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CONE BOUNDARIES FOR DYNAMIC PROBLEMS OF 3-D HALF SPACE

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SUMMARY

In this paper, the 3-D axisymmetric dynamic problems of unbounded foundations in the time domain is studied. For simulation of wave propagation due to far field effects, a cone boundary is presented for modeling 3-D half-space. The wave type considered are compression, shear or combination of the two. The cone boundary provides a powerful tool for dealing with structure-foundation interaction or wave propagation problems for irregular foundations. To analyze the semi-infinite domain of the soil numerically, a surface or zone is chosen, called interaction horizon. The constitutive associated with the nodes on this horizon represents the significant features of the far field. As body waves in 3-D case, propagate with a hemi-spherical wave front, the power of P-wave is maximum directly under the source in the dilation window and the power of S-waves is concentrated in the shear window. The rest of the half space transmits a small part of the radiated power in the far field. From this investigation, cone models could be used as a transmitting boundary for body waves in 3-D dynamic analysis of the half space medium. So, the transmitting boundary for the body waves can be modeled for each of the degree of freedom(DOF) at the boundary nodes as a bunch of cones simulated by springs and dashpots. A finite element model of soil medium is considered for 3-D axisymmetric analysis. An earthquake loading is applied either in horizontal or in vertical direction for various boundary conditions such as; free, fixed, viscous and transmitting cone boundary. The boundary effects are investigated from the displacement time history response at the nodes near the boundary for vertical loading as well as for horizontal loading. Radiation and boundary conditions can be interpreted as constitutive equations for the interaction forces between the near and far fields. The missing part of the cones from the boundary locations to infinity is modeled by a mechanical system which contains a spring and a damper with frequency dependent coefficient. This study investigates the effectiveness of transmitting cone boundary compared to other boundaries for the finite element model of axisymmetric domain. The improvement in the transient response for axisymmetric models are noticed when dashpots are combined with the stiffness of the cones at the boundary nodes. The finite element model of axisymmetric domain of size 100m by 100m is analyzed under horizontal component of a earthquake for viscous and transmitting cone boundary conditions in vertical and lateral direction. Being symmetric with respect to vertical axis, so half of the domain has to discretised under vertical loading while full domain is discretized for horizontal earthquake loading. Cone boundary is found to be effective compared to viscous boundary.

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INTRODUCTION

The well known numerical problem in dynamic soil-structure interaction analysis is how to simulate far field soil medium, the phenomena of waves that radiate outward from the excited structures towards infinity. This radiation condition leads to boundary value problem for an unbounded domain. The dynamic response of massive structures, such as nuclear power plants, high rise buildings, dams, etc.; may be influenced by the soil-structure interaction as well as the dynamic characteristics of the exciting loads and the structures. The effect of soil-structure interaction is noticeable especially for stiff and massive structures resting on relatively soft ground. It may alter the dynamic characteristics of the structural response significantly. Thus the interaction effects have to be considered in the dynamic analysis of the structures in a semi-infinite soil medium. A surface or zone is chosen for analyzing the semi-infinite domain of the soil. The constitutive associated with the nodes on this surface represents the significant features of the far field. The interaction horizon can be situated depending upon the problem. Depending on the modeling method for the soil region, method of SSI analysis can be classified into two groups: the substructure method and the direct method [Wolf, 1].

In substructure method, based on fundamental solution, which satisfy exactly the radiation condition formulated at infinity and on the principal of superposition, applies only to regular linear half space models. Thus, the procedure of the soil-structure interaction analysis becomes very simple, and effort for analysis can be minimized. However, this method is restricted to the cases for simple geometry of foundation and linear behavior of soil medium.

The direct method models numerically the structure and a region of soil in contact up to the artificial boundary. Hence the direct method has the advantages of considering a complex geometry, spatial variations of soil properties, and non-linear behavior of soil medium.

