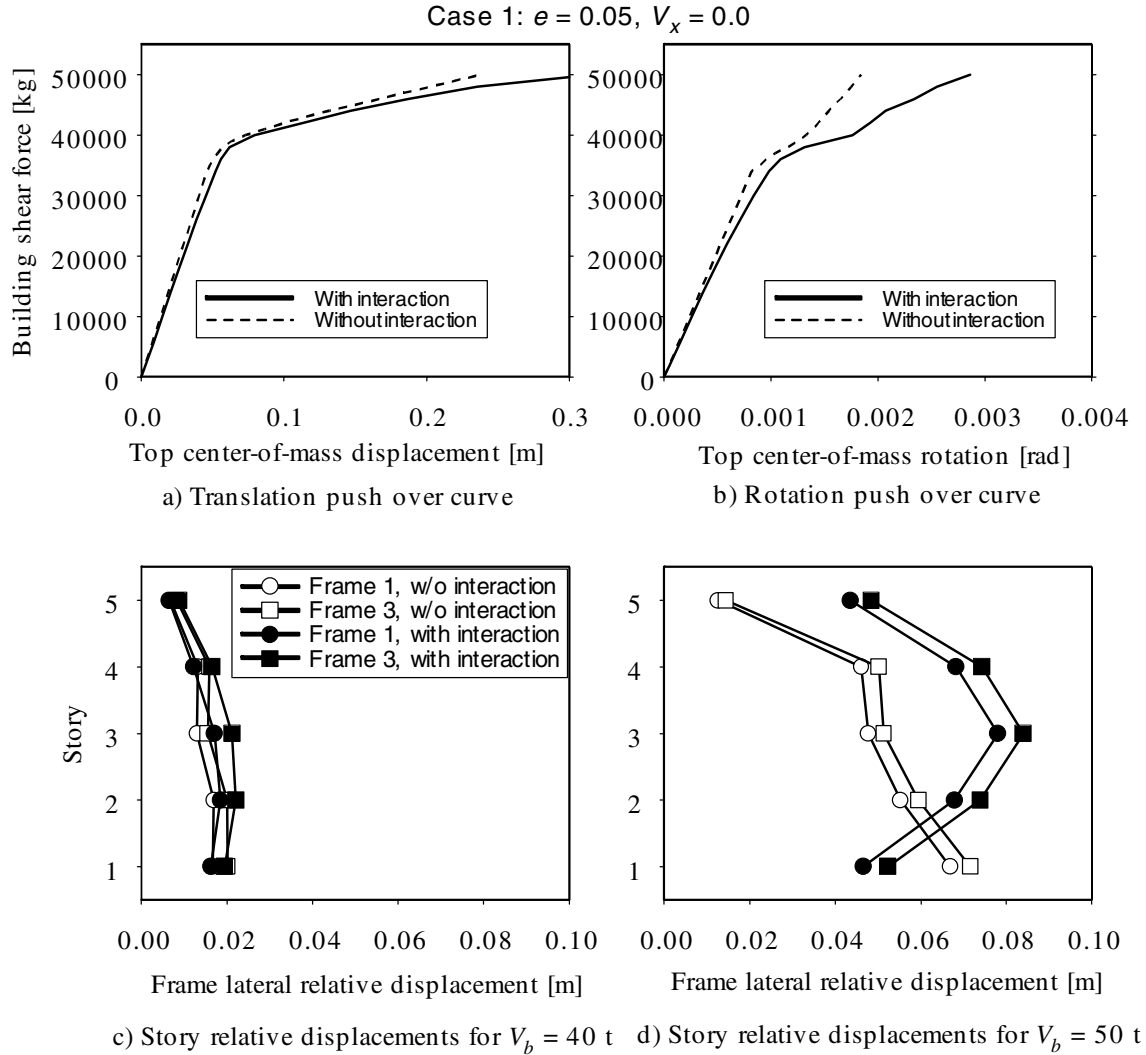


Figure 3. Push over curves and frame relative lateral displacements for *Case 1*

Fig. 3b shows rotation push over curves computed with both analyses. For this case, it is observed that column interaction becomes important at large values of base shear. The effect of column interaction on slab rotation can be also observed in Figs. 3c and 3d, which show frame lateral relative displacements for two base shear values ($V_b = 40$ t and $V_b = 50$ t), respectively. Again, interaction seems important at large values of base shear (or deformation). In Fig. 3c, the maximum drift computed with the *complete* analysis was $\Delta_{max} = 0.007$; while in Fig. 3d, $\Delta_{max} = 0.028$. The dissipated plastic energy, which was computed with the simple analysis only, is

summarised in Table 3 and corresponds to the plastic hinges shown in Fig. 4 for $V_b = 40$ t. Notice that, although percentages of energy dissipation are about the same among the three frames, frame 3 dissipates more plastic energy than frames 1 and 2 as expected.

