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CAPACITY SPECTRUM FOR STRUCTURES ASYMMETRIC IN PLAN

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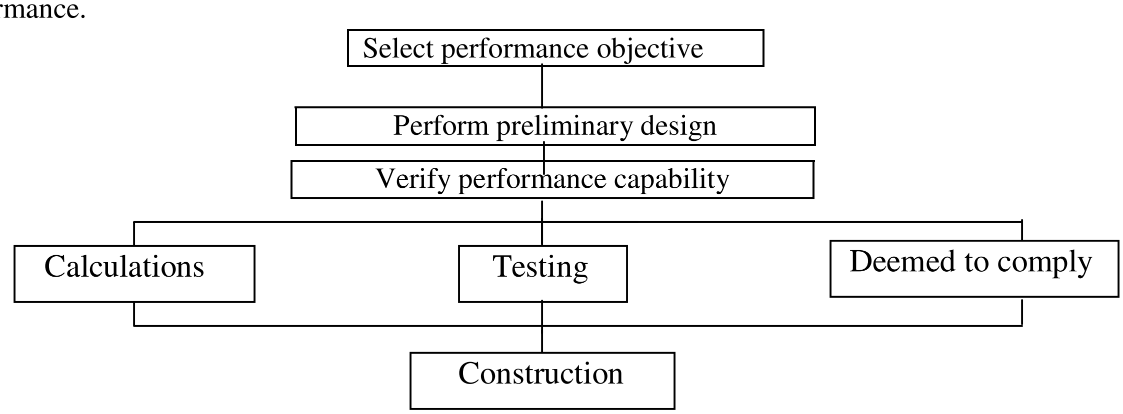
SUMMARY

Capacity spectra are obtained by pushover analysis. In the pushover analysis the six equations of motion are used to obtain the column forces due to incremental lateral forces at the mass centre. As the equations of motion contain the contribution due to eccentricities the column forces do exhibit the influence of rotations about the vertical axis. Plots of spectral acceleration Vs spectral displacement (ADRS format) are obtained from independent spectral acceleration and spectral displacement spectra for various levels of ductilities. Juxtaposing one on the other will confirm the ductility required for the given yield acceleration.

INTRODUCTION

The traditional code procedures are generally based on experience but this phase is changing and is being replaced by performance based seismic design. Performance based engineering consists of actions including site selection; development of conceptual, preliminary and final structural designs; construction and maintenance of the building over its life to ensure that it is capable of predictable and reliable seismic performance.



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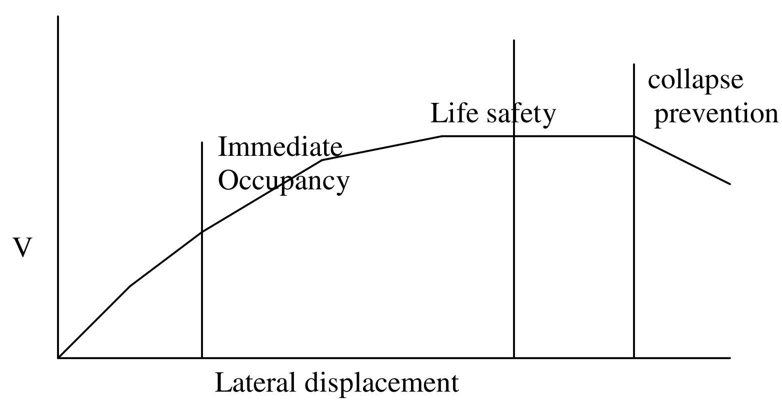
PERFORMANCE – BASED DESIGN PROCESS

There are generally three performance levels

1. Immediate occupancy level in which relatively little damage of the structure occurs
2. Collapse prevention level , in which near complete damage of the structure occurs.
3. Life safety level defined as a condition of severe damage ,but a state in which margin remains against collapse.

In principle , non linear dynamic analysis is the correct approach. However such an approach is not practical for everyday design use and for the time being ,the most rational analysis and performance evaluation methods for practical applications seem to be simplified inelastic procedures ,which combine the nonlinear static (pushover) analysis of a relatively simple mathematical model and the response spectrum approach. Freeman [9] developed a rapid evaluation method ,which can be considered as a forerunner of today's "Capacity spectrum method".Saiidi and Sozen [10] proposed to perform non-linear dynamic analyses on an equivalent SDOF system. Based on this idea, Fajfar and Fischinger [5 & 6] developed the first version of N2 method (N stands for Non-linear and 2 for two mathematical models – a SDOF and a MDOF model) .Examples of such approaches are capacity spectrum method, applied in ATC 40 [1], and the nonlinear static procedure, applied in FEMA 273 [3] and further developed in FEMA 356 [4]. The capacity spectrum method by means of a graphical procedure , compares the capacity of a structure with the demands of earthquake ground motion. The graphical presentation makes possible a visual evaluation of how the structure will perform when subjected to earthquake ground motions.The capacity of the structure is represented by a force –displacement curve obtained by nonlinear static (pushover) analysis .Pushover analysis is a term used for the non-linear static analysis of frames. In this method first a distribution for the lateral loads on the frame is assumed and is increased monotonically. Due to this the structural element yields sequentially and the structure experiences a loss in stiffness. The base shear forces and roof displacements are converted to the spectral acceleraion and spectral displacement of an equivalent single degree of freedom (SDOF) system respectively. These spectral values define the capacity spectrum. Inelastic demand spectra are determined from the elastic design spectra and is converted into acceleration displacement response spectra (ADRS) format. This is the demand spectrum and the intersection of the capacity spectrum and demand spectrum provides an estimate of the inelastic acceleration (strength) and displacement demand.



