



## **SEISMIC FRAGILITY ANALYSIS OF DEGRADING STRUCTURAL SYSTEMS**

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### **SUMMARY**

Seismic fragility of a system describes the probability of the system to reach or exceed different degrees of damage, including possible collapse. These curves serve to predict the damage level and possible collapse of a structure due to a specific earthquake ground motion. Earlier work focused on developing seismic fragility curves of systems for several values of a calibrated damage index. A damage index is a factor that represents the degree of damage of the structure, and typically ranges from 0 to 1, with the value of 1 representing complete collapse. Collapse was therefore expressed implicitly as the state of the structure when its damage index approaches a value of 1. This research work focuses on developing seismic fragility curves for a collapse criterion, in an explicit form. To perform this task, new degrading constitutive material models that represent collapse explicitly are developed. The newly developed models take into account strength softening, defined as the material degradation in strength after reaching its full capacity under static loading, and cyclic degradation in strength and stiffness under repeated reversed loading. Collapse is defined when the system completely loses its full capacity. Several degrading models were developed, a bilinear model, a modified Clough model, and a pinching model. An energy-based criterion is used in all models to estimate strength and stiffness degradation. The previously described degrading models were used to conduct the fragility study. An ensemble of recent earthquake records was used in the work, and a variety of degrading systems that cover a wide range of periods, yield values, and level of degradation were considered. Several fragility curves are developed for a collapse criterion, and conclusions are drawn for each case. The newly developed fragility curves represent a major advancement over damage index-based fragility curves.

### **INTRODUCTION**

Seismic design provisions in the United States are moving towards adopting the performance based design concept, a demand/capacity procedure that incorporates multiple performance objectives. Most codes rely on approximate methods that predict the desired seismic demand parameters. Two methods were established in that sense, the capacity spectrum method developed originally by Freeman [1] and adopted by ATC-40 [2], and the method of coefficients developed by Krawinkler [3] and used by FEMA-

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273 [4]. Both methods are similar in the sense that they are based on a nonlinear static push-over of the structure. They are different, however, in the way they estimate the maximum “target” inelastic displacement. The first method is based primarily on superimposing capacity diagram plots on demand diagram plots, and estimating the target displacement with an iterative procedure using elastic dynamic analyses, where cyclic degradation is neglected. Several modified versions were introduced to improve the originally developed method. Paret et al. [5], and Bracci et al. [6] modified the proposed procedure to account for higher mode effects. WJE [7], Reinhorn [8], Fajfar [9], and Chopra and Goel [10] further improved the procedure by using inelastic design spectra as defined by Newmark and Hall [11] rather than elastic spectra. In these later versions, inelastic dynamic analyses are performed but using simple bilinear non-degrading material models. In the second method used by FEMA-273, the target displacement  $\delta_t$  is calculated as follow:

$$\delta_t = C_0 C_1 C_2 C_3 S_a \frac{T_e^2}{4T^2} \quad (1)$$

where  $C_0$  is a modification factor that accounts for higher mode effects,  $C_1$  is factor that accounts for yielding,  $C_2$  is a factor that accounts for degradation effects,  $C_3$  is a factor that accounts for dynamic second-order effects, and  $T_e$  is the effective fundamental period of the structure. The factor  $C_2$  was derived by considering models that degrade only in strength, or in stiffness, and does not account for strength softening behavior. It is also worth mentioning that none of the previous studies attempted to investigate collapse of the structure due to degradation effects.

The main drawbacks of both methods, the capacity spectrum method and the coefficients method, is that they are based on static analysis, and in their inability to predict collapse of individual components of the structure, which might affect the overall response and possibly collapse of the entire structure. The reason is that both models use simple numerical procedures in estimating seismic demands. Replication of collapse necessitates modeling of degradation characteristics of structural elements. It is known that any material degrades in strength after reaching its full capacity under static loadings, also known as strength softening, which subsequently causes collapse. Also, any material degrades in strength and stiffness under repeated cyclic loadings, which might cause complete loss of strength and possibly dynamic material failure. An attempt to introduce direct dynamic effects in the analysis of building structures was proposed by Vamvatsikos and Cornell [12], and used by several researchers (e.g. Mehanny and Deierlein [13]). The process is named incremental dynamic analysis or dynamic pushover analysis. In this process, a dynamic load - deformation plot is determined by subjecting the structure to a specific earthquake history, and then scaling the earthquake record up several times and repeating the analysis. Although dynamic effects were included, collapse prediction was not possible since the material models used followed also very simple rules.