Several kinds of direct methods for soil-structure interaction analysis have been developed to consider the radiational damping of an unbounded soil medium. They are transmitting boundary [Lysmer, 2], boundary elements [Estorff, 3], infinite elements [Medina, 4] and system identification method [Tzong, 5]. The infinite element method, of which concept was originally introduced by Ungeles [6] and Bettess [7] about two decades ago, has one of the popular techniques, since its concept and formulation procedure are similar to those of the finite element method except for the infinite extent of the element region and shape functions. The shape functions of infinite elements are usually formulated depending on the type of the problem in order to describe the behavior of the infinite medium effectively [Yun, 8].

Numerical procedures for the dynamic SSI analysis may be classified according to the nature of the time dependence as either time harmonic or transient. In the linear case, time-harmonic solutions can be used indirectly employing Fourier transform to solve the transient problems. Still a direct time integration approach is necessary whenever nonlinearities occur and may be advantageous for some classes of linear problems. For example, in linear problems exhibiting broadband phenomena, the indirect approach may not be computationally feasible. Also, the measurements of actual performance of SSI problems are usually recorded directly in the time domain, so it may be of interest to use or compare this information with that predicted directly by the mathematical models. In this paper, local transmitting boundaries applicable for transient analysis are considered and a direct method for SSI analysis in three-dimensional (3-D) axisymmetric medium is presented in time domain. As body waves in the 3D case propagate with a hemi-spherical wave front, a wave pattern with amplitude in inverse proportion of the distance from the source to the boundary node. It is shown in Ref. [9] that these models, called translational cone models, used for foundation vibration analysis can sufficiently represent body waves in 3D analysis.

METHODOLOGY

When an impulse is acting on elastic half space medium (Fig. 1a), the energy is radiated by shear and dilational waves (S and P waves). In order for the waves to transmit energy at infinity, the displacement amplitude must die off at large distance in a special low. A radiation criterion states that radiation of energy occurs when the displacement amplitude decays at infinity in inverse proportion to the square root of the surface area at infinity. For, 3D analysis, the surface at infinity for body waves is a large hemisphere with radius tends to infinite. The amplitude of body waves can be approximated to decrease in inverse proportion to the distance from the input source to boundary node. From this investigation cone models could be used as a transmitting boundary for body waves in a 3D dynamic analysis of the halfspace medium.

The boundary stress vector at the boundary location , considering the angle of incidence can be given in general as [Kellezi, 10]:

$$\{\sigma\} = [D_k]\{u\} + [D_c]\{u_{,t}\} \quad \ldots\ldots..(1)$$

where, $\{u\}$ = displacement at the boundary location node,
$\{u_{,t}\}$ = velocity at the same location,
constitutive stiffness matrix $[D_k]$ is given as:

$$[D_k] = (\rho V_p^2/r)(\mathbf{n.r})[N] + (\rho V_s^2/r)(\mathbf{n.r})\{[I]-[N]\} \ldots.(2)$$

and the constitutive damping matrix $[D_c]$ is given as :

$$[D_c] = (\rho V_p)(\mathbf{n.r})[N] + (\rho V_s)(\mathbf{n.r})\{[I]-[N]\} \ldots\ldots.(3)$$

In Eq.(2) and Eq.(3),
ρ = mass density of soil,
V_p = velocity of P waves,
V_s =velocity of S waves

$$[N] = \text{Transformation matrix} = \begin{bmatrix} n_x^2 & n_x n_y & n_x n_z \\ n_y n_x & n_y^2 & n_y n_z \\ n_z n_x & n_z n_y & n_z^2 \end{bmatrix}$$

$[I]$ = Identity matrix of order 3,
$\mathbf{n}(n_x, n_y, n_z)$ = outward unit vector normal to the boundary surface,
$\mathbf{r}(r_x, r_y, r_z)$ =unit vector to represent the direction of wave propagation,
$r = |\mathbf{r}|$ = distance from boundary node to the source of location,
$\mathbf{n.r} = \cos\alpha$, α= angle between $\mathbf{n}$ and $\mathbf{r}$.