PERFORMANCE CONCEPT IN FEMA –273 AND ATC-40

OBJECTIVES AND SCOPE

In this paper, pushover analysis is done on a structure asymmetric in plan. While doing the pushover analysis, the 6 static equations of motion for the two-storey shear building structure is made use of to obtain the column forces. Thus by tracking the yielding of columns, the capacity spectrum is obtained. The capacity spectrum is superimposed on the demand spectra drawn in ADRS format according to the procedure available in the paper by Fajfar [8].

DEMAND SPECTRA (***AS PER FAJFAR(1999)*** **)**

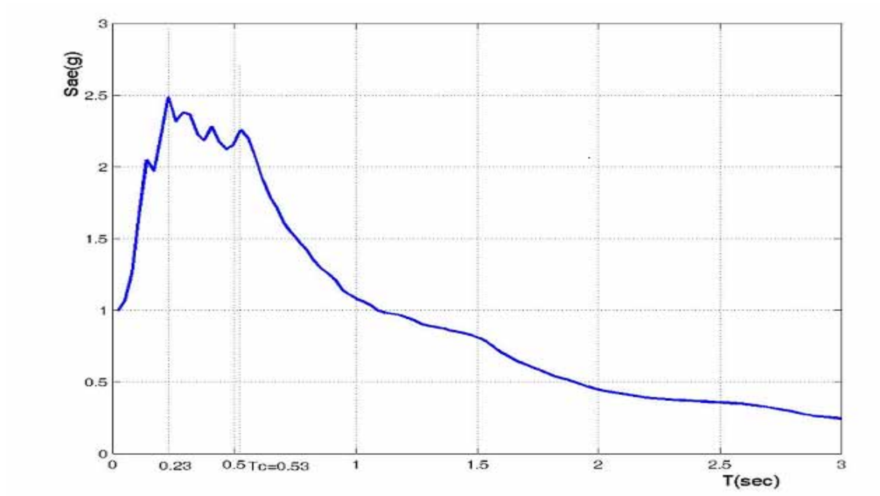


Fig-1 Elastic acceleration spectrum (S_{ae}) for 5% damping normalized to 1.0g peak Ground acceleration in traditional format

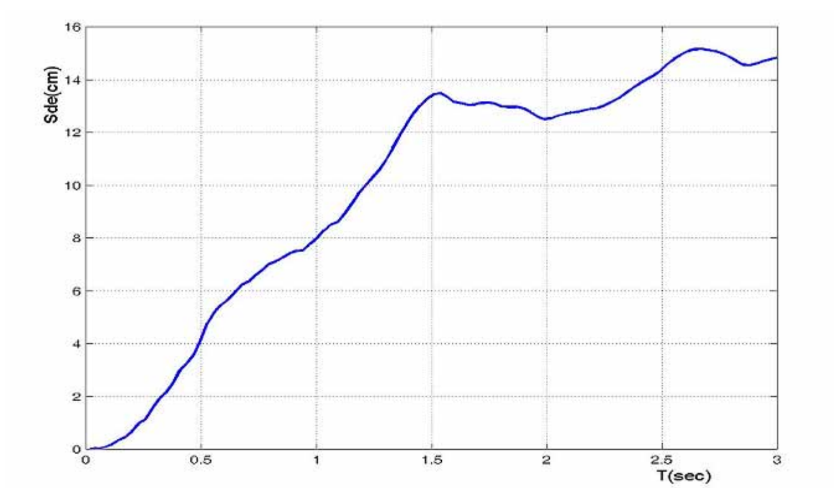


Fig-2 Displacement spectra (S_{de}) for 5% damping normalized to1.0gpeakGroundacceleration in traditional format.

For elastic single degree of freedom (SDOF) system, the following relation applies

$$S_{de} = \frac{T^2 S_{ae}}{4\pi^2} \qquad (1)$$

where S_{ae} and S_{de} are the values in the elastic acceleration and displacement spectrum respectively, corresponding to the period T and a fixed viscous damping ratio. For an inelastic SDOF system with a bilinear force-deformation relationship, the acceleration spectrum (S_a) and the displacement spectrum (S_d) can be determined as Vidic et al. 1994.

$$S_a = \frac{S_{ae}}{R_\mu} \qquad (2)$$

$$S_d = \frac{\mu S_{de}}{R_\mu} = \frac{\mu T^2 S_{ae}}{R_\mu 4\pi^2} = \frac{\mu T^2 S_a}{4\pi^2} \tag{3}$$

Where μ is the ductility factor defined as the ratio between the maximum displacement and the yield displacement, and R μ is the reduction factor due to ductility, i.e., due to hysteretic energy dissipation of ductile structures.

Several proposals have been made for the reduction factor R μ . Here for the reduction factor we will use

$$R_\mu = \frac{(\mu - 1)T}{Tc} + 1 \qquad T < Tc \tag{4}$$

$$R_\mu = \mu \qquad T \geq Tc \tag{5}$$

Where Tc is the characteristic period of the ground motion. It is defined as the transition period where the constant acceleration segment of the response spectrum passes to the constant velocity segment of the spectrum.

