

# DISPLACEMENT ESTIMATES AND COLLAPSE PREDICTION OF DEGRADING STRUCTURAL SYSTEMS

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## SUMMARY

Seismic code provisions in several countries have recently adopted the new concept of performance-based design. New analysis procedures have been developed to estimate seismic demands for performance evaluation. Most of these procedures are based on simple models, and do not take into account degradation effects, a major factor influencing structural behavior under earthquake excitations. More importantly, most of these models can not predict collapse of structures under seismic loads. This study presents a newly developed model that incorporates degradation effects into seismic analysis of structures. The newly developed model was added to the material library of commercial software for seismic analysis of structural systems. A new energy-based approach is used to define several types of degradation effects. The model also permits collapse prediction of structures under seismic excitations. The model was used to conduct extensive statistical dynamic analysis of different structural systems subjected to a large ensemble of recent earthquake records. Strength levels that subject structures to collapse were identified from the study, for collapse-prevention limit state design purposes. Results were also used to propose approximate methods for estimating maximum inelastic displacements of degrading systems, in case collapse does not occur, for use in performance-based seismic code provisions. The findings provide necessary information for the design evaluation phase of a general performance-based earthquake design process, and could be used for evaluation and modification of existing seismic codes of practice.

## **INTRODUCTION**

The seismic design provisions of building codes in the United States are moving towards adopting the general concept of performance based design. A Performance Based Earthquake Engineering (PBEE) design process is a demand/capacity procedure that incorporates multiple performance objectives. The procedure consists of four main steps. In the first step, performance objectives of a structural system at different hazard levels are defined. In the second step, a conceptual design of the structure is performed in order to meet the objectives defined in step 1. A design evaluation phase is then needed in order to evaluate the conceptual design developed in step 2. Finally a socio-economic study is needed to finalize the process. In the design evaluation phase, seismic demands of the structure need to be evaluated as

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accurately as possible at different hazard levels for demand/capacity comparison. Most codes rely on approximate methods that predict the desired demand parameters; the most common two are the method of coefficients and the capacity spectrum method. The Applied Technology Council (ATC) 40 [1] provisions use the method of capacity spectrum developed by Freeman [2]. In this simplified method, a static pushover analysis of the entire structure is conducted first. The expected maximum displacement at a specific hazard level is determined by superimposing the pushover curve on ductility-based inelastic design spectrum graphs. Static pushover analysis of the structure is then repeated up to the specified maximum displacement in order to estimate seismic demand parameters. The Federal Emergency Management Agency (FEMA) 273 [3] provisions use the method of coefficients developed by Krawinkler and his co-workers [4]. In that simplified method, the expected inelastic maximum displacement at a specific hazard level is determined by multiplying the maximum corresponding elastic displacement by a series of coefficients that account for inelastic behavior, higher mode effects, and dynamic second order effects. These coefficients were determined from extensive parameter studies conducted on different structural systems using simple hysteresis material models. A static pushover analysis is then conducted for the structure up to the specified maximum displacement in order to estimate the different seismic demand parameters.

The main drawback of both methods, the capacity spectrum method and the coefficients method, is their inability to accurately estimate maximum inelastic displacements, and to predict failure of individual components of the structure, which might affect the overall response and possibly failure of the entire structure. The reason is that both models use simple numerical procedures in estimating the maximum expected displacement during a specific earthquake excitation. In the capacity spectrum method, only static analysis is performed for non-degrading systems. It is known that any material degrades in strength after reaching its full capacity under static loadings, also known as strength softening, which subsequently causes failure. Also, any material degrades in strength and stiffness under repeated cyclic loadings, which might cause complete loss of strength and possibly dynamic material failure. Since the capacity spectrum method considers only non-degrading systems and neglects dynamic effects, it fails to predict failure accurately. The coefficients method also is mainly based on static analysis, but dynamic effects are introduced by a series of approximate factors determined from extensive statistical parameter studies of simple hysteresis material models. These models also do not account for strength softening, usually the main cause of failure, and consider only strength degradation under repeated dynamic loading. The method therefore also does not predict failure of a component accurately. An attempt to introduce direct dynamic effects in the analysis of building structures was proposed by Cornell and his co-workers [5], and used by several researchers (e.g. Mehanny and Deierlein [6]). The process is named incremental dynamic analysis or dynamic pushover analysis. In this process, a dynamic load - deformation plot is determined by subjecting the structure to a specific earthquake history, and then scaling the earthquake record up several times and repeating the analysis. Although dynamic effects were included, failure prediction was not possible since the material models used followed also very simple rules.