The main goal of the proposed research study is to develop a new numerical procedure for estimating collapse of structures under seismic excitations. Seismic fragility curves for a collapse criterion, defined as the probability of the system to collapse are developed for different structural systems. Earlier work [e.g. 14-16] focused on developing seismic fragility curves of systems for several values of a calibrated damage index. A damage index is a factor that represents the degree of damage of the structure, and typically ranges from 0 to 1, with the value of 1 representing complete collapse. Collapse was therefore expressed implicitly as the state of the structure when its damage index approaches a value of 1. In this study, seismic fragility curves for a collapse criterion are developed in an explicit form. To accomplish this task, several degrading constitutive material models including both static and dynamic degradation effects are

developed. Incremental Dynamic Analyses (IDA) of a variety of degrading systems that cover a wide range of periods, yield values, and level of degradation are performed under a large ensemble of earthquake records representing recent events, and using the newly developed degrading material models. The results are used to investigate collapse criteria of structures under earthquake excitations, and to develop seismic fragility curves. The findings provide necessary background for the evaluation of ductility capacities associated with the limit state of collapse within the performance-based earthquake design process.

The degrading material models are described first.

## DEGRADING MATERIAL MODELS

Three material models were developed. The models considered were: (a) a bilinear model, (b) a Modified-Clough type model as described in [17], and (c) a pinching model. All models include a strength softening branch, referred to as a cap, to model strength degradation under monotonic loads. An 8 parameter energy-based model was developed to model four different types of cyclic degradation: Yield (Strength) degradation, Unloading stiffness degradation, Accelerated stiffness degradation, and Cap degradation. The energy-based criterion is based on the work by Rahnama and Krawinkler [18]. A brief description of the models and the degradation criteria is described below.

The main skeleton for the bilinear, Clough, and pinching models is shown in Figs (1 - 3). It consists of an elastic branch, a strain hardening branch, and a softening branch for all models. Loading-Reloading rules under cyclic loading differ though from a model to another. For the bilinear model, initial unloading is parallel to the initial slope. The reloading curve is then bounded by the positive and negative strain hardening branches, which form two main asymptotes for the model as shown in Fig(1). For the Clough model, initial unloading is also parallel to the initial slope. The behavior under cyclic loading then targets the maximum previous displacement as shown in Fig (2). The pinching model is similar to the Clough model, except that reloading consists of two branches: First reloading is directed towards a point defined by a reduced target force (point C in Fig. 3). Thereafter, the reloading branch is directed towards the previous maximum peak point.