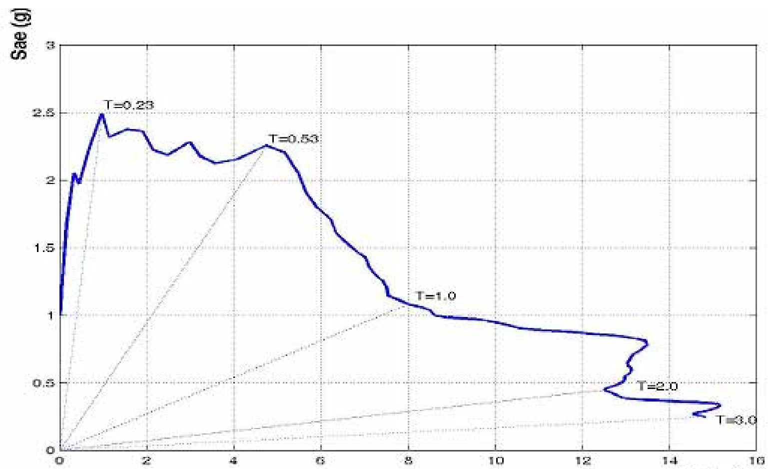


Fig-3 Elastic acceleration and displacement spectrum (S_{ae}) for 5% damping normalized to 1.0g peak ground acceleration in AD format

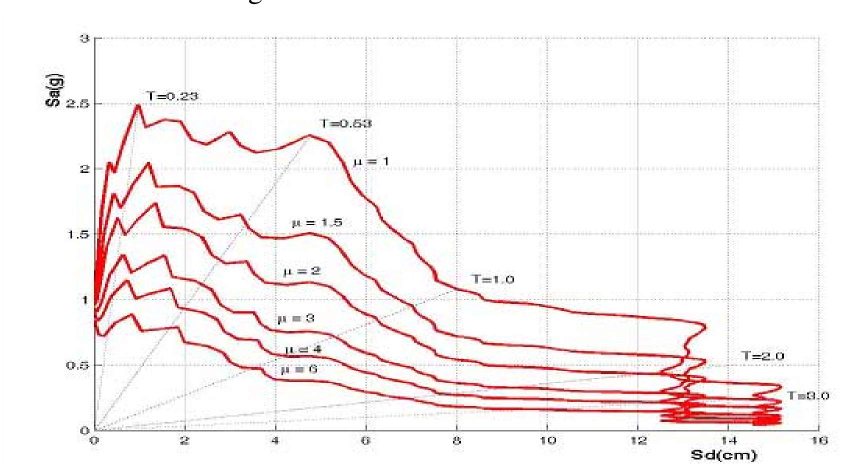


Fig-4 Demand spectra for the constant ductilities in AD format normalized to 1.0g peak ground acceleration.

Using a pushover analysis, a characteristic nonlinear force-displacement relationship of the MDOF system can be determined. In principle ,any force and displacement can be chosen. Here, base shear and roof displacement have been used as representative of force and displacement, respectively. The selection of an appropriate lateral load distribution is an important step within the pushover analysis. Figs 1 – 4 shows typical plots of spectra.

In the N2 method, the vector of the lateral loads **P** used in the pushover analysis is determined as

$$\mathbf{P} = p\boldsymbol{\psi} = p\mathbf{M}\boldsymbol{\varphi} \qquad (6)$$

Where **M** is the diagonal mass matrix. The magnitude of the lateral load is controlled by p. The distribution of load is denoted by $\boldsymbol{\psi}$. It is related to assumed displacement shape $\boldsymbol{\varphi}$. Here the load pattern is known by assuming the displacement shape, which follows that lateral force in the i-th level is proportional to the component ϕ_i of the assumed displacement shape $\boldsymbol{\varphi}$, weighted by the storey mass m_i

$P_i = p m_i \boldsymbol{\varphi}_i$

Such an approach for the determination of the distribution of lateral loads has a physical background: if the assumed displacement shape was exact and constant during ground shaking, then the distribution of lateral forces would be equal to the distribution of effective earthquake forces.

EQUIVALENT SDOF MODEL AND CAPACITY DIAGRAM

In this method, seismic demand is determined by using inelastic response spectra, so the structure should, in principle be modeled as a SDOF system. The starting point is the equation of motion of a 3D structural model (with 3N degrees of freedom) representing a multi storey building

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{R} = -\mathbf{M}\mathbf{s}\,a \qquad (7)$$

U is the vector representing displacements and rotations. Here they are 6 in number, 3 for each floor. R is a vector representing internal forces, *a* is the ground acceleration as a function of time, and *s* is a vector defining the direction of ground motion. In case of uni-directional ground motion, e.g. in the direction of x, the vector s consists of one unit sub-vector and of two sub-vectors equal to 0.

$\mathbf{s}^T = [\mathbf{1}^T, \mathbf{0}^T, \mathbf{0}^T]$

For simplicity, damping is not included in the derivation. Its influence will be included in the design spectrum. It will be assumed that the displacement shape ϕ is constant, i.e. that it does not change during the structural response to ground motion. This is the basic and the most critical assumption within the procedure. The displacement vector **U** is defined as

$$\mathbf{U} = \boldsymbol{\varphi} D_t \qquad (8)$$

Where D_t is the time-dependent top displacement. $\boldsymbol{\varphi}$ is, for convenience, normalized in such a way that the component at the top is equal to 1.

From statics it follows

$$\mathbf{P} = \mathbf{R} \qquad (9)$$

i.e., the internal forces **R** are equal to the statically applied external loads **P**.