Several researchers developed procedures for estimating maximum inelastic displacements. In most of these studies, the material models used followed simple hysteretic non-degrading rules. Few of these studies considered degradation, but still followed very simple rules. In addition, degradation effects were not based on physical reasoning. Furthermore, none of these studies considered collapse prediction of the structures. A brief summary of earlier studies in this field is given below.

The first research work in this field is the one by Veletsos and Newmark [7] who analyzed SDOF systems using 3 earthquake records. The models were assumed elasto-plastic. They concluded that in the regions of low frequency, the maximum inelastic deformation is equal to the maximum elastic deformation, which is known as the equal displacement rule. They also concluded that this rule doesn't hold true for regions of high frequency, where the inelastic displacement considerably exceeds the elastic one.

Shimazaki and Sozen [8] conducted a similar numerical study on a SDOF system using five different hysteretic models. The models used were either bilinear or of Clough type [9], and only El Centro earthquake record was used for the analysis. No degradation was considered in their study. In their work, they developed a relation between maximum inelastic displacements and corresponding maximum elastic displacements for different values of strength and period ratios. The conclusion of their work is that for periods higher than the characteristic period, defined as the transition period between the constant acceleration and constant velocity regions of the response spectra, the maximum inelastic displacement equals approximately the maximum elastic displacement regardless of the hysteresis type used, confirming the equal displacement rule. For periods less than the characteristic period, the maximum inelastic displacement exceeds that of the elastic displacement, and the amount vary depending on the type of hysteretic model and on the lateral strength of the structure relative to the elastic strength. Their conclusion was confirmed later by Qi and Moehle [10].

Miranda [11-13] analyzed over 30,000 SDOF systems using a large ensemble of 124 earthquake ground motions recorded on different soil types. He developed ratios of maximum inelastic to elastic displacements for 3 types of soil conditions. He also studied the limiting period value where the equal displacement rule applies. The material model used in his study is also elasto-plastic. Lately, Miranda and Ruiz-Garcia [14] evaluated six different methods for predicting maximum inelastic displacements. Four methods are based on equivalent linearization techniques, while two are based on multiplying maximum elastic displacements by modification factors. In all methods, cyclic degradation effects were not considered. Krawinkler and his co-workers [4,15-16] conducted similar studies to the one by Miranda. The material models used were either bilinear, Clough or of pinching type. Degradation effects were included, but in the form of strength degradation only, or stiffness degradation only. Gupta and Kunnath [17] conducted a similar study on SDOF systems subjected to 15 ground motions. They included degradation effects using a 3 parameters model.

More recently, Whittaker et al. [18] conducted a numerical study on SDOF systems using 20 earthquake records. They used the Bouc-Wen model [19] in their analysis and neglected degradation effects. They developed mean, and mean+1sigma ratio plots of maximum inelastic to elastic displacements for different strength values. Miranda [20] extended his earlier work, and developed displacement ratio plots for different earthquake magnitudes, epicenter distance, and soil conditions. His study was also on non-degrading SDOF systems. Most recently, Miranda [21] showed that maximum inelastic displacements could be related to maximum elastic displacements either through inelastic displacement ratios or through strength reduction factors. He also showed that the second method is a first order approximation of the first, and that both methods yield similar results in the absence of variability.

Several studies were also conducted on MDOF systems (e.g. Ayoub and Filippou [22-24], Saiidi and Sozen [25], Freeman [3], Fajfar and Fischinger [26], Qi and Moehle [10], and Krawinkler [4, and 15]). Most researchers concluded that the demand of MDOF systems could be estimated by appropriate modification of the response of the first mode SDOF of the system. Two methods were established in that sense, the capacity spectrum method developed originally by Freeman [3] and adopted by ATC-40, and the method of coefficients developed by Krawinkler [4] and used by FEMA-273. Both methods are similar in the sense that they are based on a nonlinear static push-over of the structure. They are different, however, in the way they estimate the maximum "target" inelastic displacement. The first method is based primarily on superimposing capacity diagram plots on demand diagram plots, and estimating the target displacement with an iterative procedure using elastic dynamic analyses, where cyclic degradation is neglected. Several modified versions were introduced to improve the originally developed method. Paret et al. [27], and Bracci et al. [28] modified the proposed procedure to account for higher mode effects. WJE [29], Reinhorn [30], Fajfar [31], and Chopra and Goel [32] further improved the procedure by using inelastic design spectra as defined by Newmark and Hall [33] rather than elastic spectra. In these later