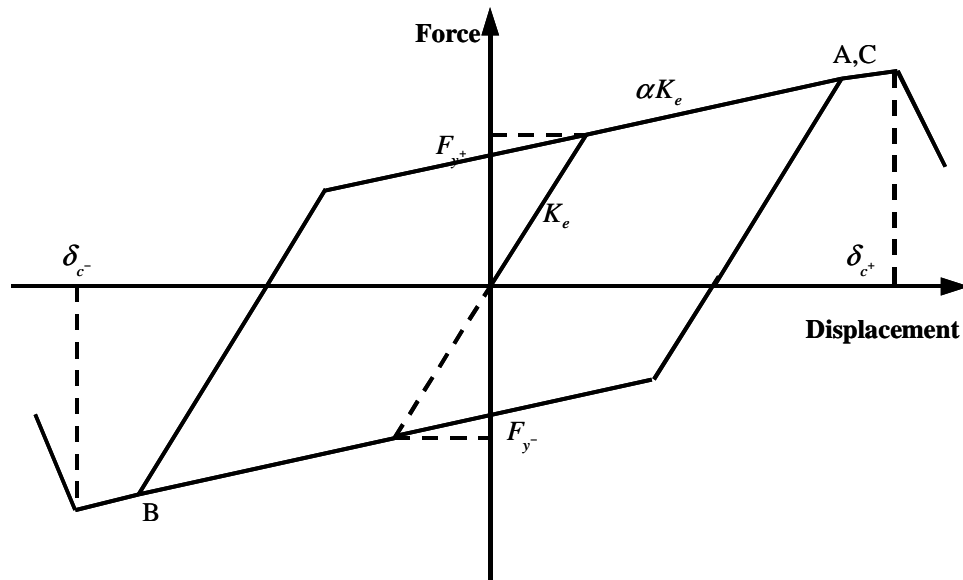
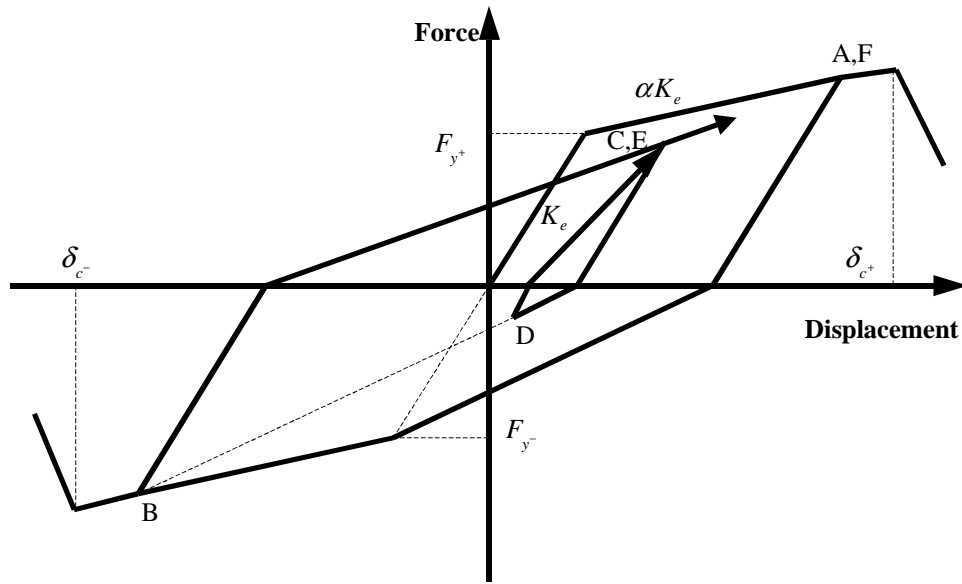
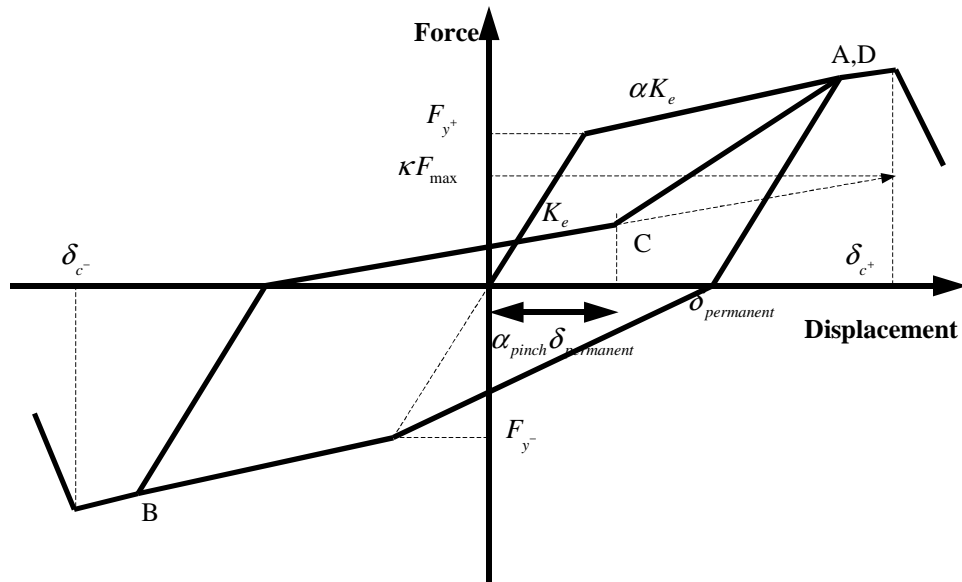


Figure (1) Bilinear Model



**Figure (2) Modified-Clough Model**



**Figure (3) Pinching Model**

### Degradation Rules

It is well known from experimental evidence that any material deteriorates as a function of the loading history. Every inelastic excursion causes damage and the damage accumulates as the number of excursions increases. Therefore it is necessary to include degradation effects in modeling hysteretic behavior. Four types of degradation are included in all three models: Strength degradation, Unloading stiffness degradation, Accelerated stiffness degradation, and Cap degradation.

### Strength Degradation

Strength degradation refers to the decrease of the yield strength value as a function of the loading history. The strength degradation parameter used in this study is energy dependent, and is derived by considering the following expression:

$$F_y^i = F_y^{i-1} (1 - \beta_{str}^i) \quad (2)$$

Where,  $F_y^i$  is the yield strength at the current excursion  $i$ ,  $F_y^{i-1}$  is the yield strength at the previous excursion  $i-1$ , and  $\beta_{str}^i$  is a scalar parameter ranging between 0 and 1, that accounts for degradation effects at the current excursion  $i$ . The parameter  $\beta_{str}^i$  is determined as follow:

$$\beta_{str}^i = \left[ \frac{E_i}{E_{capacity} - \sum_{j=1}^i E_j} \right]^{C_{str}} \quad (3)$$