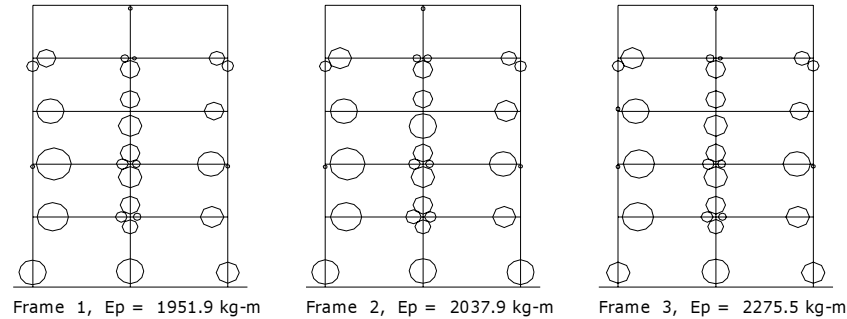


Figure 4. Distribution of plastic hinges computed with a *simple* analysis for *Case 1*.

Table 3. Computed dissipated plastic energy E_p [kg-m]

Case	Frame 1	Frame 2	Frame 3	Frame 4	Frame 5	Frame 6	Total
1	1951.9 [31%]	2037.9 [33%]	2275.5 [36%]	-	-	-	6265.3
2	1951.6	2037.9	2275.8	0.0	0.0	0.0	6265.3
3	1387.6 [12%]	1969.2 [18%]	3061.0 [27%]	2181.4 [20%]	1561.5 [14%]	944.0 [9%]	11104.7
4	1323.0 [26%]	1566.4 [31%]	2199.6 [43%]	-	-	-	5089.0
5	1322.8	1566.4	2199.9	0.0	0.0	0.0	5089.1
6	211.5 [2%]	1455.8 [14%]	4290.4 [41%]	3069.7 [29%]	1421.7 [14%]	0.0 [0%]	10449.7

Case 2: $e = 0.05$ and $V_x = 0.30V_y$

This case is similar to *Case 1* but it includes a perpendicular base shear ($V_x = 0.30V_y$) that acts simultaneously with V_y . Results obtained for this case are not shown here because they are practically equal to those shown in Fig. 3. No significant differences were found for results computed with both types of analyses. Plastic energy values did not show any significant change with respect to those computed for *Case 1* (Table 3).

Case 3: $e = 0.05$ and $V_x = V_y$

Push over curves and frame relative displacements are shown in Fig. 5. For this extreme case, Fig. 5a shows that column interaction leads to significantly larger building displacements than those computed without column interaction for high values of base shear. On the other hand, rotation push over curves resulted almost equal each other for both types of analysis. The combination of these results in the computation of relative lateral displacements of frames 1 and 3 leads to Figs. 5c and 5d. As observed before for *Cases 1* and 2, for *Case 3* displacements computed with a *simple* analysis are smaller than those computed with a *complete* analysis, particularly for high values of base shear. In this case, frame displacements computed without column interaction can be underestimated in approximately 50% with respect to those computed with column interaction at ultimate levels of load. Notice however, that for common rule

combinations of lateral forces (*Case 2*, Fig. 3) maximum frame lateral displacements are underestimated in about 20% when column interaction is neglected.