For axisymmetric case of circular boundary, r is constant for all boundary nodes and $\mathbf{n.r}$ =1.0 while for rectangular boundary of plane strain analysis , r for each boundary nodes together with $\mathbf{n}$ and $\mathbf{r}$ are considered to calculate the constitutive stiffness and damping matrices. So, for the plane strain analysis Eq.(1) can be employed as the transmitting boundary in the area where body waves propagate. The boundary or geometric stiffness and damping matrices for the whole SSI system are obtained by assembling those for the finite element (FE) boundaries. The consistent matrices locally couple the nodes along the boundary giving a more realistic implementation. The equations of motion for the visco-elastic system are :

$$[M]\ \{u_{,tt}\} + [C]\ \{u_{,t}\} + [K]\ \{u\} = \{P\}\ldots\ldots.(4)$$

where, [M] is the mass matrix,
$\{u_{,tt}\}$ is acceleration,
[C] is damping matrix, and
[P] is transient load.

So, the transmitting boundary for body waves can be modeled for each of the degree of freedom at the boundary nodes as a bunch of cones simulated by springs and dashpots attached to the boundary and connected to rigid base, Fig. 1. The apexes of the cones are equal to the radius of the hemisphere for a spherical boundary with origin at the source.

Internal damping is implemented as Rayleigh damping. The direct step by step Newmark's β method is used to solve the equations of motion for evaluating displacement at various locations in SSI system.

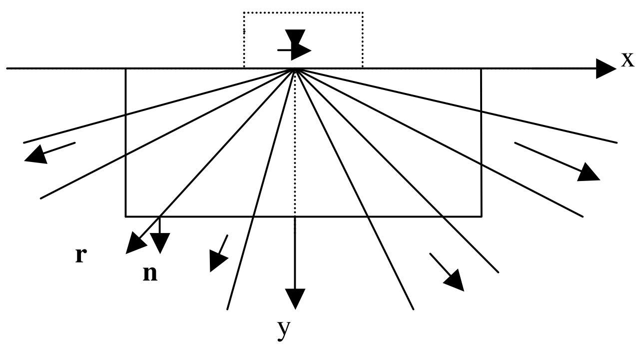

Fig. 1 Cone Transmitting Boundary

PROBLEM FOR AXISYMMETRIC ANALYSIS

Fig. 2 shows the finite element model of half of axisymmetric domain in which 100 element and 121 nodes are there. The nodes are shown on which responses are plotted under horizontal component of Earthquake, Koyna, India (Fig. 3). The lateral and bottom boundary nodes are attached by damper and stiffness to represent viscous and cone transmitting boundaries. The material properties of the soil domain are as follows :

Mass density = 2000.0 kg/cub.m
Poission's ratio = 0.25
Shear wave velocity = 224 m/s.