Where:

$E_i$  is the hysteretic energy dissipated in the current excursion  $i$ ;

$\sum_{j=1}^i E_j$  is the total hysteretic energy dissipated in all excursions up to the current one;

$E_{capacity}$  is the energy dissipation capacity of the element.  $E_{capacity}$  represents the resistance of the material to cyclic degradation. The structure is considered totally degraded when the total dissipated hysteretic energy due to cyclic loading reaches a value that equals the energy dissipation capacity.  $E_{capacity}$  is usually calculated as a function of the strain energy up to yield as follow:

$$E_{capacity} = \gamma_{str} F_y \delta_y \quad (4)$$

where  $F_y$  and  $\delta_y$  are the initial yield strength and deformation respectively, and  $\gamma_{str}$  is a constant. Finally, the parameter  $C_{str}$  in (3) is an exponent defining the rate of deterioration. The values of  $\gamma_{str}$  and  $C_{str}$  are calibrated for each material using experimental data. Recommended values for different materials are given in [2].

### Unloading Stiffness Degradation

Unloading stiffness degradation refers to the decrease of the unloading stiffness as a function of the loading history. The decrease in unloading stiffness is determined by evaluating the parameter  $\beta_{unl}^i$  at the current excursion  $i$ , using an expression similar to (3), except that different values for the scalar parameters  $c$  and  $\gamma$  are used, namely  $\gamma_{unl}$  and  $c_{unl}$ . The modified unloading stiffness is then calculated as:

$$k_{unl}^i = k_{unl}^{i-1} (1 - \beta_{unl}^i) \quad (5)$$

Where  $k_{unl}$  is the unloading stiffness.

### Accelerated Stiffness Degradation

In peak-oriented models, the reloading stiffness degrades as a function of cumulative loading. This effect can be accounted for in the analytical hysteretic model by modifying the target point to which the loading is directed. This is referred to as accelerated stiffness degradation. The accelerated stiffness degradation parameter  $\beta_{acc}^i$  is also similar to the one for strength degradation, except that different values for  $c$  and  $\gamma$  are used, namely  $\gamma_{acc}$  and  $c_{acc}$ . The displacement value of the target point is then calculated as:

$$\delta_{tar}^i = \delta_{tar}^{i-1} (1 + \beta_{acc}^i) \quad (6)$$

Where  $\delta_{tar}$  is the displacement of the target point, selected in this study as the maximum displacement in all excursions.

### Cap Degradation

It is observed from experimental results that the point of onset of softening moves inward as a result of cumulative damage. This phenomenon is referred to as cap degradation. Collapse of the system is assumed if the cap slope reaches the displacement axis. The cap degradation parameter  $\beta_{cap}^i$  used in this model is also similar to the one for strength degradation, except that different values for  $c$  and  $\gamma$  are used, namely  $\gamma_{cap}$  and  $c_{cap}$ . The point of onset of softening is then modified as follow:

$$\delta_{cap}^i = \delta_{cap}^{i-1} (1 - \beta_{cap}^i) \quad (7)$$

where  $\delta_{cap}$  is the displacement of the point of onset of softening.

### Collapse of Structural Elements

A structural element is assumed to have experienced complete collapse if any of the following two criteria is established:

- (a) The displacement has exceeded the value of that of the intersection point of the softening (cap) slope with the X-axis, which is referred to as cap failure, or
- (b) The scalar parameter  $\beta$  has exceeded a value of 1, which is referred to as cyclic degradation failure.

It is important to note that an element could fail in a direction of loading (e.g. tension), while still exhibiting resistance in the other direction of loading (e.g. compression). A reverse loading condition could always push such element into the direction that still shows resistance, and thus the element in that case should not be considered as a collapsed structure. However, in the present study, an element is considered as completely collapsed if only one direction of loading shows no resistance. Such an assumption is considered to be on the safe side from a design standpoint.

## IDA and Fragility Results

A large database set of earthquake records is used to conduct this study. The records were used by Krawinkler in several earlier studies [e.g. 19, 20], and consists of four bins representing different M (Magnitude), and R (Distance from fault) pairs. Each bin constitutes of 20 earthquake records. The records were all recorded in California, and correspond to NEHRP soil types C or D (stiff soil or soft rock). The 80 records are all scaled to their median spectral acceleration value. This prior scaling

approach has been proven to reduce the variability in results, while not affecting the median values of the response quantities, in accordance with the work by Cornell [21].