versions, inelastic dynamic analyses are performed but using simple bilinear non-degrading material models. In the second method used by FEMA-273, the target displacement  $\delta_t$  is calculated as follow:

$$\delta_{t} = C_{0}C_{1}C_{2}C_{3}S_{a}\frac{T_{e}^{2}}{4\Pi^{2}}$$
(1)

where  $C_0$  is a modification factor that accounts for higher mode effects,  $C_1$  is factor that accounts for yielding,  $C_2$  is a factor that accounts for degradation effects,  $C_3$  is a factor that accounts for dynamic second-order effects, and  $T_e$  is the effective fundamental period of the structure. The factor  $C_2$  was derived by considering models that degrade only in strength, or in stiffness, and does not account for strength softening behavior. It is also worth mentioning that none of the previous studies attempted to investigate collapse of the structure due to degradation effects.

#### **OBJECTIVE**

The main objective of the proposed research study is to develop a new numerical procedure for predicting maximum inelastic displacements, and for estimating collapse of degrading structures under seismic excitations. Several general constitutive material models including both static and dynamic degradation effects are developed. The models are used to conduct statistical analytical studies using a large ensemble of earthquake records representing recent events. The results are used to predict maximum inelastic displacements, and to investigate collapse criteria of structures under earthquake excitations. The findings provide necessary background for the design evaluation phase of a general performance-based earthquake design process.

#### **DEGRADING MATERIAL MODELS**

Three material models are developed. The models considered are: (a) a bilinear model, (b) a Modified-Clough type model as defined in [9], and (c) a pinching model. All models include a strength softening branch, referred to as a cap, to model strength degradation under monotonic loads. An 8 parameter energybased criteria is used to model four different types of cyclic degradation: Yield (Strength) degradation, Unloading stiffness degradation, Accelerated stiffness degradation, and Cap degradation. The energybased criterion is based on the work by Rahnama and Krawinkler [16]. A brief description of the models and the degradation criteria is described below.

The main skeleton for the bilinear, Clough, and pinching models is shown in Figs (1 - 3). It consists of an elastic branch, a strain hardening branch, and a softening branch for all models. Loading-Reloading rules under cyclic loading differ though from a model to another. For the bilinear model, initial unloading is parallel to the initial slope. The reloading curve is then bounded by the positive and negative strain hardening branches, which form two main asymptotes for the model as shown in Fig (1). For the Clough model, initial unloading is also parallel to the initial slope. The behavior under cyclic loading then targets the maximum previous displacement as shown in Fig (2). The pinching model is similar to the Clough model, except that reloading consists of two branches: First reloading is directed towards a point defined by a reduced target force (point C in Fig. 3). Thereafter, the reloading branch is directed towards the previous maximum peak point.



## **Degradation Rules**

It is well known from experimental evidence that any material deteriorates as a function of the loading history. Every inelastic excursion causes damage and the damage accumulates as the number of excursions increases. Therefore it is necessary to include degradation effects in modeling hysteretic behavior. Four types of degradation are included in all three models: Strength degradation, Unloading stiffness degradation, Accelerated stiffness degradation, and Cap degradation.

#### **Strength Degradation**

Strength degradation refers to the decrease of the yield strength value as a function of the loading history. The strength degradation parameter used in this study is energy dependent, and is derived by considering the following expression:

$$F_{y}^{i} = F_{y}^{i-1} (1 - \beta_{str}^{i})$$
<sup>(2)</sup>

Where,  $F_y^i$  is the yield strength at the current excursion *i*,  $F_y^{i-1}$  is the yield strength at the previous excursion *i*-1, and  $\beta_{str}^i$  is a scalar parameter ranging between 0 and 1, that accounts for degradation effects at the current excursion *i*. The parameter  $\beta_{str}^i$  is determined as follow:

$$\beta_{\rm str}^{\rm i} = \left[\frac{E_i}{E_{capacity} - \sum_{j=1}^{\rm i} E_j}\right]^{C_{\rm str}}$$
(3)