By introducing equations 6,7 and 9, and by multiplying from the left side with $\boldsymbol{\varphi}^T$, we obtain

$$\boldsymbol{\varphi}^T \mathbf{M} \boldsymbol{\varphi} \ddot{D}_t + \boldsymbol{\varphi}^T \mathbf{M} \boldsymbol{\varphi} p = -\boldsymbol{\varphi}^T \mathbf{M}\mathbf{s}\,a \qquad (10)$$

After multiplying and dividing the left hand side with ϕ^T M s, the equation of the motion of the equivalent SDOF system can be written as

$$m^* \ddot{D}^* + F^* = -m^* a$$

Where m* is the equivalent mass of the SDOF system

$m^* = \boldsymbol{\varphi}^T \mathbf{M}\,\mathbf{s}$

D* and F* are the displacement and force of the equivalent SDOF system

$$D^* = \frac{D_t}{\boldsymbol{\Gamma}} \qquad F^* = \frac{V}{\boldsymbol{\Gamma}} \qquad (11)$$

V is defined as $V = \boldsymbol{\varphi}^T \mathbf{M}\,\mathbf{s}\,p = p m^*$

It is the base shear of the MDOF model in the direction of ground motion.

$$\boldsymbol{\Gamma} = \frac{\boldsymbol{\varphi}^T \mathbf{M}\mathbf{s}}{\boldsymbol{\varphi}^T \mathbf{M} \boldsymbol{\Phi}} \qquad (12)$$

Γ is usually called the modal participation factor. Here the assumed displacement shape ϕ is normalized – the value at the top is 1. Any reasonable shape can be used for ϕ. As a special case, the elastic first mode shape can be assumed.
The same constant Γ applies for the transformation of both displacements and forces as in equation 11. As a consequence, the force – displacement relationship determined for the MDOF system (the V – D_t diagram) applies also to the equivalent SDOF system (the F* - D* diagram), provided that both force and displacement are divided by Γ.
The graphical procedure, used in the simple N2 method, requires that the post yield stiffness is equal to zero. This is because the reduction factor R_μ is defined as the ratio of the required elastic strength to the yield strength.
The elastic period of the idealized bilinear system T* can be determined as

$$T^* = 2\pi\sqrt{\frac{m^* D^*_y}{F^*_y}} \qquad (13)$$

Where F^*_y and D^*_y are the yield strength and displacement, respectively.
Finally, the capacity diagram in AD format is obtained by dividing the forces in the force – deformation (F* - D*) diagram by the equivalent mass m*

$$S_a = \frac{F^*}{m^*} \qquad (14)$$

SEISMIC DEMAND FOR THE EQUIVALENT SDOF SYSTEM

The seismic demand for the equivalent SDOF system can be determined by using the graphical procedure by plotting the demand spectra and capacity diagram in the same graph. The intersection of the radial line corresponding to the elastic period of the idealized bilinear system T* with the elastic demand spectrum S_{ae} defines the acceleration demand (strength) required for the elastic behavior and the corresponding elastic displacement demand. The yield acceleration S_{ay} represents both the acceleration demand and the capacity of the inelastic system. The reduction factor R_μ can be determined as the ratio between the accelerations corresponding to the elastic and inelastic systems.

$$R_\mu = \frac{S_{ae}(T^*)}{S_{ay}} \qquad (15)$$

Note that $R\mu$ is not the same as the reduction (behavior, response modification) factor R used in the seismic codes. The code reduction factor R takes into account both energy dissipation and the so called over strength.
If the elastic period T* is larger than or equal to Tc, the inelastic displacement demand S_d is equal to the elastic displacement demand S_{de} and ductility demand, defined as $\mu = S_d / D^*_y$, is equal to R_μ

$$S_d = S_{de}(T^*) \qquad T^* \geq Tc \qquad (16)$$

$$\mu = R_\mu \qquad (17)$$

If the elastic period of the system is smaller than Tc , the ductility can be calculated from the rearranged equation 4

$$\mu = (R_\mu - 1)\frac{Tc}{T^*} + 1 \qquad T^* < Tc \qquad (18)$$

The displacement demand can be determined either from the definition of ductility or from equation 3 and 18

$$S_d = \mu D^*_y = \frac{S_{de}}{R_\mu}\left(1 + (R_\mu - 1)\frac{Tc}{T^*}\right) \qquad (19)$$

In both cases ($T^* < Tc$ and $T^* \geq Tc$) the inelastic demand in terms of accelerations and displacements corresponds to the intersection point of the capacity diagram with the demand spectrum corresponding to the ductility demand μ. At this point, the ductility factor determined from the capacity diagram and the ductility factor associated with the intersecting demand spectrum are equal. All the steps in the procedure can be performed numerically without using the graph. However visualization of the procedure may help in better understanding the relations between the basic quantities.

GLOBAL AND LOCAL SEISMIC DEMAND FOR THE MDOF MODEL

The displacement demand for the SDOF model S_d is transformed into the maximum top displacement D_t of the MDOF system by using equation 11. The local seismic demand (e.g., storey drift, joint rotations) can be determined by pushover analysis. Under monotonically increasing lateral loads with a fixed pattern, the structure is pushed to its target top displacement D_t. It is assumed that the distribution of deformations throughout the structure in the static (pushover) analysis approximately corresponds to that which would be obtained in the dynamic analysis

PERFORMANCE EVALUATION (DAMAGE ANALYSIS)

Expected performance can be assessed by comparing the seismic demands with the capacities for the relevant performance level.