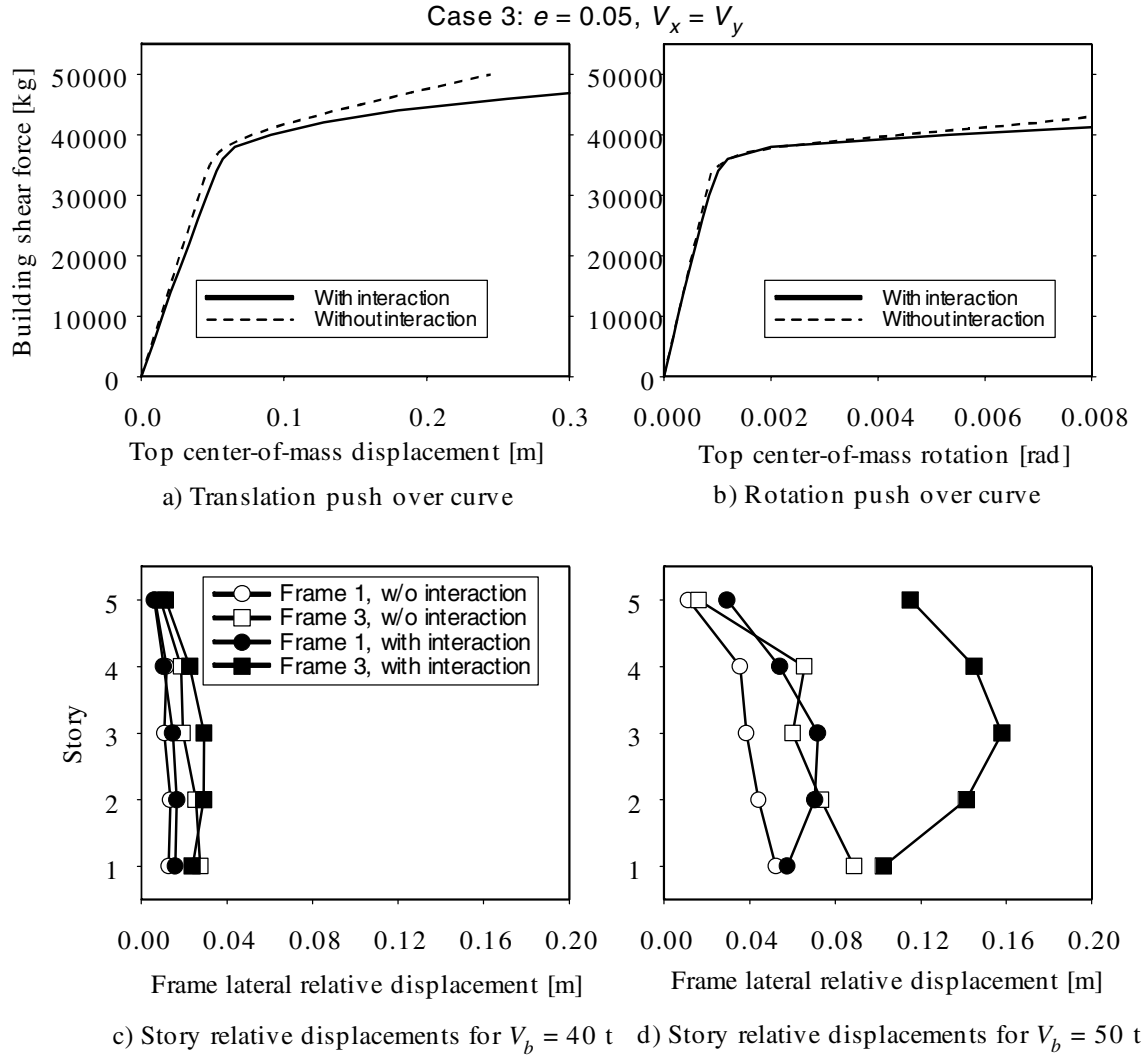


Figure 5. Push over curves and frame relative lateral displacements for *Case 3*

Plastic energy values are indicated in Table 3. It can be observed that plastic energies dissipated by frames 1 and 2 are smaller for *Case 3* than for *Cases 1* and 2. On the other hand, energy dissipated by frame 3 is larger for *Case 3* than that for *Cases 1* and 2. At first sight, these results of *Case 3* seem erroneous because structures of *Cases 1*, 2, and 3 are the same each other (same design) and base shear V_x is applied at the centre of stiffness. Moreover, if these energy values are computed with a *simple* analysis (without interaction), one would expect no energy variation in these three cases. The explanation is simple: these energy variations for *Case 3* indicate that V_x also causes torsion in the nonlinear behaviour, which arouses because the centre of strength does not coincide with the centre of stiffness. Thus, torsion originated in the nonlinear range by V_x influences the response of orthogonal frames.