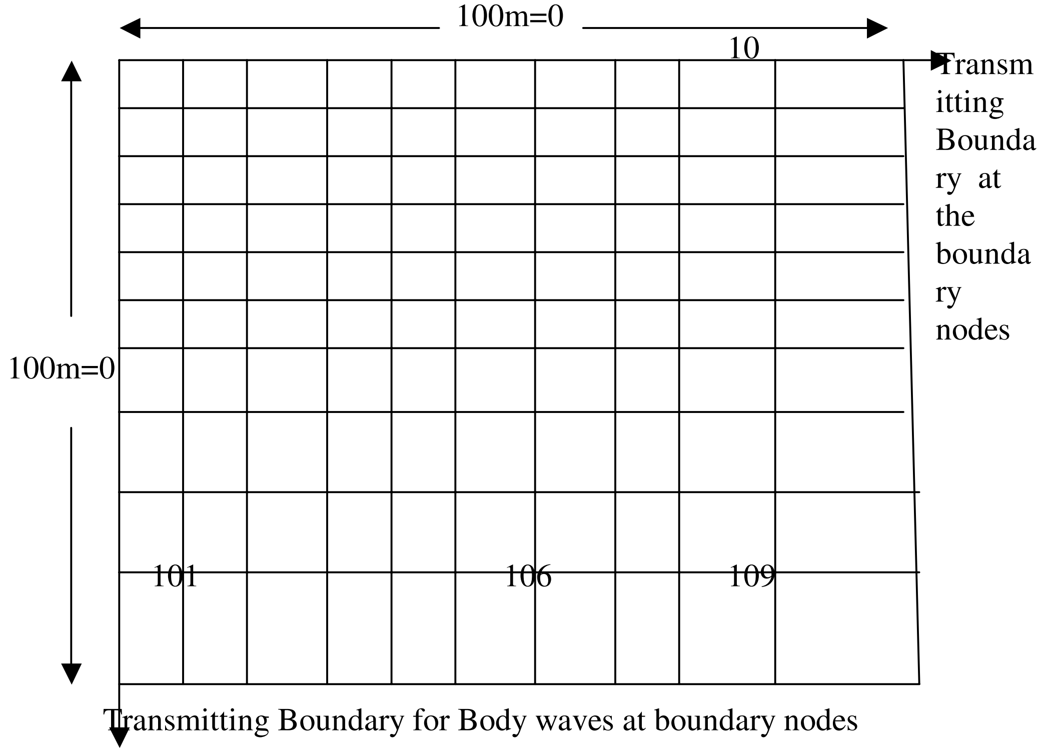

Fig. 2 Finite Element Model for Axisymmetric Analysis

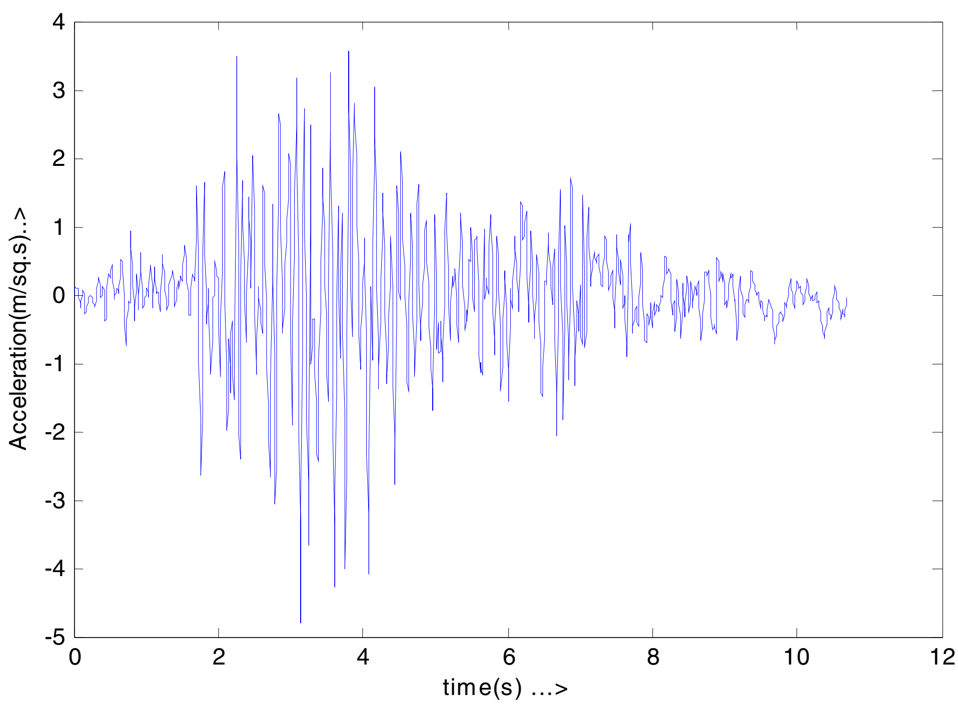

Fig. 3 Koyna, (India), Earthquake Transverse Component

ANALYSIS RESULTS

The fine element model of axisymmetric soil domain as shown in Fig.2, is subjected to horizontal earthquake (Fig. 3), and obtained the displacement time history of three components, i.e., in radial, vertical and tangential component at various nodes of the domain. These three displacements components time histories at various nodes of finite element axisymmetric model are plotted through Fig. 4 to Fig. 6.

Radial Displacement time history at various nodes

It is observed from Fig. 4 that the radial displacement response at various boundary nodes are comparative very less for the case of cone transmitting boundary from that of the case of using viscous boundary. It means that reflection from the boundary nodes towards the near field of the soil domain is t negligible small in case of using cone transmitting boundary.