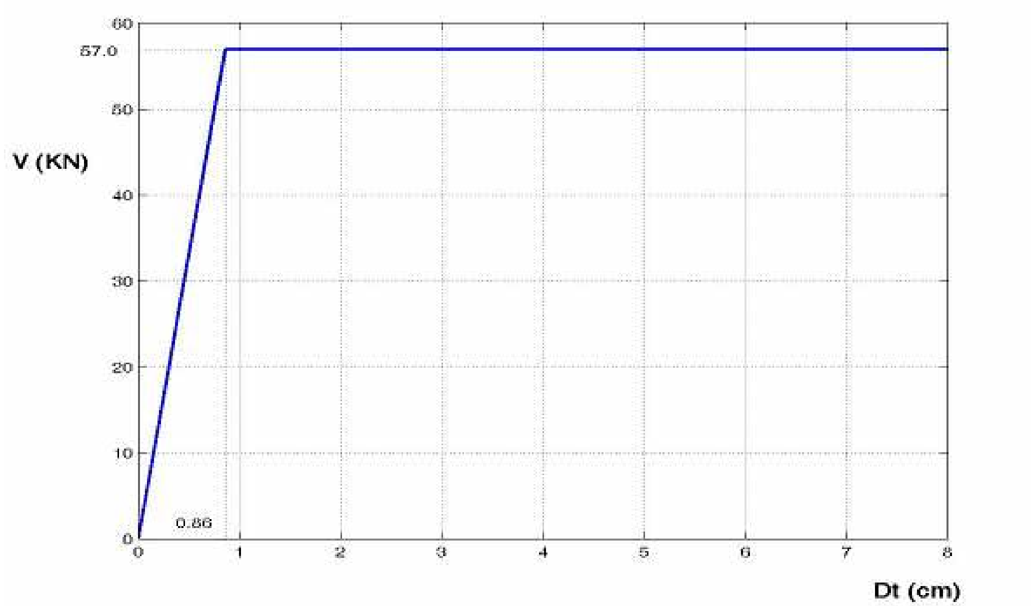


Fig-5 Pushover curve for the two storey frame with 0% eccentricity

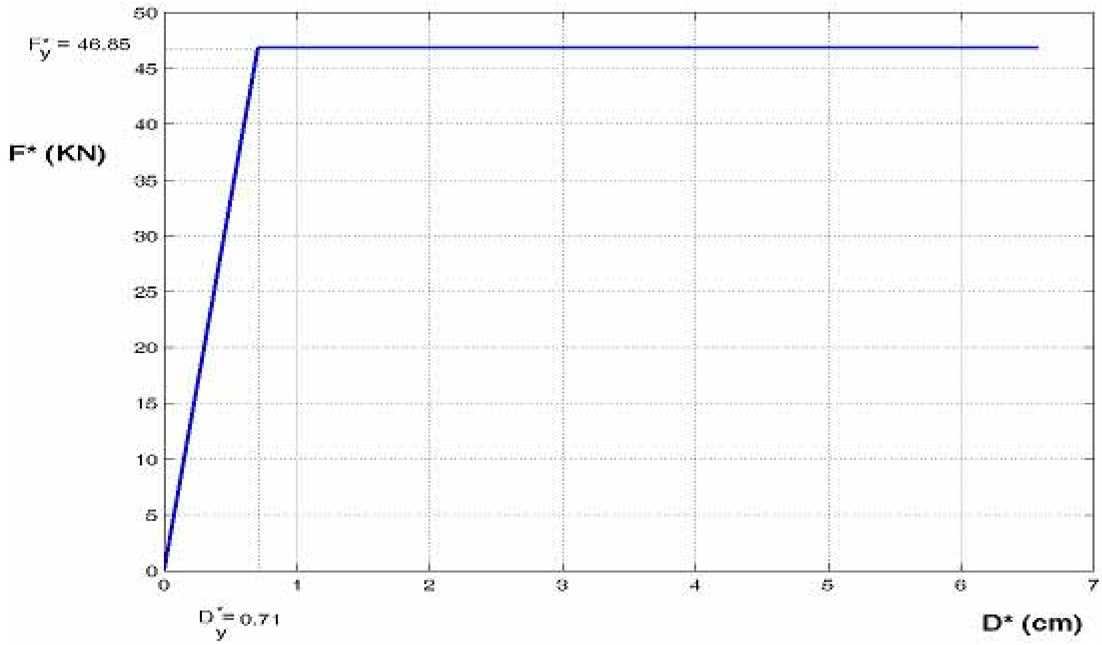


Fig-6 Pushover curve for the equivalent SDOF system with 0% eccentricity

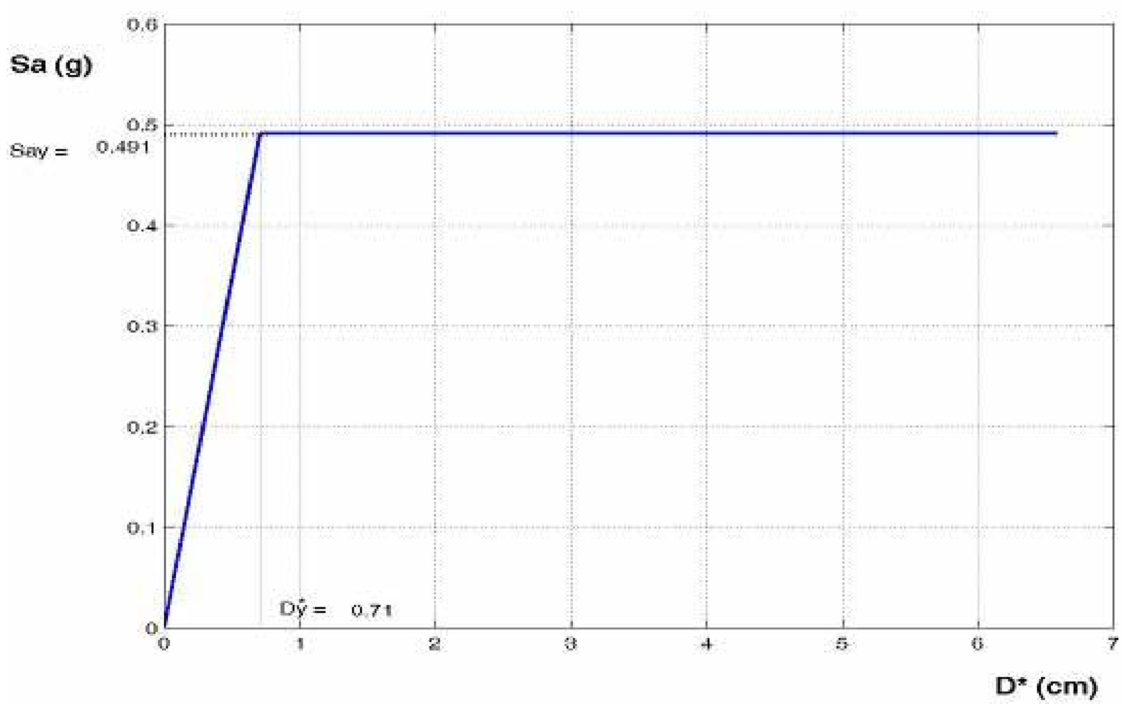


Fig-7 Capacity diagram Sa vs D* for 0% eccentricity

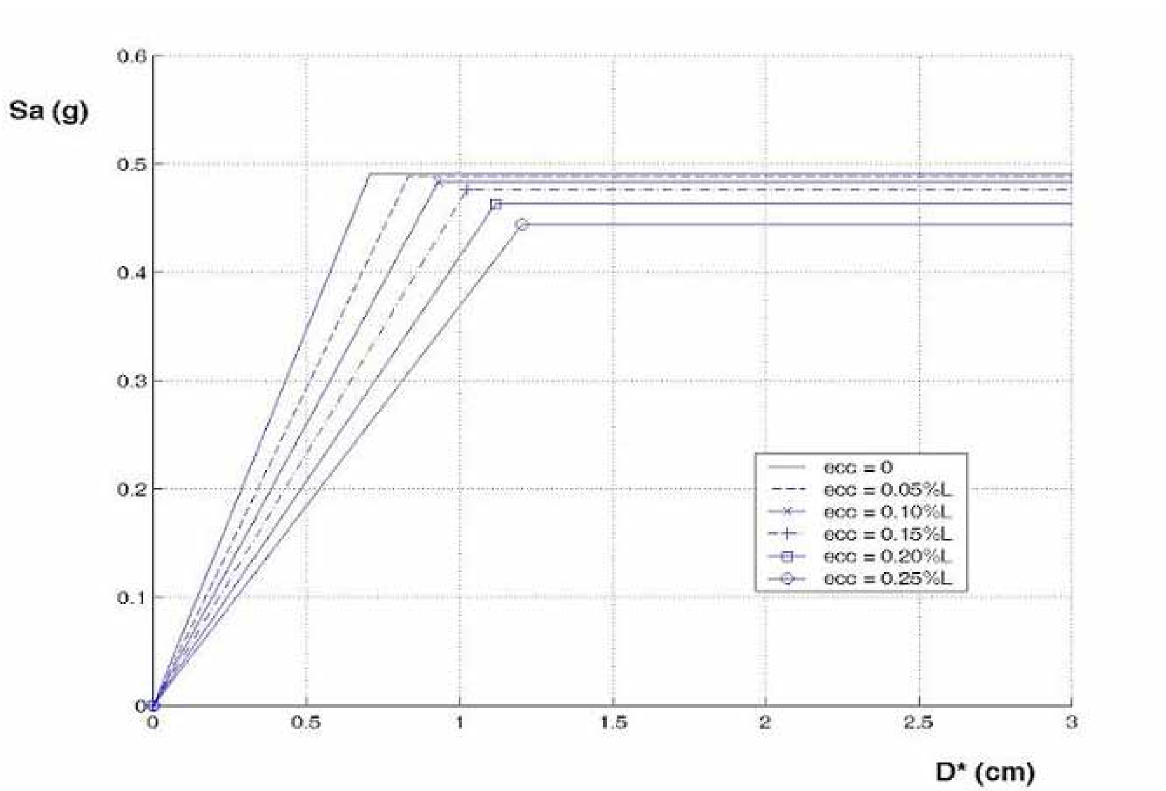


Fig-8 Capacity diagrams for different values of eccentricity

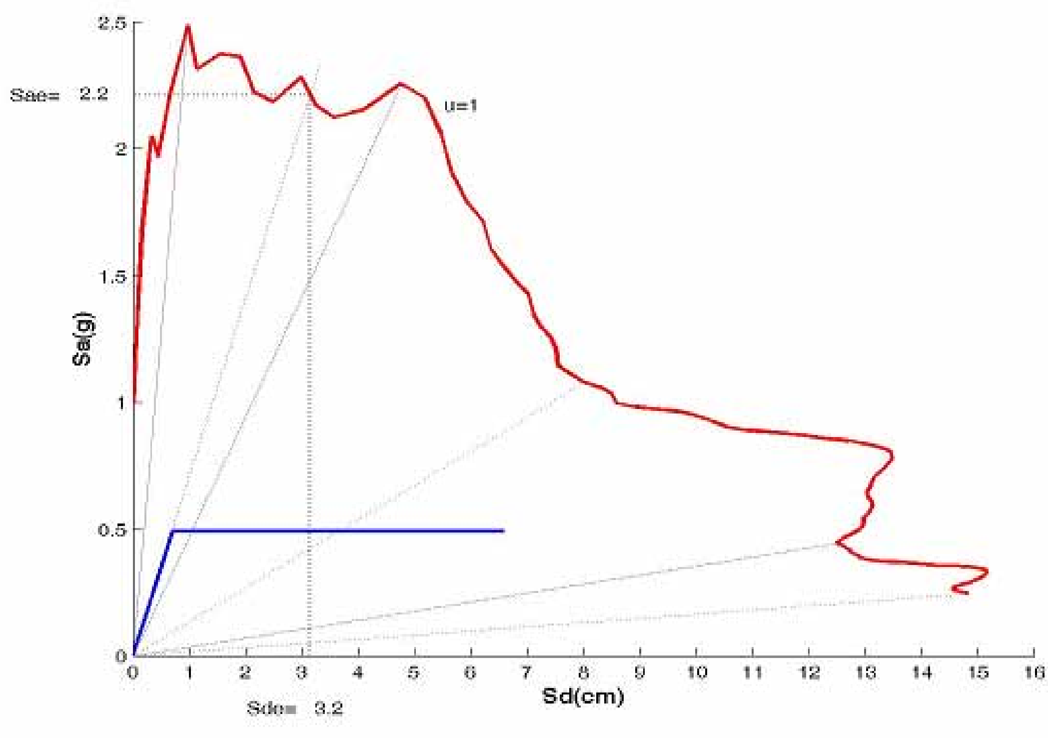


Fig-9 Demand spectra versus Capacity diagram for 0% eccentricity

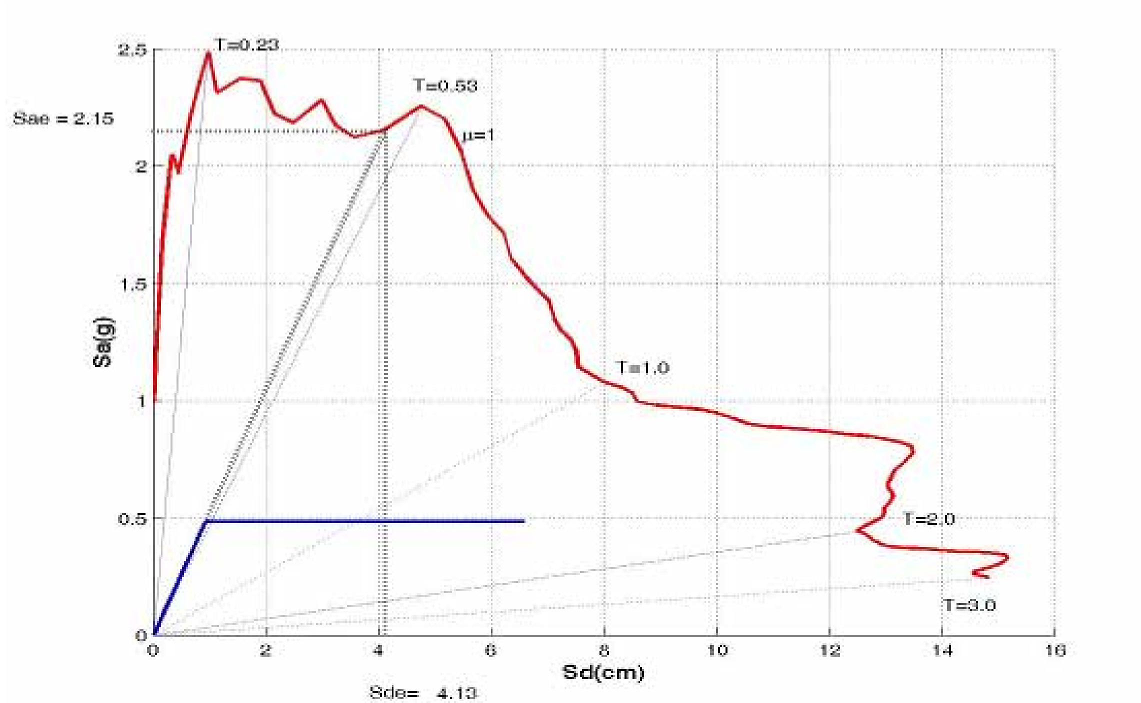


Fig-10 Demand spectra versus Capacity diagram for 10% eccentricity

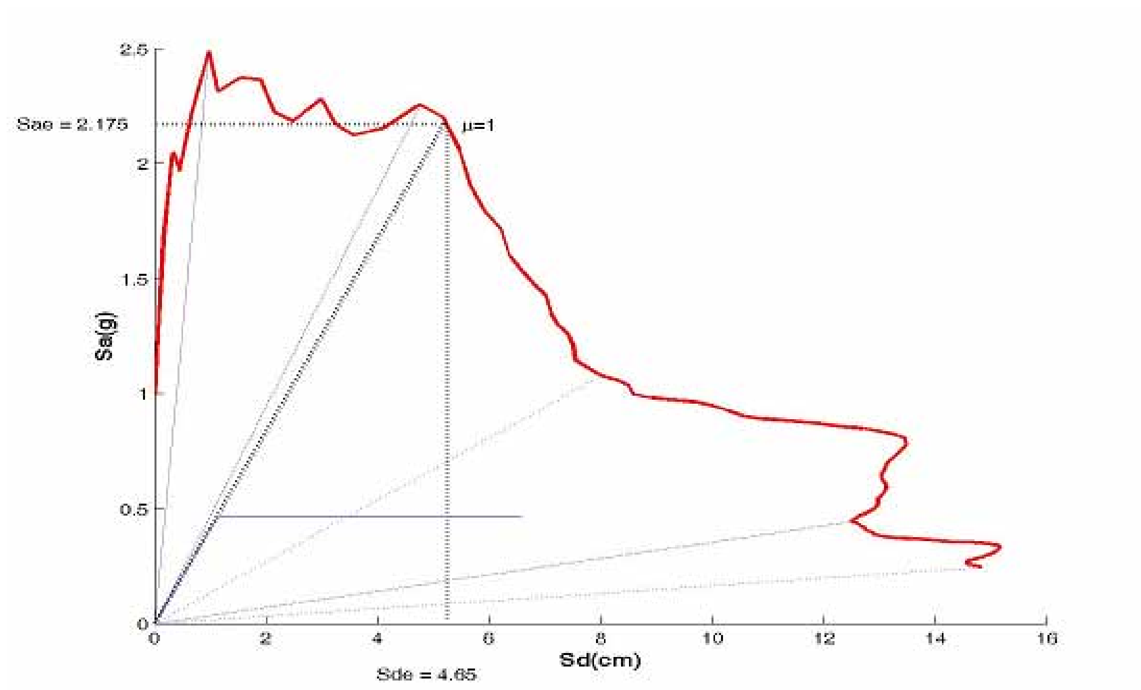


Fig-11 Demand spectra versus Capacity diagram for 20% eccentricity

RESULTS AND DISCUSSION

DETAILS OF THE STRUCTURE

The structure consists of two stories supported on four columns square in cross section. The plan of the structure is square. The columns do not have same stiffness, and therefore the center of mass does not coincide with the center of stiffness