Case 4: $e = 0.15$ and $V_x = 0.0$

This case is similar to *Case 1* but the corresponding model has a larger eccentricity due to the a right-hand shift of the centre of mass. Design of this model was carried out with the same static procedure and factors used for previous cases. Because of the larger eccentricity for cases 4, 5, and 6 (as compared with cases 1, 2, and 3), strength properties of beams and columns are different (generally larger) from those of previous cases. Notice however, that cross section geometry for beams and columns is the same for all cases (cases 1 through 6). Thus, not only the location of the centre of mass is changed for cases 4, 5, and 6 but also the centre of strength.

Fig. 6 shows the push over curves for this fourth case. By comparing Fig. 6a with that of *Case 1* (similar case but with smaller eccentricity), it can be observed that the difference between translation push over curves computed with both analyses (*simple* and *complete*) is larger for *Case 4* than for *Case 1*. It seems that larger eccentricities lead to larger differences between translation push over curves. As for rotation push-over curves (Fig. 6b), results indicate that eccentricity does not increase the difference between these curves computed with both types of analysis. Figs. 6c and 6d, which show relative story displacements, corroborate that significant differences between frame relative displacements only occur at high base shears (or building displacements).

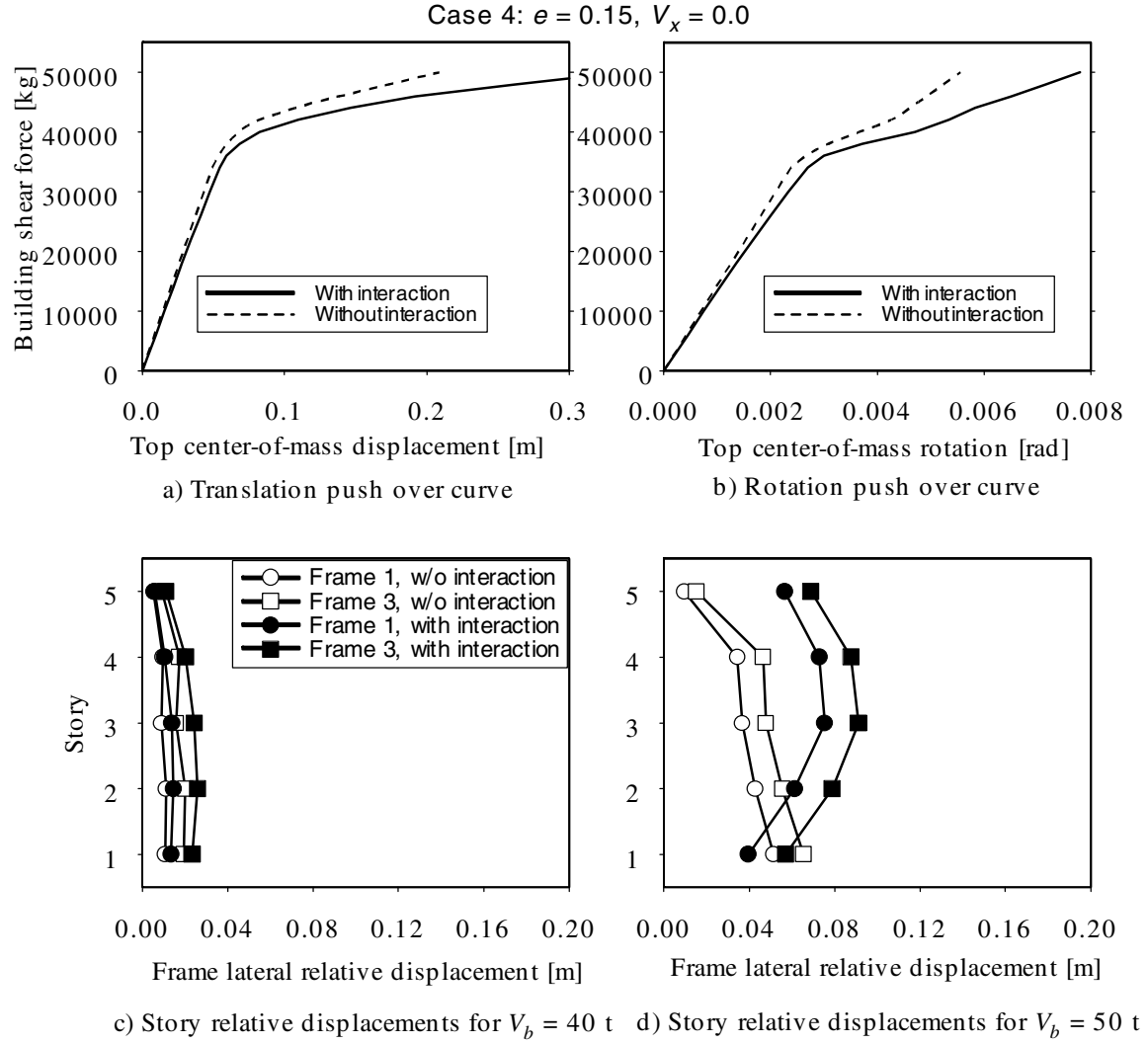


Figure 6. Push over curves and frame relative lateral displacements for *Case 4*