Figures (4-9) show the Incremental Dynamic Analysis plots and Seismic Fragility curves for a structure with period of 1 sec. Figure (4) shows an Incremental Dynamic Analysis (IDA) plot for a bilinear model. The Incremental Dynamic Analysis (IDA) is a procedure that establishes a relationship between seismic demand parameters, such as ductility or inter-story drift, and strength parameters, such as spectral acceleration (SA) or strength reduction factor  $R$  commonly used in codes of practice. In this case the  $R$ - $\mu$  plot is developed, and a '\*' represents collapse of the structure. From the plot, it is clear that the ductility capacity at collapse equals 7.8 for systems with low and moderate degradation, and 5.5 for systems with severe degradation. The corresponding  $R$  values are 7.4 and 5.8 respectively. Figure (5) shows the fragility curve for the same system for a collapse criterion. The data are smoothed using lognormal distribution functions. The 50% collapse probability point corresponds to the point identified with a '\*' in Figure (4). From the plots, the collapse rate of systems with low degradation is almost similar to the one for systems with moderate degradation. Systems with severe degradation fail with a much faster rate: A spectral acceleration that equals 2.5g produces a 90% probability of collapse for systems with severe degradation, while a spectral acceleration of 4g produces the same probability of collapse for other systems.

Figure (6) shows the IDA and fragility plots for a Clough model with period 1 sec. The ductility capacity at collapse equals 15.8, 15, and 12 for systems with low, moderate, and severe degradation respectively. The corresponding  $R$  values equal 12.2, 11, and 9.8 respectively. Figure (7) shows the fragility curve for the same system. The collapse rate of Clough models is slower than that of a bilinear model. A 90% collapse probability corresponds to a spectral acceleration of 3.5 for systems with severe degradation, and 4.5 for systems with low degradation.

Figure (8) shows the IDA plot for a pinching model with period of 1 sec. The ductility capacity and strength reduction factor at collapse are larger than the corresponding values for both bilinear and Clough models. Figure (9) shows the fragility curve for the same system. From the plot, the collapse rate of pinching models is also slower than that of the bilinear and Clough models.

## SUMMARY AND CONCLUSIONS

A numerical study to investigate collapse of degrading systems is conducted. Three degrading Bilinear, Clough, and pinching models were developed for this purpose. An energy-based criterion is used to define the degradation parameters. Four types of degradation were considered: strength degradation, unloading stiffness degradation, accelerated stiffness degradation, and cap degradation. An Incremental Dynamic Analysis (IDA) is conducted for a structure with period 1 sec. to identify the ductility capacity and strength reduction factor at collapse for several models with different degradation rates. Seismic fragility curves are plotted for a collapse criterion for the same structure. These curves are necessary for collapse-prevention limit state design purposes.