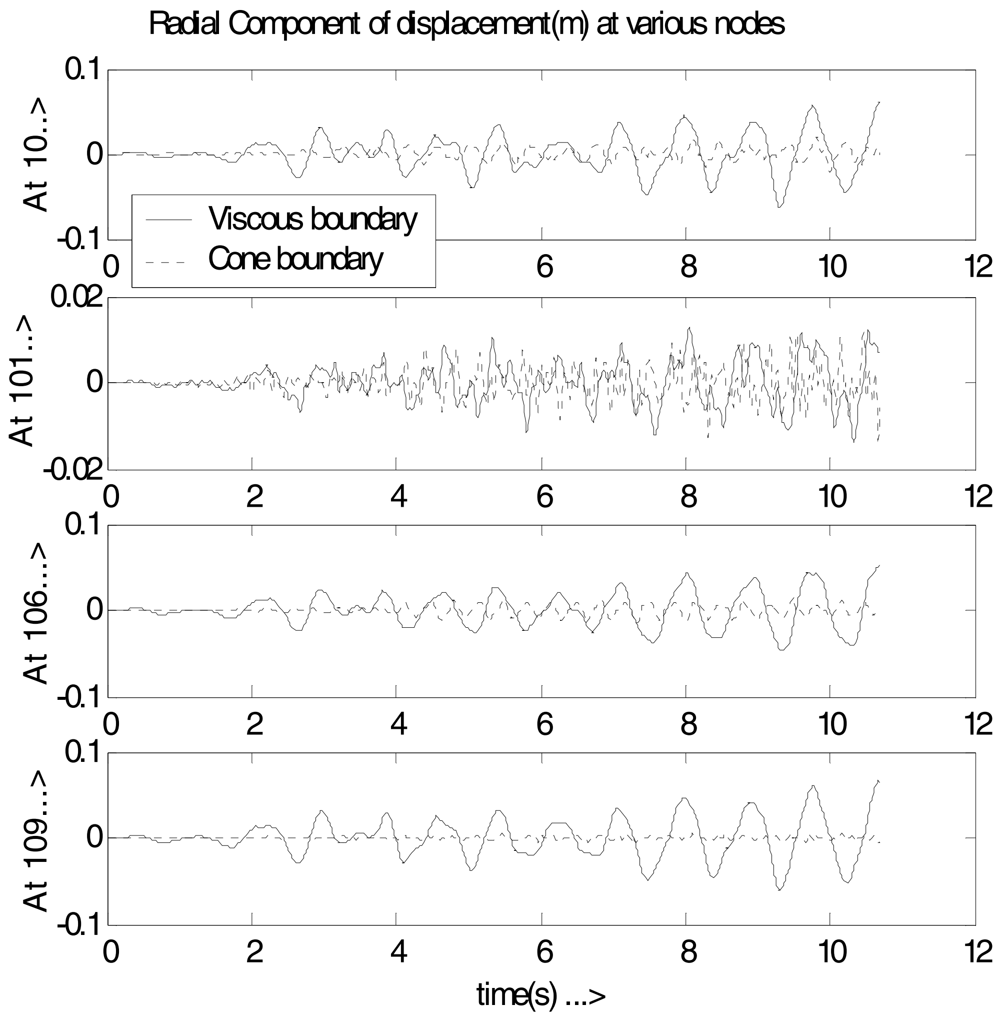

Fig. 4 Comparison of Radial component of displacement at various nodes of axisymmetric Finite Element model with Viscous and Cone boundaries under horizontal earthquake

Vertical Displacement time history at various nodes

It is observed from Fig. 5 that the vertical displacement response at various boundary nodes are comparative very less for the case of cone transmitting boundary from that of the case of using viscous boundary. It means that reflection from the boundary nodes towards the near field of the soil domain is t negligible small in case of using cone transmitting boundary.