Where:

 $E_i$  is the hysteretic energy dissipated in the current excursion i;

 $\sum_{j=1}^{i} E_j$  is the total hysteretic energy dissipated in all excursions up to the current one;

 $E_{capacity}$  is the energy dissipation capacity of the element.  $E_{capacity}$  represents the resistance of the material to cyclic degradation. The structure is considered totally degraded when the total dissipated hysteretic energy due to cyclic loading reaches a value that equals the energy dissipation capacity.  $E_{capacity}$  is usually calculated as a function of the strain energy up to yield as follow:

$$E_{capacity} = \gamma_{str} F_y \delta_y \tag{4}$$

Where  $F_y$  and  $\delta_y$  are the initial yield strength and deformation respectively, and  $\gamma_{str}$  is a constant. Degradation defined this way follows simple physical reasoning. Finally, the parameter  $C_{str}$  in (3) is an exponent defining the rate of deterioration. The values of  $\gamma_{str}$  and  $C_{str}$  are calibrated for each material using experimental data.

#### **Unloading Stiffness Degradation**

Unloading stiffness degradation refers to the decrease of the unloading stiffness as a function of the loading history. The decrease in unloading stiffness is determined by evaluating the parameter  $\beta_{unl}^i$  at the current excursion *i*, using an expression similar to (3), except that different values for the scalar parameters *c* and  $\gamma$  are used, namely  $\gamma_{unl}$  and  $c_{unl}$ . The modified unloading stiffness is then calculated as:

$$k_{unl}^{i} = k_{unl}^{i-1} (1 - \beta_{unl}^{i})$$
(5)

Where  $k_{unl}$  is the unloading stiffness.

#### **Accelerated Stiffness Degradation**

In peak-oriented models, the reloading stiffness degrades as a function of cumulative loading. This effect can be accounted for in the analytical hysteretic model by modifying the target point to which the loading is directed. This is referred to as accelerated stiffness degradation. The accelerated stiffness degradation parameter  $\beta_{acc}^i$  is also similar to the one for strength degradation, except that different values for c and  $\gamma$  are used, namely  $\gamma_{acc}$  and  $c_{acc}$ . The displacement value of the target point is then calculated as:

$$\delta_{tar}^{i} = \delta_{tar}^{i-1} (1 + \beta_{acc}^{i}) \tag{6}$$

Where  $\delta_{tar}$  is the displacement of the target point, selected in this study as the maximum displacement in all excursions.

#### **Cap Degradation**

It is observed from experimental results that the point of onset of softening moves inward as a result of cumulative damage. This phenomenon is referred to as cap degradation. Collapse of the system is assumed if the cap slope reaches the displacement axis. The cap degradation parameter  $\beta_{cap}^{i}$  used in this model is also similar to the one for strength degradation, except that different values for *c* and  $\gamma$  are used, namely  $\gamma_{cap}$  and  $c_{cap}$ . The point of onset of softening is then modified as follow:

$$\delta_{cap}^{i} = \delta_{cap}^{i-1} (1 - \beta_{cap}^{i}) \tag{7}$$

Where  $\delta_{\scriptscriptstyle cap}$  is the displacement of the point of onset of softening.

## **EARTHQUAKE RECORDS**

A large database set of earthquake records is used in this study. The records were used by Krawinkler in several earlier studies [e.g. 34,35], and consists of four bins representing different M (Magnitude), and R (Distance from fault) pairs as follows:

Bin-I: small M-small R: M<6.5 and R<30 km

Bin-II: small M-large R: M<6.5 and R>30 km

Bin-III: large M-small R: M>6.5 and R<30 km

Bin-IV: large M-large R: M>6.5 and R>30 km

Each bin constitutes of 20 earthquake records. The records were all recorded in California, and correspond to NEHRP soil types C or D (stiff soil or soft rock).