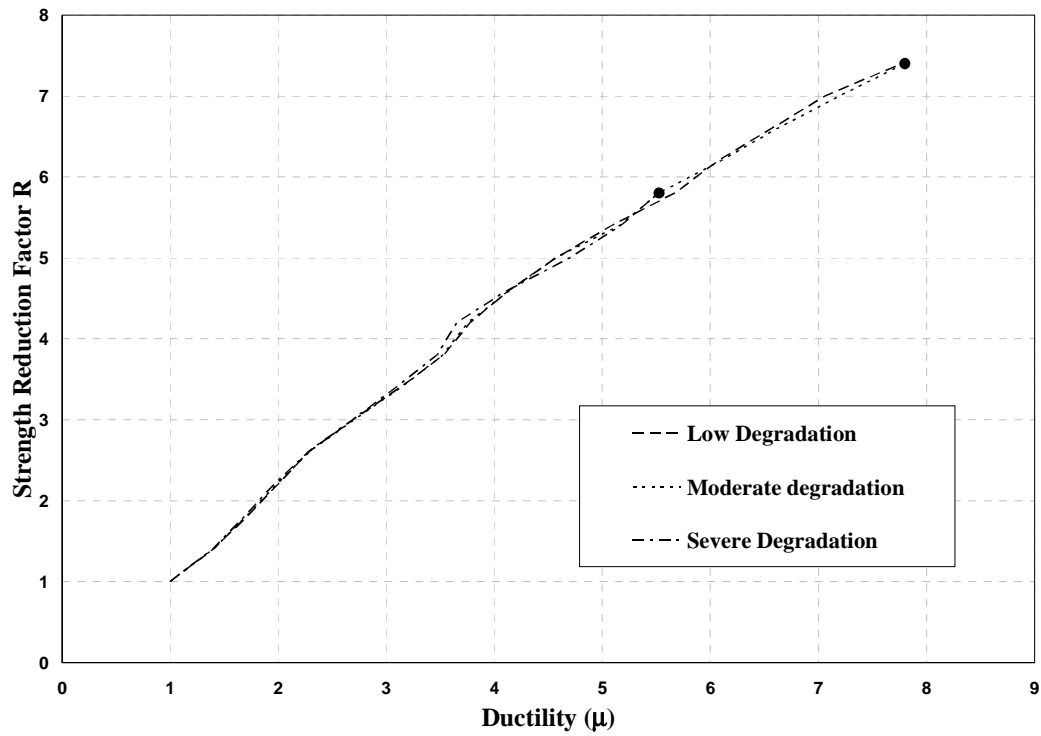


Figure (4) IDA Plot for Bilinear Model

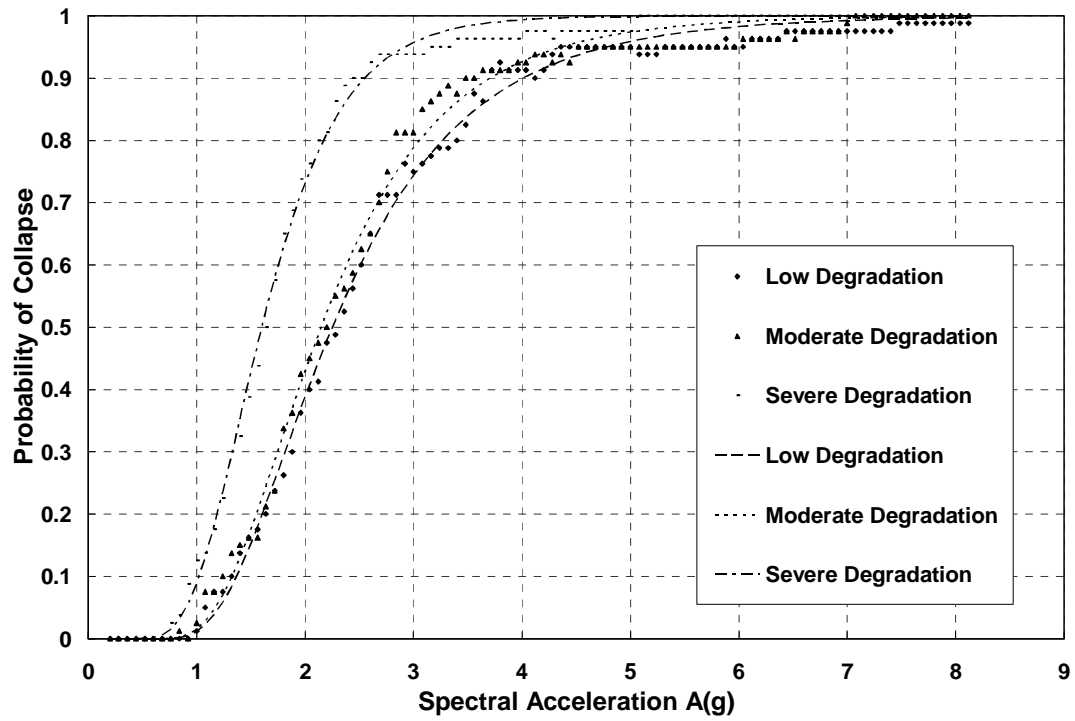