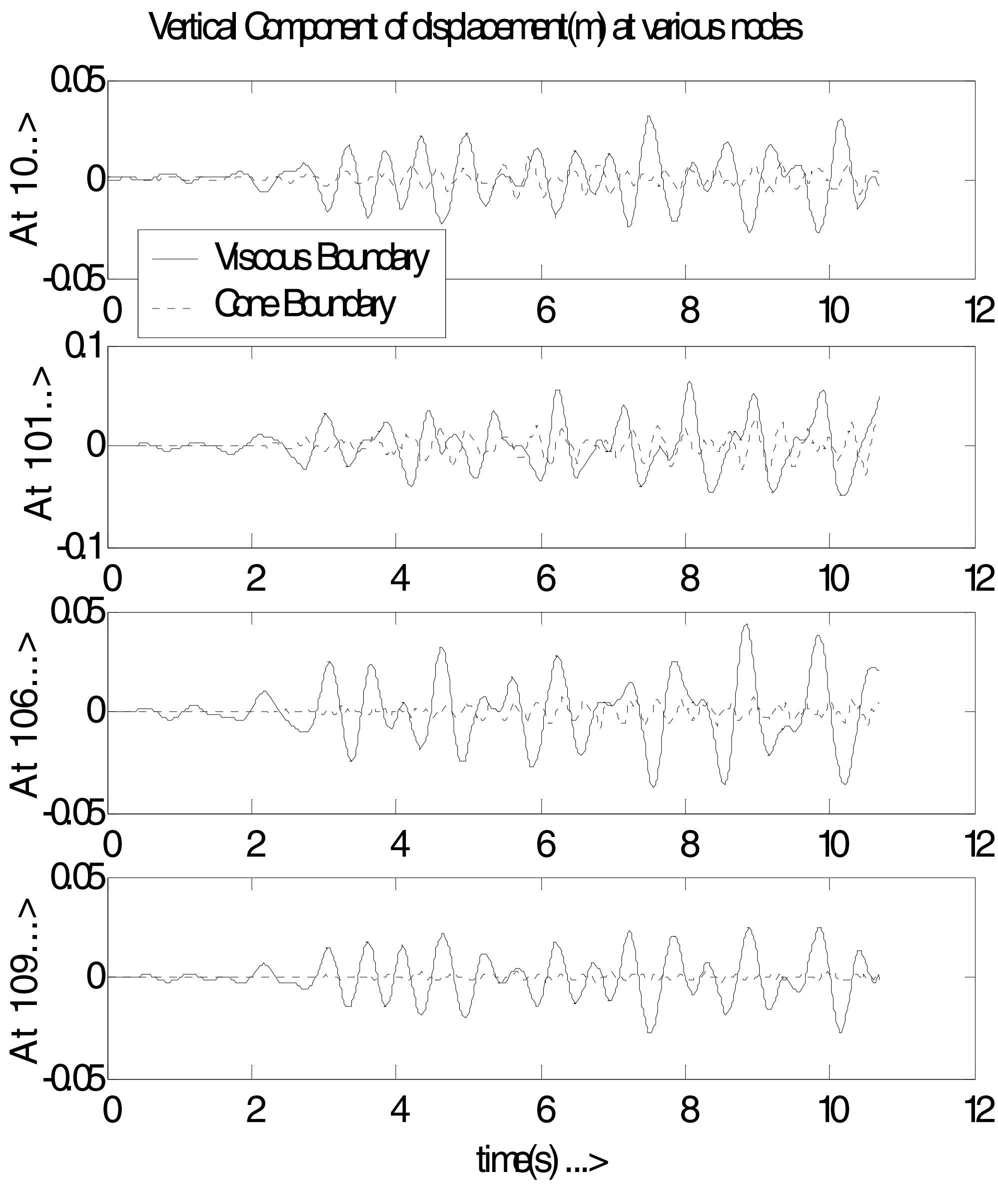

Fig. 5 Comparison of Vertical component of displacement at various nodes of axisymmetric Finite Element model with Viscous and Cone boundaries under horizontal earthquake

Tangential Displacement time history at various nodes

It is observed from Fig. 6 that the tangential displacement response at various boundary nodes are comparative very less for the case of cone transmitting boundary from that of the case of using viscous boundary. It means that reflection from the boundary nodes towards the near field of the soil domain is t negligible small in case of using cone transmitting boundary.