An earlier study by Cornell and his co-workers [36] showed that proper scaling of earthquake records does not introduce any bias to the response, and will therefore reduce the necessity of the number of analysis needed for statistical evaluation. Furthermore, proper scaling ensures that all records used fall within the same hazard level defined by codes of practice. Cornell in his study showed that scaling an ensemble of records, even if they don't fall initially within the same hazard level, to the median spectral acceleration value does not change the median values of the response quantities, but reduces considerably the variability in results. His conclusion was found also to apply to scaling to any value of spectral acceleration, higher or lower than the median value. A new study by the authors investigated the Cornell approach for different degrading material models (bilinear, Clough, and pinching), and for different degrees of degradation. The study also included failure estimation for the different systems. The conclusion is that the Cornell approach holds true for degrading systems in terms of both response measures and failure estimation. The prior scaling approach will be thus used throughout this study for all available records in order of reducing the total number of analysis required for statistical evaluations, and to ensure that all records fall within the same hazard level.

### NUMERICAL RESULTS AND DISCUSSIONS

The main research goal is to predict collapse of SDOF systems, and to provide an estimate for the maximum inelastic displacements in case collapse does not occur. The 4 bins of earthquake records recorded in California, and described earlier in this proposal are used to conduct the numerical study. Three different degradation cases for each of the three material models described earlier are considered and compared to a corresponding non-degrading system. These cases represent low, moderate, and severe degradation respectively. Plots of ratio of maximum inelastic displacements to maximum elastic displacements for different period values and for different strength reduction values R are generated for all degradation cases. The results for the case of Bins I-IV scaled to a spectral acceleration according to USGS values LA 10/50 are shown in Figures (4-9). Collapse is defined when more than 50% of the records failed. The last point before collapse of the system is identified with a '\*' in the plot, and no corresponding point for non-degraded systems exist.

The ratio of the mean maximum inelastic to maximum elastic displacements for a strength reduction factor R=4 are shown in Figures (4-6). Figure (4) shows the results for a Bilinear model, while Figures (5) and (6) show the results for a Clough and pinching models respectively. From the figures, it is clear that degradation did not affect the behavior of long period structures, and that in this range the equal displacement rule still applies even for degraded systems. The effect of degradation becomes apparent for short period structures (T<0.5 sec). In this range, degradation increases the maximum inelastic displacements for all three models. For very short periods (T<0.2 sec), degraded system typically collapse, and for periods T=0.3 sec, severely degraded systems also collapse.

The ratio of the mean maximum inelastic to maximum elastic displacements for a strength reduction factor R=8 are shown in Figures (7-9) for a Bilinear, Clough, and Pinching models respectively. For long period structures, the equal displacement rule is still preserved. For Bilinear models, severely degraded systems collapse at a period value that equals 0.8 sec, while systems with low and moderate degradation collapse at a period value that equals 0.4 sec. For Clough models, systems with severe and moderate degradation collapse at period value that equals 0.4 sec, while systems with low degradation collapse at a value of 0.3 sec. Severely degraded pinching systems collapse at a period value that equals 0.5 sec, while systems with low and moderate degradation collapse at a value of 0.3 sec.

The previous results confirm the fact that degradation has a major effect on the inelastic behavior of short period structures, and on the potential of collapse of these systems.

## CONCLUSIONS

The purpose of this study is to propose approximate methods for estimating maximum inelastic displacements of degrading systems, and to predict their potential for collapse. Three degrading Bilinear, Clough, and pinching models were developed for this purpose. An energy-based criterion is used to define the degradation parameters. Four types of degradation were considered: strength degradation, unloading stiffness degradation, accelerated stiffness degradation, and cap degradation. A suite of earthquake records scaled to a spectral acceleration according to USGS values LA 10/50 is used to conduct the study. Plots of mean values of ratio of maximum inelastic to maximum elastic displacements are developed for two strength reduction factor values R=4 and R=8. In these plots, collapse is identified when more than 50% of the records failed. The plots proved that the equal displacement rule for long period structures is

preserved even for degrading systems. Accurate estimates of maximum inelastic displacements for short period structures are determined in case collapse does not occur. The findings provide necessary information for the design evaluation phase of a general performance-based earthquake design process, and could be used for evaluation and modification of existing seismic codes of practice.



Figure (4) Displacement Estimates for Bilinear Model – R=4



Figure (5) Displacement Estimates for Clough Model – R=4







Figure (7) Displacement Estimates for Bilinear Model – R=8



Figure (8) Displacement Estimates for Clough Model – R=8



Figure (9) Displacement Estimates for Pinching Model – R=8

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