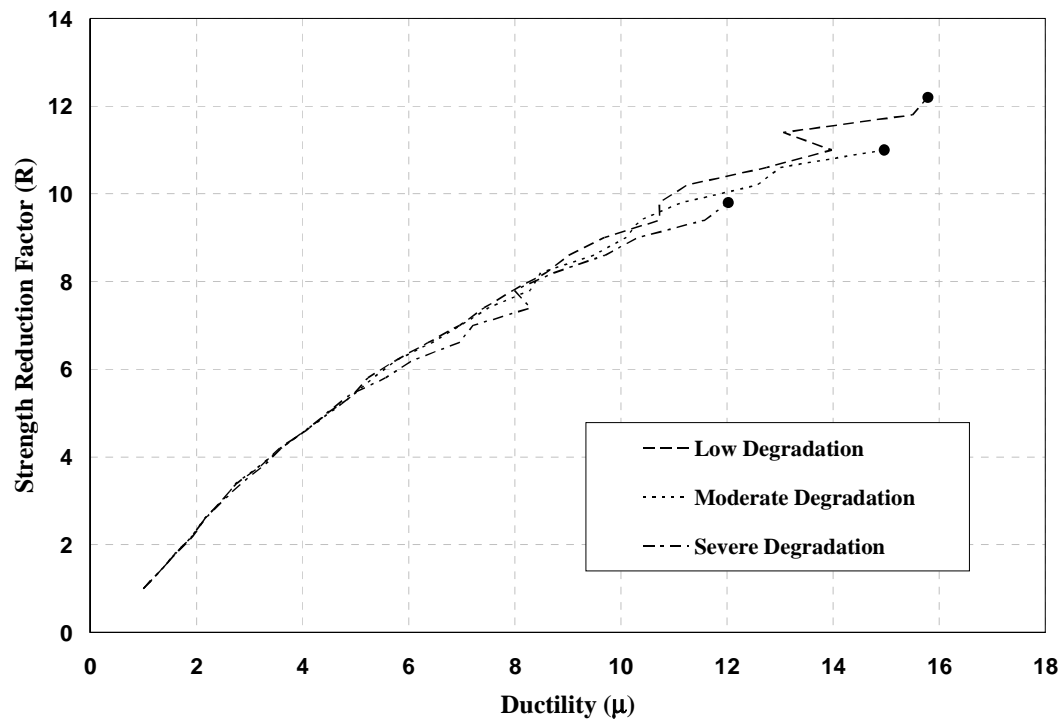
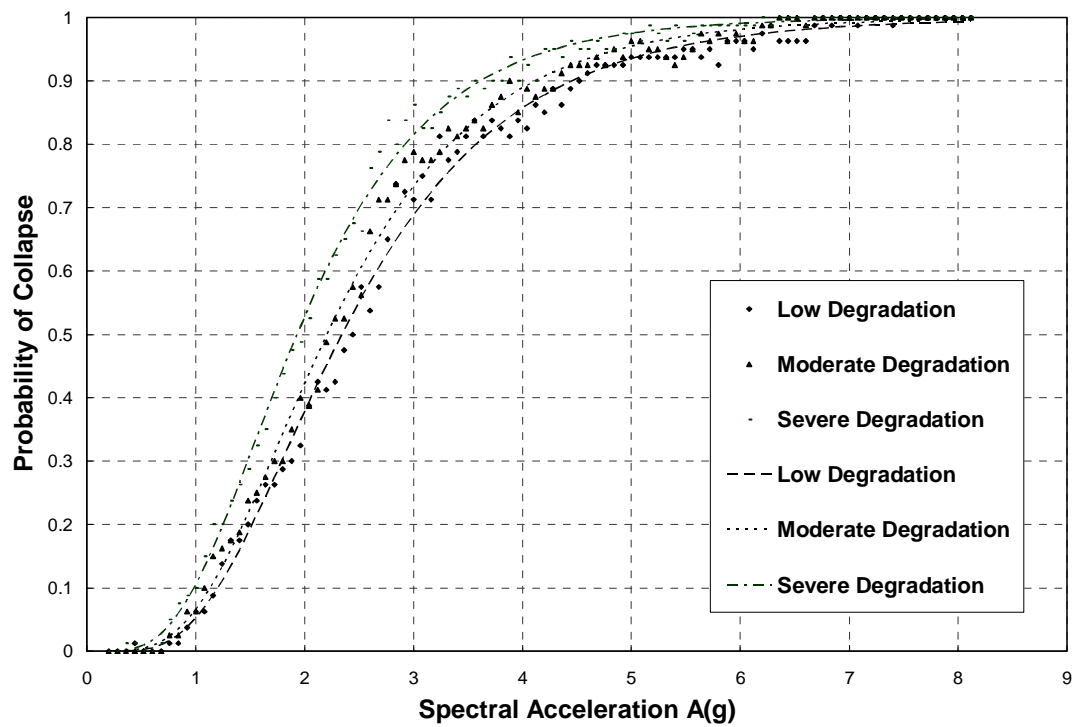