The distribution of plastic hinges is shown in Fig. 7 and plastic energies dissipated are listed in Table 3. It can be observed that the total energy dissipated in *Case 4* is smaller than the energy dissipated in *Case 1*. This suggests that torsion design leads to stronger structures. Moreover, in contrast with *Case 1*, it is interesting to observe that differences in percentages of energy dissipation among frames of *Case 4* are larger than those of *Case 1*. Thus, although buildings were torsion designed (using Equations 1 and 2), these percentages indicate that torsion unbalance is related to the dissipated plastic energy unbalance.

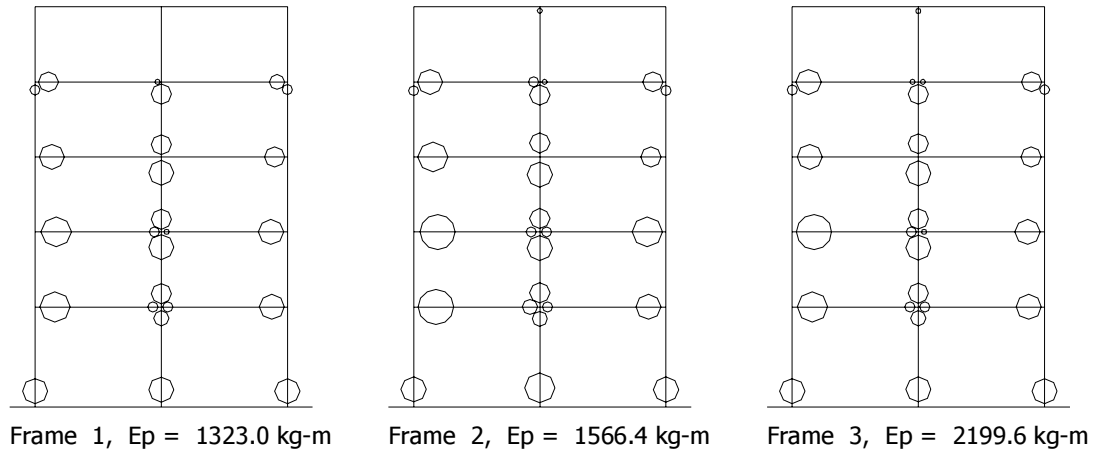


Figure 7. Distribution of plastic hinges computed with a *simple* analysis for *Case 4*.

Case 5: $e = 0.15$ and $V_x = 0.30V_y$

As for *Case 2*, where $V_x = 0.30V_y$, push over curves and frame relative lateral displacements of this case do not show any significant change with respect to *Case 4*. For this reason, results are not shown and the same commentaries of *Case 2* apply to this case.

Case 6: $e = 0.15$ and $V_x = V_y$

This case shows (Fig. 8) results similar to those observed for *Case 3* at high base shears, *i.e.*, large differences in translation push over curves and small differences in rotation push over curves. When these observations are translated to frame relative lateral displacements, small differences between displacements are observed for intermediate base shears and large differences are observed for high base shears. In both cases (3 and 6), *complete* analysis relative displacements are about 50% higher than those computed with a *simple* push over analysis at high base shears.