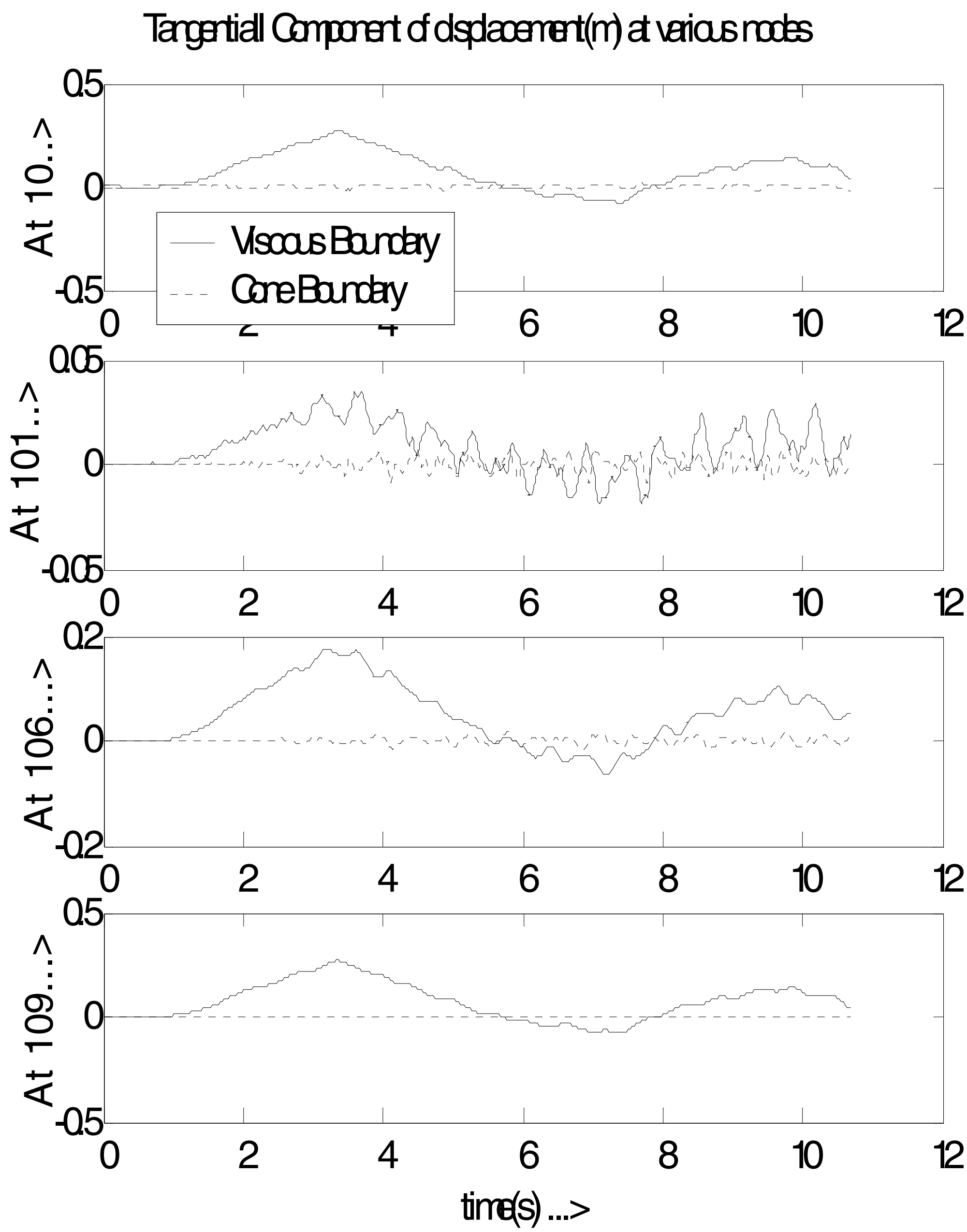

Fig. 6 Comparison of Tangential component of displacement at various nodes of axisymmetric Finite Element model with Viscous and Cone boundaries under horizontal earthquake

DISCUSSIONS AND CONCLUSIONS

It is observed from the study that radiation and boundary conditions can be interpreted as constitutive equations for the interaction forces between the near and far fields. It is concluded that there is a improvement in the seismic response at the boundary nodes when the dashpots at the boundary nodes are combined with the stiffness of the cones, that is, cone transmitting boundaries are much effective compares to the viscous boundary.

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