Figure (5) Seismic Fragility of Bilinear Model





**Figure (6) IDA Plot for Clough Model**



**Figure (7) Seismic Fragility of Clough Model**

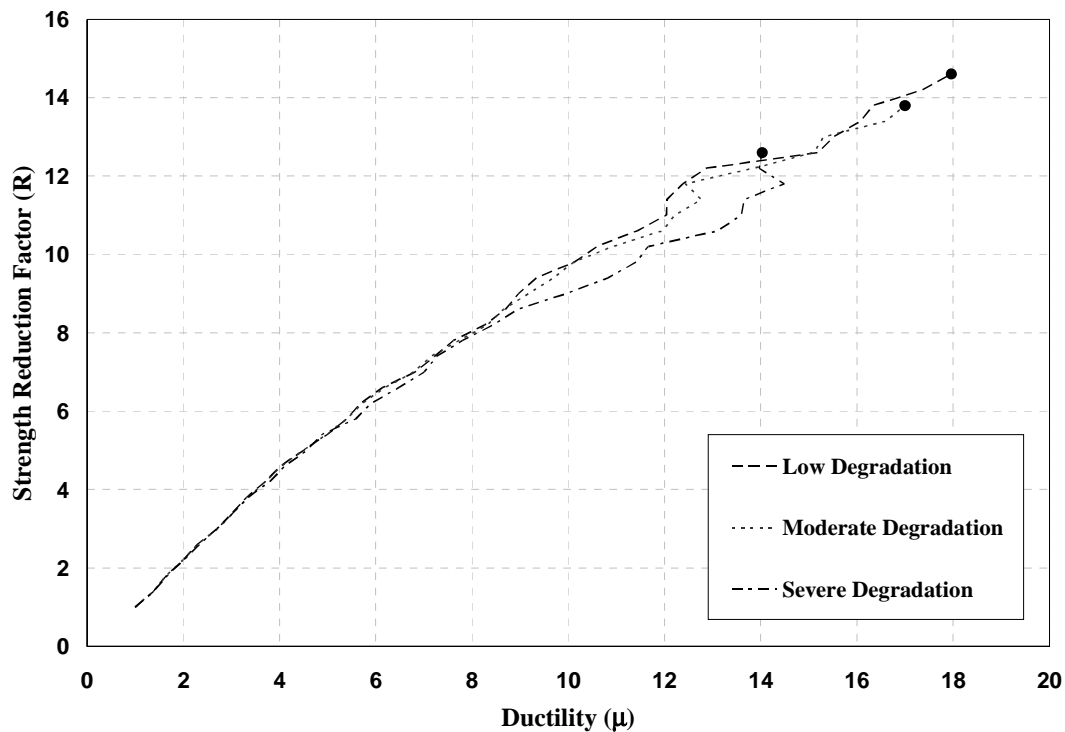


Figure (8) IDA Plot for Pinching Model

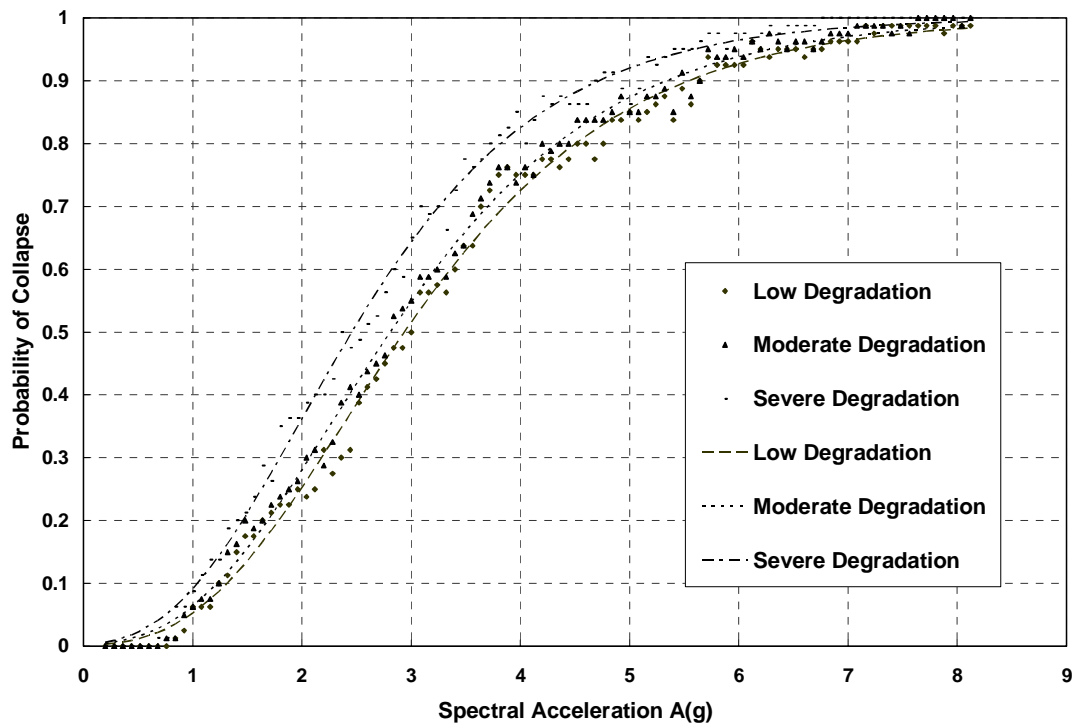


Figure (9) Seismic Fragility of Pinching Model

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