As for *Case 3*, plastic energy dissipated by frame 3 is larger than the energy dissipated by the other frames. In this particular case, however, the inelastic torsion caused by V_x is such that frame 6 remains elastic, as indicated in Table 3 and shown in Fig. 9. Obviously, if V_x were applied in opposite direction energy dissipated by frame 6 will be larger than that dissipated by frame 4, as indicated in Table 4 computed with negative values of V_x . Observe that plastic energy percentages for frames 1, 2, and 3 are almost equal to those presented in Table 3.

Table 4. Dissipated E_p for *Cases 3* and *6*, computed with negative values of V_x [kg-m]

Case	Frame 1	Frame 2	Frame 3	Frame 4	Frame 5	Frame 6	Total
3	1431.2 [13%]	1974.5 [18%]	2996.7 [27%]	1024.4 [9%]	1462.3 [13%]	2191.7 [20%]	11080.8
6	221.0 [2%]	1454.3 [14%]	4261.2 [42%]	4.6 [0%]	1087.3 [11%]	3089.7 [31%]	10118.1

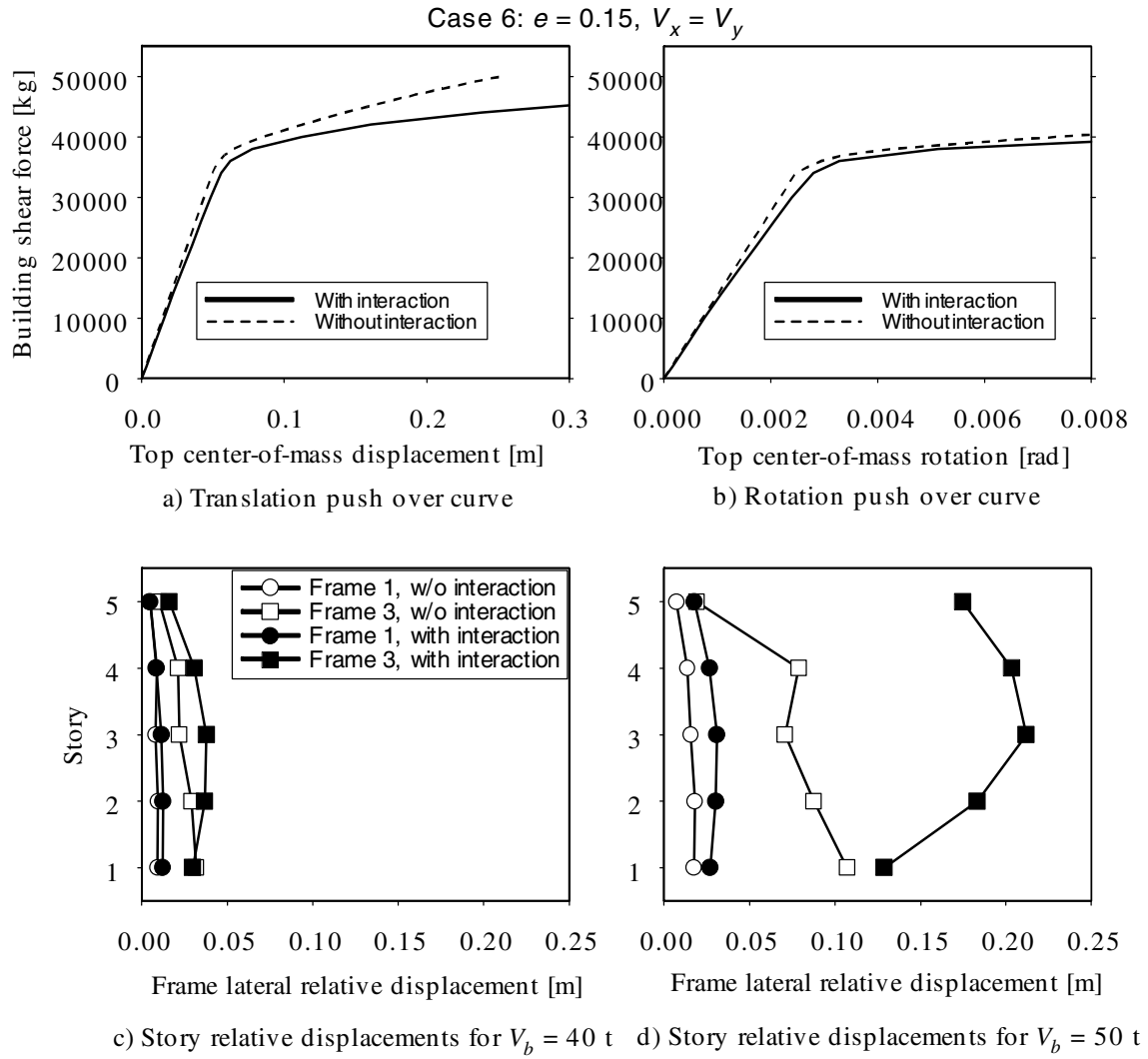


Figure 8. Push over curves and frame relative lateral displacements for *Case 6*

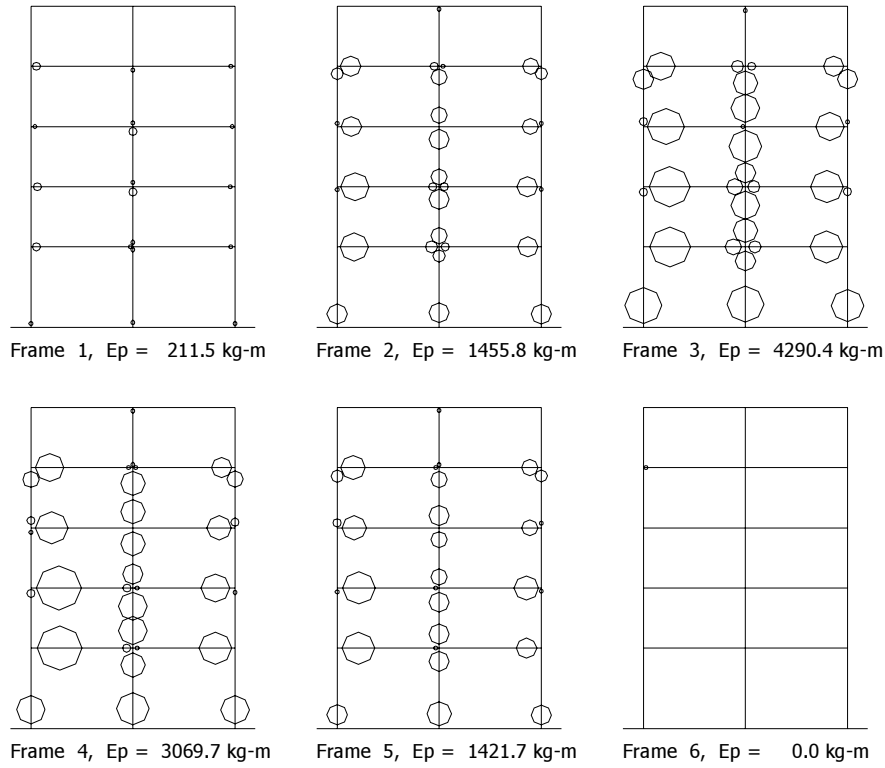


Figure 9. Distribution of plastic hinges computed with a *simple* analysis for *Case 6*.

CONCLUSIONS

Two types of 3D push over analyses were used to assess the effect of nonlinear interaction in columns (bending moments and axial loads) on the response of two concrete five-story frame buildings. Mainly, translation and rotation push over curves, as well as frame lateral relative displacements, were used as comparison outputs. From six different cases (2 eccentricity values and 3 values of the orthogonal base shear), the following conclusions can be derived.

Results indicate that for analysis where the orthogonal base shear (say V_x) is no greater than 30% of the main base shear (V_y), interaction in columns does not significantly modifies the *translation* push over curve obtained with a simple analysis that neglects interaction, particularly if results are used within the context of *capacity* and *demand* diagrams [9].

For the limited number of analyses carried out, results indicate that rotation push over curves computed with and without interaction are almost coincident each other when orthogonal base shears (V_y and V_x) grow in the same proportion. Significant differences between rotation curves were observed for systems loaded with one base shear component.

Relative lateral displacements of frames computed with interaction in columns can result up to 100% larger than those computed without column interaction, particularly for large (4 or more) building lateral displacement ductilities.

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