due to which the structure undergoes rotation about the vertical axis through the mass center in addition to translations along X and Y directions. Numerical details of the structure are given in the following table.

Width of each side of the plan	4.0 m
Height of the columns	3.0 m
Column size	0.3 m * 0.3 m
Mass of the ground floor slab	8687.5 Kg
Mass of the first floor slab	7337.5 Kg
Damping	5%

YIELD BEHAVIOUR

From the assumption that the column yields only at the ends, the column is said to be completely yielded when both ends have yielded. The columns are designed to carry biaxial moments. The yield displacements in the two orthogonal directions are equal because, the column is square in section, with equal amount of steel. Therefore the yield surface generated is circle. The equation is

$$\left(\frac{R_{iu}}{R_{iuo}}\right)^2 + \left(\frac{R_{iv}}{R_{ivo}}\right)^2 = 1$$

Riu = Resistance force of the column along the X direction (corresponding to U Displacement.)
Riuo = Yield force of the column along the X direction (corresponding to U Displacement.)
Riv = Resistance force of the column along the Y direction (corresponding to V Displacement.)
Rivo = Yield force of the column along the Y direction (corresponding to V Displacement.)

During the push over analysis, the yielding and subsequent hinge formation is confirmed by the above condition.

Pushover analysis of the two-storey shear building is performed for the various values of eccentricity. Plots of base shear V Vs D_t, (Fig 5) the top storey displacements are obtained. From these plots, plots of base shear and top storey displacement (Fig 6) for an equivalent SDOF are obtained.

The plots are of typical elasto – plastic behaviour. These are again plotted as S_a vs. D* which are then called Capacity diagrams, which is shown in the Fig.7. Fig 8 shows, S_a Vs D* diagrams for various eccentricities. As the eccentricity increases, the yield acceleration, or in other words, yield value S_a reduces. This is an important contribution in the push over analysis in the present work on a building asymmetric in plan. The above capacity diagrams are juxtaposed on demand spectrum plotted in AD format, for different values of ductility factor (Figs 9, 10 & 11) or in other words S_a Vs S_d plots. The intersection of the two diagrams is noted. They indicate the required ductility corresponding to the yield acceleration available in the structure.

CONCLUSIONS

In this work, pushover analysis is applied to a two storeyed building asymmetric in plan. The influence of eccentricity is obtained in the pushover analysis. Thus the formation of hinges based on standard yield criterion for biaxial bending can be tracked, thus leading to capacity diagrams. It can be observed that as eccentricity increases the yield level of the whole frame reduces.

Such capacity diagrams are juxtaposed on plots of spectral acceleration Vs spectral displacement for various levels of ductility. The points of intersection give the required ductility for the given yield level of the frame, undergoing both rotations and translations.

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